Relationship between the fermion dynamical symmetric model Hamiltonian and nuclear collective motion

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In this paper we investigate the role of single-particle energies on the low-lying states of ^{132}Ba, a typical O(6) nucleus in the IBM and the fermion dynamical symmetric model (FDSM). It is found that one can reproduce the physical quantities of a realistic system with nondegenerate single-particle energies using degenerate single-particle levels and a slightly different parametrization of the two-body interaction. However, if the single-particle splittings are enlarged by a factor of 1.5, the O(6)-like behavior of the nucleus is lost and a model that assumes degenerate levels cannot describe its collective structure. Contributions from interactions other than monopole and quadrupole pairing and a quadrupole-quadrupole force are found to be unimportant. Although the role of the abnormal-parity level depends on the details of the single-particle structure, its effects can be ''compensated'' by using different Hamiltonian parameters and degenerate single-particle levels in a FDSM treatment.

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I. INTRODUCTION

Through the great success of the IBM [1], it has been recognized that collective *S* and *D* pairs (or perhaps *S*, *D*, and *G* pairs in rotational nuclei) play a dominant role in the low-lying collective structure of medium-heavy nuclei. In the IBM these collective *S* and *D* pairs are approximated as *s* and *d* bosons for the sake of simplicity. A similar model, the fermion dynamical symmetry model (FDSM) [2], uses SP(6) or SO(8) symmetry-dictated *SD* pairs as building blocks of a truncated shell-model space.

Recently, an algorithm for the nucleon pair approximation of the shell model [3] was proposed. If one restricts to *SD* nucleon pairs, this algorithm is reduced to the *SD* pair approximation of the shell model, or the *S* and *D* pair shell model (SDPM). The *S* and *D* pairs of the SDPM, unlike those of the FDSM, have no restriction of dynamical symmetry in nucleon pair structure. In other words, the structure coefficients of the *SD* pairs can be arbitrary. As a consequence, the FDSM can be regarded as a special case of the SDPM. This property of the SDPM makes it a nice tool with which some basic questions concerning nuclear collectivity can be investigated. For example, (I) Can the behavior of dynamical symmetries be reproduced using nonsymmetry dictated nucleon pairs? (II) How well do the assumptions in the FDSM work for realistic nuclei? (III) How well do various boson mappings work in microscopic studies of the IBM? Answers to these questions would be helpful in providing an increased microscopic understanding of the IBM and the FDSM.

The answer to the question (I) is affirmative. Recent calculations [4,5] within the SDPM successfully reproduced the IBM (O)(6) properties of the ^{134,132}Ba and ^{132}Xe, both for the energy spectra and $E2$ transition rates. From the point of view in the shell model, the calculation of [4] is more realistic than that of [5] in several respects: (1) Reference [4] uses more appropriate signs for the neutron-proton quadrupole-quadripole interaction and for the effective neutron charge than [5]. (2) The parameters of the SDPM Hamiltonian used in [4] were in closer accord with those of frequently used interactions. (3) The overall fit with the experimental data such as binding energies, excitation energies, $E2$ transition rates, and nuclear radii in [4] were superior to the previous calculations.

A study of question (III) using the nucleon pair approximation of the shell model is in progress. In this paper we discuss question (II) by studying ^{132}Ba. In the FDSM the normal-parity single-particle levels reduce to just one or two degenerate orbits in spite of the fact that they are not degenerate in the shell-model single-particle basis; there are only $\mathcal{S}$ paris on the abnormal-parity level. The residual pairing interaction is dominant by the monopole and quadrupole terms. We use the calculation of [4] as the starting point to address the following three questions, which are related to key assumptions of the FDSM:

(i) What is the effect of the nondegeneracy of realistic single-particle energies on dynamical symmetry?

(ii) What is the effect of the abnormal parity levels on nuclear collectivity?

(iii) Is the residual pairing interaction dominated by its monopole and quadrupole pairing?

This paper is organized as follows: In Sec. II we describe how to truncate the nuclear shell model space to the *SD*-pair subspace. In Sec. III we study the effect of single-particle splittings, and the consequences of neglecting the non-$\mathcal{S}$ pairs on the abnormal-parity level are studied in Sec. IV. Up to this point, we assume that the interaction between like

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valence nucleons contains monopole and quadrupole pairing and a quadrupole-quadrupole component and that the interaction between protons and neutrons has a pure quadrupole-quadrupole form. In Sec. V a hexadecapole pairing interaction and a hexadecapole-hexadecapole interaction are introduced, and their contribution is investigated. All the calculations are carried out using the parameters of [4] as the starting point. Conclusions and a summary are given in Sec. VI.

