# Multipion coherent effects in high energy heavy-ion collisions

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Multipion production in high energy nucleus-nucleus collisions is considered within the model of the pion radiation by classical current. Strong coherent effects of narrowing of the pion longitudinal and transverse momentum distributions are predicted at RHIC energies. The coherence enhances the large pseudorapidities, producing a bump in the distribution. The growth of the average pion multiplicity and oscillation effect in the multiplicity distribution are caused by the coherence as well.

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## I. INTRODUCTION

The observation of a new physical phenomenon could be done in experiments at RHIC, which is a coherent multipion radiation in heavy-ion collisions analogous to the laser radiation [1,2] of the electromagnetic field. Besides, this phenomenon is important in connection with disoriented chiral condensate (DCC) [3] and classical pion field [4] problems. It is known that quantum statistical correlations change the multiplicity distribution and momentum spectra of identical hadrons. The effect becomes crucial in the high energy heavy-ion collisions where the mean number of pions in the unit volume of phase space ($\Delta x \Delta k$ cell) reaches the value of about a unity (or more than a unity). In this case the coherent radiation of pions can lead to the formation of a so-called pion laser [1]. Usually these effects are discussed assuming thermal emission of pions by a static source with radius $R$ [5].

On the other hand, the most popular basic model for the description of the underlying events in heavy-ion collisions (assuming no any new physics in the nucleus-nucleus scattering) is that of independent nucleon-nucleon interactions. Each pair of incoming nucleons produces secondary pions independently. In other words, each pair of colliding nucleons plays the role of a separate source placed at some point $x_i$ within the space-time domain where the beam $A_1$ and target $A_2$ ions overlap. Each source is more or less weak, though the number of sources is large ($\sim A$), so that the density of pions becomes greater than unity (in the cell of phase space). In the present paper we study how the quantum statistics (i.e., the permutations) of identical pions modifies the spectra and multiplicity distribution of secondaries in such a simple (basic) model of independent nucleon-nucleon interactions.[1]

Certain possible effects of the pion coherence were considered qualitatively within the simple model of bremsstrahlung of scalar pions with a small pion multiplicity [6]. Now we step further towards a quantitative understanding of this phenomenon considering more realistic central nucleus-nucleus collisions and accounting for true pion amplitudes, with large multiplicity in the nucleon-nucleon interaction. This can be carried out using the model of Gyulassy, Kauffmann, and Wilson [7] for the pion radiation by classical current in the nucleus-nucleus scattering. We see a strong coherent narrowing in the pion transverse and longitudinal momentum distribution for such a process. This confirms the analogous result obtained in [6]. The longitudinal momenta are strongly softened by the coherence, unlike in the model of the scalar pion bremsstrahlung where the monochromatism of pions takes place. In spite of this difference, the conclusion [6] about the enhancement of large pseudorapidities by coherence is encouragingly confirmed now. Apart from the coherence effects manifested by the kinematical variables, we have prominent coherent effects in the pion multiplicity with a large number of radiating nucleons. That is in accordance with the threshold character of the coherence as a function of the number of pion sources [6].

[1]This model can be considered as a realistic one, if there are no other new dynamical effects in the nucleus-nucleus collision.

## II. PION RADIATION BY CLASSICAL CURRENT

The average pion multiplicity in the central heavy-ion collisions at RHIC energies is of the order of $10^4$. A convenient way to perform calculations with such a number of radiated pions consists in considering the pion radiation by classical current [7]. The $S$ matrix for the emission of $m$ pions by the current $J(\mathbf{k})$ can be written as

$$|J\rangle = e^{-\bar{n}/2}\exp\left( i\int d^3k J(\mathbf{k}) a^\dagger(\mathbf{k})\right)|0\rangle, \tag{1}$$

$$\bar{n} = \int d^3k |J(\mathbf{k})|^2, \tag{2}$$

$$S_{fi} = \langle a(\mathbf{k}_1)\cdots a(\mathbf{k}_m)|J\rangle. \tag{3}$$

The coherent state $|J\rangle$ is the state with an indefinite number of quanta, the quantum-mechanical average of the pion annihilation operator $a(\mathbf{k})$ being equal to $J(\mathbf{k})$. The exclusive cross section, semi-inclusive cross section for the case of $m$ radiated pions, inclusive cross section, and pion multiplicity distribution can be easily obtained from Eqs. (1)–(3) using the permutation property of the operators $a(\mathbf{k}_i)$ and $a^\dagger(\mathbf{k}_i)$:

$$\frac{dW^{(m)}}{d\mathbf{k}_1\cdots d\mathbf{k}_m} = |J(\mathbf{k}_1)|^2\cdots|J(\mathbf{k}_m)|^2 e^{-\bar{n}}, \tag{4}$$

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$$\frac{dW^{(m)}}{d\mathbf{k}_1}=|J(\mathbf{k}_1)|^2\frac{\bar{n}^{m-1}}{(m-1)!}e^{-\bar{n}},\tag{5}$$

$$\frac{dW}{d\mathbf{k}_1}=\sum_{m=1}^{\infty}\frac{dW^{(m)}}{d\mathbf{k}_1}=|J(\mathbf{k}_1)|^2,\tag{6}$$

$$P_m=\frac{\bar{n}^m}{m!}e^{-\bar{n}}.\tag{7}$$

So we obtain the Poisson multiplicity distribution within this model, and the physical meaning of the value $\bar{n}$ [Eq. (2)] is clear from Eq. (7) as the average pion multiplicity.

One can write the nuclear current as a sum of currents of constituent nucleons [7]:

$$J(x)=\sum_{i=1}^{N}J_\pi(x-x_i),\quad J(\mathbf{k})=J_0(\mathbf{k})\sum_{i=1}^{N}e^{i\omega t_i-i\mathbf{k}\cdot\mathbf{x}_i}.\tag{8}$$

Here the space-time points $x_i$ are the coordinates of certain ''inelastic scattering centers'' where the strength of the current is localized; $\omega=\sqrt{k^2+\mu^2}$ is the energy and $\mu$ is the mass of a pion. We obtain all distributions with formulas (4)–(7), substituting the following expression for $\bar{n}$ which depends now on the instant space-time coordinates of $N$ nucleons $x_1,\ldots,x_N$:

$$\bar{n}=\bar{n}(x_1,\ldots,x_N)=\int d^3k|J_0(\mathbf{k})|^2\left(N+2\sum_{i<j}\cos k(x_i-x_j)\right).\tag{9}$$

The final cross section can be obtained by averaging the distributions of Eqs. (4)–(7) over the whole space-time region of the nucleus-nucleus collision.

