Microscopic derivation of interacting boson-fermion model Hamiltonian and its application to singly magic nuclei

N. Yoshinaga*
Department of Physics, Saitama University, Urawa, Saitama 338-8570, Japan

Y. D. Devi
RIKEN, Hirosawa, Wako-shi, Saitama 351-0198, Japan

A. Arima
House of Councilors, Sangiin-kaikan 223, Nagatacho, Chiyodaku, Tokyo 100-8962, Japan
(Received 31 January 2000; published 19 July 2000)

The interacting boson-fermion model (IBFM) Hamiltonian is derived microscopically based on the generalized seniority scheme starting from the shell-model interaction. Employing many nondegenerate levels and identical nucleon systems, systematic studies of singly magic nuclei are given in comparison with experiments. The resulting energy spectra in the IBFM are found to be in good agreement with those of the shell model and the *SD* pair plus one-particle model.

PACS number(s): 21.60.Fw, 21.60.Cs, 21.60.Ev

I. INTRODUCTION

The interacting boson model (IBM) [1] is remarkably successful in providing a unified description of quadrupole collective properties of low-lying nuclear states with its angular momentum 0 (*s*) and 2 (*d*) bosons. The success is not only limited to its analytical formulation in the three dynamical symmetry limits, but also extends to numerical formulation which makes the study of transitional nuclei tractable due to its small model space dimensions even in the medium and heavy nuclei. It has yielded many new insights and is found to be equally promising in the study of odd-mass nuclei when extended to include the fermionic degree of freedom in addition to the boson core. The resulting model is known as the interacting boson-fermion model (IBFM) which was proposed by Arima, Iachello, and Scholten [2–6].

The microscopic foundation of the IBM is quantitatively satisfactory at least around the vibrational and transitional regions [7]. That of the IBFM is less studied compared to the IBM partly because some justification is still needed in the boson space especially in deformed region and partly because its work is much involved and complicated. However, it is quite important to know the microscopic origin of the interaction if the model is to be used to systematically study the high spin states and beta decay. Recently the supersymmetric level scheme has been experimentally confirmed in the quartet ^{195,196}Au and ^{194,195}Pt [8,9]. Phenomenologically these supersymmetric nuclei can be studied in terms of the IBM, IBFM, and IBFFM and its semimicroscopic origin was partly investigated by Bijker and Scholten [10]. However, the full microscopic origin of the supersymmetry still needs to be investigated. In that sense it is very important to investigate both models (IBM and IBFM) microscopically and simultaneously on the same footing.

Historically, the microscopy of the IBFM was first investigated by Scholten [4,11] using the Otsuka, Arima, and Iachello (OAI) [12] mapping procedure and the BCS approximation. Specifically he derived the fermion exchange force coming from the strong proton-neutron quadrupole force. Talmi [13] also claimed the importance of the exchange force which was derived by the effect of the Pauli principle acting between identical nucleons on the quadrupole interaction between protons and neutrons. The exchange force was further investigated by Gelberg [14] who showed the relation between exchange and direct forces using Talmi's method. However, it was soon found that the shell model estimate of the exchange force accounted only for 1 order of magnitude less than the phenomenological values. This contradition was finally accounted for by Otsuka and others [15] who emphasized the importance of the quadrupole pairing interaction.

In this paper we carry out a microscopic derivation of the IBFM Hamiltonian in singly magic nuclei starting from any shell model interaction within the framework of the generalized seniority. We basically follow the idea of the OAI [12], but the method is extended for the application to odd-*A* nuclei with nondegenerate multi-*j* orbits. The validity of this procedure is tested by employing a shell model (SM) Hamiltonian which consists of single particle energies, monopole and quadrupole pairing and quadrupole-quadrupole interactions. The parameters are adjusted to fit experimental data in the shell model space. We treat even (IBM) and odd (IBFM) systems on the same footing, i.e., the boson core in an odd system is the same as the neighboring even system. The resulting energy spectra in the IBM (the IBFM) are compared with experiments, those in the SM and those in the truncated *SD* space which is constructed only by the angular momentum zero (*S*) collective pair and two (*D*) collective pair (*S* and *D* pairs plus one particle) in the even (odd) systems. In our previous paper [17] we have shown numerically that our scheme is successful in comparison with the SM and *SD*-pair truncated model in single *j* shells. In this paper we extend our scheme to the realistic many nondegenerate *j*

*Electronic address: yosinaga@riron.ged.saitama-u.ac.jp

shells and apply our method to singly magic nuclei using a realistic two-body interaction.

In Sec. II our mapping method is described for both even and odd systems. Application of our method to a single *j*-shell case is discussed in Sec. III. Here spherical to deformed nuclei are simulated by changing a single parameter in a midshell, where the direct force is found to vanish. In Sec. IV, our method is applied to realistic singly magic nuclei to test the validity of our present method involving many nondegenerate *j* shells. The work reported in this paper is summarized and certain conclusions are drawn in Sec. V.

