

Fission modes in the reaction $^{208}\text{Pb}(^{18}\text{O}, f)$

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Results of fission fragment mass-energy distributions of the compound ^{226}Th nucleus formed in the sub-barrier fusion reaction $^{18}\text{O}+^{208}\text{Pb}$ at an energy of ^{18}O ions $E_{\text{lab}}=78$ MeV are reported. The reaction has been studied twice using two different accelerators, and both sets of experimental data agree quite well. Performed analysis of the experimental data with the use of a new multicomponent method has shown that alongside the well-known modes, i.e., the symmetric (S) and two asymmetric modes standard-one and standard-two, a high-energy mode standard-three has manifested itself. The last named mode appears due to the influence of the close-to-sphere neutron shell with $N\approx 50$ in the light fission fragment group. Theoretical calculations of the precession shapes of the fissioning nuclei $^{224,226}\text{Th}$ confirm this conclusion.

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I. INTRODUCTION

In recent years, the concept of multimodal fission has been the generally recognized concept describing the properties of mass-energy distributions (MED's) of fission fragments in spontaneous and low-energy fission of nuclei with $A > 200$. In a large number of works (see, for example, Refs. [1–26]), it was this concept that was employed for analyzing MED's of nuclear fission fragments in the region of Pb [1,2], MED's of heavy isotopes from Ra to Ac [3,4], those of actinide nuclei from Th to Cf [5–20], and superheavy nuclei in the region of Fm-Sg [21–25]. These results have been systemized in the review work [26].

The history of the concept of modality of the fission fragment MED structure is quite interesting and didactic. It originated from the work by Turkevich and Niday [27]. In it, the authors without describing the physical nature of the phenomenon, proposed a hypothesis according to which the mass yields of fission fragments in the fission of ^{232}Th induced by reactor neutrons were a superposition of two independent kinds of distribution, namely, symmetric single-humped and asymmetric two-humped distributions. This hypothesis was named the hypothesis of two independent modes (HIM). Later, on the basis of this hypothesis some

authors (for example, Refs. [28–31]) tried to explain the experimentally observed large variety of properties of fission fragment MED's in low-energy nuclear fission. However, soon it became clear that in many cases of nuclei heavier than Th the employment of this hypothesis did not produce any satisfactory results in the description of fission fragment MED's. Soon, the interest in that purely empirical hypothesis which did not have any theoretical basis was lost, though, as time showed, there was something in it.

In the early 1980's, the asymmetric fission component was discovered in the fission of preactinide nuclei in the Pb region. Its contribution into the total yield did not exceed 0.5% [1], and it was shown that in the framework of HIM it was quite easy to describe quantitatively the main trends in the fission fragment MED's behavior assuming that there were three independent modes instead of two, since two more independent modes could be distinguished in the asymmetric mode itself [2]. The calculations by Pashkevich became the theoretical basis for this approach. First for ^{208}Pb [32], the existence of two valleys was predicted. Then for ^{213}At by means of more complex calculations the existence of three valleys at the potential energy surfaces of those nuclei [33] was predicted in the dependence on the mass-asymmetric deformation. This surface was formed by the

properties of the fissioning nucleus and those of fission fragments. The calculations agreed well with the experiment [1,2].

