Parity violation in $\gamma\vec{p}$ Compton scattering

Paulo F. Bedaque*
Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195-1550

Martin J. Savage†
Department of Physics, University of Washington, Seattle, Washington 98195-1560
and Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, Virginia 23606

(Received 19 October 1999; published 20 June 2000)

A measurement of parity-violating spin-dependent $\gamma\vec{p}$ Compton scattering will provide a theoretically clean determination of the parity-violating pion-nucleon coupling constant $h_{\pi NN}^{(1)}$. We calculate the leading parity-violating amplitude arising from one-loop pion graphs in chiral perturbation theory. An asymmetry of $\sim 5\times 10^{-8}$ is estimated for Compton scattering of 100 MeV photons.

PACS number(s): 13.60.Fz, 14.20.Dh, 11.30.Er

Precise experimental work performed during the past decades has provided a catalogue of parity-violating matrix elements in both light and heavy nuclei. Unfortunately, a complete theoretical understanding of these measurements has proved elusive thus far. In particular, from the measurements in light nuclei one would hope to be able to extract the parity-violating isovector pion-nucleon coupling constant, $h_{\pi NN}^{(1)}$, which is expected to provide the dominant source of $\Delta I=1$ parity violation in the nucleon-nucleon potential. However, a unique value of $h_{\pi NN}^{(1)}$ consistent with all measurements has not been established and, in addition, the ^{18}F measurements [1] suggest that $h_{\pi NN}^{(1)}$ is much smaller than naive estimates [2–9], or that there are significant cancellations between leading and subleading interactions [7]. This discrepancy is probably due to the difficulty of the nuclear physics component of the calculations, as opposed to a signal of new physics. Thus, the question remains as to which measurement or set of measurements will most reliably determine $h_{\pi NN}^{(1)}$. Two-nucleon observables would seem to have a distinct advantage over other multinucleon systems as the deuteron is so loosely bound. Recently, a proposal [10] has been made to precisely determine $h_{\pi NN}^{(1)}$ from the forward-backward asymmetry A_γ in the radiative capture of polarized neutrons by protons, $\vec{n}+p\rightarrow d+\gamma$. The current experimental limit is $A_\gamma=-(1.5\pm 4.8)\times 10^{-8}$ [11], while the proposed experiment expects to measure A_γ with a precision of $\pm 5\times 10^{-9}$. Theoretically, if $h_{\pi NN}^{(1)}$ is of its naively estimated size then A_γ will be dominated by the $h_{\pi NN}^{(1)}$ coupling [12–17]. If, on the other hand, $h_{\pi NN}^{(1)}$ is much smaller than estimated, the existing calculations of this asymmetry will be invalid. An alternate determination may be possible at Bates [18], by a precise measurement of deuteron spin-dependent parity-violation in electron-deuteron scattering. This process is not ideal due to contributions from direct Z^0 exchange (and higher order interactions) between the electron and the deuteron. Nonetheless, a constraint will be placed on $h_{\pi NN}^{(1)}$ from such a measurement [19,20]. An analogous measurement in scattering from nucleons does not provide the same constraint due to the much larger isovector coupling of the Z^0, which is absent (at tree-level) in the deuteron. In this work we show that a precise measurement of Compton scattering from polarized protons (and neutrons) will allow for a theoretically clean extraction of $h_{\pi NN}^{(1)}$, which contributes through one-loop pion graphs.