II. THE *SD* SUBSPACE AND HAMILTONIAN

The creation operators of collective S and D pairs are defined to be

$$S^{\dagger}=\sum_{a}\hat{j}\frac{v_j}{u_j}(C_j^{\dagger}\times C_j^{\dagger})^0,$$

$$D^{\dagger}=\frac{1}{2}[Q,S^{\dagger}], \tag{1}$$

where the u_j and the v_j, are the unoccupied and occupied amplitudes (for orbit j), respectively. They are obtained by solving the Bardeen-Cooper-Schriefer (BCS) equation. We denote this as $\hat{j}=\sqrt{2j+1}$ throughout this paper. The Q in the above equation is the quadrupole operator, defined according to

$$Q=\sum_{\alpha\gamma}\langle\alpha|r^2Y^2|\gamma\rangle C_{\alpha}^{\dagger}C_{\gamma}=\sum_{jj'}q(jj')(C_j^{\dagger}\times\tilde{C}_{j'}^{\dagger})^2. \tag{2}$$

Here, $\alpha=(nljm)$ and $\gamma=(nl'j'm')$ denote the single-nucleon states and

$$q(jj')=\frac{(-)^{j+1/2}}{\sqrt{20\pi}}\hat{j}\hat{j}'C^{20}_{j1/2,j'-1/2}\langle Nl|r^2|Nl'\rangle.$$

$C^{20}_{j1/2,j'-1/2}$ is a Clebsch-Gordan coefficient. Note that in Eqs. (1) and (2), we omit for the sake of simplicity the subscripts $\sigma=\pi$ or ν for proton and neutron, respectively. The above definition has the same form for both.

The Hamiltonian we consider is the sum of the following three terms:

$$H=H_{\pi}+H_{\nu}+\kappa Q_{\pi}\cdot Q_{\nu}. \tag{3}$$

H_{π} and H_{ν} are the Hamiltonian for like valence protons and neutrons, respectively, and take the form

$$H_{\sigma}=\sum_{j_\sigma}\epsilon_{j_\sigma}C^{\dagger}_{j_\sigma}C_{j_\sigma}+g_{\sigma}^{(0)}P_{\sigma}^{\dagger(0)}P_{\sigma}^{(0)}$$
$$+g_{\sigma}^{(2)}P_{\sigma}^{\dagger(2)}\cdot P_{\sigma}^{(2)}+\kappa_{\sigma}Q_{\sigma}\cdot Q_{\sigma}, \tag{4}$$

where

$$P_{\sigma}^{\dagger(0)}=\sum_{j_\sigma}\frac{\hat{j}_\sigma}{2}(C^{\dagger}_{j\sigma}\times C^{\dagger}_{j_\sigma})^0,$$

$$P_{\sigma}^{\dagger(2)}=\sum_{j_\sigma}q(jj')(C^{\dagger}_{j_\sigma}\times C^{\dagger}_{j_\sigma})^2. \tag{5}$$

When we consider in Sec. V the role of pairing interactions of the multipolarity $\lambda>2$ order, we will use the notation $V_{\sigma}^{\lambda}=g_{\sigma}^{(\lambda)}P_{\sigma}^{\dagger(\lambda)}\cdot P_{\sigma}^{(\lambda)}$, where

$$P_{\sigma}^{\dagger(\lambda)}=\sum_{j_\sigma j'_\sigma}y^{\lambda}_{j_\sigma j'_\sigma}(C^{\dagger}_{j_\sigma}\times C^{\dagger}_{j'_\sigma})^{\lambda},$$

$$y^{\lambda}_{j_\sigma j'_\sigma}=-\frac{\langle j_\sigma\|Y^{\lambda}\|j'_\sigma\rangle}{\hat{\lambda}}=\frac{(-)^{j_\sigma+1/2}}{\sqrt{4(2\lambda+1)\pi}}\hat{j}_\sigma\hat{j}'_\sigma C^{\lambda 0}_{j_\sigma 1/2,j'_\sigma-1/2}. \tag{6}$$

Likewise, particle-hole interactions of multipolarity $\lambda>2$ will be defined by

$$V_{QQ}^{\lambda\sigma}=\kappa_{\sigma}^{\lambda}Q_{\sigma}^{\lambda}\cdot Q_{\sigma}^{\lambda}, \tag{7}$$

with Q_{σ}^{λ} given by

$$Q_{\sigma}^{\lambda}=\sum_{j_\sigma j'_\sigma}y^{\lambda}_{j_\sigma j'_\sigma}(C^{\dagger}_{j_\sigma}\times\tilde{C}_{j'_\sigma})^{\lambda}. \tag{8}$$

III. CONTRIBUTION FROM THE SINGLE-PARTICLE SPLITTINGS

The realistic shell-model Hamiltonian contains a nondegenerate single-particle energy term, as prescribed by experimental data [7]. In the FDSM the effect from the single-particle splittings is ignored in order to generate a closed algebra. If the contribution from the single-particle splittings is important, the FDSM truncation would not be reasonable no matter how well it fits the experimental data.