II. METHOD OF MAPPING

In the medium and heavy nuclei the number of SM configurations become very large ($\approx 10^{14}-10^{18}$) and it is unrealistic to treat them in the full fledged SM. Therefore we need to truncate the model space assuming its physical importance. The amazing success of the IBM puts forward the idea that the collective *SD* fermion pairs can become the building blocks for the many-body collective states. Therefore it is natural to truncate the SM space to the collective *SD* subspace. Using the SM interaction we can obtain the corresponding interaction in the IBM and the IBFM. For that purpose we have to carry out two things, (i) the truncation of the SM space to the collective *SD* pair space (*SD* plus one-particle space) in case of even (odd) systems, (ii) the boson mapping from the *SD* pair space (*SD* plus one-particle space) to the *sd* boson space (*sd* boson plus one particle space) in the case of even (odd) systems. Throughout this paper, we deal with identical particles, but our procedure can be extended in a straightforward, but involved manner to doubly open-shell nuclei without difficulty. In the following we describe our mapping method for both even and odd systems.

Our mapping method is based on the generalized seniority starting from the SM. In the first stage we truncate the gigantic SM space to the *SD* pair space. Here nucleon pair creation operators are defined as

$$A_M^{\dagger(J)}(j_1 j_2)=[c_{j_1}^{\dagger} c_{j_2}^{\dagger}]_M^{(J)}, \tag{2.1}$$

where $c_j^{\dagger}$ stands for the creation operator of a single particle. Using this pair, the collective nucleon pair creation operators with angular momentum zero and two are defined as

$$S^{\dagger}=\sum_j \alpha_j A_0^{\dagger(0)}(jj), \tag{2.2}$$

and

$$D_M^{\dagger}=\sum_{j_1 j_2} \beta_{j_1 j_2} A_M^{\dagger(2)}(j_1 j_2). \tag{2.3}$$

In our scheme we deal with nondegenerate shells and the structure coefficients $\alpha_j, \beta_{j_1 j_2}$ should be determined to maximize the collectivity in a given system as stated below. Using *SD* collective pairs, many-body *SD* pair states are constructed as

$$|S^{N_s} D^{N_d} \gamma J\rangle_F, \tag{2.4}$$

for even systems of $2N_s+2N_d$ particles. This space is called the *SD* space hereafter. For odd systems of $2N_s+2N_d+1$ particles, we have

$$|S^{N_s} D^{N_d} c_j \gamma J'\rangle_F, \tag{2.5}$$

where c_j stands for the last particle added to the core state: $|S^{N_s} D^{N_d} \gamma J\rangle_F$. The necessary angular momentum coupling is abbreviated here for simplicity. The space made of *SD* pairs plus one-particle is called *SDp* space. Here γ's are the quantum numbers which uniquely specify the *d*-boson states $|d^{N_d} \gamma J\rangle_B$. There is a one to one correspondence between fermion states $|S^{N_s} D^{N_d} \gamma J\rangle_F$ and boson states $|s^{N_s} d^{N_d} \gamma J\rangle_B$ unless some fermion states are Pauli forbidden. These fermion states should be orthonormalized like boson states. It is assumed that higher generalized seniority states are always orthogonal to all the lower seniority states in order to secure the criterion that the creation of *D* pair always raises generalized seniority number by two. One of the methods to carry out this procedure was discussed in detail in [18].

Assuming the following correspondence:

$$|S^{N_s} D^{N_d} \gamma J\rangle_F \leftrightarrow |s^{N_s} d^{N_d} \gamma J\rangle_B, \tag{2.6}$$

our strategy for the boson mapping is to equate the matrix elements of any fermion operator $\hat{O}$ between fermion and boson states

$$\langle S^{N_s} D^{N_d} \gamma J|\hat{O}|S^{N_s} D^{N_d} \gamma J\rangle_F = \langle s^{N_s} d^{N_d} \gamma J|\hat{o}|s^{N_s} d^{N_d} \gamma J\rangle_B, \tag{2.7}$$

where $\hat{o}$ is the boson image of the fermion operator $\hat{O}$. Without breaking Hermiticity of the mapping, this necessarily results in many-body boson interactions. Therefore we should restrict this criterion to be satisfied only for matrix elements of physical importance. In the spherical and vibrational regions it is well known that the generalized seniority is a good concept [16] and the states with lower generalized seniority number are more important than those with higher seniority number. Therefore, if the boson operator has *K* unknown coefficients, we need to equate *K* independent matrix elements ascending order of seniority starting from the lowest generalized seniority state. In our mapping procedure we first determine the boson interactions (IBM part or boson core). The details were already given in the paper for the IBM part [18].