Brosa and coauthors (see Ref. [34], and references therein) also calculated the fission modes for a large group of nuclei-actinides from ^{227}Ac to element 108 and predicted the existence of three main valleys: the symmetric (S), the bottom of which was always at $A_{\text{CN}}/2$, and two asymmetric ones, namely, standard-one ($S1$) conditioned by the influence of the spherical shell in the heavy fission fragment with the mean mass 132–134 (Z and N were close to the doubly magic 50 and 82, respectively) and standard-two ($S2$) appearing due to deformed shells also in the heavy fragment with a mean mass of ~ 140 . Using this model, the authors of a large number of experimental works (for example, Refs. [5–8,11–14,17]) started to analyze MED's of actinide nucleus fission fragments proceeding from these three fission modes. The study of MED's of spontaneous fission fragments of $^{236-244}\text{Pu}$ isotopes performed by Wagemans' group (Belgium) [12–14] demonstrated most clearly the existence of two independent asymmetric modes $S1$ and $S2$ for actinide nuclei. Recently, Brosa and coauthors have published results of their systematic analysis of fission fragment MED's obtained from the fission of actinide-nuclei induced by neutrons with various energies. In their description the three-component approach was used [20]. However, in 1995, Siegler and coauthors [15] presented new theoretical calculations for ^{238}Np made in the framework of Brosa's model and found that the fourth valley manifested itself at the potential energy surface of the nucleus at a mass-asymmetric deformation which was greater than that observed in the case of $S2$. The authors introduced this valley into their MED analysis. This fission mode was named standard-three ($S3$). In Refs. [35,36] the $S3$ mode was also used in the description of the ^{252}Cf MED's. However, it should be noted that the authors [35,36] introduced it in an arbitrary way, just for getting a better description of the MED's. Ter-Akopian and coauthors in Ref. [37] came to a conclusion that one more asymmetric mode (the fifth?) appeared in the spontaneous fission of ^{252}Cf for the charges and fission fragment masses typical for the standard-two mode but with a lower kinetic energy compared with $S2$. And finally, in their recent publications [18,19] Hamsch, Oberstedt, and coauthors have revised Brosa's model and made more precise calculations of the fission valleys for the fissioning ^{239}U nucleus. It turned out that there were six valleys in the calculations. The authors [18,19] came to the conclusion that not all of the calculated fission modes were realized in the experiment. Thus, one can see that more and more fission modes are being introduced for the interpretation of the complex structure of fission fragment MED's in low-energy fission, and there is no agreement regarding their number and properties.

An entirely new method of multicomponent analysis was introduced in Ref. [38]. The method is free from any parametrization of mass distributions (MD's), and on the basis of this method it was shown that four fission modes were realized for a wide region of fissioning actinide nuclei from ^{233}Pa to ^{245}Bk .

Until recently, the only experimentally unexplored region

regarding fission modes was the region of nuclei $213 < A < 226$, which can be conventionally called intermediate. Several years ago, here at the Flerov Laboratory of Nuclear Reactions (Dubna) and at the GSI (Darmstadt), experiments were started aimed at the study of the properties of the fission process in the intermediate region of nuclei at low excitation energies. We investigated MED's of fission fragments of ^{220}Ra , ^{219}Ac , and $^{220-224}\text{Th}$ nuclei in the sub-barrier fusion reactions $^{12}\text{C} + ^{208}\text{Pb}$ [39], $^{16}\text{O} + ^{203}\text{Tl}$, $^{204,208}\text{Pb}$ [40,41]. At the GSI, they studied charge distributions of fission fragments of secondary radioactive beams of nuclei from ^{214}Ra to ^{234}U . The fission of these nuclei was the result of exciting the giant dipole resonance due to the electromagnetic interaction between the ion beam and Pb target nuclei [42–44]. The experiments [39–44] showed that with increasing A of the fissioning nucleus a transition takes place from the predominantly symmetric to predominantly asymmetric fission in this region of nuclei at low excitation energies. The analysis of these experimental data was performed by a standard procedure on the basis of decomposition of the mass distributions into three components.

For the intermediate nuclear region, theoretical calculations of the potential energy were made in works [45–50] using the method of shell corrections in dependence on the mass-asymmetric deformation. The calculations [47–49] showed that in the case of isotopes $^{220-232}\text{Th}$ symmetric and asymmetric valleys were separated by a high potential ridge and that the last-mentioned valley was also split into two components, with a low barrier between them. It was also found that the saddle points for the symmetric and asymmetric fission modes determining the population of the valleys were different in height as well as in deformation. And thirdly, it was found that with an increase in Th nuclear mass numbers from 220 to 232 the sign of the difference in the saddle point heights changed—for light isotopes it was $E_f^a - E_f^s > 0$, whereas the picture was the opposite for heavy isotopes which qualitatively agreed with experimental results [39–44]. There was no agreement between the calculations from Refs. [47–49] and those from Ref. [50], in which only two valleys were found for the isotopes $^{220,226,232}\text{Th}$.

