Level densities for nuclei with $20 \leqslant A \leqslant 41$

Po-lin Huang,* S. M. Grimes, and T. N. Massey
Ohio University, Athens, Ohio 45701

(Received 24 June 1999; revised manuscript received 5 January 2000; published 19 July 2000)

Some commonly used level density compilations are compared with one another and with data in the mass range $20 \leqslant A \leqslant 41$. A new formula is proposed which appears to be consistent with both low energy ($U < 5$ MeV) and high energy ($U \geqslant 20$ MeV) data. This new compilation is used as a benchmark for testing theoretical calculations of level densities utilizing the two-body interaction with moment methods.

PACS number(s): 21.10.Hw, 21.60.Cs, 27.30.+t

I. INTRODUCTION

Level densities are important in nuclear physics for a number of reasons. From a basic standpoint, our ability to predict the energy distribution of all of the excited levels of a nucleus provides a severe challenge to our theoretical understanding, given the size of the basis and the dependence on all facets of the interaction. A number of related areas of physics and technology are also dependent on our studies of the level density. These include nucleosynthesis studies in astrophysics, medical applications of nuclear physics, and fission and fusion reactor design.

The largest body of level density data is the set based on counting neutron resonances at low bombarding energy. Such studies yield a level density at an energy slightly above the neutron binding energy in the compound nucleus. There are some difficulties with this technique. For light nuclei, the level density is not large at this energy, if we assume that the levels observed are reached through $l=0$ absorption. At low energies this is expected to be the case, which means that a total level density can be inferred only if the parity ratio (fraction of levels with a specific parity) and the spin cutoff parameter are known. The combination of problems caused by the small number of levels of the appropriate spin and parity in the energy ''window'' at the binding energy, possible missing levels, and the need to know both the parity ratio and the spin cutoff parameter causes difficulty with compilations based solely on resonance parameters. Most of the sets [1–8] of level density parameters have been based on resonance counting at the binding energy.

To carry out a fit to these data, it has often been necessary to make an assumption to eliminate one of the two parameters of the Fermi gas model. Most often, it has been assumed that the energy shift δ in the Fermi gas formula was simply that due to pairing. This implicitly puts all of the shell effects in the level density parameter a. At high energies, one expects shell effects to anneal out of the level density, but this does not happen if there are shell effects in a. The Fermi gas level density form has

$$\rho(U) = \frac{1}{12\sqrt{2}} \frac{1}{\sigma a^{1/4}} \frac{\exp[2\sqrt{a(U-\delta)}]}{(U-\delta)^{5/4}}. \tag{1}$$

The parameter σ is the spin cutoff parameter and is $\langle J_z^2 \rangle^{1/2}$ for levels at that energy. The density of levels with spin J at the energy U is then

$$\rho(U,J) = \frac{(2J+1)}{2\sigma^2} \exp\left[\frac{-\left(J+\frac{1}{2}\right)^2}{2\sigma^2} \right] \rho(U). \tag{2}$$

In general, σ will be energy dependent. For changes in a or δ, we obtain

$$\frac{d\rho}{\rho} = \sqrt{\frac{(U-\delta)}{a}} da - \sqrt{\frac{a}{(U-\delta)}} d\delta + \frac{3}{2(U-\delta)} d\delta. \tag{3}$$

Equation (3) follows from Eq. (1) if we adopt the usual U dependence for $\sigma(\propto U^{1/4})$. At high energies, $d\rho/\rho$ is $[\sqrt{a/(U-\delta)}]d\delta$ for shifts in energy but is $[\sqrt{(U-\delta)/a}]da$ for changes in a. Thus, level densities at high energies are very sensitive to a and are not so sensitive to δ, while the reverse is true at low energies.

To obtain the level density for levels of specified J and parity p ($= +1$ or -1), the parity ratio is introduced:

$$\Pi(U) = \frac{\rho^+(U)}{\rho^+(U)+\rho^-(U)}. \tag{4}$$

To deduce level densities from resonance counting at the binding energy, it has been necessary to make assumptions about $\Pi(U)$ and $\sigma(U)$. Ericson [9] has shown that under very general assumptions the total number of levels of positive parity is nearly equal to the number of negative parity levels, but data and shell model calculations agree in indicating that at low energies there is a tendency for levels of one parity to dominate. For even-even nuclei, the positive parity density is usually larger, while for odd A nuclei either even or odd parity levels dominate depending on the parity of the single particle orbital nearest the Fermi level. The mass range of the present study ($20 \leqslant A \leqslant 41$) is predominately in the sd shell region; for all nuclei with odd A except for $A = 41$ it is expected that positive parity levels will dominate. Relatively little effort has been invested in determining the excitation energy at which the two parity level densities are equal, although it is expected that this point is reached at lower energy as A is increased.

*Present address: 94-2 Rao-an Road, Yung-an Hsiang, Kaohsiung County, Taiwan, ROC.

There are two commonly used paths to obtaining spin cutoff parameters. Ericson [9] has shown that the spin cutoff parameter should be

$$\sigma^2 = g t \langle m^2 \rangle,$$

where g is the density of single particle states, t is the temperature, and $\langle m^2 \rangle$ is the average of the square of the spin projection on the Z axis for single particle states near the Fermi level. g is connected to the level density parameter a through the equation

$$a = \frac{\pi^2}{6} g.$$

An early estimate of $\langle m^2 \rangle$ was [10]

$$\langle m^2 \rangle = 0.146 A^{2/3}, \tag{5}$$

but it was later realized [11] that this estimate was unrealistic because it included all levels and not just those at the Fermi level. The earlier estimate was utilized in one of the most frequently used tabulations [1], however. It has been proposed [5] that the expression

$$\langle m^2 \rangle = 0.24 A^{2/3} \tag{6}$$

is a better representation of the average over levels at the Fermi energy.

The difference between expressions (5) and (6) is about 50%, which would translate into a similar discrepancy in level densities derived from resonance counting.

An alternative method of determining σ^2 is to assume that the nucleus is a rigid sphere. In this case, an assumed value for the radius of $R(A) = 1.25 A^{1/3}$ fm yields a value for σ^2 of

$$\sigma^2 = 0.0145 A^{5/3} t, \tag{7}$$

where the temperature is given by

$$t = \sqrt{\frac{U-\delta}{a}}, \tag{8}$$

so the forms for σ^2 become

$$\sigma^2 = \frac{6}{\pi^2} a \sqrt{\frac{U-\delta}{a}} 0.146 A^{2/3} = 0.0888 \sqrt{a(U-\delta)} A^{2/3}, \tag{9}$$

$$\sigma^2 = 0.1461 \sqrt{a(U-\delta)} A^{2/3}, \tag{10}$$

$$\sigma^2 = 0.0145 A^{5/3} \sqrt{\frac{U-\delta}{a}}. \tag{11}$$

Equation (10) results if Eq. (6) is used instead of Eq. (5), while Eq. (11) is based on the use of Eq. (7).

For values of a and δ typically used in this mass region, the σ^2 values estimated from Eq. (11) are between those predicted by Eqs. (9) and (10), but are closer to the latter.

Other databases have also been used to determine level density parameters. One analysis [12] has been based on the use of evaporation spectra to derive level density parameters for a wide range of nuclei, and other studies [13–17] have used this technique to determine parameters for a more limited set of nuclei. A good source of information about level densities at high energies is Ericson fluctuations; one compilation [18] has been based entirely on this type of information. Others [19,20] have derived sets of parameters in this mass region which incorporate information from three different types of data and similar analyses [21,22] have been prepared for specific nuclei. A recent compilation by Iljinov *et al.* [23] has utilized a very extensive body of data over a wide range in A to derive a set of level density parameters. In general, despite many years' work, the consistency of these compiled values is not as good as would be desired and many of them do not extrapolate correctly to the level density at low energy.

II. COMPILATION OF LEVEL DENSITY PARAMETERS

In general the compilations in widest use do not include information about the level density at more than one energy. It was decided to formulate a new set of parameters which would include more information in the fit. The fit of Braga-Marcazzan and Milazzo-Colli [18] (BMMC) and that of Rohr [4] were taken as starting points. The former is one of the few which is based on information from Ericson fluctuations and hence should be reliable at 20 MeV. BMMC report a values deduced individually for a number of nuclei based on the assumption that the Gilbert-Cameron [1] pairing energies could be used as the energy shift.

