Quonic expansion and its random-phase approximation counterpart

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The usefulness of the quon expansion to treat Pauli principle violations and many-body correlations in many-fermion systems is investigated within two models, namely, the Lipkin and the two level pairing models. The q deformation parameter of the quon algebra is fixed through a dynamical criterion. The spectra obtained are compared with the random-phase approximation (RPA), usual boson expansion, and exact results. The quonic expansion is shown to take into account correlations which are not included either in the second order boson expansion or in the RPA approach.

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I. INTRODUCTION

The use of quantum (or q-deformed) algebras [1,2] and some of their partners, such as the *quon* algebra, has recently received special attention, not only from a more fundamental point of view [3], but also as a tool to improve our descriptions of many-particle systems. Some of the most important differences between the quons and quantum algebras are described in [4]. Some examples of the use of those algebras follow. In a simple application of the time dependent Hartree-Fock (TDHF) method to a toy model, it was shown that the deformation of the algebra can be associated with the correlations of the system, which had not been properly described by the determinantal solution [5]. In other interesting papers [6], the deformation parameter of the algebra was used to correctly account for dynamical effects associated with the rotation of quantum mechanical systems, like molecules and nuclei. As another example, we would like to mention the use of the quon algebra to generate deformed boson (*quonic*) expansions as an alternative way to describe many fermion systems [7–9]. The main advantage in this case comes from the fact that, since the *quons* obey commutation relations that interpolate between the usual relations followed by regular bosons and fermions, it is possible to control the value of the deformation parameter in order to improve the convergence of the expansions. For $q=+1$ $(q=-1)$, boson (fermion) statistics are recovered.

In this paper we present quonic expansions for two given many-fermion Hamiltonians, and discuss the solution of these expansions in connection with the corresponding random-phase approximation (RPA) solutions for the original fermionic systems. As was widely discussed in the literature (see, for example, [10]), the second order boson expansion image of a fermionic Hamiltonian presents an energy spectrum very close to the corresponding RPA solution of the original Hamilton operator, for interaction strength values below the so-called transition point for a reasonably large number of particles. It was also shown that, if the bosonic expansion is extended to include fourth order terms, the corresponding solution agrees very well with the exact one, eliminating the breakdown which occurs at the transition point. This procedure is indeed equivalent to introducing anharmonicities in the spectrum. From a fermionic point of view, it was also shown recently [11] that the so-called self-consistent RPA (SCRPA) can in fact correct the behavior of the system described by the pairing vibration model [12], when the transition point from the normal to the superfluid phase is reached. This improvement is achieved mainly through the introduction of ground state correlations, not taken into account properly by the usual RPA solution.

In the present work we also investigate the behavior of the solution of the second order quonic Hamiltonian, close to the above-mentioned transition point, for two schematic models: the Lipkin-Meshkov-Glick (LMG) model [13] and the two-level pairing model [14]. We propose a dynamical way to choose the value of the deformation parameter and finally compare our solutions with the exact ones and with other approaches commonly used in the literature. At this point, we would like to stress that despite the fact that in both models just mentioned the angular momentum is not relevant, our choice for the *quon* algebra instead of quantum algebras allows us to readily apply our procedure to other models for which the angular momentum is important. As was previously shown [15], it is possible to build an su(2) irreducible representation for the quon algebra, such that the usual angular momentum coupling rules are obeyed by the quons. Hence the extension of our results to other models that need to deal with tensor couplings can be easily done. We should also mention that a previous application of deformed algebras to solve the two-level pairing model in the RPA approach has already been done (see [2] and references therein). In that case, the fermionic su(2) generators were deformed from the beginning. Moreover, in quantum algebras, the deformation parameter can acquire any real or complex value and was chosen so as to fit the exact spectra. Here, as stated before, a dynamical criterion is imposed in order to fix the deformation parameter.