The strong interactions of the pions and nucleons are described at leading order[1] in heavy baryon chiral perturbation theory ($HB\chi PT$) [21] (for subsequent discussions of $HB\chi PT$ see [22]) by

$$\mathcal{L}_{st}=\frac{f^2}{8}\mathrm{Tr}\, D_\mu\Sigma D^\mu\Sigma^\dagger+\bar{N}iv_\mu D^\mu N+2g_A\bar{N}S_\mu\mathcal{A}^\mu N+\cdots, \tag{1}$$

where N is the isospin doublet of nucleon fields with four velocity v, M_N is the nucleon mass, S_μ is the covariant spin operator, $g_A\sim +1.25$ is the axial coupling constant, $f=132$ MeV is the pion decay constant, D^μ is the covariant derivative, and the ellipses represent operators involving more insertions of the light quark mass matrix, meson fields, and derivatives. The pion fields are contained in a special unitary matrix,

$$\Sigma=\xi^2=\exp\frac{2i\Pi}{f},\quad \Pi=\begin{pmatrix}\pi^0/\sqrt{2} & \pi^+\\ \pi^- & -\pi^0/\sqrt{2}\end{pmatrix}, \tag{2}$$

and the axial vector meson field is $\mathcal{A}_\mu=\partial_\mu\Pi/f+\cdots$. The Lagrange density in Eq. (1) and the Wess-Zumino term, give the leading contributions to the parity-conserving $\gamma\vec{p}$ Compton scattering amplitude T^{pc}, which has the form [23] (in the center of momentum frame)

*Electronic address: bedaque@phys.washington.edu
†Electronic address: savage@phys.washington.edu

[1]We define the leading-order contribution to any observable to be the order of the first nonzero contribution.

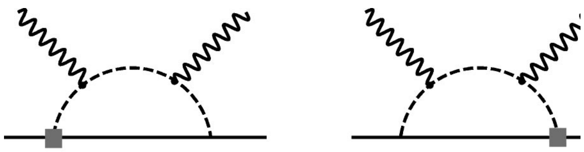

FIG. 1. The leading-order contribution to parity violation in γN Compton scattering. The solid square is the weak operator with coefficient $h_{\pi NN}^{(1)}$. Wavy lines are photons, solid lines are nucleons, and dashed lines are pions. We have not shown the crossed graphs. In addition we have not shown graphs with photons from the strong vertex, or insertion of the two-photon-pion vertex as they vanish in the $v\cdot A=0$ gauge.

$$T^{\rm pc}=\overline{N}[A_1\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}'^*+A_2\boldsymbol{\epsilon}\cdot\hat{\mathbf{k}}'\boldsymbol{\epsilon}'^*\cdot\hat{\mathbf{k}}+i2A_3\mathbf{S}\cdot(\boldsymbol{\epsilon}'^*\times\boldsymbol{\epsilon}) +i2A_4\mathbf{S}\cdot(\hat{\mathbf{k}}'\times\hat{\mathbf{k}})\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}'^*+i2A_5\mathbf{S}\cdot[(\boldsymbol{\epsilon}'^*\times\hat{\mathbf{k}})\boldsymbol{\epsilon}\cdot\hat{\mathbf{k}}' -(\boldsymbol{\epsilon}\times\hat{\mathbf{k}}')\boldsymbol{\epsilon}'^*\cdot\hat{\mathbf{k}}]+i2A_6\mathbf{S}\cdot[(\boldsymbol{\epsilon}'^*\times\hat{\mathbf{k}}')\boldsymbol{\epsilon}\cdot\hat{\mathbf{k}}' -(\boldsymbol{\epsilon}\times\hat{\mathbf{k}})\boldsymbol{\epsilon}'^*\cdot\hat{\mathbf{k}}]]N, \tag{3}$$

where $\mathbf{S}$ are the three-vector components of S_μ, and $\hat{\mathbf{k}},\hat{\mathbf{k}}'$ are unit vectors in the direction of $\mathbf{k},\mathbf{k}'$, respectively. The A_i are functions of the photon energy ω and scattering angle θ, and can be found in [23–25]. They receive contributions from tree-level and one-loop pion graphs, as well as from the Wess-Zumino term. The leading contribution to A_1 arises from the covariant derivative term in Eq. (1), and gives

$$A_1=-\frac{e^2}{M_N}. \tag{4}$$

The remaining A_i vanish at this order but receive nonzero contributions at higher order in the chiral expansion.

