Schematic model for ρ-a_1 mixing at finite density and in-medium effective Lagrangian

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Based on schematic two-level models extended to a_1-meson degrees of freedom, we investigate possible mechanisms of chiral restoration in the vector–axial-vector channels in cold nuclear matter. In the first part of this article we employ the massive Yang-Mills framework to construct an effective chiral Lagrangian based on low-energy mesonic modes at finite density. The latter are identified through nuclear collective excitations of ''meson''-sobar type such as $\pi\leftrightarrow[\Delta(1232)N^{-1}]\equiv\hat{\pi}$, $\rho\leftrightarrow[N^*(1520)N^{-1}]\equiv\hat{\rho}$, etc. In a mean-field type treatment the in-medium gauge coupling $\hat{g}$, the (axial-)vector meson masses and $\hat{f}_\pi$ are found to decrease with density indicating the approach towards chiral restoration in the language of in-medium effective fields. In the second part of our analysis we evaluate the (first) in-medium Weinberg sum rule which relates vector and axial-vector correlators to the pion decay constant. Using in-medium ρ/a_1 spectral functions (computed in the two-level model) also leads to a substantial reduction of the pion decay constant with increasing density.

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I. INTRODUCTION

The density dependence of vector-meson masses encoded in the so-called Brown-Rho (BR) scaling conjecture [1] has stimulated considerable discussion. From the experimental side, the CERES dilepton experiments [2] have provided strong evidence that the properties of ρ mesons are substantially modified in hot and dense matter. The measurements performed at the full SpS energy indicate an excess of dileptons with invariant masses below ~ 0.6 GeV, as well as missing strength in the region around the free ρ-mass. More quantitative results are expected from further runs at both 40*A* and 158*A* GeV with an additional time projection chamber (TPC) improving the mass resolution in order to discriminate the contributions of final state ω decays from the ρ-meson decays within the interacting hadronic fireball.

The simplest and most economical explanation for the observed low-mass dilepton spectra is achieved in terms of quasiparticles (both fermions and bosons) whose masses drop according to BR scaling, thereby making an appealing link to the chiral (quark) structure of the hadronic vacuum. In an alternative view to this description, Rapp, Chanfray, and Wambach (RCW) [3] showed that the excess of low-mass dileptons follows from hadronic many-body calculations using in-medium spectral functions (see, e.g., Ref. [4] for a recent review). On rather general grounds, this ''alternative'' description was in a sense anticipated as discussed by one of the authors [5]. In analogy to the quark-hadron duality in heavy-light meson decay processes, one may view BR scaling as a ''partonic'' picture while RCW as a hadronic one. Loosely speaking, on the finite density axis, the former can be thought of as a top-down approach and the latter as a bottom-up one. The link between BR scaling and the Landau quasiparticle interaction F_1 established in [6] is one specific indication for this ''duality.'' Indeed, in [7], Brown *et al.* argued that the RCW explanation encodes features of a density-dependent ρ-meson mass, calculated in a hadronic basis (in contrast to that of constituent quarks used by Brown and Rho). In particular it was suggested [7] that if one replaces the ρ-meson mass m_ρ by the mass $m_\rho^*(\rho)$,[1] at the density being considered, one would arrive at a description, in hadron language, which at high densities appeared dual to that of the Brown-Rho one in terms of constituent quarks. These developments involved the interpretation of a *collective* isobar-hole excitation as an effective vector-meson field operating on the nuclear ground state, i.e.,

$$\frac{1}{\sqrt{A}}\sum_i\,[N^*(1520)_i N_i^{-1}]^{1-}$$
$$\simeq\sum_i\,[\rho(x_i)\quad\text{or}\quad\omega(x_i)]|\Psi_0\rangle_s,\qquad(1)$$

with the antisymmetric (symmetric) sum over neutrons and protons giving a ρ-like (ω-like) nuclear excitation. The dropping vector-meson masses could then be estimated in terms of the mixing of this collective state with the elementary vector meson state [7]. In [8], the authors studied mixing of vector and axial-vector mesons at finite temperature.

In the present work, we will expand on these ideas by constructing an effective chiral Lagrangian involving effective fields for ''meson sobars'' ($\hat{\pi},\hat{\rho},\ldots$) which in a dense medium are assumed to be the relevant, lowest-lying degrees of freedom in terms of the nuclear collective states in the corresponding meson channels. Therefore we will assume that each meson field has its ''sobar'' partner,[2] that is, $\pi\leftrightarrow[\Delta(1232)N^{-1}]\equiv\hat{\pi}$, $\rho\leftrightarrow[N^*(1520)N^{-1}]\equiv\hat{\rho}$, and

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[1]We will discuss the reasoning behind this suggestion in Sec. II.

[2]In general, the sobar field would be a linear combination of N^*-hole states of the appropriate quantum numbers, but here, for simplicity, we are taking only what we consider to be the dominant component.

$a_1 \leftrightarrow [N^*(1900)N^{-1}] \equiv \hat{a}_1$. We then construct a chiral effective Lagrangian for meson-sobar fields following the procedure of the massive Yang-Mills approach [9]. An explicit a_1 meson is not necessarily required to formulate a chiral invariant Lagrangian involving ρ mesons, as is well known, e.g., from the hidden local symmetry framework [10]; in our framework, however, it provides a convenient treatment of the associated low-lying mode on an equal footing with the ρ meson, thereby facilitating the discussion of chiral restoration in the vector–axial-vector doublet. Moreover, in the mean-field analysis carried out below we find that at some density our meson-sobar fields could be described in terms of interpolating fields that are the effective fields figuring in an in-medium Lagrangian exhibiting BR scaling. One of the main differences between the fields of BR scaling and meson-sobar fields is that while in the former the full pole strength ($Z \sim 1$) is retained by the low-lying mode, the sobar fields only carry a fraction of the strength in the respective meson channel (typically $Z \sim 0.3$). It will be suggested that this discrepancy can be resolved by applying a (finite) wave-function renormalization to the sobar fields.

Our article is organized as follows: in Sec. II we review the schematic model [7] and extend it by including a_1 meson. The corresponding meson-sobar chiral Lagrangian and the ensuing finite-density results in mean-field approximation are presented in Sec. III. Using the same schematic model from Sec. II, but following the philosophy of the many-body spectral function approach [3,11], we compute in Sec. IV the density dependence of the pion decay constant employing the in-medium Weinberg sum rules. Section V contains a summary and concluding remarks.