The paper is organized as follows: we first outline the main aspects of the Marumori mapping [16] from a fermionic space to a quonic space in Sec. II. Once the deformation parameter is set equal to 1, the usual boson mapping expansion is recovered. In Secs. III and IV the LMG model and the two-level pairing model are reviewed and the corresponding Hamiltonians are mapped with the help of the procedure explained in Sec. II. Our choice for the deformation parameter is discussed in both cases. Once the corresponding second

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order quon Hamiltonians are defined for both models, we discuss the connection of our results with a possible q-deformed version of the RPA method. This is done in Sec. V, where we also compare the exact energy spectra with the ones obtained from the quonic and traditional bosonic expansions as well as with the usual RPA solution. As for the pairing model case, a comparison with the SCRPA is also presented. Finally, the main results obtained are summarized and the conclusions are drawn.

II. QUONIC MARUMORI MAPPING

In what follows we use a q-deformed version of the Marumori mapping [16] to go from a fermionic to a quonic space. This problem was already tackled before (see, for instance, [8]), so we just outline here the main results in order to apply the method to the purposes of this paper. We start from an arbitrary operator $\hat{O}$ acting on a finite fermionic space. This fermionic Hilbert space with dimension $N+1$ is spanned by a basis formed by the states $\{|n\rangle\}$, with $n=0,1,\ldots,N$. Hence,

$$\hat{O}=\sum_{n,n'=0}^{N}\langle n'|\hat{O}|n\rangle|n'\rangle\langle n|. \tag{1}$$

In order to obtain the quon operators, we map $\hat{O}\rightarrow\hat{O}_B$:

$$\hat{O}_B=\sum_{n,n'=0}^{N}\langle n'|\hat{O}|n\rangle|n')(n|, \tag{2}$$

where

$$|n)=\frac{1}{\sqrt{[n]!}}(b^\dagger)^n|0) \tag{3}$$

are the quon states [1,3] with $[n]=(q^n-1)/(q-1)$, $[n]!=[n][n-1]\cdots 1$, and $[b,b^\dagger]_q=bb^\dagger-qb^\dagger b=1$. Note that the usual brackets $\langle|\rangle$ stand for fermionic states and the round brackets $(|)$ stand for bosonic (when $q=1$) and quonic states. From the above considerations, it is straightforward to check that

$$\langle m|\hat{O}|m'\rangle=(m|\hat{O}_B|m'). \tag{4}$$

Therefore, we notice that the mapping is achieved by the equality between the matrix elements in the fermionic space and their counterparts in the quonic space. As examples, we show the expressions for the su(2) operators in the quonic space, which will be used in the next sections:

$$(J_z)_B=\sum_{n=0}^{2j}\sum_{l=0}^{\infty}(-j+n)\frac{(-1)^l q^{l(l-1)/2}}{[n]![l]!}(b^\dagger)^{n+l}b^{n+l}, \tag{5}$$

$$(J_+)_B=\sum_{n=0}^{2j}\sum_{l=0}^{\infty}\sqrt{\frac{(n+1)(2j-n)}{[n+1]}}\frac{(-1)^l q^{l(l-1)/2}}{[n]![l]!}\times(b^\dagger)^{n+l+1}b^{n+l}, \tag{6}$$

$$(J_+^2)_B=\sum_{n=0}^{2j}\sum_{l=0}^{\infty}\sqrt{\frac{(n+1)(n+2)(2j-n)(2j-n-1)}{[n+2]![n]!}}\times\frac{(-1)^l q^{l(l-1)/2}}{[l]!}(b^\dagger)^{n+l+2}b^{n+l}, \tag{7}$$

$$(J_+J_-)_B=\sum_{n=0}^{2j}\sum_{l=0}^{\infty}n(2j-n+1)\times\frac{(-1)^l q^{l(l-1)/2}}{[n]![l]!}(b^\dagger)^{n+l}b^{n+l}, \tag{8}$$

$$(J_-J_+)_B=\sum_{n=0}^{2j}\sum_{l=0}^{\infty}(2j-n)(n+1)\times\frac{(-1)^l q^{l(l-1)/2}}{[n]![l]!}(b^\dagger)^{n+l}b^{n+l}, \tag{9}$$

where $(J_-)_B=(J_+)_B^\dagger$ and $(J_-^2)_B=(J_+^2)_B^\dagger$. In deducing the above expressions we have used that [17]

$$|0)(0|=:\exp_q(-b^\dagger b):=\sum_{l=0}^{\infty}\frac{(-1)^l q^{l(l-1)/2}}{[l]!}(b^\dagger)^l b^l, \tag{10}$$

and we define the su(2) basis as usual, i.e., $|n\rangle=|jm\rangle$, with $m=-j+n$. One should bear in mind that, for $q=1$, the usual Marumori boson expansions are recovered. Notice that the above mapped expressions preserve the usual su(2) commutation relations [18].