The Rohr compilation is based on resonance counting and so should be reliable at 7 MeV. In this case the energy shifts were also taken to be those of the Gilbert-Cameron compilation but the a values were fit with the function $a = \alpha A + C$, where A is the mass number, a is the level density parameter, and α and C are fitting constants. The use of a smooth form reduces the effect of level statistics in evaluating the level density for a particular nucleus and prevents local shell effects from appearing in a.

We have used a similar fitting function to represent the individual a values compiled by BMMC. The constants derived are slightly different from those of Rohr. While Rohr based his fit on data from more nuclei, the BMMC results, because they are at higher energy, are less likely to be biased by the choice of energy shift, since the effects of changing the energy shift vanish asymptotically as the energy increases.

It was found that using the Rohr or BMMC a values produced poor fits to the level density at low energies [24] based on comparisons with known levels for excitations below 5 MeV. This is perhaps not surprising, since the role of δ at low energies according to Eq. (3) is magnified relative to that of a.

Figures 1–3 show typical results for the Rohr and BMMC parameters compared with level counting at low energy. Note that the consistent tendency is for the predictions to underestimate the data.

The a values were, therefore, fixed at the BMMC magnitudes and a least squares fit was conducted for the δ value

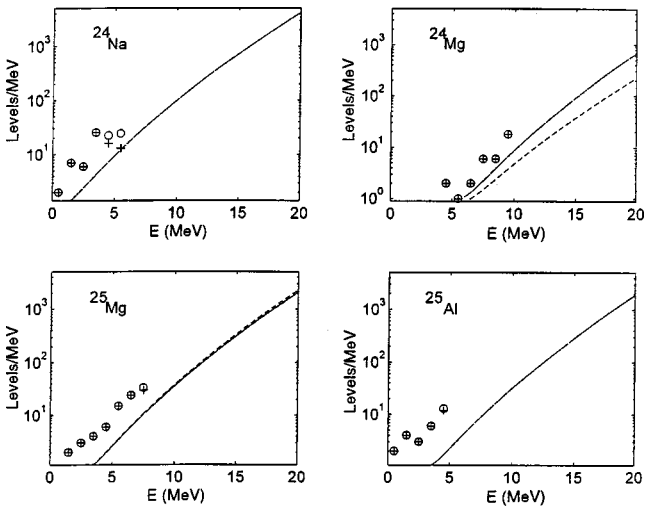

FIG. 1. Level densities for ^{24}Na, ^{24}Mg, ^{25}Mg, and ^{25}Al. In each panel, the + symbols denote the level density based on counting resolved levels of known spin and parity, the ○ symbols denote the level density based on counting all known levels, the solid line indicates the level density predicted by Rohr, and the dashed line is the prediction based on the systematics of Braga-Marcazzan and Milazzo-Colli.

needed to optimize the fit to the known levels at low energy. In all cases, reasonable fits could be achieved by modifying only the δ parameter. An effort to improve the agreement by fixing the δ values at their original magnitude and varying the a values was less successful, with numerous fits showing inappropriate slopes.

In retrospect, this result is not surprising. If, as indicated by Eq. (1), level densities at high energies have less sensitivity to δ than to a, an assumed value for δ would not have biased the a value significantly if the fitted point were at 20 MeV. The corrections to the values at 20 MeV caused by adjusting the δ values were about 30%, which is just within the estimated 30% error of the Ericson analysis. The best fit values for δ are listed in Table I. Also shown are the original

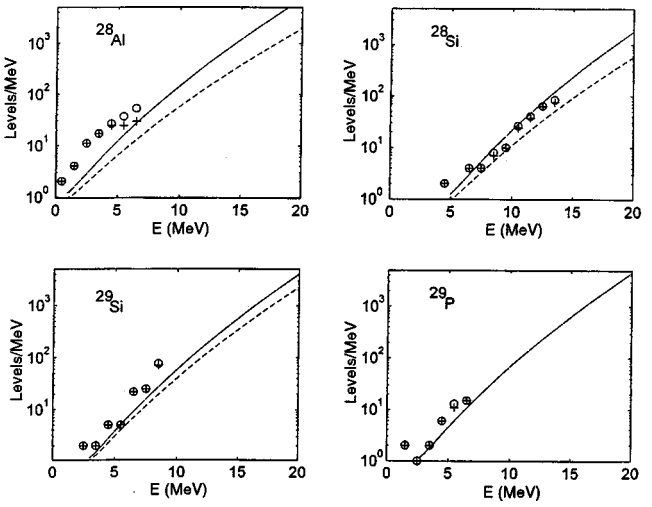

FIG. 2. Same as Fig. 1, except that the nuclei are ^{28}Al, ^{28}Si, ^{29}Si, and ^{29}P.

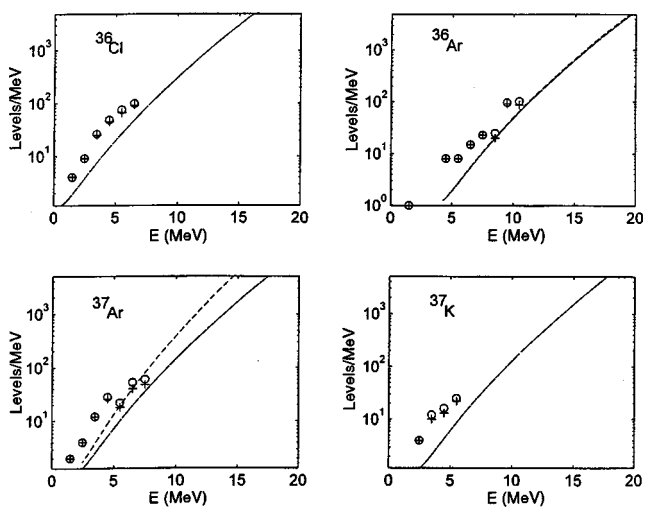

FIG. 3. Same as Fig. 1, except that the nuclei are ^{36}Cl, ^{36}Ar, ^{37}Ar, and ^{37}K.

pairing shift values used by Rohr and BMMC; these were the pairing shifts proposed by Gilbert and Cameron and are labeled GC.

To make the formula for a useful in predicting level densities, it is necessary to find a formula for predicting δ as well. Two approaches were tried. The first is based on the use of the semiempirical mass formula. If the terms corresponding to pairing and shell corrections are dropped, the expression for the nuclear binding energy is

$$B(N,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A}, \quad (12)$$

where N, Z, and A are the neutron number, proton number, and mass number of the nucleus, respectively. a_v, a_s, a_c, and a_a are fitting constants. If the actual binding energy of the nucleus is compared with the value predicted by this formula, the difference should be the shift in the ground state energy caused by pairing and shell effects. Reasonable agreement was found between the predictions for δ using this formula with the parameters of Myers and Swiatecki [25]. Somewhat better results were obtained with the parameters deduced by Grimes [26], which are also listed in Table II. The latter parameters were deduced by fitting the parameters of this equation to nuclear masses *only* in the region $20 \leqslant A \leqslant 42$.

The energy shifts in Table I under ''Myers-Swiatecki'' and ''Grimes'' should be compared with those under ''Best fit''; the deviations are slightly smaller for the parameters of Grimes than for Myers and Swiatecki.

An alternative approach is based not on experimental mass values but on the systematics of the two-body interaction. The normal assumption used in pairing models is that the only interaction is between identical nucleons in states in which they couple to 0^+. This would yield a value of 0 for the energy shift of an odd-odd nucleus. The fits gave results which were normally not close to zero for odd-odd nuclei. This phenomenon has been observed previously and is the

TABLE I. Level density energy shifts.