In the odd systems we have one-body fermion interaction H_F and the two-body boson-fermion interactions H_{FB}. Thus the IBFM Hamiltonian is written as

$$H=H_B+H_F+H_{FB}, \tag{2.8}$$

where the IBM Hamiltonian H_B is assumed to have been already determined using the above prescription. The H_F is the fermion single particle energies written as

$$H_F=\sum_{jm} \epsilon_j a_{jm}^{\dagger} a_{jm}, \tag{2.9}$$

where single particle energies ϵ_j are assumed to take the same values as appearing in the one-body term in the original SM Hamiltonian. Here $a^\dagger_{jm}$ and a_{jm} are the creation and annihilation operators of ''ideal'' fermions which commute with *s*- and *d*-boson operators. The H_{FB} is the interaction between bosons and fermions up to two-body terms and parametrized as [6]

$$H_{FB}=\sum_j w^j_{ss}\sqrt{2j+1}[[s^\dagger a^\dagger_j]^{(j)}[s\tilde{a}_j]^{(j)}]^{(0)}_0 + \sum_{i,j} w^{ij}_{sd}\sqrt{2i+1}([[s^\dagger a^\dagger_i]^{(i)}[\tilde{d}\tilde{a}_j]^{(i)}]^{(0)}_0+\text{H.c.}) + \sum_J \sum_{i,j} w^{Jij}_{dd}\sqrt{2J+1}[[d^\dagger a^\dagger_i]^{(J)}[\tilde{d}\tilde{a}_j]^{(J)}]^{(0)}_0. \tag{2.10}$$

Therefore we only need to take up to generalized seniority 3 states to determine unknown parameters. Using the fermion-boson correspondence for $2N+1$ fermions,

$$|S^N,c_j;J\rangle_F \leftrightarrow |s^N,a_j;J\rangle_B, \tag{2.11}$$

$$|S^{N-1}D,c_j;J\rangle_F \leftrightarrow |s^{N-1}d,a_j;J\rangle_B, \tag{2.12}$$

the w^j_{ss}, w^{ij}_{sd}, and w^{Jij}_{dd} parameters are determined by the following equations, respectively:

$$\langle s^N a_j|H|s^N a_j\rangle=\langle S^N c_j|V|S^N c_j\rangle, \tag{2.13}$$

$$\langle s^N a_i|H|s^{N-1}da_j\rangle=\langle S^N c_i|V|S^{N-1}Dc_j\rangle, \tag{2.14}$$

$$\langle s^{N-1}da_i;J|H|s^{N-1}da_j;J\rangle=\langle S^{N-1}Dc_i;J|V|S^{N-1}Dc_j;J\rangle. \tag{2.15}$$

Here V is the original SM Hamiltonian up to two-body interactions (including one-body term). Using fermion matrix elements, the three parameters are explicitly expressed as

$$w^j_{ss}=-\frac{1}{N}(\langle S^N c_j|V|S^N c_j\rangle-\epsilon_j-k_0), \tag{2.16}$$

$$w^{ij}_{sd}=-\frac{1}{\sqrt{N}}\langle S^N c_i|V|S^{N-1}Dc_j\rangle, \tag{2.17}$$

$$w^{Jij}_{dd}=-\langle S^{N-1}Dc_i;J|V|S^{N-1}Dc_j;J\rangle + [\epsilon_j+k_0+\epsilon_d-w^j_{ss}(N-1)]\delta_{ij}, \tag{2.18}$$

where

$$k_0=\langle S^N|V|S^N\rangle, \tag{2.19}$$

$$\epsilon_d=\langle S^{N-1}D|V|S^{N-1}D\rangle-k_0. \tag{2.20}$$

Numerical evaluation of fermion matrix elements appearing in the right-hand side (RHS) of Eqs. (2.16) to (2.20) is the most difficult task in the boson-fermion mapping. In this paper we evaluate them explicitly using a SM code.

Another parametrization of the IBFM Hamiltonian is due to Scholten [4]:

$$H_{FB}=\sum_j A_j\hat{n}_d\hat{n}_j+\sum_{i,j}\Gamma_{ij}(Q_B\cdot[a^\dagger_i\tilde{a}_j]^{(2)}) + \sum_{ijJ}\frac{\Lambda_{Jij}}{\sqrt{2J+1}}:[[d^\dagger\tilde{a}_j]^{(J)}[\tilde{d}a^\dagger_i]^{(J)}]^{(0)}_0:, \tag{2.21}$$

with

$$Q_B=s^\dagger\tilde{d}+d^\dagger s+\chi[d^\dagger\tilde{d}]^{(2)}, \tag{2.22}$$

where the second term in Eq. (2.21) is called the direct term and the last term is called the exchange term. The relation between our Hamiltonian and that of Scholten is explicitly written as

$$w^j_{ss}=A_j, \tag{2.23}$$

$$w^{ij}_{sd}=-(-)^{i+j}\sqrt{\frac{5}{2i+1}}\Gamma_{ij}, \tag{2.24}$$

$$w^{ijJ}_{dd}=5\Gamma_{ij}\chi(-)^{i+J}\begin{Bmatrix}2&2&2\\ j&i&J\end{Bmatrix}+\sum_K\Lambda_{Kij}\begin{Bmatrix}2&j&J\\ 2&i&K\end{Bmatrix}, \tag{2.25}$$

apart from a constant. Note that the direct term is directly related to the w_{sd} term.