The amplitude of the vacuum-vacuum transition $\exp[-\bar{n}(x_1,\ldots,x_N)/2]$ gives the correct cross section normalization, i.e., provides the unitarity to be fulfilled, thus taking into account the pion radiation in one point and its absorption in another one.

As an example we study the central Au-Au collisions with energy $p_0=100$ GeV per nucleon in the center-of-mass system. The number of radiating nucleons, $N=150$, is fixed. We neglect now the spin and isospin variables, considering pions of the same sign of charge only. The current $J(\mathbf{k})$ is obtained by fitting the single inclusive pion longitudinal [8] and transverse momentum distributions, with real average pion multiplicity in the nucleon-nucleon scattering.[2] To average the cross section over the radiation space-time region, we approximate this domain by the Woods-Saxon distribution with radius $R=6.3$ fm and diffusion parameter $a=0.6$ fm. The longitudinal coordinate and radiation time were averaged independently, the Lorentz contraction of the nucleus being taken into account.

[2]We are grateful to Yu.M. Shabelsky for providing us with tabulated pion distributions.

The resulting spectra of pion kinematical variables are shown in Fig. 1. It is seen that coherence strongly affects both transverse and longitudinal momentum distributions. Comparing the transverse momentum spectrum without interference [the dashed line in Fig. 1(b)] with spectra where coherence takes part [solid and dotted lines in Fig. 1(b)], we can observe the diminishing of pion transverse momenta due to coherence. The average transverse momentum in the nucleon-nucleon interaction is $\langle k_{t0}\rangle\approx0.36$ GeV; just this value corresponds to the dashed line in Fig. 1(b). Typical transverse momentum for the spectra shown in Fig. 1(b) with coherence (solid and dotted lines) is $\langle k_t\rangle\approx0.055$ GeV. This value turned out to be about the inverse nucleus radius $1/R\approx0.031$ GeV, but it is slightly higher due to the fact that the Fourier transform of the space distribution of the pion source is determined not only by the radius $R$ but by a smaller diffusion parameter $a$ as well. The obtained result corresponds to the picture of pion production in the nucleus-nucleus collision, when in the main the whole nucleus radiates pions coherently but not separate nucleons independently.

The longitudinal momentum, as shown in Fig. 1(a), also decreases because of the coherence. The solid and dotted lines show much narrower distributions than the dashed line calculated without pion interference. To estimate the characteristic energy of coherent pions with transverse momentum $\langle k_t\rangle$, let us take the value $\omega\approx0.75$ GeV corresponding to the maximum of the pseudorapidity distribution in Fig. 1(c) (solid and dotted lines). However, to estimate correctly the longitudinal range of the pion source, we must remember that the connection between the pion current $J(\mathbf{k})$ and space-time source range,

$$J(\mathbf{k})=\int d^4x\frac{e^{i\omega t-i\mathbf{k}\cdot\mathbf{x}}}{[2\omega(2\pi)^3]^{1/2}}J(t,\mathbf{x}),$$

includes the normalization factor $1/\sqrt{2\omega}$ of a pion plane wave [7]. So, to estimate this range, we must use the longitudinal momentum distribution multiplied by the pion energy (or pion longitudinal momentum, if its value is large enough). It is the invariant cross section shown in Fig. 1(a) that possesses the necessary information. We use the width of the distribution on the level of one-half of the cross section maximum as a scale for the inverse size of the pion source. We obtain the value $Q_L\approx4.2$ GeV from the solid (or dotted) line; it can be compared with $Q_L^{NN}\approx11$ GeV for the corresponding value in nucleon-nucleon interactions (dashed line). The obtained $Q_L$ turns out to be about the inverse nuclear range shortened by the Lorenz boost: $Q_L^R=p_0/m/R\approx3.3$ GeV ($m$ is the nucleon mass). This corresponds to the conclusion which has been done before in the analysis of the transverse momentum distribution about the coherent radiation of the nucleus as a whole.

The physical nature of the coherent narrowing of the transverse momentum distribution was studied in the framework of a simple model of the bremsstrahlung of a single scalar pion created in every nucleon-nucleon interaction [6]. Such an effect appeared in [6] as a consequence of pion Bose