	Myers-Swiatecki	Grimes	GC	Best fit	AIM
^{20}F	−4.39	−3.04	0.00	−2.43	−2.44
^{20}Ne	2.07	2.44	4.87	3.80	2.44
^{21}Ne	−1.84	−1.07	2.10	−0.10	−0.70
^{21}Na	−0.86	−0.96	2.77	−0.93	−0.92
^{22}Na	−2.80	−2.57	0.00	−1.95	−2.57
^{22}Ne	−0.17	1.04	4.77	1.80	2.39
^{23}Na	−1.30	−0.68	2.67	−0.10	−0.89
^{23}Mg	−0.50	−0.75	2.46	−0.70	−0.82
^{24}Na	−3.44	−2.39	0.00	−2.48	−2.57
^{24}Mg	2.97	3.06	5.13	3.60	2.57
^{25}Mg	−0.84	−0.37	2.46	−0.30	−0.82
^{25}Al	0.06	−0.33	2.67	−0.89	−0.89
^{26}Mg	0.80	1.70	4.26	1.70	2.13
^{26}Al	−1.69	−1.74	0.00	−1.50	−1.95
^{27}Al	0.02	0.36	1.80	0.50	−0.60
^{27}Si	0.70	0.19	2.09	0.00	−0.70
^{28}Al	−2.01	−1.26	0.00	−1.94	−1.95
^{28}Si	4.72	4.54	3.89	3.85	1.95
^{29}Si	1.67	1.87	2.09	0.80	−0.70
^{29}P	2.52	1.88	1.80	0.70	−0.60
^{30}P	0.63	0.32	0.00	−0.50	−1.65
^{30}Si	2.24	2.85	3.76	1.88	1.88
^{31}P	1.26	1.32	1.67	0.65	−0.56
^{31}S	1.96	1.18	1.62	0.50	−0.54
^{32}P	−1.10	−0.62	0.00	−1.35	−1.65
^{32}S	3.73	3.29	3.29	2.30	1.65
^{33}S	0.54	0.48	1.62	0.10	−0.54
^{33}Cl	1.34	0.46	1.67	0.00	−0.56
^{34}S	1.44	1.79	3.48	1.40	1.74
^{34}Cl	−0.62	−1.02	0.00	−1.25	−1.74
^{35}Cl	0.21	0.04	1.86	−0.10	−0.62
^{35}Ar	0.92	−0.08	1.62	−0.54	−0.54
^{36}Cl	−1.93	−1.70	0.00	−1.74	−1.74
^{36}Ar	2.80	2.14	3.48	1.70	1.74
^{37}Ar	−0.50	−0.79	1.62	−0.30	−0.54
^{37}K	0.30	−0.79	1.86	−0.35	−0.62
^{38}Ar	0.43	0.55	3.66	1.40	1.83
^{38}K	−1.05	−1.80	0.00	−1.20	−1.94
^{39}K	−0.18	−0.56	2.04	0.30	−0.68
^{39}Ca	0.52	−0.66	1.83	−0.20	−0.61
^{40}K	−3.46	−3.44	0.00	−1.80	−1.94
^{40}Ca	2.69	1.85	3.87	2.20	1.94
^{41}Ca	−1.26	−1.73	1.83	−2.20	−0.77
^{41}Sc	−0.25	−1.52	2.04	−0.30	−0.68

basis for the ''backshifted'' Fermi gas model [3,7,19]. This model includes an additional energy shift (beyond conventional pairing) which produces an enhancement in the level density. This change has been based on empirical systematics but is probably best justified by noting that an energy shift of this magnitude and sign is possibly serving to produce the level density enhancement that collective states produce.

TABLE II. Parameters for level density fit. Set A: $a=0.086A+1.18$. δ = binding energy $-a_vA+a_sA^{2/3}+a_cz^2/A^{1/3}+a_a(N-Z)^2/A$ Set B: $a=0.086A+0.23$. $\delta=$ binding energy $-a_vA+a_sA^{2/3}+a_cZ^2/A^{1/3}+a_a(N-Z)^2/A-2.7+0.025A$.

	Myers-Swiatecki (MeV)	Grimes (MeV)
a_v	15.56	15.598
a_s	17.23	17.97
a_c	0.697	$(0.8176–0.6457)A^{-1/3}$
a_a	23.285	24.1

It should be noted that the conventional pairing shift, which is 0 for odd-odd nuclei, $\Delta_p(\Delta_n)$ for odd-A nuclei with even $Z(N)$ and $\Delta_p+\Delta_n$ for even-even nuclei, is also ''backshifted.'' The BCS formulation [12] has a condensation energy term in the solution; if this term is ignored, it essentially represents a ''backshift'' for the conventional pairing shift model and an additional ''backshift'' for the conventional backshifted model.

Comparison of the observed ''best-fit'' energy shifts with the predictions of the simple pairing model suggested an alternative to the semiempirical mass formula for obtaining the correct energy shift. Since the shifts would be such as to make the correction for even-even and odd-odd nuclei about equal in magnitude and opposite in sign, the two cases were shifted in size by half of the traditional correction. Instead of using $\Delta_p+\Delta_n$ for even-even nuclei and 0 for odd-odd, the shifts were set to $\frac{1}{2}(\Delta_p+\Delta_n)$ and $-\frac{1}{2}(\Delta_p+\Delta_n)$, respectively, where Δ_p and Δ_n were taken from Gilbert and Cameron. To be consistent with the simple pairing model, the energy shift for even Z-odd N and odd Z-even N nuclei should be set to $\frac{1}{2}(\Delta_p-\Delta_n)$ and $\frac{1}{2}(\Delta_n-\Delta_p)$, respectively. It is found that better agreement with the observed shifts occurs if this shift is set to $-\frac{1}{3}\Delta_p$ and $-\frac{1}{3}\Delta_n$, respectively. These values are also tabulated in Table I. Note that this average interaction method (AIM) gives results which are roughly as good as those obtained with the semiempirical mass equation.

The results of the fits with modified δ values are shown in Figs. 4 –6 for typical nuclei. Note the considerably better fits shown in Figs. 4–6 compared to those in Figs. 1–3.

In order to test the effect of the modified δ values on the level densities at 20 MeV, a second set of level density parameters, denoted set B, was also deduced. For set B, the parameters of set A were adjusted so as to preserve the level density value at 5 MeV while restoring the value at 20 MeV to that of the Braga-Marcazzan parameters before the energy shift was changed. This fit has consistently lower a values and slightly lower (more negative) energy shifts as well. Table II lists the parameters of set B as well as those of set A.

III. COMPARISON WITH THEORY

The compilations are based on the macroscopic Fermi-gas model. This model was introduced by Bethe and utilized the

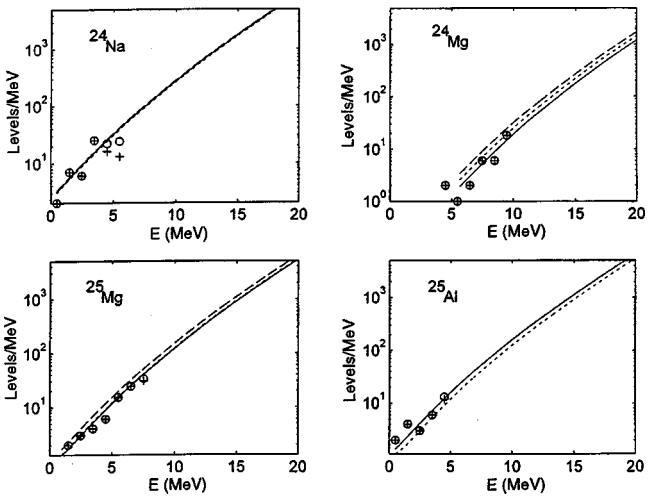

FIG. 4. Level densities of ^{24}Na, ^{24}Mg, ^{25}Mg, and ^{25}Al. The + symbols and ○ symbols are as explained in Fig. 1. The solid line is the best-fit value of δ used with the a value systematics of Table II, while the short dashed and long dashed curves use the same a value but are based on the systematics of Grimes and the AIM method, respectively.

assumption that the single-particle state density is constant with energy.

This assumption can be relaxed and the partition function evaluated for a ''realistic'' set of single-particle levels. This model is called the microscopic Fermi-gas model. It has the virtue that shell effects should automatically be included in the calculation. Further, by using the BCS [27] formalism, the effects of pairing can also be included.