III. SINGLE *j*-SHELL CASE

To clarify the contents of the previous section in a simple case, we apply the method to a single *j* shell. Some numerical investigations have been already carried out in Ref. [17]. For the single *j*-shell case we do not need to determine the structure coefficients of *S* and *D* pairs and we can make use of the seniority scheme of Racah and its reduction formula [20]. In this case the parameters appearing in Eq. (2.10) are evaluated analytically. Those were given in the appendix of Ref. [7].

According to Scholten, Γ_{ij} in Eq. (2.21) is proportional to $(u_iu_j-v_iv_j)Q_{ij}$ where u_i and v_i are the occupation amplitudes with Q_{ij} being the quadrupole matrix element. Therefore the direct term in a single *j* shell vanishes in the middle of the shell. It is interesting to see whether the direct term vanishes in the middle of the shell or not in our present scheme. Using seniority scheme we get [7] for a single *j* shell of $\Omega=j+\frac{1}{2}$,

$$w_{sd}=\frac{1}{\sqrt{N}}F\langle Dc_j|V|Sc_j\rangle, \tag{3.1}$$

with

$$F=\frac{\sqrt{(2\Omega-2N-2)2N}}{2\sqrt{\Omega-2}}\,\frac{\Omega-2N-1}{\Omega-3}. \tag{3.2}$$

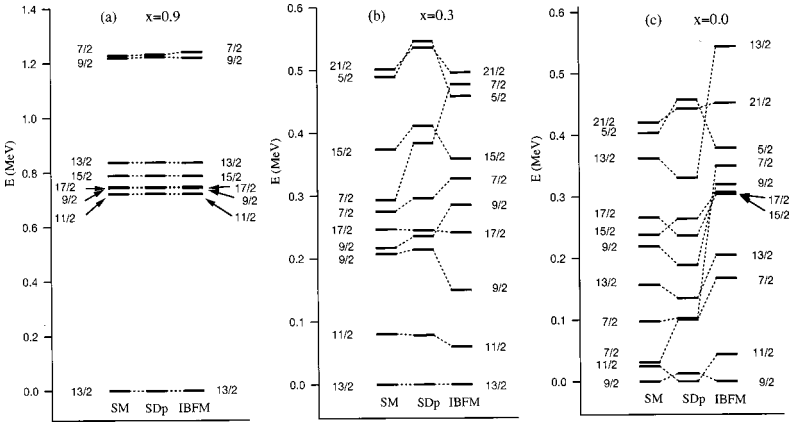

FIG. 1. Comparison of energy spectra in the SM, SDp, and IBFM for the half-filled case of $j=13/2$ shell as a function of strength parameter x [(a) $x=0.9$, (b) $x=0.3$, (c) $x=0.0$].

Therefore in our scheme the direct term also vanishes in the middle of the shell (i.e., $\Omega=N+1$) due to the factor coming from the seniority reduction formula. The direct term also vanishes when the interaction conserves seniority. This can be easily seen from the matrix element $\langle Dc_j|V|Sc_j\rangle$ in Eq. (3.1) where the RHS has the seniority 1 and the LHS has the seniority 3.

Our schematic Hamiltonian consists of pairing plus quadrupole interaction with a variable x to simulate vibrational to rotational nuclei

$$V=-xA_0^{\dagger(0)}A_0^{(0)}-(1-x):Q\cdot Q:, \tag{3.3}$$

with

$$A_0^{\dagger(0)}=\sqrt{\frac{1}{2}}[c_j^\dagger c_j^\dagger]_0^{(0)}, \tag{3.4}$$