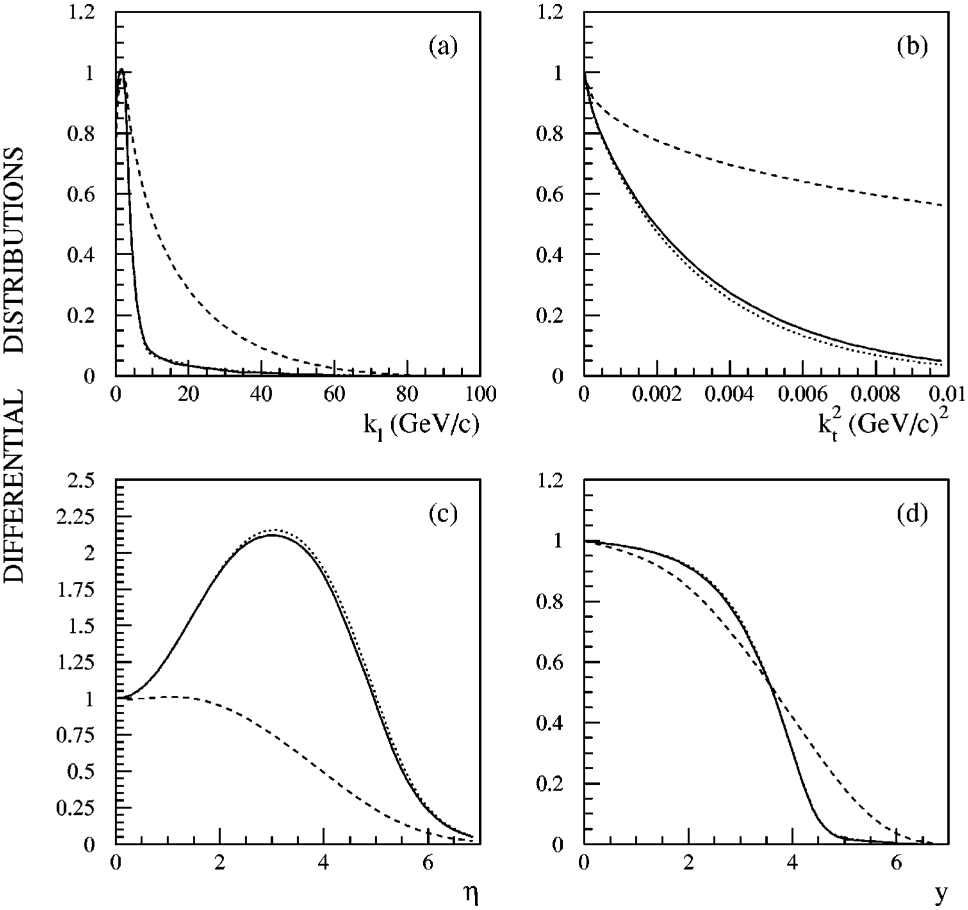


FIG. 1. Distributions of the (a) pion longitudinal momentum (multiplied by $k_l$), (b) transverse momentum squared, (c) pseudorapidity, and (d) rapidity. The solid line shows the semi-inclusive cross section with the fixed number of pions in the event $m=7000$, the dotted line shows the inclusive cross section, and the dashed line shows the inclusive cross section without the interference contribution (normalized in the maximum for $k_l$ distribution and at zero for others).

statistics. At the same time, there was also a narrowing of the longitudinal pion momentum spectrum around its characteristic value $(\mu_t/m)p_0$, where $\mu_t$ is the pion transverse mass $\sqrt{\mu^2+k_t^2}$. It is clear that the diminishing of pion longitudinal momenta due to the coherence observed now [see Fig. 1(a)] may be explained with the help of the same mechanism, where many pions are radiating analogously. The narrowing of the spectrum around the average value $(\mu_t/m)p_i$ takes place at every $i$th step of the radiation. Here $p_i$ is the momentum of the nucleon before the $i$th emission. So the summary effect of the softening of the spectrum can be produced by decreasing the spread around the average values and diminishing pion transverse momenta due to the coherence.

We see that the coherent decrease is stronger for transverse momenta than for longitudinal ones:

$$Q_L/\langle x_{F0}\rangle p_0 \approx 0.4, \quad \langle k_t\rangle/\langle k_{t0}\rangle \approx 0.15.$$

Here $\langle x_{F0}\rangle \sim 0.1$ is the average Feynman's $x$ in the nucleon-nucleon invariant cross section. We conclude from the analysis of the spectra in Figs. 1(a) and 1(b) that the source of coherent pions could be roughly approximated by a disk, radiating pions with the transverse momenta $\sim 1/R$ and longitudinal momenta $\sim Q_L^R$. One could illustrate the observed difference by the same picture of the pion radiation, for example, within the model, where the average transverse momentum of the emitted pion is $\langle k_{t0}\rangle \sim \mu$. The coherent decrease of the average longitudinal momentum in the $i$th step of a ''ladder'' is

$$\frac{Q_L^R}{\langle k_{l0}(i)\rangle} \approx \frac{p_0}{m}\frac{1}{R}\Bigg/\frac{\mu}{m}p_i \approx \frac{p_0}{p_i}\frac{1}{R\langle k_{t0}\rangle}.$$

This value is found to be higher than the decrease of transverse momenta $(1/R)/\langle k_{t0}\rangle$, as $p_i \ll p_0$ for most of the steps in the ''ladder.'' Within a more realistic multiperipheral model of pion production one obtains exactly the same result with the pion momentum $k_i$ instead of $(\mu/m)p_i$ in the denominator. As in the multiperipheral approach the typical value of $k_i \ll (\mu/m)p_0$; we have to expect here the effect discussed above as well.

So, as a consequence, large pseudorapidities must be enhanced by coherence. This effect is really seen in Fig. 1(c), the cross section being twice as much as the noncoherent one in the shoulder region having a bump there. This is mainly conditioned by the multiperipheral character of the pion production. One could suppose this effect to be enhanced with the growth of $p_0$ by virtue of a decrease of $\langle x_{F0}\rangle$ and increase of $\langle k_{t0}\rangle$.

The rapidity distribution [Fig. 1(d)] shows a coherent effect as well. However, such a kinematical variable is not convenient for this case to clear up the physics, when the pion transverse momentum is lower than the pion mass. All

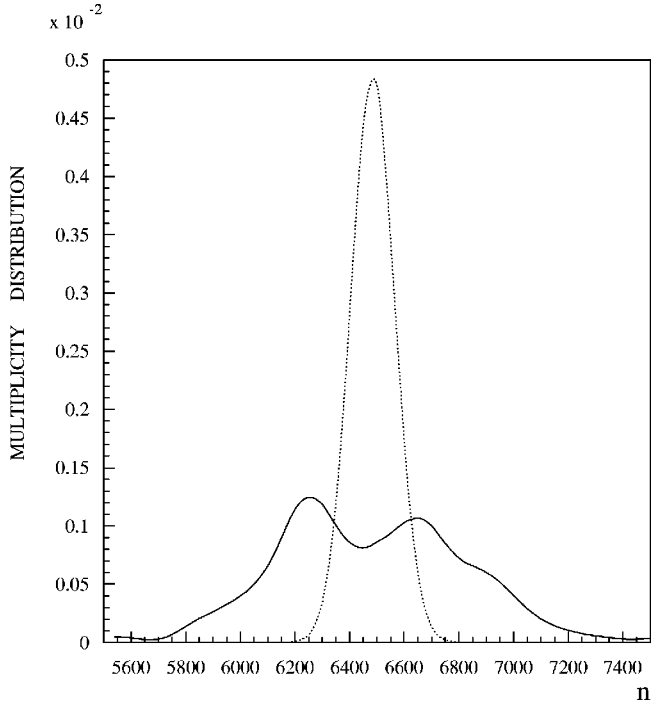


FIG. 2. The pion multiplicity distribution (the solid line) and Poisson distribution with the same average multiplicity (the dotted line).

considered inclusive distributions do not differ noticeably from the spectra where the number of pions is fixed near its mean value.