The $\Delta I=1$ flavor-conserving-parity-violating interactions, including $\gamma\vec{N}$ Compton scattering, will be dominated by the lowest order operator in the chiral Lagrangian[2]

$$\mathcal{L}_{\rm weak}^{\Delta I=1}=-\frac{h_{\pi NN}^{(1)}}{\sqrt{2}}\varepsilon^{3\alpha\beta}\overline{N}\pi^\alpha\tau^\beta N+\cdots=ih_{\pi NN}^{(1)}\pi^+p^\dagger n+\text{H.c.}+\cdots, \tag{5}$$

if its coefficient, $h_{\pi NN}^{(1)}$, is of natural size. The ellipses denote terms involving more pion fields required by chiral invariance. Explicit computation of the one-loop graphs shown in Fig. 1 gives a parity-violating amplitude in the center-of-momentum frame of the form

[2]The other operators that appear in [7] involve more derivatives and are consequently of higher order in the chiral expansion than the contribution from the operator with coefficient $h_{\pi NN}^{(1)}$ for natural size coefficients.

$$T^{pv}=\frac{e^2g_Ah_{\pi NN}^{(1)}}{2\pi^2 f}\overline{N}\tau_3[\mathcal{F}_1(\omega,\theta)\mathbf{S}\cdot(\mathbf{k}+\mathbf{k}')\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}'^*-\mathcal{F}_2(\omega,\theta) \times(\mathbf{S}\cdot\boldsymbol{\epsilon}'^*\mathbf{k}'\cdot\boldsymbol{\epsilon}+\mathbf{S}\cdot\boldsymbol{\epsilon}\mathbf{k}\cdot\boldsymbol{\epsilon}'^*) -\mathcal{F}_3(\omega,\theta)\mathbf{k}\cdot\boldsymbol{\epsilon}'^*\mathbf{k}'\cdot\boldsymbol{\epsilon}\mathbf{S}\cdot(\mathbf{k}+\mathbf{k}')]N, \tag{6}$$

where τ_3 is the isospin matrix. For general kinematics the loop functions $\mathcal{F}_i$ are somewhat complicated, and we present them as integrals over two Feynman parameters,

$$\mathcal{F}_1(\omega,\theta)=\int_0^1dx\int_0^{1-x}dy(1-2y)[\mathcal{I}(-1;x\omega,\tilde{m}^2)-\mathcal{I}(-1;-x\omega,\tilde{m}^2)],$$
$$\mathcal{F}_2(\omega,\theta)=2\int_0^1dx\int_0^{1-x}dy\,y[\mathcal{I}(-1;x\omega,\tilde{m}^2)-\mathcal{I}(-1;-x\omega,\tilde{m}^2)],$$
$$\mathcal{F}_3(\omega,\theta)=2\int_0^1dx\int_0^{1-x}dy\,y(1-x-y)(2y-1)\times[\mathcal{I}(-2;x\omega,\tilde{m}^2)-\mathcal{I}(-2;-x\omega,\tilde{m}^2)],$$
$$\tilde{m}^2=m_\pi^2+2y(1-x-y)\omega^2(1-\cos\theta), \tag{7}$$

where the functions $\mathcal{I}(\alpha;b,c)$ are defined by Jenkins and Manohar in [21]:

$$\mathcal{I}(\alpha;b,c)=\int_0^\infty d\lambda(\lambda^2+2\lambda b+c)^\alpha,$$
$$\mathcal{I}(-1;\Delta,m^2)=-\frac{1}{2\sqrt{\Delta^2-m^2+i\epsilon}}\ln\left(\frac{\Delta-\sqrt{\Delta^2-m^2+i\epsilon}}{\Delta+\sqrt{\Delta^2-m^2+i\epsilon}}\right),$$
$$\mathcal{I}(-2;\Delta,m^2)=\frac{1}{2(\Delta^2-m^2+i\epsilon)}\left(\frac{\Delta}{m^2}-\mathcal{I}(-1;\Delta,m^2)\right). \tag{8}$$

Notice that there is no contribution from the Wess-Zumino term at this order as the operator with coefficient $h_{\pi NN}^{(1)}$ couples nucleons to the charged pion field only.