and

$$Q_M=[c_j^\dagger \tilde{c}_j]_M^{(2)}. \tag{3.5}$$

Here we take $j=\frac{13}{2}$ and x is varied from 1 to 0. We take $n=7$ particles, which means that the shell is half filled. In Figs. 1(a)–1(c) the comparison of energy levels between the SM, the SDp model and the IBFM is shown as a function of x. In Fig. 1(a) a spherical nucleus is simulated with a variable $x=0.9$. It is seen that states with different seniority numbers ($v=1,3,5$) are grouped together due to a strong pairing interaction. In the spectrum of the SM, there are 16 states with seniority 3 for angular momenta $J\leqslant\frac{23}{2}$, among which five states drawn in the figure are within SDp space. Almost a perfect reproduction is obtained as expected within the SDp space. When the quadrupole-quadrupole interaction gets stronger, seniority numbers are largely mixed as seen in Fig. 1(b). The expectation value for the seniority quantum number for the ground state is $\langle v\rangle=1.0,1.2,1.4,1.8,3.5$ for $x=0.9,0.7,0.5,0.3,0.1$, respectively. In Fig. 1(c) a deformed nucleus is simulated with a pure QQ interaction ($x=0.0$). Low-lying energy levels are no longer well reproduced in the SDp model compared to the SM. Even the ground state spin is not reproduced. This is caused by neglecting higher angular momentum pairs such as G pairs. There is a large discrepancy between the spectra of SDp and IBFM in this case. This means that our boson mapping procedure based on the seniority breaks down in this deformed case. We certainly need three-body terms if we stick to the boson mapping based on the seniority. It is accidental that the ground state spin in IBFM is exactly reproduced in comparison with the SM.

It is interesting to see how the IBFM interaction strengths change as a function of x. In Table I we list the values of IBFM parameters in case of $x=1$ (pure pairing) and $x=0$ (pure QQ). Other cases with different x's are interpolated linearly using these values because our mapping procedure is linear in the single j-shell case. It is seen that w_{dd}^J with $J=j$ is the most important component compared to other w_{dd}^J's. The w_{sd} is zero for all the cases because the shell is half filled. In this particular case the direct term does not play any important role for the reproduction of energy spectra.

TABLE I. The IBFM parameters for a single j shell ($j=13/2$ and $n=7$) with $x=1.0$ (pure pairing) and $x=0.0$ (pure QQ)

	$x=1.0$	$x=0.0$
w_{ss}	0.143	0.238
w_{sd}	0.0	0.0
$w_{dd}(J=9)$	0.0	0.173
$w_{dd}(J=11)$	0.0	−0.058
$w_{dd}(J=13)$	0.0	1.090
$w_{dd}(J=15)$	0.0	0.606
$w_{dd}(J=17)$	0.0	0.198

TABLE II. Single particle energies and structure constants α_j for $N=82$ isotones and Sn isotopes. In the second (sixth) column single proton-particle (neutron-hole) energies are shown in units of MeV.

Levels	ε_j	^{134}Te	^{136}Xe	^{138}Ba	ε_j	^{130}Sn	^{128}Sn	^{126}Sn
$d_{3/2}$	2.6	0.194	0.181	0.167	0.0	0.615	0.615	0.616
$h_{11/2}$	2.5	0.179	0.167	0.154	0.05	0.564	0.568	0.571
$s_{1/2}$	2.0	0.232	0.220	0.208	0.4	0.447	0.447	0.448
$d_{5/2}$	0.6	0.448	0.446	0.447	1.3	0.251	0.246	0.240
$g_{7/2}$	0.0	0.822	0.832	0.840	1.8	0.203	0.197	0.190

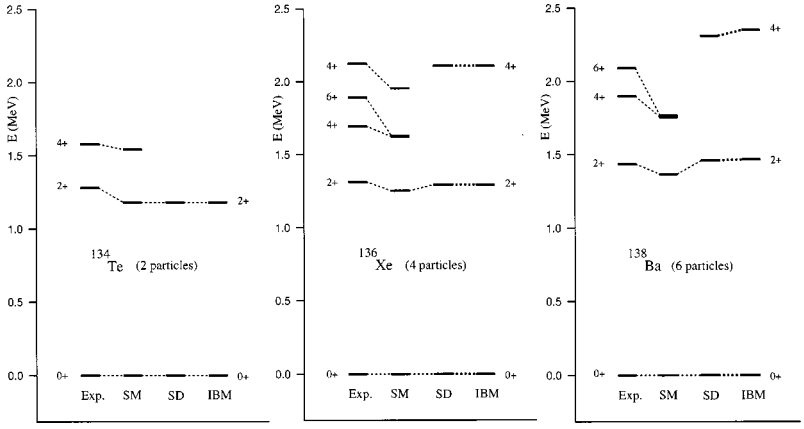

FIG. 2. Energy levels of even $N=82$ isotones in experiment (Exp), shell model (SM), SD pair space (SD), and the IBM space (IBM).