## III. MULTIPLICITY DISTRIBUTION

The existence of strong coherent effects in the pion multiplicity is suggested by the fact that laser-type pion radiation has the characteristic $N^2$ dependence of the average multiplicity on the number of nucleon sources [7]. The value $\overline{n}$ averaged over the space-time region [Eq. (9)] is given by

$$\langle \overline{n} \rangle = \overline{n}_0 [N + \epsilon N(N-1)]. \tag{10}$$

Here $\overline{n}_0 \approx 6$ is the average multiplicity of pions of the same sign of charge in the nucleon-nucleon interaction and $\epsilon$ is the probability for two pions to be at the same state (i.e., the same $\Delta x \Delta k$ cell). The strong threshold growth of coherent effects in the pion induced radiation versus the number of radiating nucleons [6] corresponds to the small overlapping of the pion distributions after averaging according to the first and second terms in Eq. (10).

We have calculated the pion multiplicity distribution, Eq. (7), and the results are shown in Fig. 2. We see that the average multiplicity is approximately 7 times larger than that in the noncoherent case. So the second term dominates Eq. (10) and we have a coherent source with thousands of pions at the same state. It is really a pion classical field. The probability $\epsilon \approx 0.04$ can be obtained using Eq. (10).

The prominent distinction of the obtained multiplicity distribution from the Poisson distribution, its large width, and oscillating shape is the second effect of the pion coherence. To study the nature of these oscillations it is necessary to consider the complicated equations (7) and (9). Nevertheless, we try to understand the main features of this effect without a detailed investigation of Eqs. (7) and (9). First, let us recall the origin of the second term in Eq. (10). It comes from averaging $\cos[k(x_i - x_j)]$ in Eq. (9). In the limit $kR \ll 1$, when, for any $i$ and $j$, $\cos[k(x_i - x_j)] = 1$, we get $\overline{n} = N^2$, i.e., $\epsilon = 1$ in Eq. (10). Vice versa, for $kR \gg 1$, as a result of the oscillations and strong cancellation, the term $\langle \cos[k(x_i - x_j)] \rangle$ is equal to zero, which leads to $\overline{n} = N$ $(\epsilon = 0)$. In reality we obtain not so small $\epsilon = 0.04$ which is larger than the naive estimate $\epsilon = (\langle k_t \rangle / \langle k_{t0} \rangle)^2 Q_L / Q_L^{NN}$ ($\epsilon$ is the part of phase space where $kR \lesssim 1$; see the end of Sec. II). This means that the major part of the multiplicity comes from the region where $\cos[k(x_i - x_j)]$ has one or few oscillations.

In some sense, the phase of $\cos[k(x_i - x_j)]$ separates all pion sources (currents) into groups. For example, there may be a group of $N_1$ nucleons, for which all the values of $\cos[k(x_i - x_j)]$ are positive (at fixed $k$). Note that they are not only the neighboring nucleons ($k\Delta x \subset [0, \pi/2]$) but currents separated by distance $\Delta x = x_i - x_j$ with $k\Delta x \subset [3\pi/2, 2\pi]$ and so on. Thus, we do not deal with a homogeneous source but with some groups of currents which produce the pions coherently. Now it is natural to expect large fluctuations in the multiplicity distribution. Such groups produce oscillations, i.e., maxima at different values of $n$.

To estimate approximately the number of nucleons in a group we suppose that the oscillations in multiplicity distributions result from the radiation by groups, in which the number of coherently radiating nucleons differs by unity in various events. Such groups produce maxima at different places of the multiplicity distribution. Fitting positions of the two maxima in the spectrum by the expressions $\overline{n}_0 \epsilon (N_1 - 1)(N_1 - 2) B$ and $\overline{n}_0 \epsilon N_1 (N_1 - 1) B$, which correspond to the two classes of events, we obtain the effective number $N_1 \approx 33$ of nucleons in the group. The next step for understanding the radiation process is to suppose the radiation of different groups to be coherent. In this case the factor $B$ is given by $N_g (N_g - 1)$, where $N_g$ is the number of groups. From the fit of the places of maxima we obtain $N_g \approx 5$, in approximate accordance with the fact that the total number of nucleons is $N = 150$.

The only known example of the oscillating multiplicity distribution is a spectrum in the multi-Reggeon cutting mechanism [9], where the peaks in the distribution correspond to the integer multiple of the single Reggeon multiplicity. The principal difference of this effect from the discussed result consists in the fact that the laser-type radiation has the characteristic $N^2$ dependence of the average multiplicity on the number of nucleons in the group. Together with the fact that the number of pions, $\overline{n}_0$, emitted by each pair of colliding nucleons [each current $J(\mathbf{k})$] increases with energy ($\overline{n}_0 \gg 1$), it provides us a reason for which contributions of different configurations are clearly separated in the spectrum.

## IV. SPIN EFFECTS

We have not considered the influence of the nucleon spin on the discussed coherent effects. The multiple pion production at high energies does not seem to have the significant spin dependence of the single inclusive pion spectra, at least in the central rapidity region or at multiperipheral pion creation. However, the property of a pion as a Goldstone particle to couple with spin (for example, with the quark spin of the nucleon), with a suppression of small transverse momenta, requires a clarification of the problem of the spin dependence of coherent effects.[3]

The first point is that being a Goldstone particle the pion with zero momentum $k=0$ (i.e., the homogeneous pion field) decouples from quarks or baryons. The next is the pion-nucleon vertex; in a sense, the pion is emitted by the spin of the fermion. It is not important for us whether the fermion is a quark or nucleon, but for the sake of simplicity we consider the pion-nucleon coupling. In the nucleon rest frame the vertex takes the form $V=g(\boldsymbol{\sigma}\mathbf{k})$.

At first sight, as a result of the Goldstone nature of pions, one might expect the inclusive cross section to tend to zero at small $k_t$, just in the region where we collect the main interference effects. However, this is not the case: (i) when the pion is emitted by a fermion, the vanishing of the vertex $V \sim k$ is compensated for by the pole of the fermion propagator and at $k \rightarrow 0$ the amplitude tends to a constant; (ii) in a central rapidity region the majority (more than half) of pions comes from the resonance $(\rho,\omega,f,\ldots)$ decay. Anyway in our calculations we have used the experimental cross section (measured in $pp$ collisions) which does not tend to zero as $k_t \rightarrow 0$.