Single-particle energy sets proposed by Nilsson [28], Seeger and Perisho [29], and Seeger and Howard [30] were used in these calculations. The level density values predicted by the model with the single-particle energies of Nilsson were too high for all of the nuclei studied. For nuclei with A below 28, the level scheme of Seeger-Perisho gave the best results. Near this value for A, both Seeger-Perisho and Seeger-

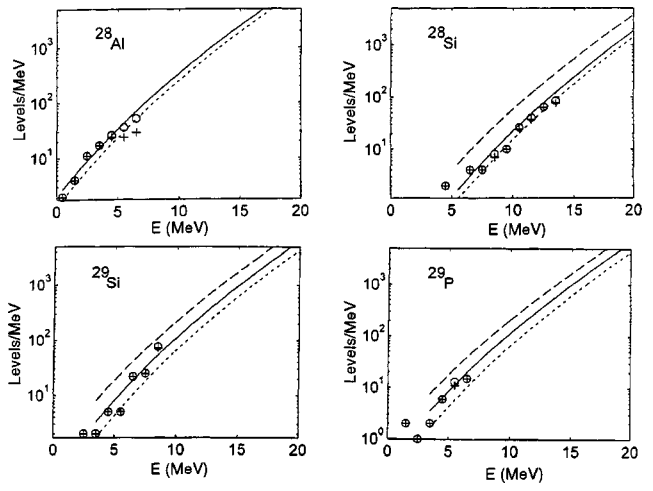

FIG. 5. Level densities of ^{28}Al, ^{28}Si, ^{29}Si, and ^{29}P. The significance of the curves and symbols is as indicated in the caption of Fig. 4.

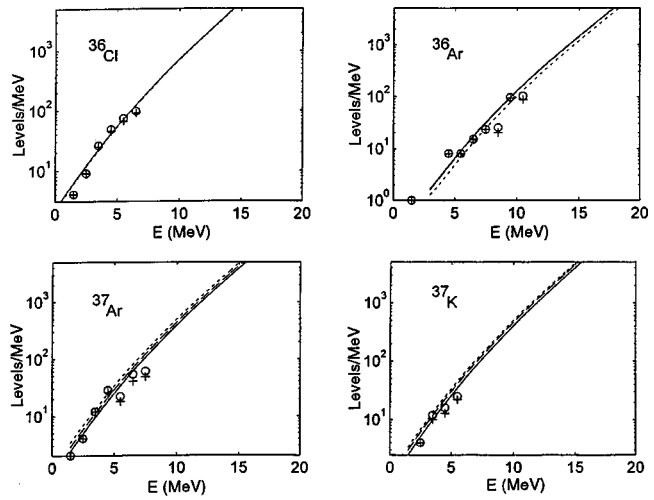

FIG. 6. Level densities of ^{36}Cl, ^{36}Ar, ^{37}Ar, and ^{37}K. The significance of the curves and symbols is as indicated in the caption of Fig. 4.

Howard levels provided a good representation of the data. Beyond $A=30$, the predictions of the model with the levels of Seeger and Howard were superior to those of Seeger and Perisho. Comparisons are shown in Figs. 7–12.

These single-particle schemes also provide reasonable representations of the spin-cutoff parameter. As has already been indicated, this quantity is especially important. It is used in Hauser-Feshbach calculations and is also necessary in interpreting the level density information provided by neutron resonance counting. A conclusive test of the spin-cutoff parameter predictions is difficult because the database for this parameter is somewhat sparse.

One additional parameter is also of interest. The parity ratio represents the fraction of the levels at a given excitation energy which have positive parity. This parameter is expected to be 0.5 at high energy but approaches 0 or 1 (depending on the orbital parity of the level closest to the Fermi level and whether the nucleus is even or odd) at low energy. As is the case for the spin-cutoff parameter, knowledge of this parameter is important in extracting information about level densities from neutron resonance counting. It has been pointed out [31] that the statistical mechanical approach, though reliable for level densities and spin-cutoff parameters, is not accurate in calculating parity ratios. This is because the formalism in evaluating the partition function produces the correct average N and Z but does have some dispersion in these quantities. Thus, the level density is an average of the level density for the nucleus with (N,Z) and small fractions of the corresponding densities for $(N-1,Z+1)$ and $(N+1,Z-1)$. Since the level densities are similar, it is possible to obtain [31] accurate values for the level density despite this averaging. The consequences of the averaging [31] are much more dramatic for the parity ratio and result in an energy dependence for the parity ratio that cannot overshoot 0.5, in contrast to more realistic calculations.

The two primary drawbacks to the Fermi-gas model are that no interactions (other than pairing) are included and that parity ratios are not reliable. This latter parameter is of par-

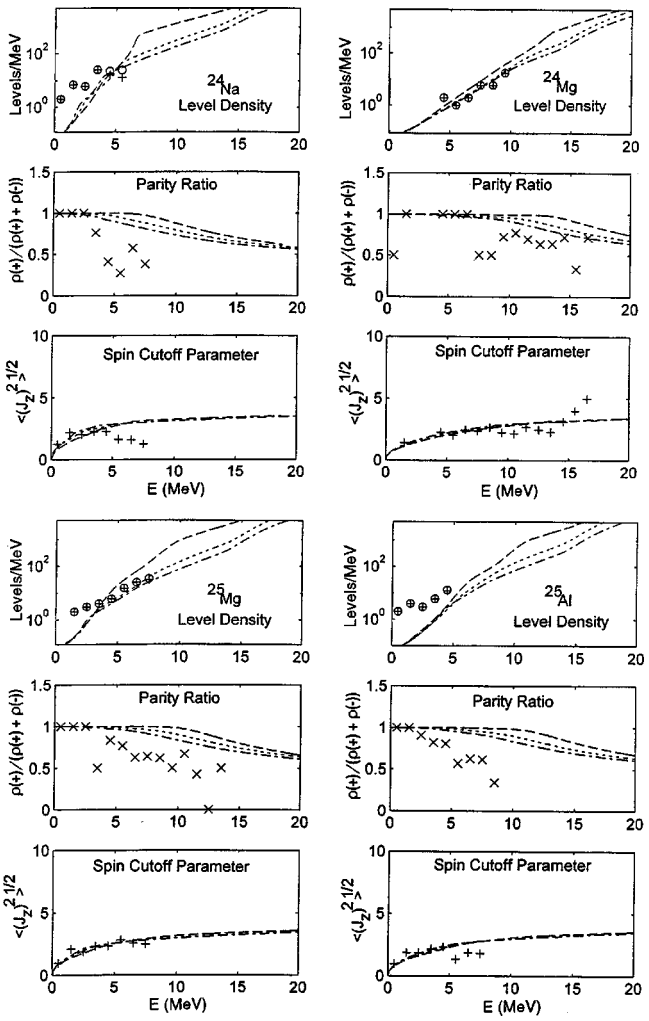

FIG. 7. Level density, parity ratio, and spin-cutoff parameter for ^{24}Na and ^{24}Mg. The long-dashed curve is the calculation based on levels proposed by Nilsson, the short-dashed curve is based on levels proposed by Seeger and Perisho, and the dot-dashed line is the result based on levels proposed by Seeger and Howard.

