Effect of in-medium hadron parameter modification on nuclear matter equation of state

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We investigate the effect on the nuclear matter equation of state (EOS) due to modification of meson and nucleon parameters in a nuclear medium as a consequence of the partial restoration of chiral symmetry. To get the EOS, we have used Brueckner-Bethe-Goldstone formalism with Bonn-B potential as a two-body interaction and QCD sum rule and Brown-Rho scaling prescriptions for modification of hadron parameters. We find that the EOS is very much sensitive to the meson parameters. We can fit, with the two-body interaction alone, both the saturation density and the binding energy per nucleon.

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It is becoming more and more clear now that the chiral symmetry of the QCD Lagrangian and its spontaneous breaking [1] play a very important role in determining the structure of low mass hadrons which are comprised of *u*, *d*, and *s* quarks, and instantons play a crucial role in hadron correlators in mediating the spontaneous chiral symmetry breaking [2,3]. Physical confinement of quarks seems to play a lesser role. The spontaneous breaking of the chiral symmetry is signaled by the nonvanishing values in the physical vacuum of the quark and gluon condensates [4–6]. Calculations based on chiral perturbation theory and the QCD sum rule (QSR) indicate that values of these condensates are reduced when the hadrons are put in a medium, hence giving rise to partial restoration of chiral symmetry [7,8]. Thus, there is a lot of interest nowadays to understand the mechanism of partial restoration in the nuclear medium of the chiral symmetry of the QCD Lagrangian [9,10], and to isolate effects arising out of it, as it provides a handle to understand nonperturbative QCD phenomena. The most important consequence of chiral symmetry restoration has been identified as modification of hadron properties in nuclear matter [11,12]. Experimental evidence towards this is believed to be a large excess of e^+e^- pairs observed in the invariant mass region around 400 MeV in the 200*A* GeV central collisions of S on Au and W by CERES [13] and HELIOS [14] groups, respectively. This has been explained [11,15] by arguing that the rho meson mass in nuclear matter is reduced at the densities created in the collision. Theoretical studies towards this were triggered after pointing out by Brown and Rho [16], based on the restoration of scale invariance of QCD, that masses of hadrons would scale in nuclear medium as

$$\frac{m_N^*}{m_N} \approx \frac{m_V^*}{m_V} \approx \frac{m_\sigma^*}{m_\sigma} \approx \frac{f_\pi^*}{f_\pi}, \tag{1}$$

where the density dependent quantities are denoted by asterisks. m_N, m_V, and m_σ denote the masses of nucleon, vector mesons (rho, omega), and sigma mesons, respectively. f_π^* is the in-medium pion decay constant, which is expected to vanish at high density when chiral symmetry is completely restored. Therefore, the masses in Eq. (1) should decrease with increasing density. Quantitative estimations of the dependence of masses of hadrons with respect to density have been made in the quark meson coupling (QMC) model [17] and using the QSR approach [18]. In both of these two approaches the behavior of vector meson mass variation has been approximated by a term linear in density as

$$\frac{m_{\rho,\omega}^*}{m_{\rho,\omega}} = 1 - \alpha\tilde{\rho} = s_1, \tag{2}$$

where $\tilde{\rho} = (\rho/\rho_0)$ with ρ_0 as the nuclear matter saturation density. In both approaches, the value of α has been found to lie around 0.17–0.18.