IV. APPLICATION OF THE METHOD TO SN ISOTOPES AND $N=82$ ISOTONES

In this section we apply our method to singly magic nuclei. Our SM Hamiltonian consists of the single particle energies and monopole- and quadrupole-pairing interaction and quadrupole-quadrupole interaction

$$V=\sum_j \epsilon_j \hat{n}_j - G_0 P^{\dagger(0)} P^{(0)} - G_2 P^{\dagger(2)} \cdot P^{(2)} - \kappa : Q \cdot Q : . \tag{4.1}$$

The precise definition of the multipole interactions is given in Refs. [7,18]. In many j shells, pair structures should be determined in a proper way. First we determine the collective S pair variationally

$$\delta \langle S^N | V | S^N \rangle = 0, \tag{4.2}$$

and using the same S pair, we determine the D pair by requiring

$$\delta \langle S^{N-1} D | V | S^{N-1} D \rangle = 0. \tag{4.3}$$

This method is expected to work well in the spherical and the vibrational regions. In this numerical work we assume $G_2 = \kappa$ partly because of simplicity and partly because seniority of Racah is exactly conserved [18,19] if single particle energies are almost degenerate. It is one of our purposes to investigate how single particle energy splitting breaks seniority in realistic situations. In this analysis we have assumed that the single particle energies and two-body interaction strengths are constant for all Sn isotopes and $N=82$ isotones. For $N=82$ isotones we take $G_0=0.18$ and $G_2=\kappa=0.018$ MeV. For the Sn isotopes we take $G_0=0.16$ and $G_2=\kappa=0.018$ MeV. In Table II we show single particle energies and structure constants α_j for $N=82$ isotones and Sn isotopes. It is seen that the α_j's are rather constants as a function of pair numbers.

In Fig. 2 we show energy spectra in the SM, SD model, and the IBM for even $N=82$ isotones. In Fig. 3 we show energy spectra in the SM, SDp model, and the IBFM for odd $N=82$ isotones. These are compared with experimental data for $N=82$ isotones where we have valence protons. In ^{134}Te (two-particle system), ^{135}I (three-particle system), and ^{136}Xe (four-particle system) energy spectra of SD model (SDp model) and sd boson (sd boson plus one-particle) are identical in each case. This is because our boson mapping scheme is exact up to the generalized seniority 4. In ^{134}Te and ^{136}Xe it should be reminded that the first 4^+ states of experimental data cannot be compared directly to the first 4^+

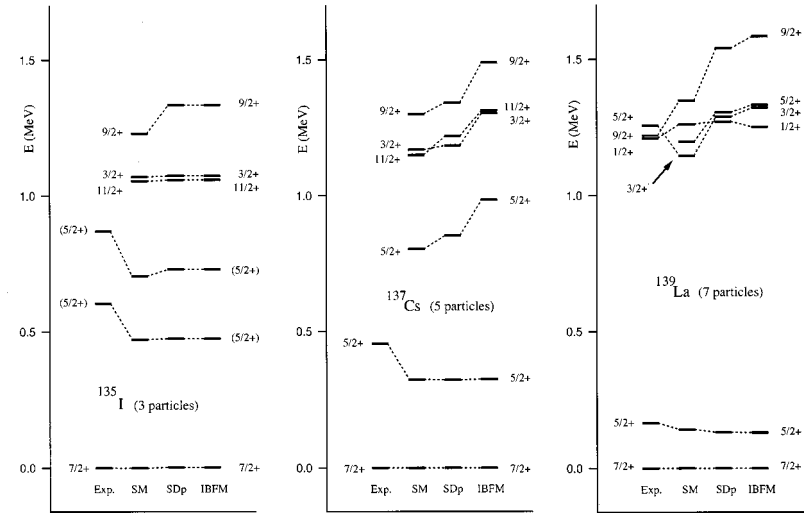

FIG. 3. Energy levels of odd $N=82$ isotones in experiment (Exp), shell model (SM), SDp pair space (SDp), and the IBFM space (IBFM).

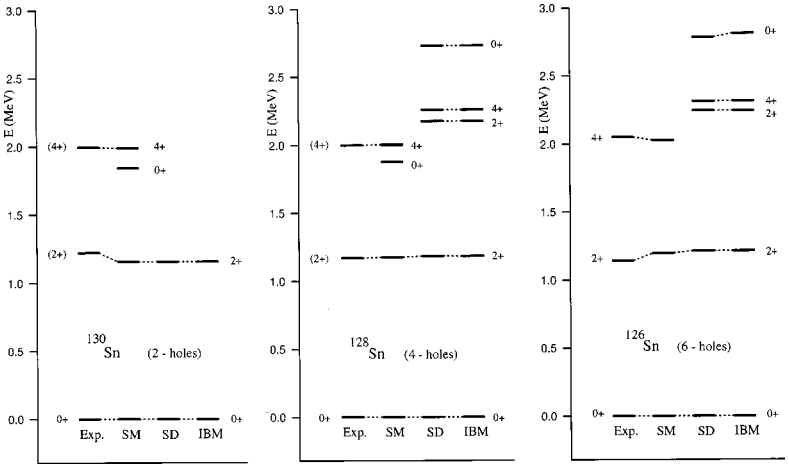

FIG. 4. Energy levels of even Sn isotopes in experiment (Exp), shell model (SM), SD pair space (SD), and the IBM space (IBM).