III. LIPKIN-MESHKOV-GLICK MODEL

The LMG model has been often used because it has many important physical features present in realistic models and at the same time is a relatively simple and exactly solvable model. It is a valuable tool to analyze approximations and methods for many-body systems and to study critical phenomena in quasispin systems. The LMG model describes a two N-fold degenerate level system with energies $\frac{1}{2}\epsilon$ and $-\frac{1}{2}\epsilon$, respectively. The states in the upper level are denoted by the quantum numbers $i=1,\ldots,N$, the states in the lower level by $-i$.

The many-body LMG Hamiltonian is

$$H=\frac{\epsilon}{2}\sum_{i=1}^{N}(a_i^\dagger a_i-a_{-i}^\dagger a_{-i})-\frac{V}{2}\sum_{i,i'=1}^{N}(a_i^\dagger a_{i'}^\dagger a_{-i}a_{-i'}+a_{-i}^\dagger a_{-i'}^\dagger a_i a_{i'}), \tag{11}$$

where $a_i^\dagger$ $(a_{-i}^\dagger)$ creates a particle in the upper (lower) level, a_i (a_{-i}) annihilates a particle in the upper (lower) level, and V is the strength of the interaction. The Hamiltonian in terms of the quasispin operators is given by

$$H=\epsilon J_z-\frac{V}{2}(J_+^2+J_-^2), \tag{12}$$

with

$$J_z=\frac{1}{2}\sum_{i=1}^{N}(a_i^\dagger a_i-a_{-i}^\dagger a_{-i}),\quad J_+=\sum_{i=1}^{N}a_i^\dagger a_{-i},$$

$$J_-=(J_+)^\dagger. \tag{13}$$

The above quasispin operators obey the su(2) algebra. The operators $J_\pm$ are particle-hole and hole-particle excitation operators while J_z is related to the number of excited particle-hole pairs (half the difference between occupied states in the upper and lower levels). In the expressions above and below, $j=N/2$. Distributing N particles on the two levels, for $V=0$, means that the ground state consists of the lower level completely occupied. Once the interaction is turned on, the ground state becomes a mixture of the previous configuration, but J^2 remains a good quantum number.

Now we proceed by mapping the operators entering the Lipkin Hamiltonian. Substituting Eqs. (5) and (7) into Eq. (12) and truncating the expressions up to fourth order, the mapped Hamiltonian reads

$$\begin{aligned}H_B=&\epsilon\left[-j+b^\dagger b+\left(\frac{2}{[2]}-1\right)b^\dagger b^\dagger bb\right]\\&-\frac{V}{2}\left[2\sqrt{\frac{j(2j-1)}{[2]!}}b^\dagger b^\dagger+2\left(-\sqrt{\frac{j(2j-1)}{[2]!}}\right.\right.\\&\left.\left.+\sqrt{\frac{3(2j-1)(j-1)}{[3]!}}\right)b^\dagger b^\dagger b^\dagger b+\text{H.c.}\right].\end{aligned} \tag{14}$$

Notice that this expression is identical to the one given in [10] once q is set equal to 1.

The basis states chosen for diagonalizing the above Hamiltonian are simply given by Eq. (3), where the quon operators act as usual:

$$b^\dagger|n)=\sqrt{[n+1]}|n+1),\quad b|n)=\sqrt{[n]}|n-1). \tag{15}$$

Our criterion for fixing q

It is already well known that the number operator within the quon algebra is given by an infinite sum in powers of the deformation parameter q [3,8,9,15]. Hence, in Eq. (14), we substitute the diagonal term simply by the number operator $\hat{n}$ and then fix q by eliminating its last term, which is a fourth order number nonconserving operator. This procedure yields the following second order quonic Hamiltonian:

$$H_q=\epsilon(-j+\hat{n})-\frac{V}{2}\left(2\sqrt{\frac{j(2j-1)}{[2]!}}b^\dagger b^\dagger+\text{H.c.}\right), \tag{16}$$

where

$$q=\frac{-1+\sqrt{9-12/j}}{2}. \tag{17}$$

For a large number of particles q goes to $+1$, which means that the usual bosonic expansion is a good one for the Lipkin model, even if just the first few terms are kept, as far as the number of particles is large enough. This property is proved and extensively discussed in [10].