For forward scattering, $\mathbf{k}=\mathbf{k}'$, the amplitude in Eq. (6) collapses to $T^{pv}\sim\omega\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}'S\cdot k$, which is clearly parity violating. As there is no nonderivative parity-violating coupling between the Δ and the nucleon, contributions from Δ intermediate states are suppressed in the chiral expansion, unlike the situation for many other observables. Therefore, the one-loop contribution in Eq. (6) is enhanced by two powers of the pion mass compared to the naive size of local counterterms, whose size is set by Λ_χ, the scale of chiral symmetry breaking. However, there will be contributions at next order in the chiral expansion that are suppressed by a single power of m_π/Λ_χ or ω/Λ_χ, compared to the contribution in Eq. (6), from both strong interactions and higher dimension weak interactions [7]. Therefore, we do not pursue this calculation beyond leading order.

In the energy regime where $\omega \ll m_\pi$, the dominant part of the parity-violating amplitude in Eq. (6) is reproduced by a Lagrange density of the form

$$\mathcal{L}^{(pv)}_{\gamma\gamma} = \overline{N}(W^{(0)}_{\gamma\gamma} + W^{(1)}_{\gamma\gamma}\tau^3) v_\mu S_\nu N F^{\mu\alpha} F^{\nu}_{\ \alpha}, \tag{9}$$

where $W^{(0),(1)}_{\gamma\gamma}$ are the isoscalar and isovector dimension seven coupling constants

$$W^{(0)}_{\gamma\gamma} = 0, \quad W^{(1)}_{\gamma\gamma} = \frac{e^2 g_A h^{(1)}_{\pi NN}}{12\pi^2 f m_\pi^2}. \tag{10}$$

The differential cross section for $\gamma\vec{p}$ Compton scattering resulting from the amplitudes in Eqs. (3) and (6) is, to leading order in the chiral expansion and weak interaction,

$$\begin{aligned}\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2M_N^2}\Bigg[& 1 + \cos^2\theta + \eta \frac{g_A h^{(1)}_{\pi NN} M_N}{12\pi^2 f m_\pi^2} \\ & \times \omega^2 (1+\cos\theta)\,\mathrm{Re}\Bigg(\tilde{\mathcal{F}}_1(\omega,\theta)(1+\cos^2\theta) \\ & + \tilde{\mathcal{F}}_2(\omega,\theta)\cos\theta(1-\cos\theta) \\ & - \tilde{\mathcal{F}}_3(\omega,\theta)\frac{\omega^2}{15 m_\pi^2}\cos\theta(1-\cos^2\theta)\Bigg)\Bigg],\end{aligned} \tag{11}$$

where $\eta = +1(-1)$ for the proton spin-polarized parallel (antiparallel) to the direction of the incident photon. The functions $\tilde{\mathcal{F}}_i$ are

$$\begin{aligned}\tilde{\mathcal{F}}_1(\omega,\theta) &= -\frac{6m_\pi^2}{\omega}\mathcal{F}_1(\omega,\theta) \to 1 + \frac{\omega^2}{15m_\pi^2}(3+\cos\theta) + \cdots, \\ \tilde{\mathcal{F}}_2(\omega,\theta) &= -\frac{6m_\pi^2}{\omega}\mathcal{F}_2(\omega,\theta) \to 1 + \frac{2\omega^2}{15m_\pi^2}\cos\theta + \cdots, \\ \tilde{\mathcal{F}}_3(\omega,\theta) &= \frac{90m_\pi^4}{\omega}\mathcal{F}_3(\omega,\theta) \to 1 + \frac{2\omega^2}{7m_\pi^2}\cos\theta + \cdots,\end{aligned} \tag{12}$$

and have been normalized such that $\tilde{\mathcal{F}}_i(\omega \to 0,\theta) \to 1$ and are slowly varying functions of ω.