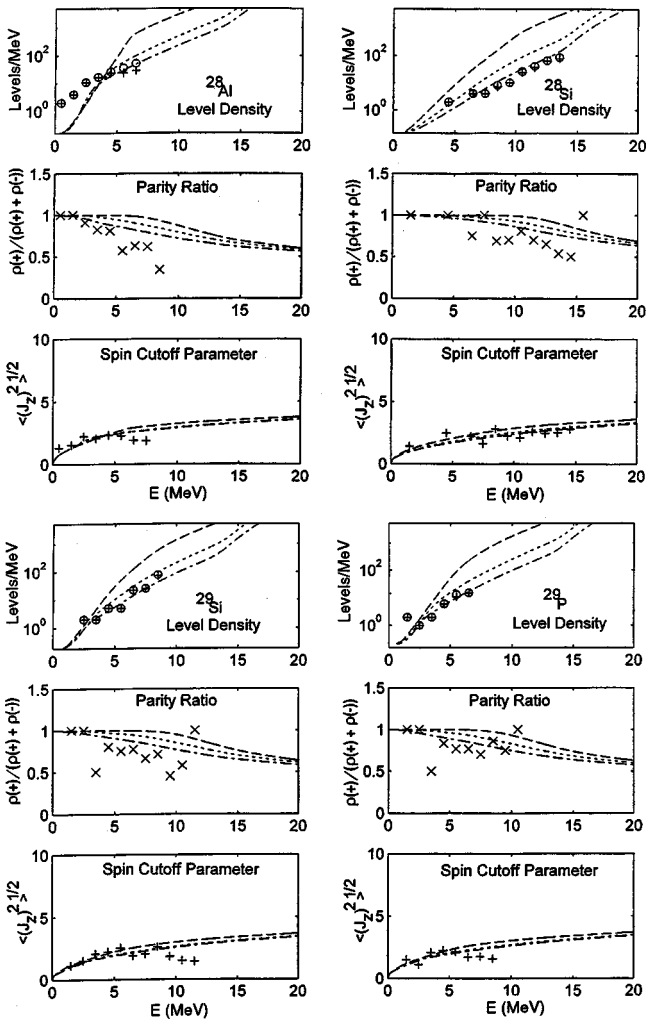

FIG. 8. Same as Fig. 7, except that the nuclei are ^{25}Mg and ^{25}Al.

ticular interest for light nuclei, since there is considerable evidence that it has not reached its asymptotic value of 0.5 at energies at which neutron resonance counts are made.

A formalism has been developed which allows full inclusion of the two-body force in such calculations. The first such calculations included the expectation values of $\langle H \rangle$ and $\langle H^2 \rangle$ in an expansion of the level density [32–36], but more recent work has also included moments as high as $\langle H^4 \rangle$ [37–39]. These calculations utilize the result of unitarity to allow calculation of moments of the Hamiltonian in the nondiagonal shell model basis. With values for these moments, an expansion for the level density itself can be calculated.

One approach to making such calculations is to use Hermite polynomial expansions. These allow a quick and efficient calculation of expansion coefficients and are consistent with asymptotic Gaussian behavior. Unfortunately, they also produce distributions which are not positive definite. Often, the negative excursions come in the low-excitation-energy region, making it impossible to use the calculations in the region of most physical interest.

One possible solution to this problem is to expand the moment information in the form of delta functions [40–42]. These obviously do not have negative excursions but have the drawback of being discrete instead of continuous. Nonetheless, the use of many Slater determinants to make the expansion produces a quasicontinuous distribution. The resulting distribution is, however, somewhat too narrow in the tail region.

A recent paper [43] has suggested an alternative procedure. If the Gaussian function is replaced by an exponential with polynomial argument, the resulting distribution is both positive definite and continuous. The present results are based on the use of the exponential with polynomial argument (EPA) technique.

Input parameters needed for such calculations include single-particle energies and two-body matrix elements. Finding suitable two-body matrix elements is somewhat complicated. In some cases, sets of two-body matrix elements are available which have been derived from fitting shell model calculations to low-lying levels. These are not suited to the present calculations because they normally do not have a procedure for extending the calculation to neighboring orbits.

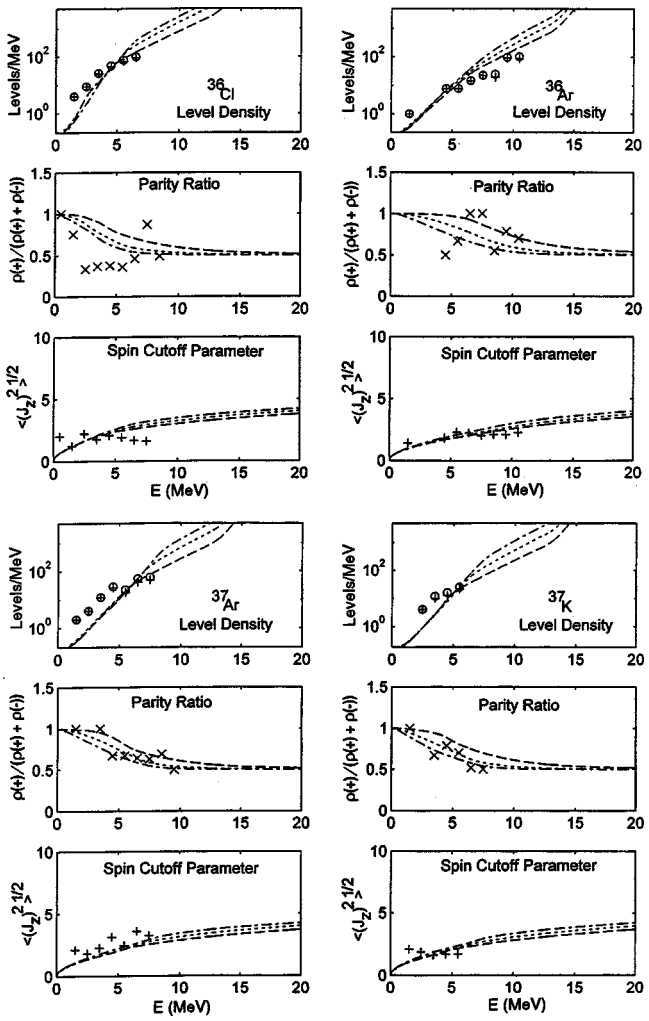

FIG. 9. Same as Fig. 7, except that the nuclei are ^{28}Al and ^{28}Si.

A set of matrix elements based on specified parameters for a potential is required. The Petrovich-McManus-Madsen-Atkinson (PMMA) set proposed by Petrovich *et al.* [44] was chosen. It was found that they had a high correlation coefficient with the empirically derived set of Warburton and Brown [45]. The PMMA parameters are fit to Yukawa potentials in the even states only (the triplet and singlet even potentials are nonzero; the triplet and singlet odd potentials are set to zero). The PMMA potential has been used with some success in shell model calculations [46].

There are three particular challenges in finding the appropriate input for these calculations. Single-particle energies are supposed to include the effects of the interaction with ''core'' particles and the kinetic energy; for these calculations, the core is much smaller than in conventional shell model calculations. This would argue for single-particle energies which are not as spread out as those in a conventional calculation. The authors of a similar study [47] to this one carried out in the mass region near $A=60$ have found it necessary to make this kind of adjustment. A second complication is in the strength of the two-body force. Since our basis is much larger than that in a conventional shell model calculation, the interaction strength should be somewhat weaker. This change is expected because the interaction in a

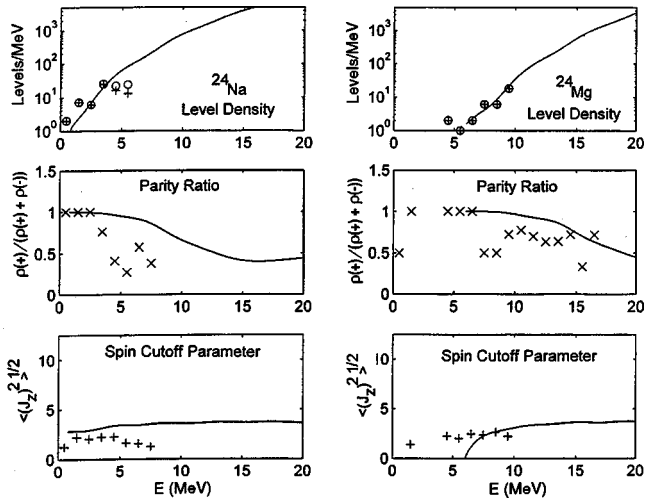

FIG. 10. Same as Fig. 7, except that the nuclei are ^{29}Si and ^{29}P.

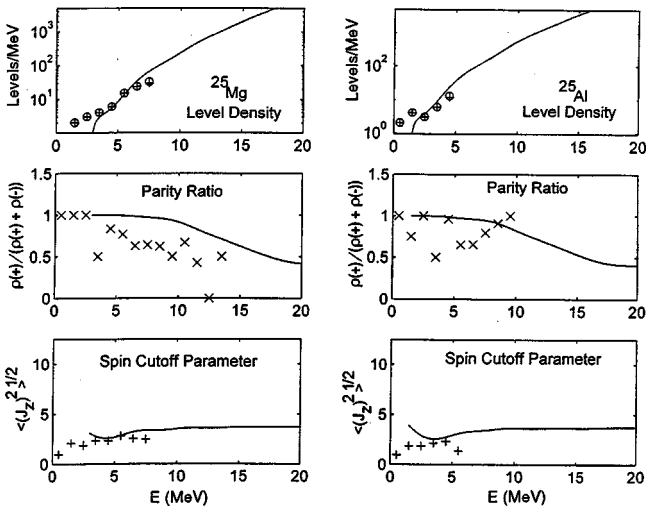

FIG. 11. Same as Fig. 7, except that the nuclei are ^{36}Cl and ^{36}Ar.