IV. PAIRING MODEL

Next, we apply the quonic expansion to the pairing interaction model [19], which consists of two N-fold degenerate levels, whose energy difference is ϵ. The lower level has energy $-\epsilon/2$ and its single-particle states are labeled j_2m_2 and the upper level has energy $\epsilon/2$ and its single-particle states are labeled j_1m_1. The pairing Hamiltonian reads [14]

$$\begin{aligned}H=&\frac{\epsilon}{2}\sum_m(a_{j_1m}^\dagger a_{j_1m}-a_{j_2m}^\dagger a_{j_2m})\\&-\frac{g}{4}\left(\sum_j\sum_m a_{jm}^\dagger a_{j\bar{m}}^\dagger\sum_{j'}\sum_{m'}a_{j'\bar{m}'}a_{j'm'}\right),\end{aligned} \tag{18}$$

where $a_{j\bar{m}}=(-1)^{j-m}a_{j-m}$. Introducing the quasispin su(2) generators

$$S_+=S_-^\dagger=\frac{1}{2}\sum_{m_1}a_{j_1m_1}^\dagger a_{j_1\bar{m}_1}^\dagger,$$

$$S_z=\frac{1}{2}\sum_{m_1}a_{j_1m_1}^\dagger a_{j_1m_1}-\frac{N}{4},$$

$$L_+=L_-^\dagger=\frac{1}{2}\sum_{m_2}a_{j_2m_2}^\dagger a_{j_2\bar{m}_2}^\dagger,$$

$$L_z=\frac{1}{2}\sum_{m_2}a_{j_2m_2}^\dagger a_{j_2m_2}-\frac{N}{4},$$

one sees that the pairing interaction has an underlying $\mathrm{su}(2)\otimes\mathrm{su}(2)$ algebra. With the help of these operators, Eq. (18) can be rewritten as

$$H=\epsilon(S_z-L_z)-g(S_+S_-+L_+L_-+S_+L_-+L_+S_-). \tag{19}$$

In what follows, the number of particles (which are fermions) N will be even and $\Omega=N/2$. Notice, however, that the degeneracy of each level J is $\Omega=(2J+1)/2$, for $J=j_1,j_2$.

The basis of states used for the diagonalization of the above Hamiltonian is [19] $|S=\Omega/2,\ S_z=-\Omega/2+m\rangle|L=\Omega/2,\ L_z=\Omega/2-m\rangle$, where m runs from 0 to Ω, in such a way that, for $m=\Omega$, $S_z=\Omega/2$ and $L_z=-\Omega/2$. In close analogy to the LMG model we obtain, for the fourth order Hamiltonian [18],

$$H_q = -\Omega(\epsilon+g) + (\epsilon - g\Omega)b_1^\dagger b_1 + [\epsilon - g(\Omega-2)]b_2^\dagger b_2 - g\Omega(b_1^\dagger b_2^\dagger + b_2 b_1) + \left[\left(\frac{2}{[2]}-1\right)\epsilon - g\left(-\Omega + \frac{2(\Omega-1)}{[2]}\right)\right] b_1^\dagger b_1^\dagger b_1 b_1 + \left[\left(\frac{2}{[2]}-1\right)\epsilon - g\left(2(1-\Omega) + \frac{3(\Omega-2)}{[2]} + \frac{\Omega}{2}q\right)\right] \times b_2^\dagger b_2^\dagger b_2 b_2 - g\left(\sqrt{\frac{2\Omega(\Omega-1)}{[2]}} - \Omega\right)(b_2^\dagger b_1^\dagger b_1^\dagger b_1 + b_1^\dagger b_2^\dagger b_2^\dagger b_2 + b_1^\dagger b_1 b_1 b_2 + b_2^\dagger b_2 b_2 b_1). \tag{20}$$