The parity-violating asymmetry, defined by the difference in cross section for $\eta = +1$ and $\eta = -1$ normalized to the sum, is

$$\begin{aligned}A^{\gamma\gamma}(\omega,\theta) = & \frac{g_A h^{(1)}_{\pi NN} M_N \omega^2}{12\pi^2 f m_\pi^2}\,\frac{1+\cos\theta}{1+\cos^2\theta} \\ & \times \mathrm{Re}\Bigg[\tilde{\mathcal{F}}_1(\omega,\theta)(1+\cos^2\theta) \\ & + \tilde{\mathcal{F}}_2(\omega,\theta)\cos\theta(1-\cos\theta) \\ & - \tilde{\mathcal{F}}_3(\omega,\theta)\frac{\omega^2}{15m_\pi^2}\cos\theta(1-\cos^2\theta)\Bigg].\end{aligned} \tag{13}$$

For a numerical estimate of the magnitude of the asymmetry, we consider forward scattering, $\theta = 0$, where

$$\begin{aligned}A^{\gamma\gamma}(\omega,0) = & \frac{g_A h^{(1)}_{\pi NN} M_N \omega^2}{6\pi^2 f m_\pi^2}\tilde{\mathcal{F}}_1(\omega,0) = 1.5\times 10^{-9}\left(\frac{h^{(1)}_{\pi NN}}{5\times 10^{-7}}\right) \\ & \times \left(\frac{\omega}{20\ \mathrm{MeV}}\right)^2 \tilde{\mathcal{F}}_1(\omega,0).\end{aligned} \tag{14}$$

The asymmetry for 100 MeV photons is $A^{\gamma\gamma}(100,0) \sim 5 \times 10^{-8}$ assuming the naive value for $h^{(1)}_{\pi NN}$, which is comparable to the forward-backward asymmetry expected in $\vec{n} + p \to d + \gamma$.

In conclusion, we have computed the leading contribution to parity-violation in $\gamma\vec{N}$ scattering. It arises from one-loop pion graphs with one insertion of the parity-violating pion-nucleon interaction described by $h^{(1)}_{\pi NN}$, and scales like $1/m_\pi^2$ in the chiral limit. The absence of a $\gamma\gamma Z^0$ interaction means that a measurement of this asymmetry will provide a background free determination of $h^{(1)}_{\pi NN}$, up to corrections suppressed by m_π/Λ_χ and ω/Λ_χ, i.e., $\sim$15% for photon energies below the pion photoproduction threshold. While this asymmetry, along with all other parity-violating asymmetries in the few-nucleon sector, is small $\sim 10^{-8}$, the high intensity photon sources that are currently in operation (such as the FEL at Duke), or may come on-line in the future, provide hope that this asymmetry can be measured.

This work was supported in part by the U.S. Department of Energy under Grants No. DE-FG03-97ER4014 and DOE-ER-40561.

[1] C. A. Barnes, M. M. Lowry, J. M. Davidson, R. E. Marrs, F. B. Moringo, B. Chang, E. G. Adelberger, and H. E. Swanson, Phys. Rev. Lett. **40**, 840 (1978); P. G. Bizetti, T. F. Fazzini, P. R. Maurenzig, A. Perego, G. Poggi, P. Sona, and N. Taccetti, Nuovo Cimento A **29**, 167 (1980); G. Ahrens, W. Harfst, J. R. Kass, E. V. Mason, H. Schrober, G. Steffens, H. Waeffler, P. Bock, and K. Grotz, Nucl. Phys. **A390**, 496 (1982); S. A. Page, H. C. Evans, G. T. Ewan, S. P. Kwan, J. R. Leslie, J. D. MacArthur, W. McLatchie, P. Skensved, S. S. Wang, H. B. Mak, A. B. McDonald, C. A. Barnes, T. K. Alexander, and E. T. H. Clifford, Phys. Rev. C **35**, 1119 (1987); M. Bini, T. F. Fazzini, G. Poggi, and N. Taccetti, Phys. Rev. Lett. **55**, 795 (1985).