II. SCHEMATIC MODEL

Let us first briefly review the main features of the schematic model for the in-medium ρ meson as used in [7] and then extend it to the a_1 channel.

The starting point is the ρ-meson propagator in nuclear matter given by

$$D_\rho(q_0,\vec{q}) = \frac{1}{q_0^2 - \vec{q}^2 - m_\rho^2 + i m_\rho \Gamma_{\pi\pi}(M) - \Sigma_{\rho N^* N}(q_0,q)}, \tag{2}$$

where $\Gamma_{\pi\pi}$ denotes the vacuum ρ decay width, and the real part of the self-energy has been absorbed into the free (physical) mass m_ρ. The entire density dependence resides in the in-medium ρ self-energy $\Sigma_{\rho N^* N}$ induced by $N^*(1520)N^{-1}$ excitations. It is calculated from the interaction Lagrangian

$$\mathcal{L}_{\rho N N^*(1520)} = \frac{f_\rho}{m_\rho} \psi_{N^*}^\dagger (q_0 \vec{S} \cdot \vec{\rho}_a - \rho_a^0 \vec{S} \cdot \vec{q}) \tau_a \psi_N + \text{H.c.}, \tag{3}$$

where the coupling constant can be estimated from the measured branching of the $N^*(1520) \rightarrow \rho N$ decay (as well as information on the radiative decay [13]). In what follows we will for simplicity concentrate on the limit of vanishing three-momentum where the longitudinal and transverse polarization components become identical. Because of the rather high excitation energy of $\Delta E = M_{N^*} - M_N = 580$ MeV, one can safely neglect nuclear Fermi motion to obtain

$$\Sigma_{\rho N^* N}(q_0) = \frac{8}{3} f_{\rho N^* N}^2 \frac{q_0^2}{m_\rho^2} \frac{\rho_N}{4} \left(\frac{2(\Delta E)}{(q_0 + i\Gamma_{tot}/2)^2 - (\Delta E)^2} \right) \tag{4}$$

(ρ_N is the nucleon density). If the widths of the ρ and $N^*(1520)$ are sufficiently small, one can invoke the mean-field approximation (as employed in Sec. III) and determine the quasiparticle excitation energies from the zeros in the real part of the inverse propagator. In particular, for $\vec{q}=0$ the in-medium ρ mass is obtained by solving the dispersion relation

$$q_0^2 = m_\rho^2 + \text{Re}\, \Sigma_{\rho N^* N}(q_0). \tag{5}$$

The pertinent spectral weights of the solutions are characterized by Z factors defined through

$$Z = \left(1 - \frac{\partial}{\partial q_0^2} \text{Re}\, \Sigma_{\rho N^* N} \right)^{-1}. \tag{6}$$

Within a chirally invariant framework, we need to include the chiral partners of the ρ and its nuclear excitation, i.e., the a_1 and a suitable N^* resonance with spin-3/2 and positive parity. A possible candidate is the $N^*(1900)$ state, and the interaction Lagrangian is taken in analogy to (3) as[3]

$$\mathcal{L}_{a_1 NN(1900)} = \frac{f_{a_1}}{m_{a_1}} \psi_{N^*}^\dagger (q_0 \vec{S} \cdot \vec{A}_a - A_a^0 \vec{S} \cdot \vec{q}) t_a \psi_N + \text{H.c.} \tag{7}$$

with A denoting the axial-vector a_1 field. The coupling constant f_{a_1} can in principle be estimated from the partial decay width $\Gamma_{N^*(1900) \rightarrow a_1 N}$. Although the corresponding three-pion final state has not been explicitly measured, the observed [14,15] one- and two-pion final states leave room for up to 30% branching of the total $N^*(1900)$ width of ~ 500 MeV into the $a_1 N$ channel. Using the baryon decay width formula from Ref. [4],

[3]We could also consider coupling terms of the type $a_1 N^*(1520)N$ as well as $\rho N^*(1900)N$, which in their relativistic version involve an additional γ_5 as compared to the ones used here; in the nonrelativistic reduction this leads to self-energies of P-wave nature, being proportional to $\vec{q}$, and can therefore be neglected in the zero-momentum limit considered here.

$$\Gamma_{N(1900)\to a_1 N}(\sqrt{s}) = \frac{f_{a_1}^2}{4\pi m_{a_1}^2}\frac{2m_N}{\sqrt{s}}\frac{2I_a+1}{(2J_B+1)(2I_B+1)} SI(a_1 NN^*) \times \int_{3m_\pi}^{\sqrt{s}-m_N}\frac{M}{\pi} dM A_a^0(M) q_{cm} F_{ANB}(q_{cm}^2)^2 (M^2+2q_0^2), \tag{8}$$

with a standard monopole form factor $F_{a_1NB}(q_{cm}^2) = \Lambda_{a_1}^2/(\Lambda_{a_1}^2+q_{cm}^2)$ ($\Lambda_{a_1}=600$ MeV as for the ρ), we find $f_{a_1}\simeq 25.3$ for 20% and $f_{a_1}\simeq 17.8$ for 10% branching ratio. This range of values will be used to indicate the inherent uncertainties in our numerical results presented below.

Before we address the construction of the full chiral Lagrangian in the next section, we compute the density dependence of the masses and Z factors corresponding to the low-lying ρ- and a_1-sobar states when self-consistently solving the dispersion relation (5). Using the self-energy as given in Eq. (4) (and the analogous expression for the a_1 sobar) results are depicted by the dashed lines in Fig. 1 (corresponding to the RCW approach in Ref. [7]). When neglecting the baryon resonance widths, one finds a moderate simultaneous decrease of both $\hat{m}_\rho$ and $\hat{m}_{a_1}$ associated with pole strengths of 10–20 %. Upon inclusion of resonance widths $\Gamma_{tot}=\Gamma_0+\Gamma_{med}$ (with vacuum values $\Gamma_0=120$ [500] MeV for $N^*(1520)[N^*(1900)]$ and medium corrections $\Gamma_{med}\simeq 300\rho_N/\rho_0$ MeV as inferred in Refs. [12,13]) the collectivity is suppressed and little density dependence is observed. However, since large widths also imply that the quasiparticle (mean-field) approximation becomes less reliable, it would be premature to conclude from the behavior of the masses alone that there is no approach towards chiral restoration. We will come back to this issue in Sec. IV.