This Hamiltonian is equivalent to the one obtained in [19] when q is set equal to unity. The second order Hamiltonian is easily read off from the above equation by omitting all terms containing four quon operators. Diagonalizing Eq. (20) is a simple task and for this purpose the basis used is

$$|n_1 n_2) = \frac{1}{\sqrt{[n_1]![n_2]!}}(b_1^\dagger)^{n_1}(b_2^\dagger)^{n_2}|0) \tag{21}$$

and

$$b_1{}^\dagger|n_1) = \sqrt{[n_1+1]}|n_1+1), \quad b_1|n_1) = \sqrt{[n_1]}|n_1-1),$$

$$[\hat{n}_1] = b_1^\dagger b_1,$$

with similar expressions for the b_2 and $b_2^\dagger$ operators.

Our criterion for fixing *q*

The Hamiltonian (20) is somewhat different in nature from the Lipkin Hamiltonian of Eq. (14). Nevertheless, for the pairing case, we have also opted to eliminate all fourth order nondiagonal terms. Again, the diagonal terms coming from the *quonic* image of S_z and L_z are substituted by their corresponding number operators and the imposition that the number nonconserving fourth order terms vanish yields the value for the q parameter. The Hamiltonian becomes

$$H_q = -\Omega(\epsilon+g) + \epsilon(\hat{n}_1 + \hat{n}_2) - g\Omega[\hat{n}_1] - g(\Omega-2)[\hat{n}_2] - g\Omega(b_1^\dagger b_2^\dagger + b_2 b_1), \tag{22}$$

where

$$q = \frac{2(\Omega-1)}{\Omega} - 1. \tag{23}$$

Again, for a large number of particles ($\Omega \to \infty$), $q=1$ is recovered. We should notice at this point that, differently from the Lipkin model, not all the fourth order terms are removed by the above choice for q and the introduction of the the number operators. The term proportional to $g b_2^\dagger b_2^\dagger b_2 b_2$ still survives. However, the contribution from this term to the final spectrum can be safely disregarded for the assigned q value, as has been numerically checked for all the g values used in this calculation.

V. CONNECTION WITH THE RPA AND NUMERICAL RESULTS

As is well known [10,19], the second order boson expansion Hamiltonian gives the RPA solution if we diagonalize it in an unrestricted basis. This can be seen if we remember that it is possible to diagonalize the boson Hamiltonian (up to second order) by a simple linear transformation of the Bogoliubov-Valatin type [20] and that this corresponds to finding the RPA solution for the original fermionic problem. This unrestricted diagonalization leads, however, to violations of the Pauli principle. This is partially remedied in the boson expansion method through the projection onto the physical subspace. In what follows we define the q-RPA method as the diagonalization of the second order quon Hamiltonian in an unrestricted basis and compare those results with the ones obtained through projection onto the physical subspace (the subspace chosen in such a way that there is a one to one correspondence with the fermionic subspace). We also remark that because of the different properties of quonic operators as compared with the regular bosons, it is not possible to define a linear transformation of the Bogoliubov-Valatin type in this case.

We follow the above-mentioned scheme for the models described in the previous sections and for each one we get the spectrum for two numbers of fermions in the system, $N=6$ and $N=40$. In the numerical calculation, we start by diagonalizing the second order quonic Hamiltonian in the physical basis. For example, in the Lipkin model the size of the physical basis is $N+1$ quons whereas in the pairing model, it is $\Omega+1$. The results obtained from the diagonalization procedure yield the second order quonic expansion (SQE) spectrum. Exactly the same calculation performed for $q=1$ leads to the well-known second order boson expansion (SBE) results. We then increase the number of quons in the basis (q is fixed by our dynamical criteria) until convergence is achieved. As already mentioned, we have named the resulting spectrum q-RPA.