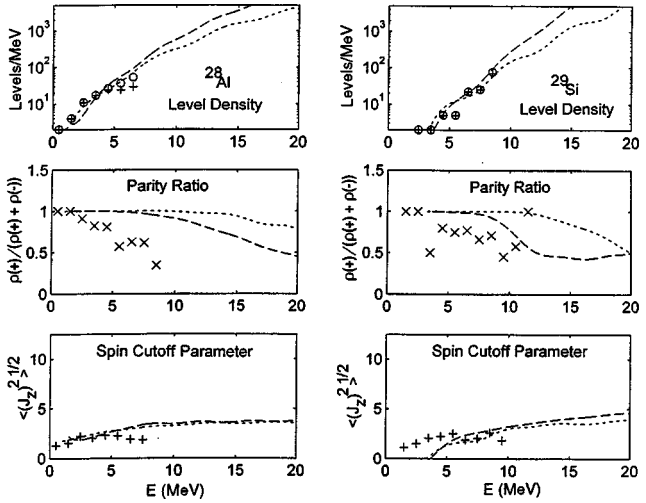

FIG. 12. Same as Fig. 7, except that the nuclei are ^{37}Ar and ^{37}K.

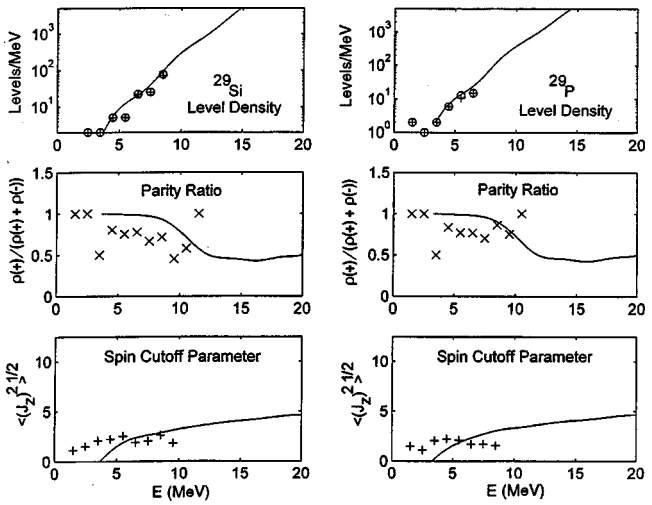

FIG. 13. Level density, parity ratio, and spin-cutoff factors for ^{24}Mg and ^{24}Na. The calculated values curves are based on use of a two-body force with the moment method; experimental points are based on known levels and their spin and parity values.

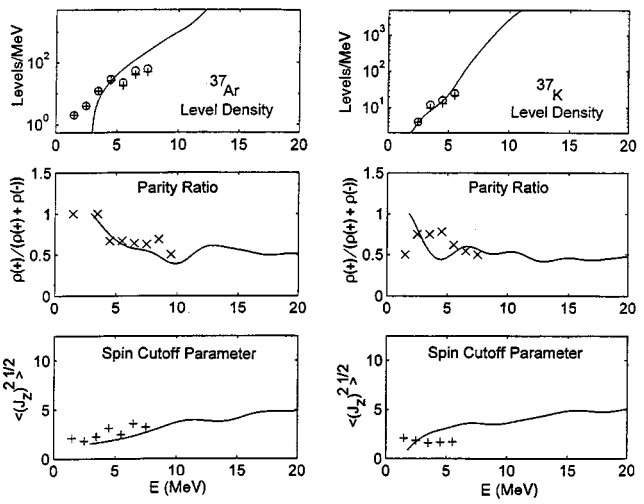

FIG. 14. Same as Fig. 13, except that the nuclei are ^{25}Mg and ^{25}Al.

small space must be renormalized to compensate for missing orbits. To obtain proper fragmentation of states, the interaction needs to be strengthened compared to the appropriate strength in a large basis. We must, therefore, undo this renormalization. Finally, most shell model calculations have focused on calculating levels of one parity. This leaves out some information about the interaction.

Initial calculations with the PMMA force found that the force was too strong. A reduction factor of 0.7 was found to be appropriate. This reduction still did not resolve all problems in achieving a fit to the data. A simultaneous fit to the level density and spin cutoff parameter required adjustment of the single-particle energies as well as the PMMA force. Even these adjustments did not bring the parity ratio into agreement with data. This parameter was not included in the study of Kar and Majumdar [47], although the interaction used by these authors was not the PMMA interaction.

To obtain a simultaneous fit to the level density, spin-cutoff factor, and parity ratio, it was necessary to add a component of the two-body force in odd states. This magnitude was set to $\frac{3}{7}$ of the force strength of the even-state interaction and it was given the opposite sign.

Results of these calculations are shown in Figs. 13–17. A generally good representation of the level density, parity ratio, and spin cutoff factor is obtained. Table III lists the input parameters.

As has been pointed out in the Introduction, an understanding of the variation of both the spin-cutoff factor and parity ratio with energy and with A is central to a level density study based on neutron resonance counting. Both the spectral distribution method and the microscopic Fermi-gas formalism seem to give reasonable results for the level density and spin-cutoff factor for these nuclei. Only the spectral distribution method can give reliable predictions for the parity ratio. Comparison of the present calculations with the compiled values discussed in Sec. II shows reasonable agreement. The calculated level densities are normally within 35% of the predicted values over the energy range 3–15 MeV when compared with the set A parametrization. The predictions of this compiled set are typically above results of the moment method level density calculation. set B parameters (Table II) give very similar results to those of set A for energies below 10 MeV but show a less rapid rise above this energy. They show a slightly better agreement with the moment method calculation over the 3–15 MeV range than does set A.

IV. COMPARISON WITH OTHER COMPILATIONS

As discussed in Sec. I, the most extensively used compilations do not always predict level densities which are consistent for particular nuclei. Two of the leading compilations, Gilbert and Cameron [1] and Rohr [4], made different assumptions about the A dependence of σ, while both assumed that the value of $\Pi(U)$ was 0.5 at the binding energy. To see how these assumptions affected their analysis, a correction

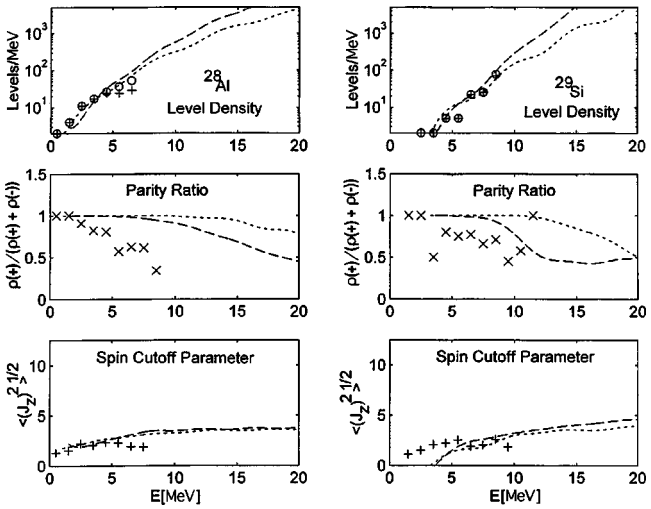

FIG. 15. Same as Fig. 13, except that the nuclei are ^{28}Al and ^{28}Si.