The situation quantitatively changes if one adopts the suggestion put forward in Ref. [7] to replace in the self-energy expressions the $1/m_\rho^2[1/m_{a_1}^2]$ factor by the (self-consistent) in-medium mass $1/(m_\rho^*)^2[1/(m_{a_1}^*)^2]$ (BR approach; solid lines in Fig. 1).[4] The dispersion relation then takes the form

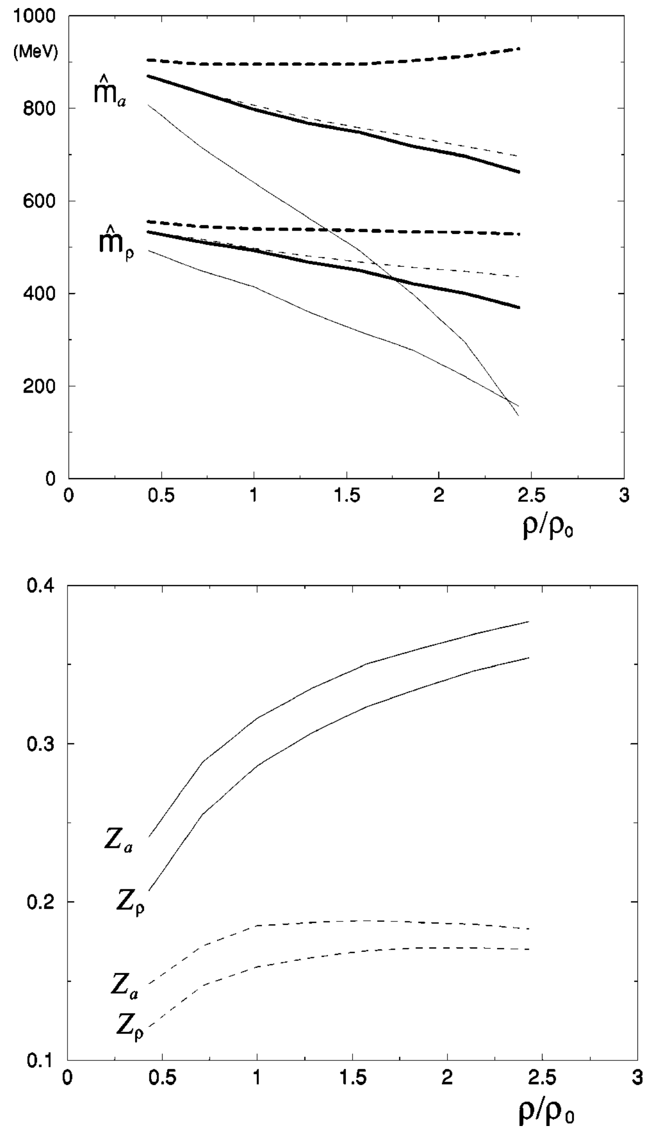

FIG. 1. Upper: In medium rhosobar and a_1-sobar masses in the RCW (dashed lines) and BR (solid lines) approach. Thick lines represent in-medium masses with $\Gamma_{tot}=\Gamma_{tot}(\rho)$ and thin lines are for $\Gamma_{tot}=0$. Lower: Z factors for rhosobar (Z_ρ) and a_1-sobar (Z_a) in the RCW (dashed lines) and BR (solid lines) approach for $\Gamma_{tot}=0$.

$$q_0^2 = m_\rho^2 + f_{\rho N^* N}^2 \frac{4}{3}\rho_N \frac{\Delta E\left[q_0^2-(\Delta E)^2-\frac{1}{4}\Gamma_{tot}^2\right]}{\left[q_0^2-(\Delta E)^2-\frac{1}{4}\Gamma_{tot}^2\right]^2+\Gamma_{tot}^2 q_0^2}. \tag{9}$$

Here, we shall assume that in the medium ΔE remains unchanged (which can be argued to hold to the leading order in $1/N_c$) while the width Γ_{tot} may be affected by density (as before). With $f_{a_1}\simeq 17.8$ (corresponding to 10% branching ratio) the mean-field results ($\Gamma_{tot}=0$) lead to chiral symmetry restoration around $\rho = 2.4\rho_0$; again, inclusion of finite widths decelerates the drop in masses in this schematic treatment (in the more extreme case with $f_{a_1}\simeq 25.3$ the in-

[4]We have no compelling argument for the validity of this procedure. It is based on the following conjecture, the proof of which would require going beyond the framework we are developing here. The free ρN^*N coupling is of the form $(f/m)q_0$ with a dimensionless constant f and q_0 the fourth component of the the ρ meson momentum. Writing this as Fq_0 with $F=f/m$ the medium renormalization of the constant F will certainly depend on density ρ. In order for the vector meson mass to go to zero at some high density so as to match BR scaling, it is required that $F(\rho)q_0\to\text{const}\neq 0$. For $q=|\vec{q}|\approx 0$, which we are considering, this can be satisfied if $F(\rho)\sim m^{*-1}$, modulo an overall constant. This is the essence of the proposal of Ref. [7].

medium a_1-meson mass drops to zero around $\rho\sim 1.2\rho_0$ for $\Gamma_{tot}=0$). The ''BR approach'' in evaluating the ρ- and a_1-sobar self-energies is not based on rigorous arguments, but it successfully reproduces BR scaling leading to chiral restoration through $\hat{m}_\rho, \hat{m}_{a_1}$ merging together close to zero somewhat below 3 times the nuclear matter density.