Given the above scenario of the modification of hadron parameters in a nuclear medium due to nonperturbative QCD effects, we investigate its effect on the equation of state (EOS) of symmetric nuclear matter. In other words, we want to probe if, in the context of a two-body force model, the empirical EOS is enough to constraint the extent of medium modification of the hadron parameters due to partial restoration of chiral symmetry. Symmetric nuclear matter is defined as an infinite uniform system of nucleons with equal neutron and proton densities interacting through the strong force alone. It is characterized by its EOS, that is, the energy per nucleon as a function of the nucleon density. We have empirical information about the EOS through the saturation density ρ_0, the energy per particle, $\mathcal{E}(\rho_0)$, at the saturation density, and the incompressibility

$$K\left(= 9\rho_0^2 \left[\frac{\partial^2[\mathcal{E}(\rho)]}{\partial \rho^2} \right]_{\rho_0} \right),$$

of saturated nuclear matter. Values of these quantities have been found to be $\rho_0 = 0.17 \pm 0.02\,\mathrm{fm}^{-3}$, $\mathcal{E}(\rho_0) = -16 \pm 1$ MeV, and $K = 210 \pm 30$ MeV [19]. There are several formalisms available now to compute the nuclear matter EOS. As we have used Brueckner-Bethe-Goldstone (BBG) formalism for getting the nuclear matter EOS, we describe here briefly the main features of this theory. In the BBG formalism, one first calculates the crucial quantity known as the Brueckner reaction matrix $G(\rho,\omega)$ at a density ρ related to the Fermi momentum k_F as $\rho = 2k_F^3/(3\pi^2)$. The G matrix satisfies the Bethe-Goldstone equation

$$G(\rho,\omega) = v_{NN} + v_{NN} \sum_{k_1 k_2} \frac{|k_1 k_2\rangle Q(k_1,k_2) \langle k_1 k_2|}{\omega - \epsilon(k_1) - \epsilon(k_2) + i\eta} G(\rho,\omega), \tag{3}$$

where v_{NN} is the two-body nucleon-nucleon (NN) interaction potential and ω is energy of the two scattering nucleons. $Q(k_1,k_2)$ in Eq. (3) is the Pauli operator which prevents two nucleons in the intermediate states from scattering into states in the already occupied Fermi sea. The Pauli operator is written as

$$Q(k_1,k_2)=[1-n(k_1)][1-n(k_2)], \tag{4}$$

with $n(k)$ denoting the Fermi distribution function, which at zero temperature is given by the step function $\theta(k-k_F)$. The single particle energy $\epsilon(k)$ in Eq. (4) is written as

$$\epsilon(k)=\frac{\hbar^2k^2}{2M}+U(k), \tag{5}$$

with $U(k)$ as the single particle potential, which is written (in the standard choice) as

$$U(k)=\sum_{k'} n(k')\langle kk'|G(\rho,\omega)|kk'-k'k\rangle. \tag{6}$$

By solving self-consistently Eqs. (3), (5), and (6), for a given NN interaction potential v_{NN}, one determines the G matrix. The G matrix can be considered as an in-medium effective interaction between two nucleons. The renormalization of the bare two-body interaction occurs by the surrounding nucleons through Pauli blocking and the nuclear mean field. The nuclear matter equation of state is then obtained by computing, at a density ρ, the energy per nucleon, $\mathcal{E}(\rho)$, using the following expression:

$$\mathcal{E}(\rho)=\frac{1}{A}\sum_{k\leqslant k_F}\frac{\hbar^2k^2}{2M}+\frac{1}{2A}\times\sum_{k,k'\leqslant k_F}\langle k\,k'|G(\rho,\omega)|k\,k'-k'k\rangle. \tag{7}$$

Over the years, a large number of calculations of the nuclear matter EOS have been done by various groups using both phenomenological and microscopic two-body NN interaction potentials [19–27]. It has been observed that calculations using the nonrelativistic approach and only a two-body interaction potential fail to reproduce the saturation observables. To improve the situation, two paths have been followed, namely, (1) doing a nonrelativistic calculation with two-body and an additional three-body potential with a few adjustable parameters [28–30] or (2) doing relativistic Dirac-Brueckner calculation with two-body potential alone [19,31–33]. Considerable success has been achieved in reproducing the saturation observables in both of these paths. Thus, the debate remains unresolved whether the improvement in reproducing the saturation properties is due to the inclusion of a three-body force or it is a relativistic effect.