states in the SD model and the IBM. Because the former have nature of one-phonon structure such as a G pair excitation while the latter have nature of two-phonon excitations such as $|DD,J=4\rangle$. In all the present examples of even (odd) systems the ground states are found to have generalized seniority 0 (1) almost precisely. For instance, in ^{139}La case expectation value of generalized seniority is 1.12 in the SDp model. Namely, this state has the structure $|S^N c_j, J=j\rangle$. In ^{135}I, ^{137}Cs, and ^{139}La all the low-lying excited states in figures have generalized seniority 3 except the first $J=\frac{5}{2}^+$ states, which have generalized seniority 1. Another exception is the first $J=\frac{1}{2}^+$ state in ^{139}La. In this case first $J=\frac{1}{2}^+$ state is very close to the second $J=\frac{5}{2}^+$ state because the single particle energy $\epsilon_{s_{1/2}}$ is rather high. It is seen that the low-lying experimental levels are reproduced well in the SM although we still need some improvement with respect to the SM interaction. Especially the ground state spins are reproduced for all the cases. We have a good agreement among the SM, the SD model (SDp model), and IBM (IBFM).

In Fig. 4 we show energy levels for even Sn isotopes where we have neutron holes. In Fig. 5 we show energy levels for odd Sn isotopes. Such as $N=82$ isotones, energy spectra of SD model (SDp model) and sd boson (sd boson plus one particle) are identical in each case for up to generalized seniority 4. Again all the ground states have generalized seniority 0 (1) approximately in the present examples of even (odd) systems. In ^{129}Sn, ^{127}Sn, and ^{125}Sn, the first $J=\frac{3}{2}^+$, $J=\frac{11}{2}^-$, and $J=\frac{1}{2}^+$ states have generalized seniority 1. The first $J=\frac{3}{2}^+$ and $J=\frac{11}{2}^-$ states are almost degenerate in odd isotopes, but the ordering is not reproduced only in ^{127}Sn for the SM, SDp and IBFM. Considering the extent of degeneracy of $d_{1/2}$ and $h_{11/2}$ levels, it is not surprising that there is a small flip in the ordering of these levels in ^{127}Sn. However, it should be noted that this is the case with all the models considered and just not in the IBFM. It is remarkable that a single schematic interaction is able to describe the systematics of low-lying collective states in both even and odd systems. We have achieved this consistently in these two sets (Sn isotopes and $N=82$ isotones) of singly magic nuclei with proper ordering of levels in all the odd nuclei under consideration with the exception of ^{127}Sn discussed above. The reproduction of the ordering of levels in odd Sn isotopes is a nontrivial task [21]. Throughout this analysis we have seen that the IBFM Hamiltonian derived here reproduces low-lying energy levels very well. This means our sd boson description of odd nuclei is as good as that of the IBM.

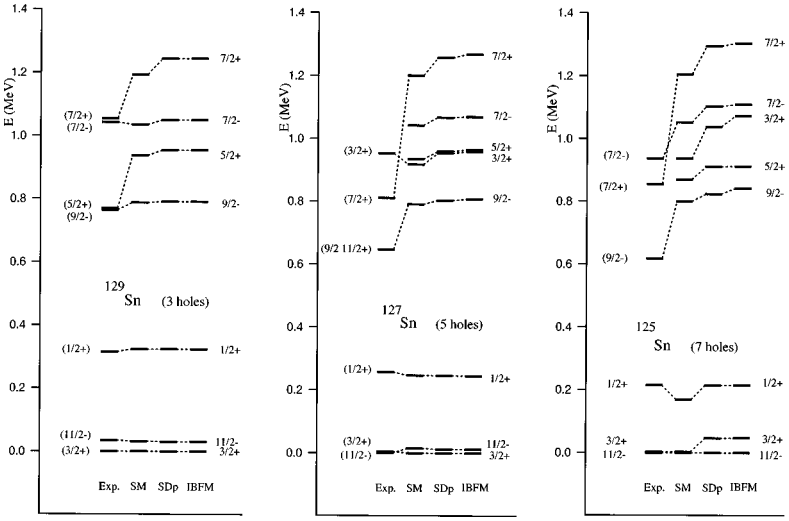

FIG. 5. Energy levels of odd Sn isotopes in experiment (Exp), shell model (SM), SDp pair space (SDp), and the IBFM space (IBFM).