III. EFFECTIVE LAGRANGIAN FOR MESON SOBARS

A. Mean-field model and in-medium pion decay constant

Our subsequent analysis in this section will be based on the following two main assumptions: (i) each meson field has its ''sobar'' partner, i.e., $\pi\leftrightarrow[\Delta(1232)N^{-1}]\equiv\hat{\pi}$, $\rho\leftrightarrow[N^*(1520)N^{-1}]\equiv\hat{\rho}$, and $a_1\leftrightarrow[N(1900)N^{-1}]\equiv\hat{a}_1$, and (ii) chiral symmetry persists in the meson-sobar subspace. Before spelling out the chiral Lagrangian, let us elaborate on the pertinence of these propositions.[5] In the context of our two-level schematic model in Sec. II, a meson Green's function $\langle\phi\phi^+\rangle$ can develop a new pole (i.e., excitation) $\langle\hat{\phi}\hat{\phi}^+\rangle$ with associated pole strength Z, due to the effects of a many-particle medium; at the same time, the original (''elementary'') excitation $\langle\phi_e\phi_e^+\rangle$, will be shifted from its original pole position and carry the residue $1-Z<1$. There appears no mixed Green's function $\langle\phi_e\hat{\phi}^+ + \text{H.c.}\rangle$ since the mixing matrix of the nuclear collective state (resonance-hole state $\approx\hat{\phi}$) and the elementary meson ($\approx\phi_e$) is diagonalized. This implies that

$$\mathcal{L}(\phi)\approx\mathcal{L}(\phi_e)+\mathcal{L}(\hat{\phi})+\mathcal{L}(\hat{\phi},\phi_e)\approx\mathcal{L}(\phi_e)+\mathcal{L}(\hat{\phi}) \tag{10}$$

as far as kinetic and mass terms are concerned; i.e., if the original Lagrangian $L(\phi)$ possesses a (linearly realized) chiral symmetry, then the noninteracting part of $\mathcal{L}(\hat{\phi})$ preserves it. For the interaction parts, as well as for a nonlinear realization of chiral symmetry, it is not so clear how to demonstrate the existence of chiral symmetry in our meson-sobar Lagrangian using the arguments given above.

Employing the massive Yang-Mills (MYM) framework [9], we can write a chiral effective Lagrangian with $SU(2)_L\times SU(2)_R$ symmetry for meson-sobar fields with minimal couplings,

$$\mathcal{L}=\frac{1}{4}\hat{f}_\pi^2\,\mathrm{Tr}(D_\mu\hat{U}D^\mu\hat{U}^\dagger)+\frac{1}{4}\hat{f}_\pi^2\mathrm{Tr}[M(\hat{U}+\hat{U}^\dagger-2)]$$
$$-\frac{1}{2}\mathrm{Tr}(\hat{F}_{\mu\nu L}\hat{F}_L^{\mu\nu}+\hat{F}_{\mu\nu R}\hat{F}_R^{\mu\nu})+\hat{m}_0^2(\hat{A}_{\mu L}\hat{A}_L^\mu+\hat{A}_{\mu R}\hat{A}_R^\mu), \tag{11}$$

where

$$\hat{U}=e^{\frac{2i\hat{\pi}}{\hat{f}_\pi}},\quad \hat{\pi}=\hat{\pi}^a\frac{\tau^a}{2},$$
$$\hat{F}_{\mu\nu}^{L,R}=\partial_\mu\hat{A}_\nu^{L,R}-\partial_\nu\hat{A}_\mu^{L,R},\quad \hat{A}_\mu^{L,R}=\hat{V}_\mu\pm\hat{A}_\mu,$$
$$D_\mu\hat{U}=\partial_\mu\hat{U}-igA_{\mu L}\hat{U}+igA_{\mu R}\hat{U},$$
$$M=m_\pi^2\mathbf{1}. \tag{12}$$

Note that we have the same mass matrix M as appearing in Ref. [9]. It remains to fix the free parameters in Eq. (12) in terms of f_π, g and the masses of the original MYM Lagrangian [9].

Following the procedure in [9], the physical mass difference between $\hat{a}_1$ and $\hat{\rho}$ is obtained after diagonalizing the quadratic piece of the Lagrangian (11) to remove the spurious $A_\mu\partial^\mu\pi$ mixing term,[6]

$$\hat{m}_{a_1}^2-\hat{m}_\rho^2=\frac{\hat{g}^2\hat{f}_\pi^2}{2}\frac{\hat{m}_{a_1}^2}{\hat{m}_\rho^2}, \tag{13}$$

where $\hat{m}_\rho=\Delta E=1520-940=580$ MeV and $\hat{m}_{a_1}=1900-940=960$ MeV. Rewriting Eq. (13) as

$$\hat{f}_\pi^2=\frac{2}{\hat{g}^2}\frac{\hat{m}_\rho^2}{\hat{m}_{a_1}^2}(\hat{m}_{a_1}^2-\hat{m}_\rho^2), \tag{14}$$

we notice that knowledge of the (in-medium) values of $\hat{g}$, $\hat{m}_\rho$, and $\hat{m}_{a_1}$ allows us to infer the density dependence of $\hat{f}_\pi$.

The masses will be taken from the lower nuclear branches of the schematic two-level model discussed in Sec. II. Note that the additional meson interaction vertices do not induce further medium dependences at zero temperature. To estimate the value of the gauge coupling constant $\hat{g}$, we use the relation $\hat{g}_{\rho\pi\pi}=(3/4\sqrt{2})\hat{g}$ [9], and assess $\hat{g}_{\rho\pi\pi}$ by evaluating the diagram shown in Fig. 2. With the standard phenomenological interaction Lagrangians

$$\mathcal{L}_{\pi N\Delta}=\frac{f_{\pi N\Delta}}{m_\pi}\overline{\psi}_{\Delta,\mu}\vec{T}\psi_N\cdot\partial^\mu\vec{\pi}+\text{H.c.},$$

[5] To see how to identify, e.g., the $\hat{\pi}$ field, consider the equation of motion for pions with $\mathcal{L}_{\pi N\Delta}$ in Eq. (15), $(\partial_\mu\partial^\mu-m_\pi^2)\vec{\pi}=(f_{\pi N\Delta}^*/m_\pi)\partial^\mu(\overline{\psi}_{\Delta,\mu}\vec{T}\psi_N)+\text{H.c.}$ with $\psi_{\Delta,\mu}$ denoting the Δ field. Taking the expectation value and ignoring the kinetic term, we have $\langle\vec{\pi}\rangle=(1/m_\pi^2)\langle(f_{\pi N\Delta}^*/m_\pi)\partial^\mu(\overline{\psi}_{\Delta,\mu}\vec{T}\psi_N)+\text{H.c.}\rangle$. Thus the π-sobar field can be identified as

$$\hat{\vec{\pi}}=\frac{1}{m_\pi^2}\left\langle\frac{f_{\pi N\Delta}^*}{m_\pi}\partial^\mu(\overline{\psi}_{\Delta,\mu}\vec{T}\psi_N)+\text{H.c.}\right\rangle.$$

[6] In the following our parameters and fields will always correspond to the physical ones, denoted by $\tilde{f}_\pi$, $\tilde{\pi}$, and $\tilde{A}_\mu$ in Ref. [9].