The validity of neglecting contribution from the single-particle nondegeneracy has been checked for two cases. Kirson and Leviatan [9] showed that introducing a realistic nondegeneracy of single-particle levels has only a small effect on the SO(5)×SU(2) dynamical symmetry. Wu *et al.* [10] showed that the SU(3) symmetry is preserved quite well even for highly nondegenerate single-particle splittings. For no other cases than these has the importance of the single-particle splittings been investigated.

Since the O(6) properties are successfully reproduced in [4] with the inclusion of single-particle splittings, we can investigate in that model the extent to which these dynamical symmetry properties are preserved under artificial (but not outrageous) adjustments of the single-particle splittings. If the variation of these splittings has only a small effect on the calculated results, which furthermore can be compensated by a reasonable adjustment of the parameters of the Hamiltonian, one can conclude that the omission of the single-particle splittings is *safe*. On the other hand, if the O(6) behavior cannot be restored, even with a fairly large adjustment

TABLE I. Single-particle levels for protons (particlelike) and neutrons (holelike) adopted from [6].

j	$g_{7/2}$	$d_{5/2}$	$d_{3/2}$	$h_{11/2}$	$s_{1/2}$
ϵ_π (Mev)	0	0.963	2.69	2.76	2.99
ϵ_ν (Mev)	2.434	1.655	0.000	0.242	0.332

of the parameters. This would imply that the splittings could not be ignored, and that for such cases the FDSM truncation could not provide a good approximation to the shell model.

To address this issue, we define adjusted single-particle energies to be $\alpha\epsilon_{j\sigma}$ in terms of a single parameter α, where $\epsilon_{j\sigma}$ are single-particle energies used in [4] and are given in Table I. $\alpha=0$ corresponds to degenerate single-particle energies, and $\alpha=1.0$ to the realistic single-particle splittings. Figure 1 shows the general trend of the calculated energy spectra, with the experimental data in panel (a). The results in panels (b)–(f) assume $G_\pi=-0.180$ MeV, $G_\nu=-0.131$ MeV, $\kappa=0.06$ MeV$/r_0^4$, $\kappa_\pi=\kappa_\nu=0.045$ MeV$/r_0^4$, $G_\pi^2=G_\nu^2=-0.03$ MeV$/r_0^4$, and $r_0^2=\hbar/M_\sigma\omega_0=1.012A^{1/3}$ fm^2. Those in panel (g) assume $G_\pi=G_\nu=-0.160$ MeV, with all other parameters the same. The values of α assumed for each panel are given in the caption. The corresponding results for $E2$ transition rates are given in Table II.

We see that the calculated spectra compress smoothly and slowly with the decreasing of α. It is interesting to note that the calculated $B(E2)$ values also change very ''slowly'' with α in panels (b)–(f) even when we use the same set of parameters in the two-body interaction. It means that the general behavior of the IBM O(6) properties are preserved even with fairly significant changes in the single-particle splittings change drastically (α decreasing from $1\to0$). The regular and slight changes in the energy levels that arise by ignoring the single-particle splittings ($\alpha=0$) can be restored using $G_\sigma=-0.160$ MeV, as shown in panel (g). Note that the fit is good if one empirically keeps $G_\pi=G_\nu$ when $\alpha=0$, or $G_\pi\sim G_\nu$ when α is very small.

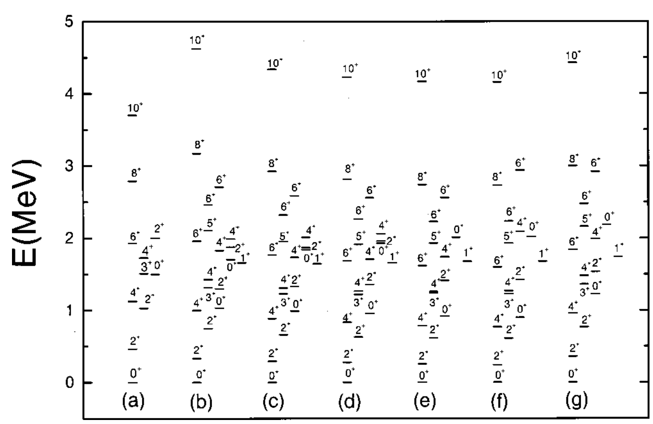

FIG. 1. Excitation energy levels changing with α. (a) Experimental data; (b) $\alpha=1.0$; (c) $\alpha=0.7$; (d) $\alpha=0.5$; (e) $\alpha=0.2$; (f) $\alpha=0.0$; (g) $\alpha=0.0$. We use the same set of parameters ($G_\pi=-0.180$ MeV, $G_\nu=-0.131$ MeV, $\kappa=0.06$ MeV$/r_0^4$, $\kappa_\pi=\kappa_\nu=0.045$ MeV$/r_0^4$, $G_\pi^2=G_\nu^2=-0.03$ MeV$/r_0^4$) for (b)–(g) except that $G_\sigma=-0.16$ MeV in (g).