V. SUMMARY AND CONCLUSIONS

In this article we have successfully demonstrated the validity of our procedure to derive microscopically IBM and IBFM Hamiltonians based on the generalized seniority in a consistent and a straightforward manner. We treat even systems (IBM) and odd systems (IBFM) on the same footing, namely, the boson core in the IBFM has the same structure as in the IBM. The resulting Hamiltonian is tested in both single *j*-shell and many *j*-shell cases using a realistic interaction.

In a single *j*-shell case the spectra in the IBFM have been compared with those in the SM, SDp. In the spherical and vibrational regions a good correspondence between the SM, the SDp, and IBFM is obtained. Since our boson mapping scheme is based on the seniority, in the deformed region the ordering of energy levels of the SM is not reproduced well in the SDp model and IBFM. It should also be expected that higher angular momentum pairs such as a *G* pair play an important role in deformed region. The direct term in the IBFM also vanishes in the middle of the shell as in the Scholten's case. However, it does not harm low-lying states at least in spherical and vibrational regions where the seniority scheme is assumed to be valid.

As examples of many *j*-shell cases, by employing a monopole and quadrupole pairings plus quadrupole quadrupole interaction which simulates the vibrational to rotational transition, we have applied our method to singly magic nuclei. The resulting spectra in the IBM (the IBFM) are compared with those in SM and *SD* pair (SDp) spaces and experiments. In these cases generalized seniority is found to be a good concept. All the ground state spins are reproduced except in ^{127}Sn. It is also found to be a good approximation that the low-lying excited states in odd systems consist of generalized seniority 1 and 3. The results clearly indicate that our mapping procedure provides a good approximation both in the IBM and the IBFM at least in spherical and vibration regions. Our main concern in this paper has been the treatment of the spherical and vibrational odd-nuclei since our method is based on the generalized seniority. For the description of deformed nuclei, it is apparent that we need a more appropriate method. Recently, Elliott and Evans proposed a mapping procedure based on SU(3) rather than seniority [22]. This might provide us with another microscopic method for deriving IBFM interaction in deformed nuclei. This is an open question.

ACKNOWLEDGMENTS

We thank Professor N. Yoshida for discussions and also for kindly providing us the PBOS computer code. One of us (Y.D.D.) would like to acknowledge the support from the STA, RIKEN Accelerator Research Facility, and the hospitality of Department of Physics, Saitama University. A part of the computational work reported in this paper was done using the RIKEN supercomputer VPP700E.

[1] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University, Cambridge, 1987).
[2] A. Arima and F. Iachello, Phys. Rev. C **14**, 761 (1976).
[3] F. Iachello and O. Scholten, Phys. Rev. Lett. **43**, 679 (1979).
[4] O. Scholten, Ph.D. thesis, University of Groningen, 1980.
[5] O. Scholten, Prog. Part. Nucl. Phys. **14**, 189 (1985).
[6] F. Iachello and P. Van Isacker, *The Interacting Boson-Fermion Model* (Cambridge University, Cambridge, 1991).
[7] N. Yoshinaga, T. Mizusaki, A. Arima, and Y.D. Devi, Prog. Theor. Phys. Suppl. **125**, 65 (1996).
[8] A. Mets *et al.*, Phys. Rev. Lett. **83**, 1542 (1999).
[9] J. Jolie, J. Phys. G **25**, 889 (1999).
[10] R. Bijker and O. Scholten, Phys. Rev. C **32**, 591 (1985).
[11] O. Scholten and A.E.L. Dieperink, in *The Interacting Bose-Fermi Systems in Nuclei*, edited by F. Iachello (Plenum, New York, 1981), p. 343.
[12] T. Otsuka, A. Arima, and F. Iachello, Nucl. Phys. **A309**, 1 (1978).
[13] I. Talmi, in *The Interacting Bose-Fermi Systems in Nuclei* [11], p. 329.
[14] A. Gelberg, Z. Phys. A **310**, 117 (1983).
[15] T. Otsuka, N. Yoshida, P.V. Isacker, A. Arima, and O. Scholten, Phys. Rev. C **35**, 328 (1987).
[16] I. Talmi, Nucl. Phys. **A172**, 1 (1971).
[17] Y.D. Devi, A. Arima, and N. Yoshinaga, Phys. Lett. B **418**, 13 (1998).
[18] N. Yoshinaga and D.M. Brink, Nucl. Phys. **A515**, 1 (1990).
[19] A. Arima, in *Microscopic Models in Nuclear Structure Physics*, edited by M.W. Guidry, J.H. Hamilton, D.H. Feng, N.R. Johnson, and J.B. McGrory, (World Scientific, Singapore, 1989), p. 356.
[20] A. Arima and M. Ichimura, Prog. Theor. Phys. **296**, 36 (1966).
[21] F. Iachello, Nucl. Phys. **A570**, 145c (1994).
[22] J.P. Elliott and J.A. Evans, J. Phys. G **25**, 2071 (1999).