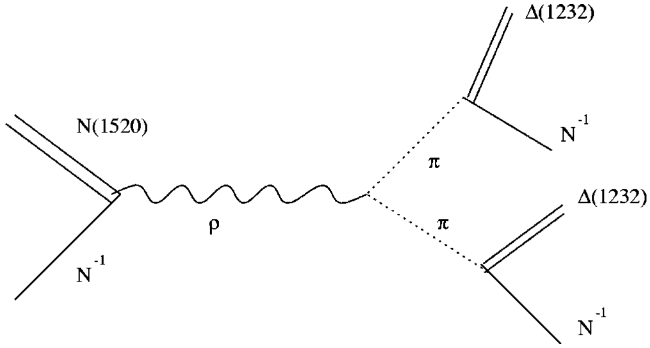

FIG. 2. The diagram for the decay process $\hat{\rho} \to \hat{\pi}\hat{\pi}$ of the low-lying sobar modes determining the effective decay constant $\hat{g}_{\rho\pi\pi}$.

$$\mathcal{L}_{\rho NN*} = \frac{f_{\rho NN*}}{m_\rho} \overline{\psi}_{N*,\nu} \gamma_\mu \vec{\tau} \psi_N \cdot (\partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu) + \text{H.c.},$$

$$\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi} \vec{\rho}_\mu \cdot (\vec{\pi} \times \partial_\mu \vec{\pi}), \tag{15}$$

we obtain in the ρ-meson rest frame, where $q^\mu = (\Delta E, \vec{0})$,

$$\mathcal{L}_{\hat{\rho}\hat{\pi}\hat{\pi}} = \hat{g}_{\rho\pi\pi} \hat{\vec{\rho}}_i \cdot \partial^i \hat{\vec{\pi}} \times \hat{\vec{\pi}}, \tag{16}$$

with[7]

$$\hat{g}_{\rho\pi\pi} = g_{\rho\pi\pi} \frac{1}{\Delta E^2 - m_\rho^2} \left(\frac{1}{(\Delta E/2)^2 - \vec{k}^2 - m_\pi^2} \right)^2 m_\rho^2 m_\pi^4 \tag{17}$$

and ∂^i corresponding to the spatial components.[8] A complication arises from the fact that $\hat{g}_{\rho\pi\pi}$ depends on the momentum of the pions and, consequently, so will $\hat{f}_\pi$. Even worse, as written in Eq. (17), $\hat{g}_{\rho\pi\pi}$ diverges for a single-pion energy of $\omega_k = \Delta E/2$, which essentially coincides with typical π-sobar energies. The latter problem can be remedied by accounting for (in-medium) widths for both pion and rho propagators. Within the isobar-hole model the former takes the standard form

$$D_\pi(\omega = \Delta E/2, \vec{k}) = \frac{1}{(\Delta E/2)^2 - m_\pi^2 - \vec{k}^2 [1 + \chi_0/(1 - g' \chi_0)]}, \tag{18}$$

where the pion susceptibility χ is given by [16]

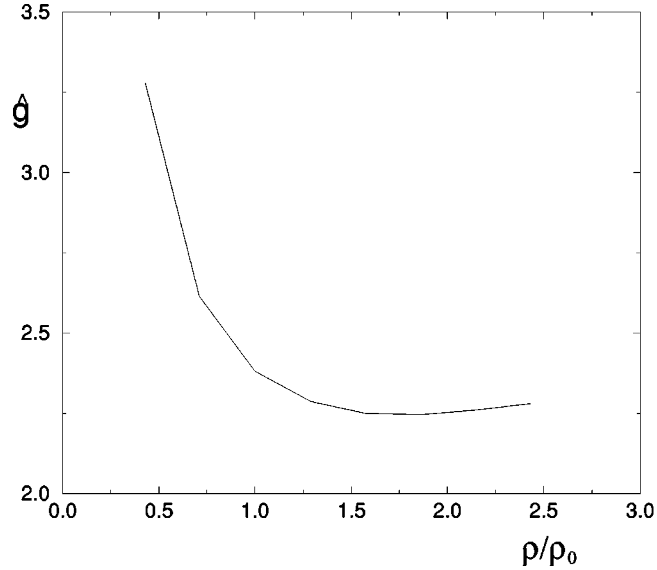

FIG. 3. Density dependence of $\hat{g}$.

$$\chi(\omega, \vec{k}) \simeq \frac{8}{9} \left(\frac{f_\pi^*}{m_\pi} \right)^2 \frac{\Delta E_\pi + \vec{k}^2/2m_\Delta}{\left(\omega + i \frac{1}{2} \Gamma_\Delta \right)^2 - (\Delta E_\pi + \vec{k}^2/2m_\Delta)^2} F_\pi(\vec{k}) \rho_N, \tag{19}$$

with $\Delta E_\pi = M_\Delta - M_N \simeq 300$ MeV and a hadronic form factor F_π. For the ρ meson, we have

$$D_\rho = \frac{1}{(\Delta E)^2 - m_\rho^2 - \Sigma_{\rho N*N}(\Delta E) + i m_\rho \Gamma_\rho}, \tag{20}$$

as before. At a given density, we then average the momentum dependence of $\hat{g}$ over an appropriate range according to $\hat{g} = \int_0^{k_c} g(k) dk/k_c$ with $k_c \simeq 700$ MeV. The final result for the density dependence of $\hat{g}$ is displayed in Fig. 3 where we have neglected any medium dependences of m_ρ, ΔE, m_π, ΔE_π, and the decay widths. We find that $\hat{g}_{\rho\pi\pi}$ initially decreases quickly with density and stabilizes beyond nuclear saturation density.

We are now in position to assemble the main result of this section. Evaluating Eq. (14) with the additional input of the density-dependent $\hat{\rho}$ and $\hat{a}_1$ masses as found in Sec. III (for the $\Gamma_{tot} = 0$ case) results in an in-medium pion decay constant $\hat{f}_\pi(\rho)$ as shown in Fig. 4. As anticipated from the upper panel of Fig. 1, the BR prescription leads to a vanishing value at about $2.5\rho_0$, whereas in the RCW prescription comparatively little density dependence is observed. However, as mentioned before, the latter might still encode an approach towards chiral restoration, albeit in a somewhat different fashion which cannot be deduced from the behavior of the quasiparticle masses. This issue will be addressed in Sec. IV.