The role of spin is a more delicate question. First, we have to emphasize that in the central region the pions are produced mainly due to the resonance decay and only a small part of the $\pi$ mesons is promptly created by fermions. Nevertheless, let us discuss the interference between two identical pions (with momenta $k_1$ and $k_2$) emitted by two nucleons with coordinates $x_1$ and $x_2$. The amplitude reads

$$A=g[(\boldsymbol{\sigma}_1\mathbf{k}_1)e^{ik_l x_1}(\boldsymbol{\sigma}_2\mathbf{k}_2)e^{ik_2x_2}+(\boldsymbol{\sigma}_1\mathbf{k}_2)e^{ik_2x_1}(\boldsymbol{\sigma}_2\mathbf{k}_1)e^{ik_1x_2}], \tag{11}$$

where we keep only the terms (and factors) important for our discussion. To calculate the cross section one has to square the amplitude, Eq. (11), and average over the nucleon polarizations. This leads to

$$\frac{dW}{d\mathbf{k}_1 d\mathbf{k}_2} \sim g^2[\mathbf{k}_1^2\mathbf{k}_2^2+2(\mathbf{k}_1\mathbf{k}_2)^2 e^{i(k_1-k_2)(x_1-x_2)}+\mathbf{k}_2^2\mathbf{k}_1^2]. \tag{12}$$

Here the second term corresponds to the interference of the amplitudes, where the pions $k_1$ and $k_2$ are emitted by nucleons 1 and 2, correspondingly, and vice versa. After averaging over polarizations, the product $(\boldsymbol{\sigma}_1\mathbf{k}_1)(\boldsymbol{\sigma}_1\mathbf{k}_2)$ gives the value $(\mathbf{k}_1\mathbf{k}_2)$. Thus, unlike the spinless case with $dW \sim \{1+\exp[i(k_1-k_2)(x_1-x_2)]\}$, we obtain an extra cosine square $(\mathbf{k}_1\mathbf{k}_2)^2/\mathbf{k}_1^2\mathbf{k}_2^2=\cos^2\theta$. Taking the spin into account, we get $dW \sim \{1+\cos^2\theta \exp[i(k_1-k_2)(x_1-x_2)]\}$. Note that for $\mathbf{k}_1=\mathbf{k}_2$ the interference is still as strong as before. The only consequence of this $\cos\theta$ is a tiny decrease of the effective volume of the elementary cell $(\Delta x \Delta k)$, where interference does take place. In other words, taking the fermion spin into account we diminished the value of $\epsilon$ a little bit. It should be stressed here that in the region of interest $|\eta| \sim 3$ the pion momenta $k_1, k_2$ are not too small. In the nucleon rest frame we deal with the values of $|\mathbf{k}_1|,|\mathbf{k}_2| \sim 1$ GeV. On the other hand, in the region of the interference peak the difference $\Delta k=|\mathbf{k}_1-\mathbf{k}_2|$ is of the order of $1/R \approx 30$ MeV. So the typical values of $\cos^2\theta \approx 1-(\Delta k)^2/k^2 \approx 0.998$ are very close to 1.

To demonstrate the role of spin we have considered numerically the production of two pions emitted by a couple of nucleons in the deuteron-deuteron collision [10], the $\pi NN$ vertex being taken as $\gamma_5$. Here the Goldstone nature of the pion reveals itself as much as possible. We do observe a certain effect (of about 3.5%) but it is too small to change our previous result noticeably.

## V. PION FINAL STATE INTERACTION

The natural question one could inquire about is the interaction of pions during the short time, when nuclei overlap, and pion interaction in the final state. The importance of this point is related to the fact that coherence could be diminished because of pion rescattering. Corresponding amplitudes must be added to the $S$ matrix, Eqs. (1)–(3), producing random phases in the waves, Eq. (8). In such a situation the production of pions of large multiplicity is rather questionable, because it is based on the strong constructive interference of pion formation amplitudes in Eqs. (1)–(3). Moreover, the strong coherent effects in the pion spectra, Fig. 1, could be smeared by pion scattering.

The important circumstance which saves the pion coherence consists in the fact that a time is necessary to pass (hadronization time), when the quark-antiquark pair created from the parton ladder transforms into an interacting pion. We estimate this time in the pion rest frame as $t_0=0.5$ fm, which is close to the pion radius. It is natural to suppose that there are no small $x$ pions formed (i.e., in the central plateau region) inside the fast moving nucleon because the intrinsic transverse momentum of partons in the nucleon is large ($\sim 2-3$ GeV [11]). Its inverse value (with the corresponding Lorentz $\gamma$ factor) gives the characteristic time for the parton fluctuation in the nucleon. This is much lower than necessary for pions with the corresponding energy to be formed.

The time delay for the beginning of pion interactions allows one to produce a coherent pion state, Eq. (1). In order to understand the minimal distance from the source along the three-axis, where the coherent pion wave has already been created (the formation length of the wave), let us consider the space picture of pion radiation by the classical current $J(x)$. Using Eq. (8) for the current, one can obtain the following solution of the Klein-Gordon equation for the Fourier

[3]We wish to thank B.L. Ioffe for drawing our attention to this important point and for an interesting discussion.