TABLE II. The relative $B(E2)$ values for the even Ba isotopes. The experimental data are taken from [8]. Refer to the text for the explanation of each case.

$J_i\to J_f$	Expt.	O(6)	(b)	(c)	(d)	(e)	(f)	(g)
$0_2^+\ \to\ 2_2^+$	100	100	100	100	100	100	100	100
$\to\ 2_1^+$	0	0	1.2	<0.1	0.5	2.1	3.0	<0.1
$2_2^+\ \to\ 2_1^+$	100	100	100	100	100	100	100	100
$\to\ 0_1^+$	0.2	0	0.1	<0.1	0.8	3.1	4.6	<0.1
$3_1^+\ \to\ 2_2^+$	100	100	100	100	100	100	100	100
$\to\ 4_1^+$	73	40	34.2	36.0	37.3	38.4	39.0	35.1
$\to\ 2_1^+$	0.2	0	<0.1	0.9	4.4	12.6	16.5	<0.1
$4_2^+\ \to\ 2_2^+$	100	100	100	100	100	100	100	100
$\to\ 3_1^+$		0	<0.1	0.6	2.3	5.4	7.1	<0.1
$\to\ 4_1^+$	75	91	94.1	102.1	100.1	83.2	75.0	94.3
$\to\ 2_1^+$	2.2	0	1.2	1.9	2.1	1.8	1.5	<0.1
$5_1^+\ \to\ 3_1^+$	100	100	100	100	100	100	100	100
$\to\ 4_2^+$		46	53.7	53.2	47.2	37.0	33.6	33.2
$\to\ 6_1^+$		45	41.2	41.5	38.2	32.1	30.9	27.3
$\to\ 4_1^+$		0	0.5	2.7	6.0	10.5	12.3	<0.1

It is also interesting to see the critical value of α for which the general features of the O(6) symmetry disappear. Figure 2(a)–2(e) shows the calculated results for $\alpha=1.0,1.3,1.5,1.8,2.2$, assuming the two-body interaction parameters to be as in Figs. 1(b)–1(f). Clearly, when $\alpha>1.5$, the O(6) behavior is quickly lost (Table III shows the corresponding results for $E2$ transition rates). Even with a fairly large adjustment of the two-body parameters, it is not possible to describe the $E2$ rates and energy levels with a description that ignores the single-particle splittings. This result indicates that the contribution of the single-particle term in the Hamiltonian would be very important in the low-lying properties of ^{132}Ba [an O(6)-symmetric nucleus in the IBM and the FDSM] were the single-particle splittings more than 1.5 times larger than for a realistic Hamiltonian.

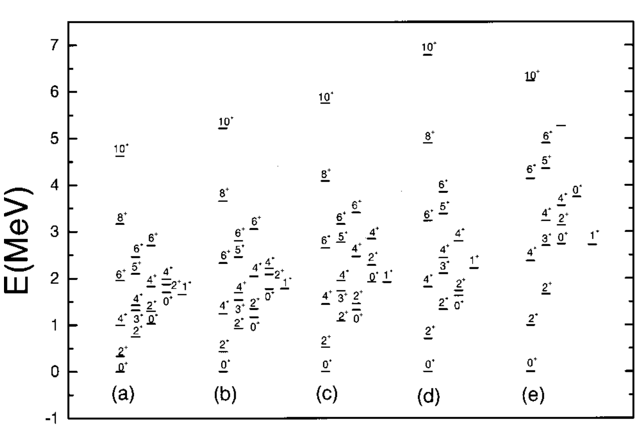

FIG. 2. Excitation energy levels changing with α. Left-hand side: experimental data; (a) $\alpha=1.0$; (b) $\alpha=1.2$; (c) $\alpha=1.5$; (d) $\alpha=1.8$; (e) $\alpha=2.2$. We use the same set of parameters [case (a) of Fig. 1] in the Hamiltonian for (a)–(e).

TABLE III. The relative $B(E2)$ values for the even Ba isotopes.