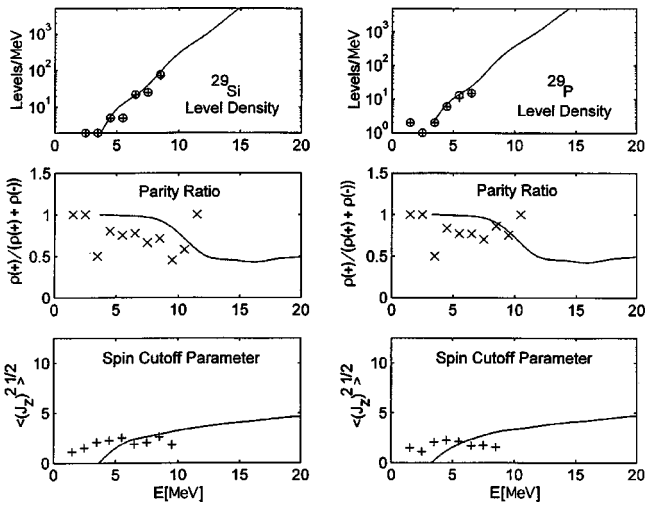

FIG. 16. Same as Fig. 13, except that the nuclei are ^{29}Si and ^{29}P.

factor was calculated based on the calculated σ and Π at the binding energy based on the moment method results. These factors are tabulated in Table IV and indicate that the predictions of Gilbert and Cameron should be increased by about 30% and those of Rohr decreased by 10% in this mass region. Other differences with compilations based on neutron resonances, including assumptions about energy shifts and poor statistics on level counting, cannot be corrected in as direct a fashion.

In Table V we present a comparison of level densities predicted by a number of compilations for specific nuclei. The column labeled ''present results'' lists level densities (set B); the other columns give the ratio of the level density predicted by that compilation to the present results. Note that in many cases deviations of 50% or more are common.

Examination of the comparison in Table V indicates that the present results (set B) give a consistently slower increase of level density with energy than the results of Rohr. This is consistent with the fact that the present results are ''backshifted.'' A correlated observation is that Rohr did not include information about level densities above 10 MeV in his fit. Comparison with Gilbert and Cameron is challenging, since those authors present a set of ''systematic'' parameters which vary between spherical and deformed nuclei. In addition, they also present specific a and δ values for some nuclei, which are not consistent with the ''systematics.'' The values indicated as Gilbert-Cameron 1 are from systematics; those labeled Gilbert-Cameron 2 are specifically listed for that nucleus. Again, because the Gilbert-Cameron values are not ''backshifted,'' they predict level densities which rise more rapidly than the present values. The predictions of Beckerman [20] agree better with the present results, perhaps because high-energy data were also included in the fit.

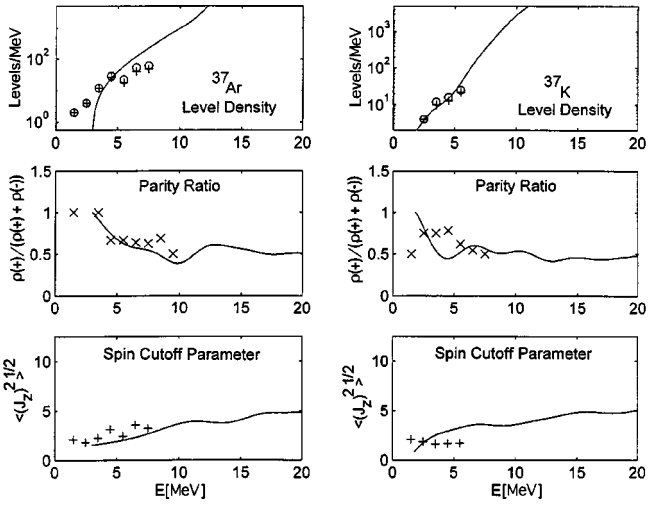

FIG. 17. Same as Fig. 13, except that the nuclei are ^{37}Ar and ^{37}K.

TABLE III. Single-particle energies.

$$E(n,L) = \frac{17.4}{A^{1/3}}(n+1) + 0.4L - 24.7 + \frac{2.82}{A^{1/3}} \times (-L) \text{ for } J = L + \frac{1}{2} \times (+L+1) \text{ for } J = L - \frac{1}{2}$$

$n=2$ for $p_{1/2}$ $p_{3/2}$ shell
$n=3$ for $d_{5/2}$ $s_{1/2}$ $d_{3/2}$ shell
$n=4$ for $f_{7/2}$ $p_{3/2}i$ $p_{1/2}$ $f_{5/2}$ shell

Not shown in the table but in better agreement with the present results are level densities predicted by Gadioli and Zetta [19] and Iljinov *et al.* [23]. The former is slightly above the present results at energies above 12 MeV. Iljinov *et al.* have lower values for a but include a rotational enhancement. The net effect of these two changes is to produce level densities which are generally within 20% of the present results.

Recent compilations have included specific correction factors for level density enhancements due to collective effects [23,48]. Many earlier compilations did not. It may seem surprising that although the enhancement factors are substantial, the fits without the factors can also describe the data. Indeed, one of the compilations [23] presents fits both with and without enhancement factors.

Although the enhancement factors are substantial, they apparently do not vary rapidly with energy. A study of the rotational enhancement factor [49] gives a very slow reduction in magnitude as the energy increases, with the enhancement vanishing at energies above 40 MeV. The nearly constant enhancement can be represented by increasing a or reducing δ, if knowledge of the detailed energy dependence

TABLE IV. Correction factors.

A	Gilbert-Cameron	Rohr
20	1.3	0.9
24	1.2	0.85
28	1.15	0.8
32	1.3	0.9
36	1.35	0.95
40	1.3	0.9

TABLE V. Comparison of level density compilations.

Nucleus	Excitation energy (MeV)	Level density present results	Ratio to present results			
			Rohr	Beckerman	Gilbert-Cameron 1	Gilbert-Cameron 2
^{24}Na	5	22	0.4	1.4	1	–
	10	140	0.7	1.6	2	–
	15	690	1.0	2.0	3.8	–
	20	3020	1.4	2.2	6.1	–
^{24}Mg	5	1.5	–	0.8	–	–
	10	14	0.6	1.2	1.1	0.7
	15	94	1.0	1.35	1.9	1.2
	20	500	1.3	1.4	3.3	1.8
^{25}Mg	5	10	0.3	1.0	0.3	0.3
	10	75	0.5	1.2	1.0	1.0
	15	460	0.7	1.3	1.6	1.6
	20	2040	1.0	1.6	2.7	3.7
^{25}Al	5	13	0.2	0.8	0.2	–
	10	88	0.4	1.0	0.5	–
	15	515	0.6	1.1	0.7	–
	20	2406	0.8	1.3	1.0	–
^{30}Si	5	4.2	0.4	1.22	0.4	0.3
	10	55	0.5	1.0	0.6	0.6
	15	420	0.8	1.0	0.8	1.0
	20	2530	1.0	1.0	1.1	1.5
^{30}P	5	24	0.6	0.6	0.4	0.5
	10	210	0.8	0.6	0.4	0.8
	15	1360	1.1	0.8	0.45	1.0
	20	7270	1.4	0.9	0.5	1.3
^{31}P	5	10	0.5	0.9	0.5	0.6
	10	106	0.8	1.0	0.6	1.2
	15	780	1.1	1.1	1.0	1.8
	20	4620	1.3	1.4	1.3	2.4
^{32}S	5	4	0.5	0.8	0.5	0.4
	10	56	0.7	0.8	0.5	0.5
	15	470	1.0	0.9	0.5	0.6
	20	3070	1.2	0.8	0.5	0.7

of the level density is not extensive. Early research on level densities in this mass region found a need to ''backshift'' the level density in order to match both the magnitude of the level density at the binding energy and the slope and magnitude of the level density at lower energies; a similar result was found in the present analysis. It is plausible that collective effects are the cause of this shift. In addition to the fact that collective effects enhance the level density, which is the same direction of change caused by the energy shift, the energy shift is correlated with ground state binding energies. One expects the shifts in these numbers to enhance the level density between closed shells relative to that at closed shells; the same behavior is expected for collective effects.