The present work continues the experimental studies of fission modes in the transitional region of nuclei. Here we report our results on MED measurements of Th neutron-deficient fission fragments in the reaction $^{18}\text{O} + ^{208}\text{Pb}$ at the energy of ^{18}O ions $E_{\text{lab}} = 78$ MeV and consider these results on the basis of a new multicomponent method proposed in Ref. [38]. The present study extends the possibilities of the proposed method, and we are making an attempt to answer the question: what fission modes are realized in the fission of ^{226}Th ? The results have turned out to be quite unexpected. In this work we also consider theoretical aspects of the problem.

In our experiments we also studied the multiplicity of γ quanta and neutrons accompanying fission as well as energy and angular distributions of neutrons. The properties of γ quanta and neutrons will be discussed in other works. The experimental results were briefly reported in Refs. [48,49,51,52].

II. METHODS OF MEASUREMENTS AND DATA PROCESSING

The experiments were carried out with the beams from the tandem LNS accelerators in Catania (Italy) and the Vivitron device in Strasbourg (France). A homogeneous layer of ^{208}Pb $220 \mu\text{g}/\text{cm}^2$ in thickness deposited on a carbon $50 \mu\text{g}/\text{cm}^2$ backing was used as a target. For the registration of fission fragments, we used a well-known method of kinematic coincidences [53–55].

At the LNS, the above mentioned method was realized using a two-arm time-of-flight spectrometer DEMAS-3 [56]. Each arm of the spectrometer consisted of a small size (3×4 cm) “start” parallel plate avalanche counter (PPAC) situated at a distance of 4 cm from the target. The counter was made of two Mylar foils $140 \mu\text{g}/\text{cm}^2$ in thickness which were mounted rigidly onto a frame. Each foil had a homogeneous layer of gold $80 \mu\text{g}/\text{cm}^2$ in thickness; the gap of 3 mm between the foils was filled with gas. Each arm also had two “stop” position-sensitive parallel plate avalanche detectors (PSPPAD) 30×20 cm in size. The minimal flight path was 40 cm. The position resolution of the spectrometer was 0.2° , the time resolution was 250 ps.

In the experiment at the Vivitron accelerator a two-arm time-of-flight spectrometer CORSET [57] was used. Each arm of the spectrometer consisted of a start detector composed of microchannel plates with an electrostatic mirror; an emitter of electrons made of Mylar foil $130 \mu\text{g}/\text{cm}^2$ in thickness with homogeneous layers of gold ($30 \mu\text{g}/\text{cm}^2$ in thickness) on it, and two stop position sensitive (x, y -sensitive) detectors, also composed of micro-channel plates 6×4 cm in size. The start detectors were located at a distance of 3 cm from the target. The minimal start-stop flight path was 12 cm. Thus, the spectrometer consisted of two start and four stop detectors and detected events within a solid angle of 360 msr. The position resolution of the stop detectors was 0.4 mm, the time resolution of the spectrometer was 150 ps [57].

The velocities and coordinates of pair fission fragments were measured by both spectrometers. The mass resolution was checked by the MED's of ^{252}Cf spontaneous fission fragments and, in our opinion, it was 3–5 amu [25,48,49]. The obtained MED's of ^{252}Cf fission fragments repeated all the structure peculiarities of the MED from work [58] in which it was measured using semiconductor surface-barrier detectors (SSBD's). According to our measurements, in the mass distribution of ^{252}Cf the peak/valley ratio was ~ 20 . In our opinion, the mass and energy resolution of both spectrometers was not lower than that of SSBD's.

The data processing was performed according to the standard two-body process procedure, its description is presented in Refs. [25,55]. The fission fragment energy losses in the target layer, backing and start detectors were taken into account. Special attention was paid to the angular folding correlations in the plane of the reaction as well as outside. Only those of recorded events were selected and analyzed which corresponded to the two-body process with complete linear momentum transfer.