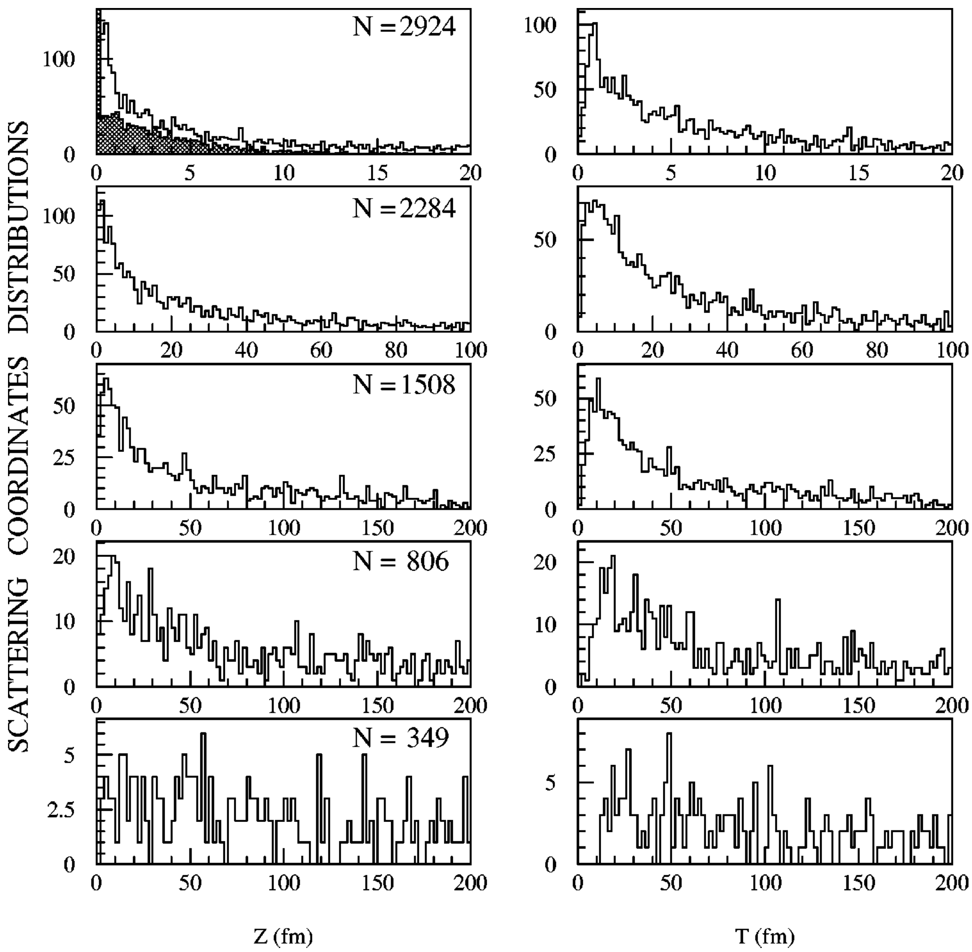


FIG. 3. Distributions of longitudinal (left) and time (right) coordinates of pion scattering for five iterations of Monte Carlo simulations (from upper to lower, light histograms). The formation length of the pion wave is shown by the dark histogram.

transform of the pion field with frequency $\omega$:

$$\phi(\omega,\mathbf{x})=\frac{1}{4\pi}\sum_{i=1}^{N} e^{i\omega t_i}\int d\mathbf{x}'\frac{e^{ik|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|}J_{\pi}(\omega,\mathbf{x}'-\mathbf{x}_i). \tag{13}$$

Here $k=\sqrt{\omega^2-\mu^2}$, and $J_\pi(\omega,\mathbf{x})$ is the Fourier transform of the current $J_\pi(x)$. As our source is a thin disk, with a thickness $d$ much smaller than its transverse size, we can imagine all radiating centers placed on the transverse plane at $x_3=0$ (we consider the waves with $kd\ll 1$ only). The sum in Eq. (13) is approximated by an integral over the plane with the density of the centers $\rho(\mathbf{b})$ ($\mathbf{b}$ is a current coordinate on the plane), and we obtain the usual plane wave moving along the three-axis vector $\mathbf{i}_3$ near the disk:

$$\phi(\omega,\mathbf{x})=\frac{1}{4\pi}\int d\mathbf{y}J_{\pi}(\omega,\mathbf{y})\int d\mathbf{b}\frac{e^{ik|\mathbf{x}-\mathbf{b}-\mathbf{y}|}}{|\mathbf{x}-\mathbf{b}_i-\mathbf{y}|}\rho(\mathbf{b})$$
$$=\frac{i\rho}{2k}\sqrt{2\omega(2\pi)^3}J_0(k\mathbf{i}_3)e^{ikx_3} \tag{14}$$

[here $J_0(\mathbf{k})$ is given by Eq. (8)]. For applicability of such an approximation, the density of sources (i.e., colliding $NN$ pairs) must be large enough in the main part of the integration path. The phase of the exponent changes from 0 to $\pi$ within the interval $\Delta b$, which can be estimated from the equation $k\sqrt{x_3^2+\Delta b^2}=kx_3+\pi$. So the average separation $r_0=R/\sqrt{N}$ between elementary sources on the plane must be $r_0\ll\Delta b$. With $\Delta b=nr_0$ we obtain the following value for the minimal $x_3$, where the plane wave has already been formed:

$$x_3=\frac{k}{2\pi}(nr_0)^2-\frac{\pi}{2k}. \tag{15}$$

Taking $n=3$, we obtain a formation length equal to $x_3\approx 1$ fm for the wave with the characteristic energy $\omega=0.75$ GeV. It is smaller than the distance $l\approx 2.6$ fm where pions with hadronization time $t_0$ start to interact. So pion scattering does not prevent significantly the production of the coherent state, Eq. (1), because the former has switched on only after this state has been created.

To understand in more detail this problem as well as possible distortions of pion spectra by scattering, we performed a Monte Carlo simulation of the multipion creation. Initial pion spectra have been simulated according to the transverse momenta and rapidity distributions of Fig. 1. The initial space coordinates of pions were obtained with the Woods-Saxon distribution for the transverse coordinates and normal distribution with $\sigma=1/0.75\ \mathrm{GeV}^{-1}$ for the longitudinal and time coordinates (the central pion). After that five other pions from one nucleon-nucleon interaction were smeared normally around this point according to the nucleon size.