In Fig. 1 the excitation energy $E_1 - E_0$ as a function of the interaction strength is shown for the Lipkin model and for $N=6$ particles. Together with the exact result we also show the results for the diagonalization of the second order bosonic Hamiltonian (SBE) and the usual RPA result, which ceases to be valid after the transition point. Moreover, the

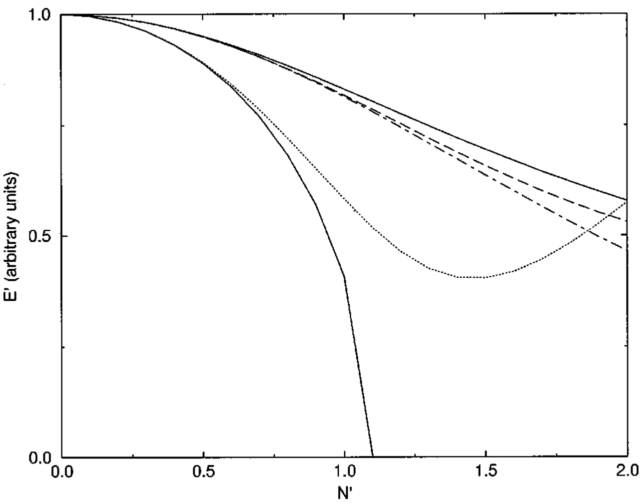

FIG. 1. Lipkin model. The first excited state minus the ground state energy $E' = E_1 - E_0$ is plotted as a function of $N' = NV/\epsilon$, from top to bottom, for the exact result (solid line), the quonic expansion result (SQE) for $q = 0.618$ obtained from the diagonalization in a physical basis (long-dashed line), the quonic expansion result (q-RPA) for $q = 0.618$ obtained from the diagonalization in an infinite basis (dot-dashed line), the second order expansion result (SBE) for $q = 1$ (short-dashed line), and the RPA result (solid line). The number of particles is six.

diagonalization of our second order quonic Hamiltonian is also presented in the same figure, where we recognize two curves: one corresponding to the diagonalization in a basis for which the number of quons is restricted by the number of physical particles (SQE) and another for which this restriction is disregarded and corresponds to the q-RPA. At the same time that these last two solutions approximate drastically to the exact one, their difference becomes very small when compared with the difference between the second order bosonic and the usual RPA solutions. Actually this difference practically does not exist when we move to a system of 40 particles, as can be seen in Fig. 2, for which the same comparisons are performed. The breakdown suffered by the RPA solution is completely overcome with the introduction of the deformation. For the sake of comparison with the q values obtained from our citerion, we have also calculated the optimal value for the deformation parameter through a

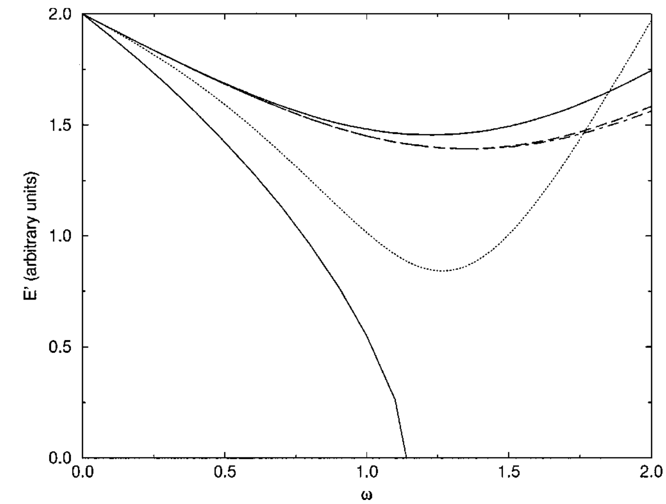

FIG. 3. Pairing model. The first excited state minus the ground state energy $E' = E_1 - E_0$ is plotted as a function of the interaction strength $\omega = 2g\Omega$, from top to bottom, for the exact result (solid line), the SQE result for $q = 0.333$ (long-dashed line), the q-RPA for $q = 0.333$ (dot-dashed line), the SBE result (dotted line), and the RPA result (solid line). Ω is 3, $\epsilon = 1$.