Note once again that the present experiment was carried out in Catania and Strasbourg at the same ^{18}O projectile

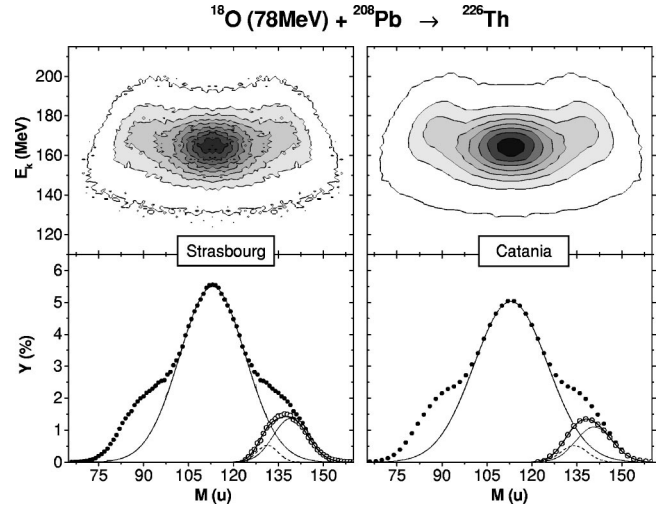


FIG. 1. Upper panel: two-dimensional matrices of fission fragments ($E_k - M$) for ^{226}Th measured at Strasbourg and Catania. Lower panel: experimental yields Y of fission fragment masses (filled symbols), the extracted asymmetric component (open symbols), and the description of the distributions by the Gaussians (the curves). The dashed curve is the Gaussian of mode S1.

energy $E_{\text{lab}} = 78$ MeV. As is shown below, this fact increases sufficiently the reliability of the experimental results and data analysis.

III. EXPERIMENTAL RESULTS

Figure 1 shows the fission fragment MED's measured in Catania and in Strasbourg. Two-dimensional matrices of fission fragments ($E_k - M$) are presented in the upper panel of the figure; the experimental mass distributions (MD's) (filled symbols), the extracted asymmetric component (open symbols), and the decomposition of the MD spectra into three Gaussians (for heavy fission fragments only) are shown in the lower panel. It is clearly seen in Fig. 1 that at the slopes of the predominant symmetric fission the “shoulders” appear due to the asymmetric mode. Note that the decomposition of the experimental MD into components with the help of the n number of Gaussians has been to the present day a standard procedure of obtaining the characteristics of fission modes, which is shown in Fig. 1 for both discussed cases. Table I shows integral characteristics of the fission fragment MED's from which it follows that the mass distribution variance σ_M^2 measured at Catania is wider than that measured at Strasbourg. It is also seen in Fig. 1. In our opinion, this effect

TABLE I. Experimental values of the mass variance σ_M^2 , average total kinetic energy \bar{E}_k , and average kinetic energy variance σ_E^2 for ^{226}Th fission fragments at $E_{\text{lab}} = 78$ MeV.

	Strasbourg	Catania
σ_M^2 (u^2)	245 ± 5	265 ± 4
\bar{E}_k (MeV)	164.8 ± 0.5	165.5 ± 0.4
σ_E^2 (MeV^2)	134 ± 4	137 ± 3

is conditioned by the uncontrolled fission fragment energy (velocity) losses in the start detectors of the DEMAS-3, since the combined thickness of foils in the start detectors of this spectrometer is twice as large as that of the CORSET setup, and no correction for straggling has been introduced into the data processing. However, as seen in Fig. 1, all the peculiarities of the fission fragment MED's are practically repeated in both cases, and the results of decomposition of the extracted asymmetric component Y_a into two Gaussians obtained as a difference between the experimental yield Y_{exp} and the Gaussian of the symmetric mode Y_G , $Y_a = Y_{\text{exp}} - Y_G$, are almost the same.

Thus in two independent experiments aimed at the study of fission fragment MED's in the fusion-fission reaction at energies below the Coulomb barrier we obtained two sets of experimental data which agree quite well.