$J_i \rightarrow J_f$		(a)	(b)	(c)	(d)	(e)
0_2^+	$\rightarrow 2_2^+$	100	100	100	100	100
	$\rightarrow 2_1^+$	1.2	6.3	15.2	79.2	4883.7
2_2^+	$\rightarrow 2_1^+$	100	100	100	100	100
	$\rightarrow 0_1^+$	0.1	0.5	0.9	2.6	15.9
3_1^+	$\rightarrow 2_2^+$	100	100	100	100	100
	$\rightarrow 4_1^+$	34.2	31.1	27.2	21.6	17.1
	$\rightarrow 2_1^+$	<0.1	0.5	1.1	2.7	8.1
4_2^+	$\rightarrow 2_2^+$	100	100	100	100	100
	$\rightarrow 3_1^+$	<0.1	<0.1	<0.1	0.4	5.1
	$\rightarrow 4_1^+$	94.1	82.6	68.0	44.1	20.4
	$\rightarrow 2_1^+$	1.2	<0.1	0.3	3.7	8.9
5_1^+	$\rightarrow 3_1^+$	100	100	100	100	100
	$\rightarrow 4_2^+$	53.7	51.2	50.4	47.3	51.0
	$\rightarrow 6_1^+$	41.2	38.2	35.5	30.1	20.4
	$\rightarrow 4_1^+$	0.5	<0.1	0.7	3.2	7.5

The results support the conclusions of [10] that realistic single-particle splittings do not produce much symmetry breaking. On the other hand, they also suggest that for single-particle splittings not that different from those of a realistic Hamiltonian, significant breaking of the O(6) symmetry would occur for nuclei in the (50-82) shell.

In [10] it was suggested that in the (126-182) shell only when the single-particle splittings were enormous (roughly ten times the observed splittings) would the effect of SU(3) dynamical symmetry breaking be significant. Our results, however, set a much smaller upper limit ($\approx$1.5) in order to preserve the O(6) symmetry approximately. We note here without further details that the above single-particle splitting effects would be smaller if we artificially enlarge the pairing interaction parameters.

IV. CONTRIBUTION FROM THE ABNORMAL-PARITY LEVEL

The relative contributions of nucleons in the normal-parity and abnormal-parity states in a major shell of the low-lying structure of medium-heavy nuclei is another topic of great contemporary interest. In the early works of the FDSM [2] and the pseudo-SU(3) model [11], nucleons in the abnormal parity high-j state were usually approximated to be coupled to seniority 0 states and the remaining nucleons in normal-parity states of a particular valence shell are permitted to deform. Although this shunting of nucleons in the abnormal-parity states to a very minor role can be compensated for by increasing the overall ''normalization'' constant, this success may not be used to validate this simplification. A more essential issue is how the omission of the non-$\mathcal{S}$ pairs in the abnormal-parity level affects the level structures of low-lying collective states and the $E2$ transitions between them. This is not a trivial issue, since the omission of non-$\mathcal{S}$ pairs from the abnormal-parity level may ''destroy'' a dynamical symmetry, as we saw for the single-particle splittings. In this section we show that if one omits non-$\mathcal{S}$ pairs in the abnormal parity level with the single-particle levels untouched, it may be difficult to restore the original physics, because the contribution from these non-$\mathcal{S}$ pairs in the abnormal-parity state depends sensitively on the structure of the single-particle levels. On the other hand, if we use degenerate single-particle levels, the omission of the non-$\mathcal{S}$ pairs in the abnormal-parity level can be compensated by the use of slightly different parameters in the two-body interaction. This suggests that the approximation of neglecting the non-$\mathcal{S}$ pairs from the abnormal-parity level is rather safe for

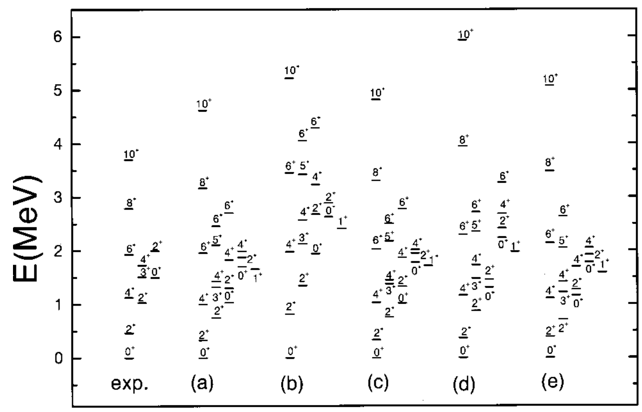

FIG. 3. Comparison of excitation energy levels with the non-$\mathcal{S}$ pairs in the abnormal-parity level and neglecting these non-$\mathcal{S}$ pairs. (a) $\alpha=1.0$ (with the non-$\mathcal{S}$ pairs); (b) $\alpha=1.0$ (without the non-$\mathcal{S}$ pairs both for proton and neutron); (c) $\alpha=1.0$ (without the non-$\mathcal{S}$ pairs only for proton); (d) $\alpha=0.0$ (without the non-$\mathcal{S}$ pairs both for proton and neutron); (e) $\alpha=0.0$ (without the non-$\mathcal{S}$ pairs both for proton and neutron). For case (e), we use $G_\sigma=-0.150$ MeV, $\kappa=0.06$ MeV$/r_0^4$, $\kappa_\sigma=-0.038$ MeV$/r_0^4$, $G_\sigma^2=-0.034$ MeV$/r_0^4$. For other cases, the parameters in the Hamiltonian are the same as those (a) of Fig. 1.