We take the paradigm that nucleons interact through exchange of mesons. The parameters of these mesons are modified inside a nuclear medium due to partial restoration of the chiral symmetry of the QCD Lagrangian, which is a nonperturbative effect. As the amount of modification of meson parameters is density dependent, the two-body NN interaction potential inside the nuclear medium, thus, is expected to be density dependent and very much different to that when the nucleons are free. We give this modified two-body density dependent NN interaction potential in the BBG formalism as input to obtain a density dependent G matrix and compute the EOS with this G matrix.

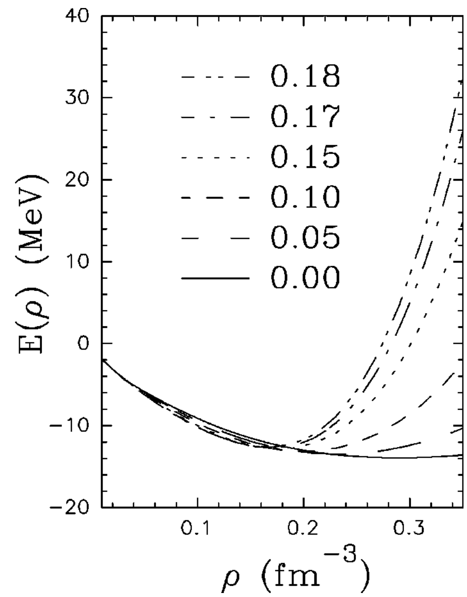

FIG. 1. Dependence of the equation of state for symmetric nuclear matter on the parameter α. Values of α are as shown in the figure ($\beta=1.0$). Calculations have been done with the Bonn-B potential as a two-body interaction.

The NN interaction potential which is based explicitly on meson exchange has been developed by the Bonn group and is known as the Bonn potential [19]. Therefore, this potential will be ideal to investigate the effect of in-medium meson parameter modification on the nuclear matter EOS. The phase shifts for free nucleon-nucleon scattering and deuteron ground state properties have been described very well by this potential. In this potential, exchange of pseudoscalar (0^-) mesons, pion and eta, vector (1^-) mesons, rho and omega, and scalar mesons, (0^+) sigma and delta, have been considered. The meson parameters include masses (m), coupling constants (g), and cutoff masses (Λ) of the form factor at the meson-nucleon interaction vertices. The masses and coupling constants of a meson determine, respectively, the range and strength of the potential contributed by exchange of that meson to the total potential. The cutoff masses are phenomenological parameters in the form factors, which are generally of monopole or dipole form, introduced at the interaction vertices to take care of the off-shell nature of the exchanged mesons and the finite size of the interaction vertices. Representing the interaction vertices as uniform density spheres, the cutoff masses are inversely related to the radius of the sphere for monopole form factors. The form factors, in a way, make the coupling constants dependent on the energy and momentum carried by the mesons at the vertices.

We modify the masses of the vector mesons using QMC and QSR prescriptions given in Eq. (2) and use Brown-Rho scaling law given in Eq. (1) to modify the nucleon mass, masses of other mesons, and the cutoff masses as

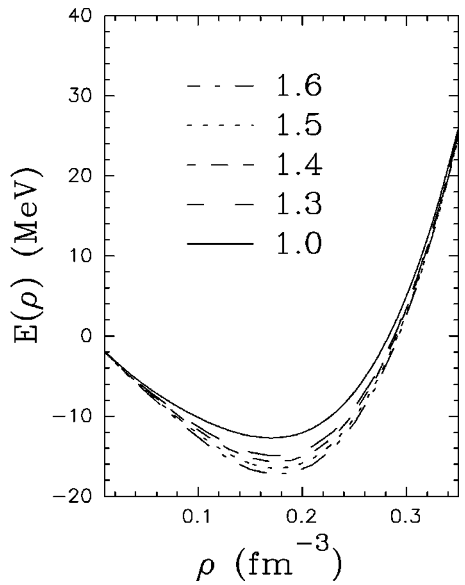

FIG. 2. Dependence of equation of state for symmetric nuclear matter on the parameter β. Values of β are as shown in the figure. Calculations have been done with the Bonn-B potential as a two-body interaction and taking $\alpha=0.17$.