IV. ANALYSIS AND DISCUSSION

A. Emission of precession neutrons and effective excitation energy of the fissioning nucleus

As was shown by our preliminary experimental results [52] in the discussed reaction an average of about 1.5 precession neutrons $\langle \nu_{\text{pre}} \rangle$ were emitted at $E_{\text{lab}} = 78$ MeV. First, it means that in reality it is not the ^{226}Th nucleus that undergoes fission but the nucleus close to ^{224}Th , and, secondly, that the fissioning nucleus near the scission point has the excitation energy that is much lower than the initial one. The initial excitation energy is $E^* = 26$ MeV, whereas the excitation energy E^* after the emission of an average of $\langle \nu_{\text{pre}} \rangle$ calculated according to [39] is ~ 13 MeV, assuming that all the neutrons $\langle \nu_{\text{pre}} \rangle$ have been emitted before the saddle point. Thus, provided that the changes in A and E^* of the fissioning nucleus (fission chances) are effectively taken into account, our results presented in Fig. 1 may agree well with the data from Refs. [43,44] in which charge distributions of ‘‘the first chance’’ fission fragments of the same Th isotopes, obtained as a result of the giant dipole resonance excitation, were investigated. However, as it is shown below, taking or not taking $\langle \nu_{\text{pre}} \rangle$ into account practically does not influence our further analysis and conclusions. That is why for simplicity we will talk about the fission of the initial nucleus ^{226}Th .

B. The model and results of analysis

Characteristics of independent fission modes are extracted from the multicomponent analysis of fission fragment MED's. As mentioned above, in most cases it is the three-component description (for example, Refs. [5–8,11–14,17]) with a standard set of modes, i.e., S , $S1$, and $S2$. Note once again that almost all currently existing methods of analysis of experimental fission fragment MED's have a common feature, namely, relative mass yields of separate modes are described by the Gaussian distributions as is shown in Fig. 1. However, in our opinion this assumption is not quite justified. Indeed, at high excitation energies when nuclear shell properties can be neglected, MD's of the symmetric mode S are well described by the Gaussians

[1,16,34,41,48,49,55,59–61]. However, at low excitation energies the S mode deviates quite strongly from the Gaussian shape due to the influence of weakly deformed shells [1,62,63]. As for the MD of the asymmetric modes $S1$ and $S2$, this assumption appears to be purely intuitive rather than empirical since the modes cannot be experimentally observed in the uncombined state. That is why strict parametrization of mass yields of separate fission modes may lead to inexact estimation of the extracted parameters.

The indicated shortcoming was overcome in Ref. [38] in which a new free from any mass yield parametrization method of multicomponent MED's analysis was proposed. According to this method, when the mass of a fission fragment is fixed, the yields $[Y_i(M)]$ of all fission modes are found from the solution to the system of three equations. These equations are easily derived assuming the existence of independent fission modes. According to this assumption, experimentally observed distribution of the total kinetic energy of fission fragments with a given mass M $[Y_{\text{exp}}(M, E_k)]$ is a superposition of the $[Y_i(M, E_k)]$ distributions of three independent modes. Using the known expressions for the moments of composed distributions, experimental values of the yields $Y_{\text{exp}}(M)$, those of the mean kinetic energies $\bar{E}_{k,\text{exp}}(M)$, and kinetic energy variance $\sigma_{E,\text{exp}}^2(M)$ can be expressed via characteristics of the independent fission modes [1,2]

$$\begin{aligned}
 Y_{\text{exp}}(M) &= \sum_i Y_i(M), \\
 \bar{E}_{k,\text{exp}}(M) &= \sum_i \frac{Y_i(M)}{Y_{\text{exp}}(M)} \bar{E}_{k,i}(M), \\
 \sigma_{E,\text{exp}}^2(M) &= \sum_i \left\{ \frac{Y_i(M)}{Y_{\text{exp}}(M)} \sigma_{E,i}^2(M) \right. \\
 &\quad \left. + \sum_j \frac{Y_i(M)Y_j(M)}{Y_{\text{exp}}^2(M)} [\bar{E}_{k,i}(M) - \bar{E}_{k,j}(M)]^2 \right\}, \quad (1)
 \end{aligned}$$

where the indices i and j correspond to the fission modes $S1$, $S2$, or S .

It is obvious that for determining the values of $Y_i(M)$ from this system of equations it is necessary to set the dependences $\bar{E}_{k,i}(M)$, and $\sigma_{E,i}^2(M)$ for all independent modes. The mode of setting those dependences was proposed in work [38] and is given below in much detail. The parameter values of such description are found from the fitting of the total matrix of fragments $Y_{\text{exp},M}(E)$.