[7]Within the spirit of the mean-field description pursued here we neglect in our estimate further vertex corrections of, e.g., the $\rho\pi\pi$ coupling which are in principal necessary to ensure gauge invariance of the vector current.

[8]Since Lorentz invariance is broken in the medium, we will encounter noncovariant terms in our effective Lagrangian. Our $\hat{g}$ and $\hat{f}_\pi$ may thus be considered as the space components of the in-medium gauge coupling and pion decay constant.

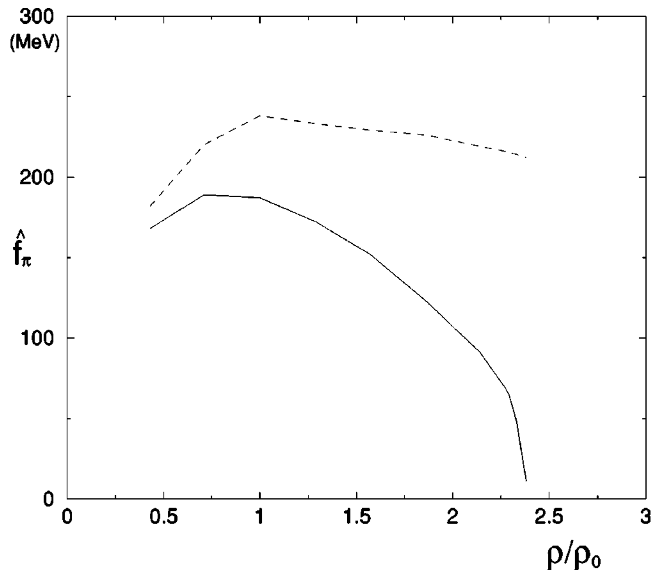

FIG. 4. In-medium $\hat{f}_\pi$ in the RCW (dashed lines) and BR (solid lines) framework.

B. Relation to BR scaling

Let us now comment on the (plausible) connection between our sobar field and effective fields of BR scaling. As was shown in Sec. II, the pole strength (Z factor) of the sobar fields is less than unity while the effective fields of BR scaling carry full pole strength. This discrepancy can be (partly) reconciled by a finite wave-function renormalization (rescaling) of the sobar fields [which somewhat resembles the (infinite) wave-function renormalization of free fields]. Consider, e.g., the $\hat{\pi}$ field. Expanding $\hat{U}$ in terms of pisobar fields, one has

$$\mathcal{L}_{\hat{\pi}}=\tfrac{1}{2}\partial_\mu\hat{\pi}\partial^\mu\hat{\pi}-\tfrac{1}{2}m_\pi^2\hat{\pi}^2+c_1 1/\hat{f}_\pi^2(\partial_\mu\hat{\pi})(\partial^\mu\hat{\pi})\hat{\pi}\hat{\pi}$$
$$+c_2 m_\pi^2/\hat{f}_\pi^2\hat{\pi}^4+\cdots, \tag{21}$$

with coefficients c_1 and c_2. Since the corresponding two-point function (propagator) is given by

$$\langle\hat{\pi}\hat{\pi}^\dagger\rangle\approx\frac{Z_{\hat{\pi}}}{k^2-m_\pi^2}, \tag{22}$$

we rescale the field according to $\hat{\pi}=Z_{\hat{\pi}}^{1/2}\pi^*$, so as to obtain a residue of 1 at $k^2=m_\pi^2$. This leads to

$$\mathcal{L}_{\hat{\pi}}=\frac{1}{2}Z_{\hat{\pi}}\partial_\mu\pi^*\partial^\mu\pi^*+c_1\frac{1}{\hat{f}_\pi^2}Z_{\hat{\pi}}^2(\partial_\mu\pi^*)(\partial^\mu\pi^*)\pi^*\pi^*$$
$$-\frac{1}{2}Z_{\hat{\pi}}m_\pi^2\pi^{*2}+c_2Z_{\hat{\pi}}^2\frac{m_\pi^2}{\hat{f}_\pi^2}\pi^{*4}+\cdots. \tag{23}$$

A redefinition of the coupling constant and mass,

$$m_\pi^*=m_\pi Z_{\hat{\pi}}^{1/2},\quad f_\pi^*=\hat{f}_\pi Z_{\hat{\pi}}^{-1/2}, \tag{24}$$

then gives

$$\mathcal{L}_{\pi^*}=\frac{1}{2}Z_{\hat{\pi}}\partial_\mu\pi^*\partial^\mu\pi^*+c_1Z_{\hat{\pi}}\frac{1}{f_\pi^{*2}}(\partial_\mu\pi^*)(\partial^\mu\pi^*)$$
$$\times\pi^*\pi^*-\frac{1}{2}m_\pi^{*2}\pi^{*2}+c_2\frac{m_\pi^{*2}}{f_\pi^{*2}}\pi^{*4}+\cdots. \tag{25}$$

A proper normalization of the kinetic term can be recovered by introducing a scale transformation

$$x\to x'=Z_{\hat{\pi}}^{-1/2}x,$$

so that we obtain

$$\mathcal{L}_{\pi^*}=\frac{1}{2}\partial_\mu\pi^*\partial^\mu\pi^*+c_1\frac{1}{f_\pi^{*2}}(\partial_\mu\pi^*)(\partial^\mu\pi^*)$$
$$\times\pi^*\pi^*-\frac{1}{2}m_\pi^{*2}\pi^{*2}+c_2\frac{m_\pi^{*2}}{f_\pi^{*2}}\pi^{*4}+\mathcal{L}_{sb}+\cdots. \tag{26}$$

However, since our sobar Lagrangian cannot be expected to be fully scale invariant (as opposed to a Lagrangian exhibiting BR scaling [1]), there appear both terms corresponding to those of BR scaling as well as stemming from broken scale invariance (denoted by $\mathcal{L}_{sb}$). Carrying out the same procedure for vector mesons, we find

$$m_\rho^*=Z_\rho^{1/2}\hat{m}_\rho,\quad m_a^*=Z_a^{1/2}\hat{m}_a,$$
$$g_{\rho\pi\pi}^*=\hat{g}_{\rho\pi\pi}Z_\rho^{1/2}Z_{\hat{\pi}}^{1/2}. \tag{27}$$

Using Eqs. (24) and (27) and neglecting L_{sb}, Eq. (14) finally becomes

$$f_\pi^{*2}\approx\frac{2}{g^{*2}}\frac{m_\rho^{*2}}{m_a^{*2}}(m_a^{*2}-m_\rho^{*2}), \tag{28}$$

where we have approximated $Z_{a_1}\approx Z_\rho$ (cf. lower panel of Fig. 1). Note that $m_\pi^*=m_{\hat{\pi}}Z_{\hat{\pi}}^{1/2}$ does not imply large changes for the in-medium pion mass, since $Z_{\hat{\pi}}$ does not vary appreciably with density (see, e.g., Ref. [17]).