$$\frac{m_N^*}{m_N}=\frac{m_\rho^*}{m_\rho}=\frac{m_\omega^*}{m_\omega}=\frac{m_\sigma^*}{m_\sigma}=\frac{m_\delta^*}{m_\delta}=\frac{\Lambda^*}{\Lambda}=s_1. \tag{8}$$

We have not changed the masses of pseudoscalar mesons, pion and eta, as those are supposed to be Goldstone bosons of spontaneous chiral symmetry breaking and the origin of the masses of these mesons is related to explicit chiral symmetry breaking due to masses of *u*, *d*, and *s* quarks in the QCD Lagrangian. We have modified the cutoff masses (Λ) of all the interaction vertices. Reduction of the cutoff masses inside the nuclear medium means that the spatial extensions of the interaction vertices are increased. Also, this introduces some kind of dependence of coupling constant on the medium density through the form factor, though the coupling constants have not been modified explicitly.

At a given density ρ and value of α, we get modified meson parameters from Eq. (8), which generate a modified *NN* interaction potential with which nucleons are interacting inside a nuclear medium. We then compute the nuclear matter EOS, solving the set of equations (3)–(7) and keeping both α and ρ_0 of Eq. (2) as parameters. The results are shown in Fig. 1 for $\rho_0=0.17\,\mathrm{fm}^{-3}$. The curve for $\alpha=0$ represents the nuclear matter EOS when the meson parameters have not been modified. In this case, the saturation density is about $0.29\,\mathrm{fm}^{-3}$ which is quite away from the empirical value. We see in Fig. 1 that the saturation density decreases with the increase of the value of α, with $\alpha=0.17$ prescribed by QMC and QSR calculations; the saturation density is $0.17\,\mathrm{fm}^{-3}$ which is the empirical saturation density. However, the energy per particle increases with the increase of α. We see the energy per particle from the EOS with $\alpha=0$ to be -13.5 MeV. This value is higher than the empirical value. For $\alpha=0.17$, the energy per particle is -12.72 MeV which is even higher. For other values of ρ_0 in Eq. (2), the trend of the result is similar; i.e., for increasing values of α the minima of the EOS curves shift towards lower values of density; however, for no value of ρ_0 and α could the binding energy be brought down to its empirical value. The feature that the binding energy is greater at higher values of α is due to the increase of the kinetic energy term in Eq. (7) as a consequence of the decrease of the nucleon mass governed by Eq. (2). So we find that with the prescription given in Eq. (8) and with the Bonn-B potential, it is not possible to bring both the saturation density and energy per particle to their respective empirical values.

To solve this problem we take two different medium dependences for the usual masses and cutoff masses. This means that the radii of the interaction vertices do not scale in the same way as the masses do. There are indications of this from the strict limits on the possible variation of the size of the nucleon in nuclear matter [34]. We take the density dependence of the usual masses and cutoff masses as

$$\frac{m_N^*}{m_N}=\frac{m_\rho^*}{m_\rho}=\frac{m_\omega^*}{m_\omega}=\frac{m_\sigma^*}{m_\sigma}=\frac{m_\delta^*}{m_\delta}=s_1,$$

$$\frac{\Lambda^*}{\Lambda}=s_1-e^{-\tilde{\rho}^2}[s_1-s_1^\beta]=s_2. \tag{9}$$