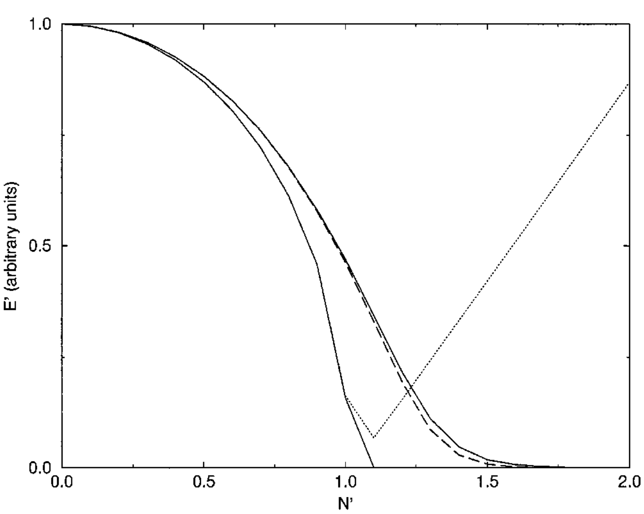

FIG. 2. Lipkin model. The first excited state minus the ground state energy $E' = E_1 - E_0$ is plotted as a function of $N' = NV/\epsilon$ from top to bottom, for the exact result (solid line), the SQE result for $q = 0.9491$ which coincides with the result for q-RPA (long-dashed line), the SBE (dotted line), and the RPA result (solid line). The number of particles is 40.

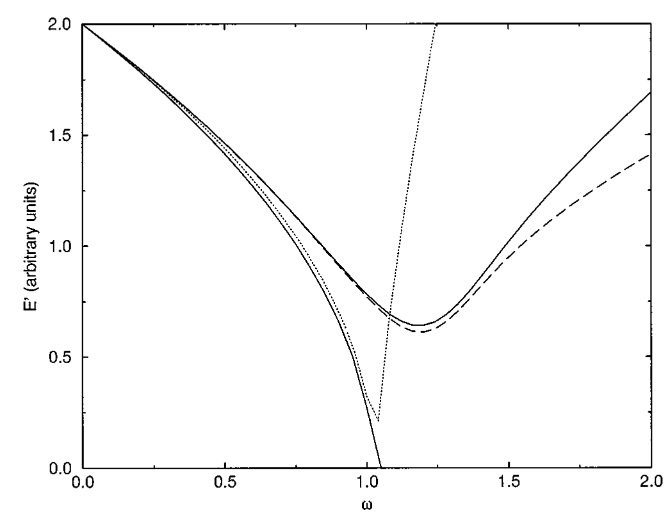

FIG. 4. Pairing model. The first excited state minus the ground state energy $E' = E_1 - E_0$ is plotted as a function of the interaction strength $\omega = 2g\Omega$, from top to bottom, for the exact result (solid line), the SQE and q-RPA results for $q = 0.9$ (long-dashed line), the SBE result (dotted line), and the RPA result (solid line). Ω is 20; $\epsilon = 1$.

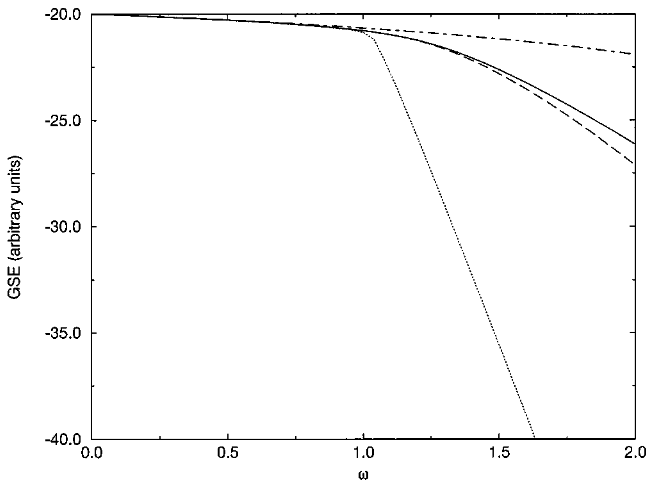

FIG. 5. Pairing model. The ground state energy (GSE) E_0 is plotted as a function of the interaction strength $\omega=2g\Omega$, for the exact result (solid line), the q-RPA result for $q=0.9$ (dashed line), the SBE result (dotted line), and the SCRPA result (dot-dashed line). Ω is 20; $\epsilon=1$.

best fitting procedure. For $N=6$, the criterion yields $q=0.618$ and the best fitting yields $q=0.55$. For $N=40$, the values are $q=0.9491$ and $q=0.94$ for the criterion and the best fitting values, respectively.