These observations indicate that the effective fields of BR scaling could be identified with a coherent linear combination of the resonance-nucleon-hole degrees of freedom at some relevant density. Similar arguments have been presented in Ref. [18] where the authors identified the sigma field in the Walecka model with the degrees of freedom associated with the $\Delta(1232)$ in the context of quantum hadrodynamics. However, we repeat that—as opposed to a Lagrangian leading to BR scaling [1]—our effective meson-sobar Lagrangian is not invariant under scale transformation symmetry. A possible extension to restore scale invariance might be the introduction of appropriate dilaton fields.

IV. IN-MEDIUM WEINBERG SUM RULES

In this section we will pursue a somewhat different strategy to assess the degree of chiral restoration realized within the schematic model of Sec. II. Rather than imposing the mean-field approximation, we here account for the full spectral shape of both ρ and a_1 spectral functions. A rather direct way of doing so is provided by the Weinberg sum rules [19], one of which relates the pion decay constant (being one of the order parameters of chiral restoration) to the (integrated) difference in the spectral shapes between the vector (ρ_V) and axialvector (ρ_A) correlators, i.e.,

$$f_\pi^2 = \int_0^\infty \frac{ds}{s} [\rho_V(s) - \rho_A(s)]. \tag{29}$$

A second Weinberg sum rule applies to the next higher moment of the correlator difference according to

$$0 = \int_0^\infty ds [\rho_V(s) - \rho_A(s)]. \tag{30}$$

Whereas the first one, Eq. (29), is firmly established in its given form with no known corrections, the second one, Eq. (30), is likely to be modified away from the chiral limit [20] and due to $U_A(1)$ symmetry breaking [21]. From a practical point of view, it is clear that within the scope of our low-energy effective model, the first sum rule is the more relevant one.

In the low-mass region the correlators can be accurately saturated within the (axial-)vector dominance model (VDM) by the spectral function of the lowest-lying resonances ρ and a_1 according to

$$\rho_{V,A}(q_0,q) = -\frac{(m_{V,A}^{(0)})^4}{\pi g_{V,A}^2} \,\mathrm{Im}\, D_{V,A}(q_0,q), \tag{31}$$

with $m_{V,A}^{(0)}$ denoting the bare pole masses and $g_{V,A}$ the (axial-)vector coupling constants. However, the ρ and a_1 spectral functions account for two- and three-pion states only. Therefore, at higher masses, the contributions from $2n$- and $(2n+1)$-pion states (with $n>1$) have to be included. This can be done by an appropriate continuum ansatz characterized by an onset threshold and (asymptotic) plateau value governed by perturbative QCD, e.g., [22],

$$\rho_{V,A}^{cont}(s) = \frac{s}{8\pi^2} \frac{1}{1+\exp[(E_{V,A}^{thr} - \sqrt{s})/\delta_{V,A}]} \times \left(1 + \frac{0.22}{\ln[1+\sqrt{s}/(0.2\mathrm{GeV})]}\right), \tag{32}$$

with $\delta_V = \delta_A = 0.2$ GeV and $E_V^{thr} \simeq 1.3$ GeV, $E_A^{thr} \simeq 1.45$ GeV. The merging of the continuum plateaus into their perturbative form is crucial to ensure the convergence of the sum rules (one-gluon exchanges do not distinguish between vector and axial-vector channels).

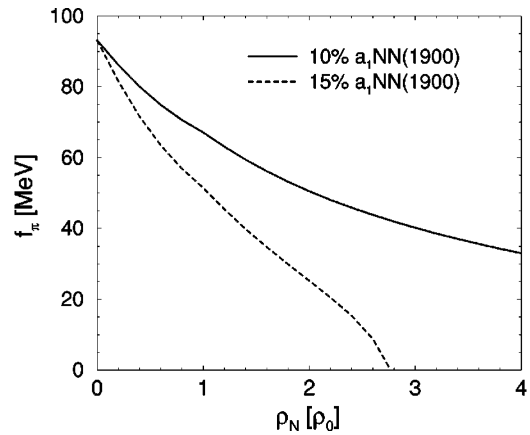

FIG. 5. Density dependence of f_π as extracted from the first Weinberg sum rule using in-medium ρ and a_1 spectral functions within the schematic two-level model; the two curves correspond to branching ratios of 10% (solid line) and 15% (dashed line) for the $N^*(1900) \to N a_1$ decay.

The in-medium extensions of these sum rules have been derived in Ref. [23]. In the zero-momentum limit to which we restrict ourselves here, one has

$$f_\pi^{*2} = \int_0^\infty \frac{dq_0^2}{q_0^2} [\rho_V(q_0,\vec{q}=0) - \rho_A(q_0,\vec{q}=0)] \tag{33}$$

and an analogous expression for the second one. The right-hand side (RHS) of Eq. (33) is now readily evaluated employing the ρ and a_1 spectral functions. As off-shell properties are of potential relevance, we calculate the free ρ self-energy microscopically using experimental data on the pion electromagnetic form factor and p-wave $\pi\pi$ phase shifts [3]. Similarly, we obtain the (imaginary part of the) free a_1 self-energy from decays into $\rho\pi$ in terms of the free ρ spectral function [3,24] [where possible energy dependences of the real part are neglected; that is, we take $(m_{a_1}^{(0)})^2 + \mathrm{Re}\,\Sigma_{a_1}(s) \equiv m_{a_1}^2$; the remaining free parameter, $g_{a_1} = 7.8$, has been fixed via Eq. (29) by requiring $f_\pi \simeq 93$ MeV]. The in-medium resonance-hole self-energies are then introduced as described in Sec. II, i.e., Eq. (4) and the analogous expression for the $a_1 N^*(1900) N^{-1}$ excitation [with microscopic free widths for both $N^*(1520)$, $N^*(1900)$, and including in-medium broadening].