TABLE IV. The relative $B(E2)$ values for the even Ba isotopes. The experimental data are taken from [8].

$J_i \rightarrow J_f$		(a) $\alpha=1.0$	(b) $\alpha'=1.0$	(c) $\alpha=1.0$	(d) $\alpha'=0$	(e) $\alpha_y=0$	Expt.
0_2^+	$\rightarrow 2_2^+$	100	100	100	100	100	100
	$\rightarrow 2_1^+$	1.2	162.5	1.0	5.6	<0.1	0.2
2_2^+	$\rightarrow 2_1^+$	100	100	100	100	100	100
	$\rightarrow 0_1^+$	0.1	5.4	<0.1	11.3	<0.1	0
3_1^+	$\rightarrow 2_2^+$	100	100	100	100	100	100
	$\rightarrow 4_1^+$	34.2	25.1	34.1	20.6	31.8	73
	$\rightarrow 2_1^+$	<0.1	3.8	<0.1	37.0	<0.1	0.2
4_2^+	$\rightarrow 2_2^+$	100	100	100	100	100	100
	$\rightarrow 3_1^+$	<0.1	14.5	0.7	100.1	<0.1	-
	$\rightarrow 4_1^+$	94.1	54.2	106.0	95.6	124.9	75
	$\rightarrow 2_1^+$	1.2	6.1	3.4	37.2	<0.1	2.2
5_1^+	$\rightarrow 3_1^+$	100	100	100	100	100	100
	$\rightarrow 4_2^+$	57.3	122.3	53.6	104.3	77.6	-
	$\rightarrow 6_1^+$	41.2	39.8	41.8	15.3	61.4	-
	$\rightarrow 4_1^+$	0.5	3.5	2.0	60.7	<0.1	-

^{132}Ba, the O(6) nucleus on which we are focusing. The reason is that the FDSM neglects *both* non-$\mathcal{S}$ pairs in the abnormal-parity level and single-particle splittings.

In Fig. 3 [panels (a) and (b)] and Table IV [columns (a) and (b)], we compare the calculated results including the non-$\mathcal{S}$ pairs in the abnormal-parity level with those that neglect them. We are unable to reproduce the relative $E2$ transition rates among the states listed in Table IV even if all the parameters are adjusted over a fairly vast range. This behavior may be understood as follows. From Table I, we see that the abnormal-parity level ($nlj = 55\frac{11}{2}$) of the valence neutron holes lies rather low in energy. As a consequence, the structure amplitude $y_\nu(\frac{11}{2}\ \frac{11}{2}\ 0)$ of the S pair is rather large [the largest among the $y_\nu(j_1 j_1 0)$'s]. Similarly, from Eq. (1), we see that $y_\nu(\frac{11}{2}\ \frac{11}{2}\ 2)$ must be large compared to the other $y_\nu(j_1 j_2 2)$'s. If one takes $y_\nu(\frac{11}{2}\ \frac{11}{2}\ 2)$ to be zero artificially, the D pair structure changes *too much* for the same physics to be restored. Thus, the pair structure, i.e., the truncation of the subspace, is very important for the symmetry behavior of the *SD* pair shell-model calculations.

To make this point clearer, we consider the case of proton single-particle levels (refer to Table I), where the abnormal-parity level $h_{11/2}$ is rather high compared to the normal-parity single-particle levels, so that $y_\pi(\frac{11}{2}\ \frac{11}{2}\ 2)$ is relatively small. The calculated results are given in Table IV and Fig. 3. We see that, in this case, omitting non-$\mathcal{S}$ pairs [in the $h_{11/2\pi}$] (column c) does not change the results as much as omitting the non-$\mathcal{S}$ neutron abnormal-parity pairs [(b) in Fig. 3 and Table IV]. Therefore, for nuclei with both $h_{11/2\pi}$ and $h_{11/2\nu}$ high lying in the valence shell (e.g., the 50-82 shell) and in which both the valence proton and neutron numbers are below Ω [$\Omega = \Sigma_j (j+\frac{1}{2})$], the symmetry behavior should be well preserved even if non-$\mathcal{S}$ pairs in the abnormal-parity level are omitted. Indeed, this symmetry behavior under such circumstances would be well maintained even if the same set of parameters in the Hamiltonian are taken and one uses the above method to construct the *SD* subspace.