Note that the reliability of the yields $Y_i(M)$ strongly depends on the errors of the experimental $Y_{\text{exp}}(M)$, $\bar{E}_{k,\text{exp}}(M)$, and $\sigma_{E,\text{exp}}^2(M)$ values. That is why this method works well for the matrices of fission fragments measured at a low background and with high statistical precision. Other-

wise we propose that a more general method should be used in which the yields $Y_i(M)$ are found from the condition of the functional minimum

$$\chi^2(M) = \sum_E \left[\varepsilon(E, M) \left(\sum_i^n \eta_i(M) Y_{i,M}(E) - Y_{\text{exp},M}(E) \right) \right]^2, \quad (2)$$

where $Y_{i,M}(E)$ is the normalized energy distribution of the i th mode (the mode of setting these functions is described in much detail in Ref. [38]), $\eta_i(M)$ is the relevant weight factor for the i th mode, and $\varepsilon(E, M)$ is the value which is in inverse proportion to the total error of the $Y_{\text{exp},M}(E)$ measurement. It is obvious that $\chi^2(M)$ is minimal when all the three conditions are fulfilled: $\partial\chi^2(M)/\partial\eta_i(M) = 0$.

In our simple case when the derivatives are in linear dependence on the relevant parameters the finding of the optimal values of $\eta_i(M)$ is reduced to solving the system of three equations

$$\sum_E \left[\varepsilon(E, M) Y_{i,M}(E) \left(\sum_j^n \eta_j(M) Y_{j,M}(E) - Y_{\text{exp},M}(E) \right) \right] = 0. \quad (3)$$

It is clearly seen from these relations that in contrast to the approach described in Ref. [38] the proposed method allows one to take into account correctly the errors in experimental data. Indeed, some checking of the calculations showed that in the case of sufficient statistics Eqs. (1) and (3) yield the same results. But on the whole, the solutions on the basis of Eq. (3) are more stable to experimental errors and can be used at a less statistical accuracy of experimental data. It is this circumstance that motivated the use of this system of equations in the present work.

Thus, in our analysis the following assumptions have been used.

There are three independent modes, one symmetric and two asymmetric, which contribute to the fission fragment MED's of the indicated nuclei. Relative mass yields of independent modes were calculated with the use of Eq. (3).

Dependence of the average total kinetic energy on the fission fragment mass for each mode and the ratio between the squared average total kinetic energy and the variance of kinetic energy for each mode can be presented as follows [1,38,55]:

$$\begin{aligned} \bar{E}_k(M) &= \bar{E}_k(A/2)(1 - \mu^2)(1 + \alpha\mu^2), \\ \bar{E}_k^2(M)/\sigma_E^2(M) &= \text{const}, \end{aligned} \quad (4)$$

where $\mu = 1 - 2M/A$, and parameter α characterizes the degree of deviation of $\bar{E}_k(M)$ from the parabolic dependence suggested by Nix and Swiatecki in Ref. [64].

Distribution of the total kinetic energy of fission fragments with fixed masses for each mode can be described by the Sharlie function [38,65]

$$\begin{aligned} f(E_k) &= \frac{1}{\sigma_E} \left[\varphi(u) - \frac{\gamma_1}{6} \varphi^{\text{III}}(u) + \frac{\gamma_2}{24} \varphi^{\text{IV}}(u) \right], \\ u &= \frac{E_k - \bar{E}_k}{\sigma_E}, \quad \varphi(u) = \frac{1}{\sqrt{2\pi}} \exp(-u^2/2), \end{aligned} \quad (5)$$

where $\gamma_1(M) = \langle (E_k - \bar{E}_k(M))^3 \rangle / \sigma_E^3$ is the dyssymmetry coefficient, $\gamma_2(M) = \langle (E_k - \bar{E}_k(M))^4 \rangle / \sigma_E^4 - 3$ is the excess coefficient. Both coefficients characterize the degree of deviation of the distribution from the normal one. At $\gamma_1 = \gamma_2 = 0$ the Sharlie distribution identically turns into a normal one.

The type of Eqs. (4) and (5) was determined from the analysis of a great totality of experimental data from the fission of nuclei in the range from ^{186}Os to ^{235}U [65,66]. The applicability of the data to the description of shell modes was convincingly demonstrated in Ref. [38].