Figure 5 shows that the pion decay constant indeed decreases rather rapidly starting from low densities reaching a 30–40 % reduction at normal nuclear density, depending on the precise value for the $a_1 N^*(1900) N$ coupling constant. The uncertainty quickly increases towards higher densities, resulting in either a leveling-off or even a dropping to zero. However, the point here is not so much to predict quantitatively the critical density for chiral restoration but to demonstrate *qualitatively* that in-medium spectral functions based on many-body excitations in the nuclear environment encode an approach towards chiral restoration. To gain more insight into the underlying mechanism we show in Fig. 6 the pertinent vector and axial-vector spectral distributions in vacuum

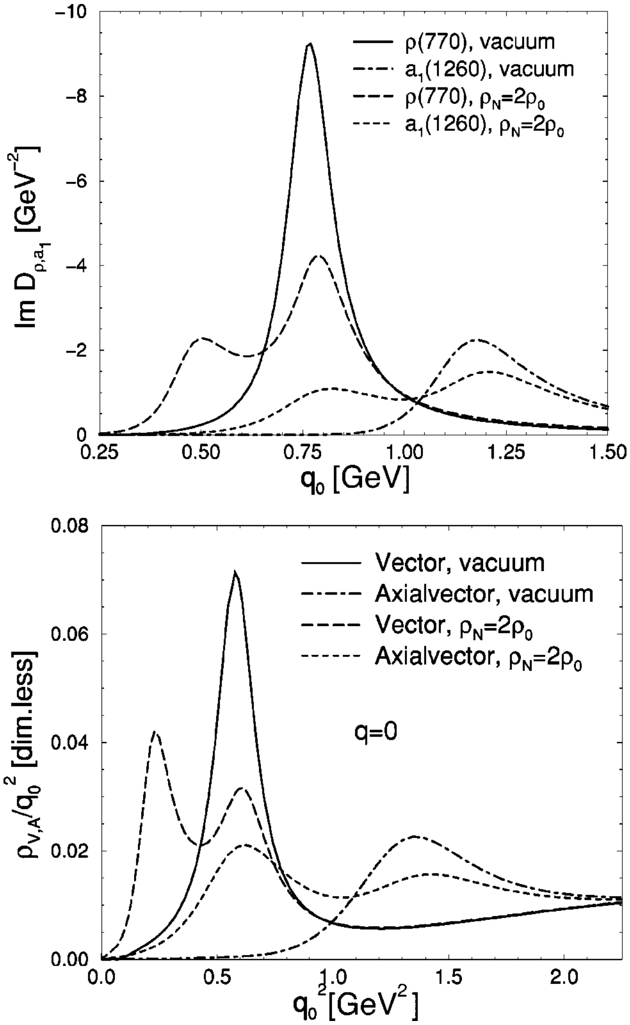

FIG. 6. ρ and a_1 spectral functions (upper) and V/A correlators divided by q_0^2 (lower) within the schematic resonance-hole model: solid lines, free ρ (V); dotted lines, free a_1 (A): long-dashed and dashed lines, ρ (V) and a_1 (A) at $\rho_N = 2\rho_0$.

and at twice normal nuclear density: clearly, the chiral breaking in vacuum, mainly constituted through different ρ and a_1 pole positions, is much reduced in the medium due to the appearance of low-lying excitations as well as a smearing of the elementary ρ/a_1 peaks. The schematic two-level model employed here certainly lacks accuracy, but it is conceivable that the inclusion of further excitations will reinforce the tendency towards degeneration of the spectral densities.

V. SUMMARY AND CONCLUSION

The identification of experimental signatures as well as theoretical mechanisms associated with chiral symmetry restoration in hot and dense matter has become an important issue in (nonperturbative) strong interaction physics. In this article we have focused on the behavior of the vector and axial-vector correlators in cold nuclear matter which have to degenerate at the critical density. Using phenomenologically well-motivated—albeit somewhat schematic—(two-level) models based on low-lying resonance-hole (''sobar'') excitations, we have pursued two approaches to assess possible avenues towards chiral restoration. In addition to the well-established π-sobar and ρ-sobar states, we have furthermore included the a_1 sobar based on the $N(1900)N^{-1}$ excitation as a candidate for the chiral partner of the $N(1520)N^{-1}$ state.

In the first part, we constructed a chiral effective Lagrangian for the ''meson-sobar'' fields supposed to be valid in the medium, incorporating vector and axial-vector degrees of freedom within the massive Yang-Mills framework. The parameters entering this Lagrangian have been estimated from the schematic model. Within a mean-field approximation, and imposing a general matching condition inherent in the MYM formalism, we found that the in-medium gauge coupling $\hat{g}$ and the (axial-)vector meson masses decrease with density. Performing a rescaling of the meson-sobar fields, we argued that the latter might be identified with the fields of BR scaling. *Our proposition is that these are the degrees of freedom that figure in effective Lagrangian field theory endowed with chiral symmetry and other flavor symmetries and that are probed in the dilepton production experiments at high density.* The fields that figure in BR scaling are therefore collective fields carrying the quantum numbers of mesons and baryons treated as local fields and hence must necessarily be approximate, with their range of validity depending on kinematics.

In the second part we have made use of the in-medium extension of the first Weinberg sum rule to evaluate the density dependence of the pion decay constant. Employing in-medium many-body spectral functions for both the ρ and a_1 as arising from the two-level model, together with appropriate (density-independent) high energy continua, $f_\pi(\rho_N)$ exhibits an appreciable reduction of $\sim 30\%$ at normal nuclear matter density. The driving mechanism in this calculation has been identified as a growing overlap of the V- and A-spectral distributions due to both the appearance of the low-lying ''sobar'' excitations as well as an accompanying resonance broadening.

Although our modeling of the various effective interactions at finite density is far from complete, we expect that the qualitative features of our results will persist in a more elaborate treatment. The latter might also provide a better estimate for the critical density with most of the uncertainty likely to reside in the a_1 channel where there is little empirical information available. QCD-based models [e.g., with instanton or Nambu–Jona-Lasinio- (NJL-) type interactions] typically give a rather low value for ρ_c (around $2\rho_0$), which might well be due to the neglect of interactions at the composite (hadronic) level. So far these are more reliably addressed in effective hadronic models as the one presented here.

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