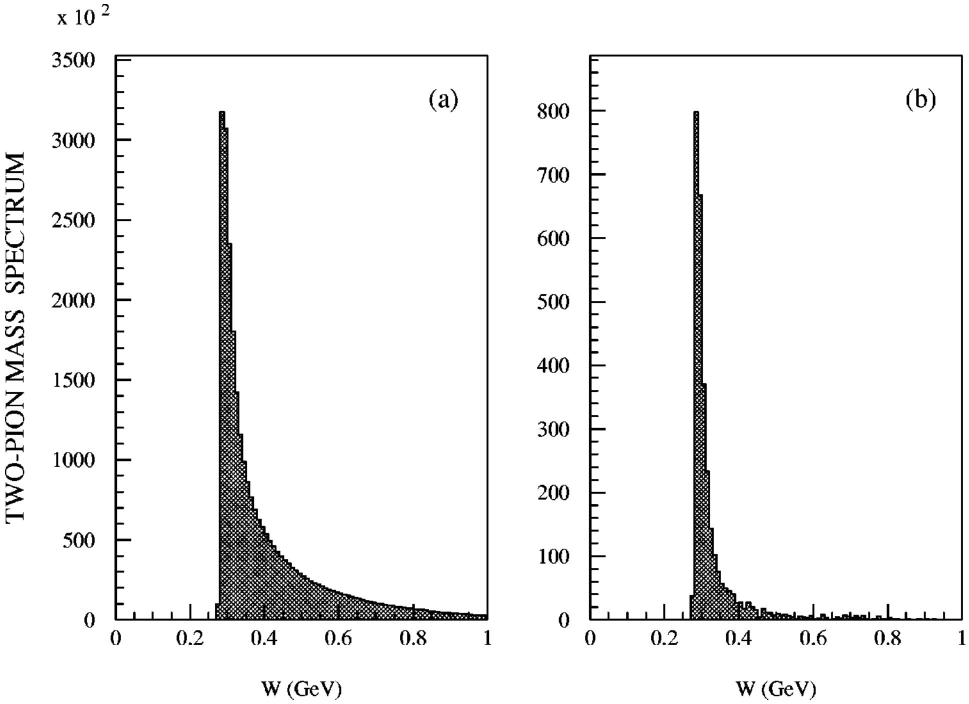


FIG. 4. The two-pion mass spectrum for (a) all pairs in the initial pion sample and (b) pions of the first scattering only.

The total number of simulated pions equals $N_\pi \approx 6500$. This number corresponds to the mean multiplicity of pions of the same sign of charge. Now we consider the scattering of $\pi^0$ mesons by charged mesons, as this interaction is the strongest one. To simulate $\pi^0$ scattering, we represent the differential $\pi\pi$ cross section by the $\rho$ resonance. When the distance between two pion trajectories becomes smaller than $\sqrt{\sigma/\pi}$, we obtain the space-time point of the interaction and simulate final particles in accordance with the differential cross section. To take into account the time of the hadronization of pions, the cross section is taken in the form $\sigma(t/t_0)^2$, when $t < t_0$. Here $t$ is the minimal value of the times of flight of two interacting pions, calculated in their rest frames.[4] After finishing the first iteration, when all pairs of $N_\pi$ pions have been considered, we use the new coordinates and momenta of the pions instead of the initial ones and go onto the next iteration. Such a classical consideration allows us to trace the further behavior of the pion cloud after the short-time production period.

In Fig. 3 we show distributions of the $x_3$ coordinate and time of pion scattering for the first five iterations; the number of interacting pairs is shown for every iteration as well. The space-time development of the process of pion scattering in the final state can be traced now. These distributions differ considerably from one iteration to another mostly in the scale of the horizontal axis, being approximately similar in the spectrum shapes. One may conclude that significant scattering takes place in the wide space-time region up to the distance $\sim 100$ fm. At the same time, the average number of interactions is not so high. We observe convergence of the iteration process (see the number of interacting pairs); on an average, every pion has no more than five final state interactions.

The distribution of the formation length, Eq. (15), of the pion wave is shown in the first picture of Fig. 3 by the dark histogram; the spectrum is normalized on the total number of interacting pairs. The large number of events occupies the first bin of the histogram; these are low energy pions which have formed the wave at once after the emission. Comparing the dark and light histograms we find that the first distribution shows smaller distances than the second one. That is in accordance with the estimate which has been done before in the analysis of Eq. (15). So one can conclude that the pion wave as a rule has been formed before the final state interaction.

The time delay of the first pion scattering due to the pion hadronization results in an important feature of the pion final state interaction. Two-pion effective mass spectra are shown in Fig. 4 both for all initial pairs of mesons and for mesons of the first scattering only. We see a strong narrowing of the mass spectrum of interacting pions near the two-pion threshold. This effect is connected with the fact that the time delay allows for the fast moving pion to interact only with a partner outside the region, where the heavy ions overlap. This could be only a pion moving fast in the same direction. Thus, the relative energy of the colliding $\pi\pi$ pair should be rather low.

This feature results in two important consequences for the process of coherent multipion creation. At first, the final state interaction does not produce any visible changes in the momenta of pions because of the small relative energy of interacting particles. So all coherent effects in the pion spectra of Fig. 1 continue to take place after such interactions. The second consequence is the fact that the produced pion cloud has features of an almost ideal Bose gas [12]. As the scatter-

[4]This $\sigma \sim t^2$ dependence is motivated by the color transparency effect $\sigma \sim \Delta r^2$, as the distance between the quarks (in $qq$ pairs), $\Delta r$, increases proportional to $t$ (at small $t$).

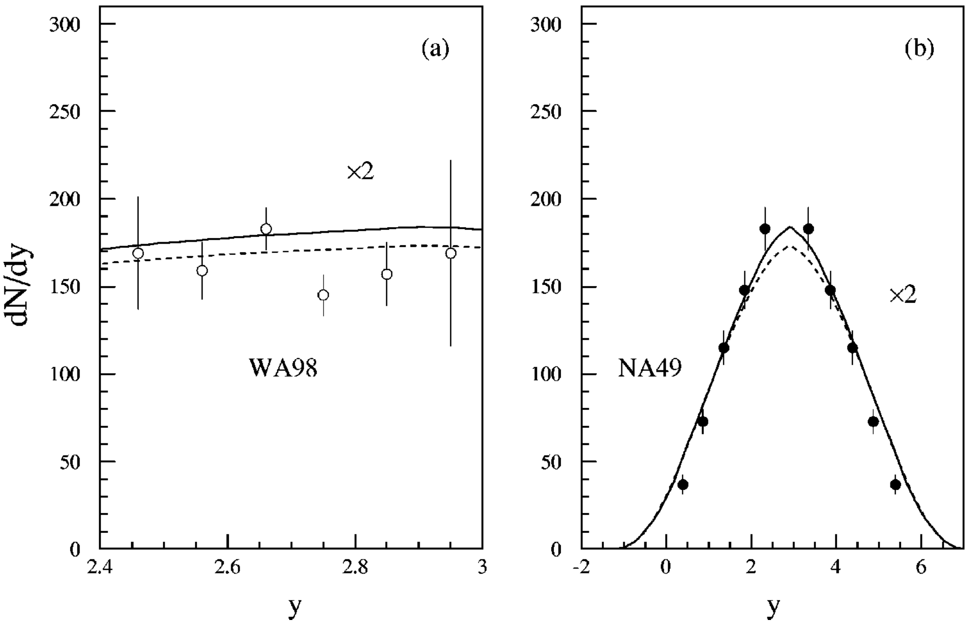


FIG. 5. Inclusive pion rapidity distributions (the average number of particles per event) with (the solid line) and without (the dashed line) the interference contribution. The data are for (a) $\pi^0$ mesons and (b) negative hadrons. The theoretical values are multiplied by a factor of 2 (see the text).