The above results suggest that the symmetry behavior of the *SD* pair approximation is rather dependent on the single-particle structure. In the FDSM, non-$\mathcal{S}$ pairs in the abnormal-parity level are omitted *together with* the use of degenerate single-particle energies. It is interesting, therefore, to see the effect of neglecting the non-$\mathcal{S}$ pairs in the abnormal-parity level when using degenerate single-particle levels. In (d) and (e) of Fig. 3 and Table IV, we give the results that omit the non-$\mathcal{S}$ pairs in the abnormal-parity level under the restriction to $\alpha = 0$. In column (d), we use the same parameters as in column (a). The agreement with the results in column (a) are not good, especially for the $E2$ transition rates. In column (e), the parameters are adjusted slightly, using the empirical restriction of symmetric parameters between the proton and neutron degrees of freedom. This is seen to improve the fit dramatically. The O(6) behavior of ^{132}Ba is reproduced very well in (e).

V. CONTRIBUTION FROM THE MULTIPOLE PAIRING AND $Q^\lambda \cdot Q^\lambda$ ($\lambda = 4$)

In the FDSM it is assumed that pairing interaction is dominated by its monopole and quadrupole components. In [4,12], the residual interaction between like nucleons consisted of monopole pairing, quadrupole pairing, and a quadrupole-quadrupole force. In a recent study [14] of the microscopic foundation of the IBM, the residual interaction between like nucleons included a hexadecapole pairing and a hexadecapole-hexacapole [Eqs. (6) and (7), $\lambda = 4$] interaction as well. In [5] either a surface delta interaction (SDI) or monopole pairing were used for the residual interaction between like valence nucleons. A comparison of [4] and [13] shows that monopole pairing is dominant, but that the contribution from quadrupole pairing is not negligible. The quadrupole-quadrupole force between like nucleons does not contribute in a significant manner to the excitation energies, but it does have an important effect on the $E2$ transition rates. In this section we show that the contribution from hexadecapole pairing and a hexadecapole-hexadecapole force is rather small. This indicates that the Hamiltonian in [4,12] is sufficient to describe the low-lying excitations of nuclei in the $A \sim 130$ region.

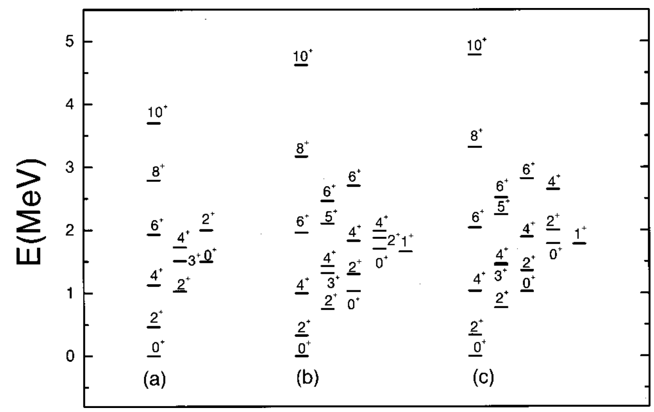

FIG. 4. Excitation energy levels with the hexadecapole pairing and hexadecapole-hexadecapole force and results without these terms. $G_\sigma^4 = -0.4$ MeV, $\kappa_4 = -0.3$ MeV, other parameters are the same as those of Fig. 1. (a) Experimental data; (b) calculational results without the hexadecapole terms; (c) calculational results with these hexadecapole terms.

In Fig. 4 and Table V we compare the results of the $E2$ transition rates and excitation energy levels with hexadecapole pairing and the hexadecapole-hexacapole interaction either included or omitted in the Hamiltonian. The hexadecapole terms are given in Eqs. (6)–(8). Here we use the same G_σ, G_σ^2, κ, and κ_σ as in [4] in both cases. We choose the strengths of the hexadecapole terms to be of the same order as in [14], namely $G_\sigma^4 = 0.4$, $\kappa_\sigma^4 = 0.3$ (both in units of MeV). As is clear, the calculated results change very little, and the contributions from these two terms can be neglected when compared with those from the other terms. As an example, the binding energy increases by only ~0.6 MeV when we include these two terms. When we include the hexadecapole pairing and hexadecapole-hexadecapole force separately, the same conclusions emerge. This can be used to validate the Hamiltonian of [4,12] for low-lying excitations in nuclei of the $A \sim 130$ region. Note that this conclusion is true only for low-lying excitations in medium heavy nuclei. For other cases it still needs to be examined.

TABLE V. The relative $E2$ transition rates with the hexadecapole pairing and the hexadecapole-hexadecapole force and the results without these hexadecapole terms.