An additional approach has been taken to deal with shell effects. Ignatyuk *et al.* [48] have proposed the use of an a which varies with energy:

$$a = a_0\left(1 - \frac{\delta}{E}(1 - e^{-\gamma E})\right). \tag{13}$$

In this expression a is the (energy-dependent) level density parameter, a_0 is the asymptotic value of the level density parameter at high energies, δ is a shell shift, and γ is a convergence parameter ($\gamma \approx 0.05$). For typical values of the energy shift ($|\delta| \leqslant 2$ MeV), this form yields a value for a which is reduced (increased) about 10% for δ positive (negative) at low energy. It has moved about halfway to the asymptotic value at $E = 25$ MeV. Thus, this approach yields a very slowly energy dependent a. As is the case for collective enhancement, the resulting energy dependence is similar to that of the conventional Fermi gas.

V. SUMMARY

An examination of the assumptions of some level density compilations has led to the development of a new compilation. This set of parameters has as its basis level density values derived from Ericson fluctuation measurements at energies above 20 MeV, but parameter adjustments produced a reasonable description of data below 5 MeV as well. The values for level density parameters from the present compilation were used in Hauser-Feshbach calculations which were compared with data by Bateman *et al.* [50]; reasonable agreement was found.

This set of parameters is compared with calculations based on BCS quasiparticles and one including the two-body force. In both cases, additional information helpful in testing the calculation included spin-cutoff parameters and parity ratios. Although these parameters are also useful in interpreting experimental information and in making Hauser-Feshbach calculations, the database is not as extensive as would be desirable.

Reasonable fits to the data were achieved with the two-body force calculations. It was possible to use the comparison with data to refine the input parameters.

The present results confirm a previous analysis which showed that including two-body forces in a large basis required a ''compression'' of the single-particle spectrum. Furthermore, comparison of parity ratio predictions with data gave indication of a need for an odd component to the two-body force.

More extensive study of this formalism will be needed in order to make it useful in *a priori* predictions.

[1] A. Gilbert and A. G. W. Cameron, Can. J. Phys. **43**, 1446 (1965).
[2] W. Dilg, W. Schantl, H. Vonach, and M. Uhl, Nucl. Phys. **A217**, 269 (1973).
[3] H. Vonach and M. Hille, Nucl. Phys. **A127**, 289 (1969).
[4] G. Rohr, Z. Phys. A **318**, 299 (1984).
[5] U. Facchini and E. Saetta-Menichella, Energ. Nucl. (Milan) **15**, 54 (1967).
[6] A. V. Ignatyuk, G. N. Smirenkin, and A. S. Tishin, Yad. Fiz. **21**, 485 (1975) [Sov. J. Nucl. Phys. **21**, 255 (1975)].
[7] J. R. Huizenga, A. N. Behkami, J. S. Swintek, and R. W. Atcher, Nucl. Phys. **A223**, 577 (1974).
[8] J. R. Huizenga, A. N. Behkami, R. W. Atcher, J. S. Swintek, H. C. Britt, and H. Freiesleben, Nucl. Phys. **A223**, 589 (1974).
[9] T. Ericson, Adv. Phys. **9**, 425 (1960).
[10] J. H. D. Jensen and J. M. Luttinger, Phys. Rev. **86**, 907 (1952).
[11] D. W. Lang, Nucl. Phys. **77**, 545 (1966).
[12] S. M. Grimes, J. D. Anderson, J. W. McClure, B. A. Pohl, and C. Wong, Phys. Rev. C **10**, 2373 (1974).
[13] J. Benveniste, G. Merkel, and A. Mitchell, Phys. Rev. **174**, 1357 (1968).
[14] C. C. Lu, L. C. Vaz, and J. R. Huizenga, Nucl. Phys. **A190**, 229 (1972).
[15] L. C. Vaz, C. C. Lu, and J. R. Huizenga, Phys. Rev. C **5**, 463 (1972).
[16] J. C. Pacer, J. Wiley, C. R. Lux, and N. T. Porile, Nucl. Phys. **A226**, 413 (1974).
[17] J. Wiley, J. C. Pacer, C. R. Lux, and N. T. Porile, Nucl. Phys. **A212**, 1 (1973).
[18] G. Braga-Marcazzan and L. Milazzo-Colli, Energ. Nucl. (Milan) **15**, 186 (1968).
[19] E. Gadioli and L. Zetta, Phys. Rev. **167**, 1016 (1968).
[20] M. Beckerman, Nucl. Phys. **A278**, 333 (1977).
[21] V. Mishra, N. Boukharouba, C. E. Brient, S. M. Grimes, and R. S. Pedroni, Phys. Rev. C **49**, 750 (1984).
[22] F. B. Bateman, S. M. Grimes, N. Boukharouba, V. Mishra, C. E. Brient, R. S. Pedroni, and T. N. Massey, Phys. Rev. C **55**, 133 (1997).
[23] A. S. Iljinov, M. V. Mebel, N. Bianchi, E. Di Sanctis, C. Guaraldo, V. Lucherini, V. Muccifora, E. Polli, A. R. Reolon, and P. Rossi, Nucl. Phys. **A543**, 517 (1992).
[24] The level density at low energies was deduced by summing the number of levels listed in P. M. Endt, Nucl. Phys. **A521**, 1 (1990), in 1 MeV bins. This same database was used to calculate $\sigma(=\langle J_z^2\rangle^{1/2})$ and Π in each bin.
[25] W. D. Myers and W. J. Swiatecki, Nucl. Phys. **81**, 1 (1966).
[26] S. M. Grimes (private communication).
[27] J. N. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1125 (1957).
[28] S. G. Nilsson, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. **29**, 16 (1955).
[29] P. A. Seeger and R. C. Perisho, Los Alamos Scientific Laboratory Report No. LA-3751, 1967.
[30] P. A. Seeger and W. M. Howard, Nucl. Phys. **A238**, 491 (1975).
[31] S. M. Grimes, Phys. Rev. C **38**, 2362 (1988).
[32] H. Banerjee and J. B. French, Phys. Lett. **23**, 245 (1966).
[33] J. B. French, Phys. Lett. **23**, 248 (1966).
[34] F. S. Chang, J. B. French, and K. F. Ratcliff, Phys. Lett. **23**, 251 (1966).
[35] J. B. French and K. F. Ratcliff, Phys. Rev. C **3**, 94 (1971).
[36] K. F. Ratcliff, Phys. Rev. C **3**, 117 (1971).
[37] G. D. Lougheed and S. S. M. Wong, Nucl. Phys. **A243**, 215 (1975).
[38] S. M. Grimes, S. D. Bloom, R. F. Hausman, Jr., and B. J. Dalton, Phys. Rev. C **19**, 2378 (1979).
[39] S. M. Grimes, S. D. Bloom, H. K. Vonach, and R. F. Hausman, Phys. Rev. C **27**, 2893 (1983).
[40] B. Strohmaier, S. M. Grimes, and H. Satyanarayana, Phys. Rev. C **36**, 1604 (1987).
[41] B. Strohmaier, S. M. Grimes, and H. Satyanarayana, Phys. Rev. C **36**, 1604 (1987).
[42] B. Strohmaier and S. M. Grimes, Z. Phys. A **329**, 431 (1988).
[43] S. M. Grimes and T. N. Massey, Phys. Rev. C **51**, 606 (1995).
[44] F. Petrovich, H. McManus, V. A. Madsen, and A. J. Atkinson, Phys. Rev. Lett. **22**, 895 (1969).
[45] E. K. Warburton and B. A. Brown, Phys. Rev. C **46**, 923 (1992).
[46] C. Wong, S. D. Bloom, S. M. Grimes, R. F. Hausman, Jr., and V. A. Madsen, Phys. Rev. C **18**, 2052 (1978).

[47] V. K. B. Kota and D. Majumdar, Nucl. Phys. **A604**, 129 (1996).

[48] A. V. Ignatyuk, K. K. Istekov, and G. N. Smirenkin, Yad. Fiz. **29**, 875 (1979) [Sov. J. Nucl. Phys. **29**, 450 (1979)].

[49] G. Hansen and A.S. Jensen, in ''Proceedings of the IAEA Advisory Group Meeting on Basic and Applied Problems of Nuclear Level Densities,'' Brookhaven National Laboratory, Upton, NY, 1983, Brookhaven National Lab Report No. BNL-NCS-51694, 1983, p. 161.

[50] F. B. Bateman, R. C. Haight, M. B. Chadwick, S. M. Sterbenz, S. M. Grimes, and H. K. Vonach, Phys. Rev. C **60**, 064609 (1998).