[2] B. Desplanques, J. F. Donoghue, and B. R. Holstein, Ann. Phys. (N.Y.) **124**, 449 (1980).

[3] E. G. Adelberger and W. C. Haxton, Annu. Rev. Nucl. Part.

Sci. **35**, 501 (1985).
[4] R. D. C. Miller and B. H. J. McKellar, Phys. Rep. **106**, 169 (1984).
[5] V. M. Dubovik and S. V. Zenkin, Ann. Phys. (N.Y.) **172**, 100 (1986).
[6] E. M. Henley, W-Y. P. Hwang, and L. S. Kisslinger, Phys. Lett. B **367**, 21 (1996); **440**, 449 (1998).
[7] D. B. Kaplan and M. J. Savage, Nucl. Phys. **A556**, 653 (1993).
[8] U-G. Meissner and H. Weigel, Phys. Lett. B **447**, 1 (1999). N. Kaiser and U-G. Meissner, Nucl. Phys. **A489**, 671 (1988); **A499**, 699 (1989).
[9] W. Haeberli and B. R. Holstein, in *Symmetries and Fundamental Interactions in Nuclei*, edited by W. C. Haxton and E. M. Henley (World Scientific, Singapore, 1995), ISBN 981-02-2057-x.
[10] W. M. Snow *et al.*, nucl-ex/9804001.
[11] J. Alberi *et al.*, Can. J. Phys. **66**, 542 (1988).
[12] G. S. Danilov, Phys. Lett. **35B**, 579 (1971); Yad. Fiz. **14**, 788 (1972) [Sov. J. Nucl. Phys. **14**, 443 (1972)].
[13] D. Tadic, Phys. Rev. **174**, 1694 (1968).
[14] B. Desplanques, Nucl. Phys. **A242**, 423 (1975); **A335**, 147 (1980).
[15] B. Desplanques and J. Missimer, Nucl. Phys. **A300**, 286 (1978).
[16] B. H. J. McKellar, Nucl. Phys. **A254**, 349 (1975).
[17] D. B. Kaplan, M. J. Savage, R. P. Springer, and M. W. Wise, Phys. Lett. B **449**, 1 (1999).
[18] The SAMPLE Collaboration, M. Pitt and E. Beise (co-spokespeople), MIT Bates proposal #94−11; B. Mueller *et al.*, Phys. Rev. Lett. **78**, 3824 (1997); D. Spayde *et al.*, nucl-ex/9909010.
[19] M. J. Savage and R. P. Springer, Nucl. Phys. **A644**, 235 (1998); nucl-th/9807014(E); nucl-th/9907069.
[20] I. B. Khriplovich and R. V. Korkin, nucl-th/9904081.
[21] E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991); Lectures given at the *Workshop on Effective Field Theories of the Standard Model*, Dobogoko, Hungary, 1991; in proceedings of Effective Field Theories of the Standard Model, Dobogoko, 1991, pp. 113–137.
[22] V. Bernard, N. Kaiser, and U-G. Meissner, Int. J. Mod. Phys. E **4**, 193 (1995); T. R. Hemmert, B. R. Holstein, and J. Kambor, J. Phys. G **24**, 1831 (1998).
[23] T. R. Hemmert, B. R. Holstein, J. Kambor, and G. Knochlein, Phys. Rev. D **57**, 5746 (1998).
[24] X. Ji, C.-W. Kao, and J. Osborne, hep-ph/9908526.
[25] V. Kumar, J. A. McGovern, and M. C. Birse, hep-ph/9909442.