$J_i \rightarrow J_f$	(a)	(b)	Expt.
$0_2^+ \rightarrow 2_2^+$	100	100	100
$\rightarrow 2_1^+$	1.2	0.9	0.2
$2_2^+ \rightarrow 2_1^+$	100	100	100
$\rightarrow 0_1^+$	0.1	0.5	0.2
$3_1^+ \rightarrow 2_2^+$	100	100	100
$\rightarrow 4_1^+$	34.2	33.6	73
$\rightarrow 2_1^+$	<0.1	0.2	0.2
$4_2^+ \rightarrow 2_2^+$	100	100	100
$\rightarrow 3_1^+$	<0.1	<0.1	-
$\rightarrow 4_1^+$	94.1	89.9	75
$\rightarrow 2_1^+$	1.2	0.6	2.2
$5_1^+ \rightarrow 3_1^+$	100	100	100
$\rightarrow 4_2^+$	57.3	51.2	-
$\rightarrow 6_1^+$	41.2	39.1	-
$\rightarrow 4_1^+$	0.5	0.2	-

VI. SUMMARY AND CONCLUSION

In this paper we have investigated several key assumptions of the fermion dynamical symmetry model, by making use of the *SD* pair shell model. We focused on the nucleus ^{132}Ba, a typical O(6) nucleus in the IBM and FDSM, and used the Hamiltonian parameters of [4] as the starting point.

We first investigated the effect of the single-particle splittings, which is omitted from the Hamiltonian of the FDSM. We find that the physics of low-lying states in ^{132}Ba which is derived from a realistic set of single-particle energies can be reproduced using degenerate single-particle levels if the two-body interaction parameters are adjusted in a not unreasonable manner. On the other hand, if the splittings are artificially enlarged by a factor of 1.5 or more, the O(6) behavior of the nucleus is destroyed. This suggests that one should be careful in applying the FDSM to an arbitrary realistic nucleus, since one cannot be sure in advance that it will produce the correct collective structure.

We then considered the effect of omitting non-$\mathcal{S}$ pairs in the abnormal-parity orbitals for neutrons and protons, another assumption of the FDSM. We find that the effect of these non-$\mathcal{S}$ abnormal-parity pairs depends sensitively on the precise single-particle structure of the nucleus. Nevertheless, the O(6) behavior of ^{132}Ba is maintained in calculations that ignore the non-$\mathcal{S}$ pairs, as long as one uses degenerate single-particle levels at the same time, as indeed is done in the FDSM.

Finally, we considered the possible contributions of interactions other than monopole and quadrupole pairing and quadrupole-quadrupole force. Such interactions are found to be unimportant in this nucleus.

Overall, therefore, our calculations suggest that the assumptions made in the FDSM are quite reasonable for the O(6) nucleus ^{132}Ba. These results encourage one to go further. It is possible to derive the FDSM Hamiltonian parameters, which are rather schematic and taken as more or less freely in previous studies, from the shell-model Hamiltonian. This problem will be discussed in a forthcoming paper [15].

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[1] A. Arima and F. Iachello, Ann. Phys. (N.Y.) **99**, 253 (1976); **111**, 201 (1978); **123**, 468 (1979); for a review see F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University, Cambridge, 1987).

[2] C. L. Wu, D. H. Feng, X. G. Chen, J. Q. Chen, and M. Guidry, Phys. Lett. **168B**, 313 (1986); Phys. Rev. C **36**, 1157 (1987).

[3] J. Q. Chen, Nucl. Phys. **A626**, 686 (1997); Y. M. Zhao, N. Yoshinaga, S. Yamaji, J. Q. Chen, and A. Arima, Phys. Rev. C **62**, 014304 (2000).

[4] Y. M. Zhao, S. Yamaji, N. Yoshinaga, and A. Arima, Phys. Rev. C **62**, 014315 (2000).

[5] J. Q. Chen and Y. A. Luo, Nucl. Phys. **A639**, 615 (1998).

[6] B. Fogelberg and J. Blomqvist, Nucl. Phys. **A429**, 205 (1984); W. J. Baldridge, Phys. Rev. C **18**, 530 (1978).

[7] M. Sakai, At. Data Nucl. Data Tables **31**, 399 (1984).

[8] R. F. Casten and P. Von Brentano, Phys. Lett. **152B**, 22 (1985).

[9] M. Kirson and M. Leviatan, Nucl. Phys. **A240**, 358 (1984).

[10] C. L. Wu, D. H. Feng, M. W. Guidry, H. T. Chen, and X. W. Pan, Phys. Rev. C **51**, R1086 (1995).

[11] See, e.g., J. P. Draayer, Nucl. Phys. **A520**, 259c (1990).

[12] N. Yoshinaga, T. Mizusaki, A. Arima, and Y. D. Devi, Prog. Theor. Phys. Suppl. **125**, 65 (1996).

[13] Y. M. Zhao, Ph.D. thesis, Nanjing University, 1995; Y. M. Zhao and J. Q. Chen, High Energy Phys. Nucl. Phys. **21**, 356 (1997) (in Chinese); *ibid.* **21**, 438(1997) (in Chinese).

[14] T. Mizusaki and T. Otsuka, Prog. Theor. Phys. Suppl. **125**, 97 (1996).

[15] Y. M. Zhao, N. Yoshinaga, S. Yamaji, and A. Arima (unpublished).