We have chosen the form of s_2 in Eq. (9) with the following considerations. Looking at the form for s_1 in Eq. (2), we find that masses of the hadrons go to zero at density ρ_0/α. It seems that at that density there could be a phase transition from nuclear matter to quark matter. So at that density the radii of hadrons and the spatial extensions of the interaction vertices should be very large, infinite so to say. As Λ is inversely related to the spatial extensions of the interaction vertices, its dependence with respect to the density should be similar to the dependence of masses with respect to the density at the high density domain. We have fixed the form of s_2 at the high density region from the above considerations. At $\rho=0$, the constraint is $\Lambda^*=\Lambda$. With these two constraints and taking only one parameter β, we have chosen the form for s_2 as given in Eq. (9). By construction s_2 should not disturb the EOS curve at the high as well as the low density region; the job it is to do is only to bring down the binding energy at the saturation density to the experimental value. The dependence s_2 is mildly nonlinear below and near the saturation density and takes the same form as s_1 at higher densities. The value of the parameter β and the Gaussian factor in front of the parentheses in Eq. (9) determine the extent of the nonlinearity of s_2. For $\beta=1$, we see from Eq. (9) that s_2 goes over to s_1. The ratio of the in-medium radius of the interaction vertices, $\langle r\rangle^*$, to that in free space, $\langle r\rangle$, is related to the cutoff parameters as $\langle r\rangle^*/\langle r\rangle=\Lambda/\Lambda^*$. For nonlinear parametrization the radius of the interaction vertices increases faster and tends to have a plateau near the saturation density in comparison to the monotonic increase in the case of linear parametrization.

We now use Eqs. (8) and (9) to find the hadron parameters at a given density and values of α and β. With these modified hadron parameters we calculate the interaction potential and then compute once again the EOS solving the set of equations (3)–(7) keeping α, β, and ρ_0 as parameters. We find that the binding energy cannot be brought down to its experimental value except for $\rho_0 = 0.17\,\mathrm{fm}^{-3}$ in Eqs. (2) and (9). The results are shown in Fig. 2 for $\alpha = 0.17$ and various values of β. We see that there is very little effect of β on the saturation density. However, the energy per particle is sensitive to the value of β and reduced with the increase of β. We see that for $\beta = 1.5$ we get -16.5 MeV as the energy per particle and the saturation density has changed to $0.175\,\mathrm{fm}^{-3}$ from $0.17\,\mathrm{fm}^{-3}$. Thus, we find that with the prescription given in Eq. (9) and with the Bonn-B potential, it is possible to bring both the saturation density and energy per particle to their respective empirical values with $\alpha = 0.17$ and $\beta = 1.5$.

For increasing values of α and β, the EOS becomes more and more stiff. We find the incompressibility K of the EOS corresponding to $\alpha = 0.17$ and $\beta = 1.5$ to be 530 MeV in comparison to the incompressibility $K = 194$ MeV of the curve with $\alpha = 0.0$ and $\beta = 1.0$ (in which case the meson parameters have not been modified). We get a value of incompressibility K much higher than the values quoted before, obtained [35] in the framework of nonrelativistic Hartree-Fock random phase apaproximation (RPA) models with effective interaction.

The high value of K in our calculation is because of two reasons. It is due to, first, the low value of the nucleon mass at nonzero density given by Eq. (2), and, to second, the interaction potential becoming more attractive only around the saturation density through Eq. (9) to reproduce the experimental binding energy. So it is not possible in this framework to get a softer EOS. We feel, however, that inclusion of a three-body force may decrease the value of the incompressibility.

At the end, we mention that there are several versions of the Bonn potential and using those in the BBG formalism with the continuous choices for the single particle potential one gets different values of the saturation density and energy per particle. Therefore, it is expected that one would require different set of parameters α and β to bring the saturation density and energy per particle to their respective empirical values for EOS's obtained using those potentials. That will provide some kind of a limit on these parameters. Moreover, one would like to know the effect of meson parameter modification on the EOS obtained using a relativistic Dirac-Brueckner formalism or nonrelativistic BBG formalism with the Bonn potential as a two-body force and microscopic three-body force based on the meson exchange formalism [36]. We are presently investigating these issues.

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