In Figs. 3 and 4 we do the same type of analysis for the two-level pairing model and once more for 6 and 40 particles, respectively. Again we may conclude that the quon expansion approaches the exact result. For this model a best fitting analysis was also performed. For $N=6$, the criterion yields $q=0.333$ and the best fitting $q=0.34$. For $N=40$, the values are $q=0.9$ and $q=0.94$ for the criterion and the best fitting values, respectively.

We stress that our emphasis was not in fitting or adjusting the q parameter. The results were obtained with the choice made using the dynamical criteria in Eqs. (17) and (23) for the Lipkin and pairing models, respectively. The comparison between the q-RPA and SQE curves shows that the importance of the projection onto the physical subspace is greatly reduced, even for a small number of particles. In other words, independently of the number of particles considered, either for the Lipkin or the pairing model, the great difference between the RPA and second order boson expansion results practically disappears once deformation is turned on. Moreover, the behavior of the curves turns out to be the same as the exact one.

It is a well-known fact that the fourth order boson ($q=1$) Hamiltonian diagonalization reproduces the exact result in the LMG model [10] and gives results very close to the exact ones in the pairing model [19]. This can be interpreted as an effect of the introduction of anharmonicities in the Hamiltonian as compared with the second order one. On the other hand, one can see from [3,21,22] that a quonic oscillator is equivalent to a regular oscillator with anharmonic terms and we strongly believe that this is the reason why our quonic solution approaches so well the exact ones. Of course, the degree of anharmonicity introduced by the deformation depends on the chosen value for the deformation parameter. In this respect, our criteria, as explained in the previous sections, seem to be very reasonable.

Finally, we would like to make quantitative comparisons with previous improvements made in the RPA solution, like the SCRPA, for instance. One crucial difference between the RPA and SCRPA is that the last one introduces important ground state correlations [12], which allow for following the system across the transition point [11]. Therefore, the comparison with the SCRPA can give us a measure of the amount of correlations taken into account by the quon approach. A numerical comparison of our results for the ground state energy E_0 with the ones obtained from the SCRPA, taken from [12], for the pairing model is seen in Fig. 5. Our q-deformed RPA result is much closer to the exact values than the SCRPA after the transition point. However, as also pointed out in [12], it is expected that the SCRPA solution deteriorates as the interaction strength increases and this can be remedied by the generalization to a quasiparticle description after the transition point [SC quasiparticle RPA (SCQRPA)]. In our case the solution also is more and more inaccurate as the interaction strength g increases, but the results are in much better agreement as compared with the exact ones.

VI. CONCLUSIONS

In this work we propose a way to improve the RPA solution for a system of interacting fermions based on a Marumori type of quon expansion of the original Hamiltonian. First, we choose the deformation parameter such that the effects of the fourth order terms in the quonic Hamiltonian are minimized. Second, we notice that the use of the definition of the number operator in the quon space automatically sums up a whole class of terms in the expansion. Finally, we solve numerically the second order Hamiltonian in an *infinite* basis quonic space, following the known result that the usual RPA solution can be recovered if we diagonalize the second order boson ($q=1$) expansion in an infinite boson basis. This scheme was applied for the simple LMG and pairing two-level models, showing that the introduction of the deformation in the RPA method allows us to avoid the well-known collapse. Besides, our choice for the deformation parameter gives results very close to the exact ones, pointing to the possibility of a unified description of the two regions (before and beyond the transition point) without any redefinition of the vacuum.

The extension of this investigation to other schematic models for which angular momentum coupling is important can be done in a straightforward manner, as pointed out in the Introduction. For example the quadrupole-quadrupole plus pairing interaction is a good candidate. Based on our previous experience on *quonic* expansions [9,15] of that type of interaction in terms of *s*, *d*, and *g* q-deformed bosons, we can devise some applications that may give us important hints about the type of improvements which can be obtained in more realistic cases.

ACKNOWLEDGMENT

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