ing length $a=a_2/\sqrt{2}\approx -0.06$ fm is small enough in comparison with the average distance between the pions, $r\approx 0.4$ fm (in the rest frame of the pion cloud), the condition of a gas approximation, $a/r\ll 1$, is satisfied. In our Monte Carlo simulation we can trace the trajectories of all pions. We do not present here the corresponding figures, but it is interesting to note that the cloud moving, say, in the direction of positive $x_3$ has longitudinal size $L\approx 1$ fm after the flight time $t=1$ fm and $L\approx 4$ fm at $t=40$ fm, the transverse size coinciding with the nucleus one.

Thousands of pions occupy here the same elementary cell of the phase space. At the same time, the speed of sound in this gas is very high. A nonrelativistic naive estimate gives $u=(4\pi|a|/\mu^2 r^3)^{1/2}\approx 4.7$, which is larger than the speed of light. Of course the true value of $u$ is $\leqslant 1$. However, this naive estimate indicates that the speed of sound in the gas is close to unity due to the high density of particles. So the classical pion field seems to have enough time to be formed before the gas expands considerably. This encourages us to expect the creation of a classical pion field wave of Anselm's type [4]. Taking into account the restricted size of the wave packet, we could suppose that this field bears the features of a pion brither [13]. This is the oscillating quasistable solution of the sine-Gordon equation and it decays slowly, emitting radial waves. Such a field was discussed in [13] as a possible decay product of the DCC bubble. When radiated, pions appeared to have some characteristic momentum in this case.

It must be stressed here that the delay of the final state interaction for the pion hadronization time results in the coherence of some part of the pions, which have been hadronized before the formation of the plane wave, Eq. (14). If one twice decreases the time $t_0$, the effective mass spectrum in Fig. 4(b) for mesons of the first scattering remains practically unchanged. So the threshold interaction dominates in this case too. By virtue of the small scattering length (the gas approximation) pion final state interactions do not add random phases to the radiated pion waves. They produce a pion optical potential which changes the wavelength of a pion not eliminating the coherence.

## VI. COMPARISON WITH DATA

In spite of the fact that the model of a pion radiation by classical current is rather crude and our results have mostly qualitative character, we compare them with the existing experimental data in Fig. 5. We use the results of two CERN experiments WA98 and NA49 on a Pb-Pb interaction at the energy 158*A* GeV [14]. The high centrality of the event was achieved there using the forward veto calorimeter, though this is somewhat lower than the value corresponding to $N=150$. As the region of low transverse momenta of pions was not accessed in both experiments, some approximation for the cross section dependence on $k_t$ was done there for obtaining the rapidity distribution. We reproduced the experimental procedure in our calculations with an accuracy $\sim$20% by taking a constant cross section in the region $k_t<0.2$ GeV. It should be noted here that the missing low $k_t$ domain is exactly the place of the most prominent manifestation of coherent phenomena due to the fact that these effects are mainly situated in the region $k_t\sim 1/R$, i.e., at $k_t<0.1$ GeV.

Another effect we have to account for in comparing with the data is the quark structure of a nucleon. In a proton-proton collision mainly one beam quark participates in the inelastic interaction with the target. Two other valence quarks act as spectators. Contrarily, in the case of a central collision with a nucleus, two or even all three quarks from a beam nucleon interact inelastically. We mean the pion production in the additive quark-type model, when one, two, or three multiperipheral ladders could be developed independently of the valence quarks of the fast moving proton. It is a twofold or threefold increasing of the number of ladders that leads to the strong growth of the pion multiplicity. Models of such a type explain the existing experimental observation that the pion multiplicity in the proton-nucleus interaction is approximately twice as large as that in the proton-proton interaction [15]. This effect is not connected with the coherence. Thus, based on the calculations made in [15], we have multiplied our results by a factor of 2.

As is seen in Fig. 5 the experimental data do not contradict the discussed coherent effects both for the pion multiplicity and for the shape of the rapidity distribution. Note that the experimental spectrum in Fig. 5(b) is even narrower than the theoretical one. This fact encourages us and we consider it as an argument in favor of the important role of coherence effects. Future experiments, measuring small pion transverse momenta, could answer the question about pion coherence in heavy-ion collisions.

## VII. CONCLUSIONS

In high energy central nucleus-nucleus collisions a lot of pions are produced coherently, being at the same state. At RHIC energies the effect of coherence deforms essentially the momentum spectra of secondary pions and enlarges the multiplicity in comparison with naive (without the coherence) conventional estimates based on the model of independent nucleon-nucleon interactions.

The crucial point is the longitudinal Lorentz contraction of colliding nuclei. Therefore almost all pions produced with $k_t \lesssim 1/R$ are emitted coherently.

We have demonstrated that the final state interaction of secondaries does not destroy the coherence effects. Because of a nonzero formation (hadronization) time of secondary pions, only a pair of pions moving at the same direction with almost equal momenta has enough chance to interact. Thus, final state rescattering practically does not alter the pion momentum distribution, and all coherent effects continue to take place.

The only way to reduce the effect shown in Figs. 1 and 2 is to say that the pions are formed much after the collision, for example, if at the first stage the quark gluon plasma (QGP) would be created and then (after the expansion and cooling of QGP) the pions would be produced from the domain with the rather large longitudinal size $\Delta z \sim 10$ fm. In this case the effect of coherence would be strongly suppressed.

So the absence of prominent coherent effects at RHIC may be considered as an argument in favor of QGP (or another new phase) formation; the pions are emitted from a large size domain after the ''decay'' of this new phase. Of course, in reality there will be competition between the direct production of a classical (coherent) pion field and the formation of another (like QGP) phase.


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