In this work as well as in Ref. [38] it is assumed that γ_1 and γ_2 do not depend on the fission fragment mass. For the energy distribution of the symmetric mode, γ_1 was equal to -0.1 and γ_2 was equal to 0 , in the case of asymmetric modes it was $\gamma_1 = \gamma_2 = -0.2$ [38,65]. Parameter α was the same for all the fission modes.

The procedure of the analysis was an iteration process realized within a standard computer code MINUIT [67]. For a given set of parameters $\bar{E}_{k,i}(A/2)$, $\sigma_{E,i}^2(A/2)$, and α from Eq. (3) the yields $Y_i(M)$ were determined, proceeding from which a fit matrix for the fission fragments was calculated. Then using the least-square method (LSM) the degree of agreement between the experimental MED and the fit matrix was checked and new values for $\bar{E}_{k,i}(A/2)$, $\sigma_{E,i}^2(A/2)$, and α were determined. In all considered cases, the LSM procedure converged when the χ^2 values per a degree of freedom were close to 1.

The abovementioned method of MED decomposition was used for the analysis of matrices of ^{226}Th fission fragments presented in Fig. 1. The results are shown in Fig. 2. There experimental data on $Y_{\text{exp}}(M)$, $\bar{E}_{k,\text{exp}}(M)$, $\sigma_{E,\text{exp}}^2(M)$, and $\gamma_1(M)$ are designated by the open circles. From the figure it is seen that the data from both experiments agree well. Some insignificant differences can be explained by the complexity of taking into account corrections for the thickness of the target and start detector.

Figure 2 also presents the decomposition results. It is seen that our descriptions in both cases agree well with the experimental dependences. Figure 3 demonstrates the quality of the data description more clearly and one can see the description of energy distributions of fragments with fixed masses taken from the experimental matrix $Y(M, E_k)$ (Strasbourg) and the contributions of independent modes shown for those fission fragments which yielded the most interesting results. It can be seen that practically throughout the entire E_k range the calculations agree with experiment, and only in the case of $E_k < 130$ MeV the data points lie somewhat higher than the calculated curves. In our opinion, this discrepancy is connected with single random events (or events distorted by scattering) which were present in both experiments as negligible background events. Note that the

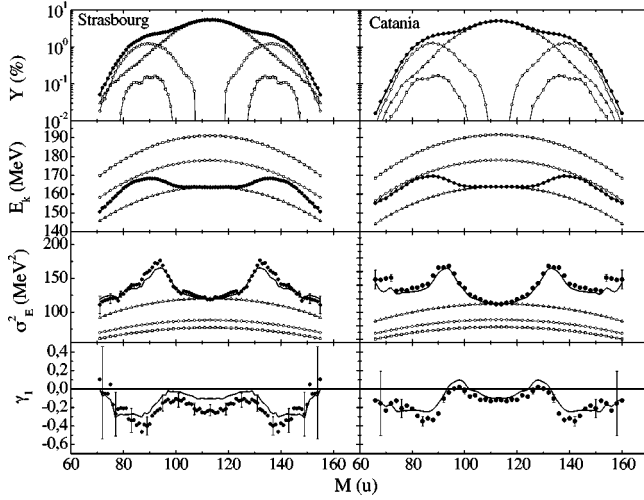


FIG. 2. From top to bottom: Experimental yields Y of fission fragment masses (filled symbols) obtained in the two sets of experiments and results of decomposition (open symbols), the triangles correspond to the symmetric mode S , the circles: mode $S2$, squares: modes $S1+S3$ (see the text). The smooth line through the data points is to guide the eye. Distributions of the total kinetic energy $E_k(M)$ in dependence on the fission fragment mass and its decomposition (the same designations). The smooth line through the data points is to guide the eye. Dependence of the variance $\sigma_E^2(M)$ of the fission fragment total kinetic energy on the mass, its decomposition and description (the solid curve). Dependence $\gamma_1(M)$ of the asymmetry of the kinetic energy distributions on the fission fragment mass and its description (the solid curve).

contribution of this background is very small but its manifestation is clearly seen in the previous figure in which calculated dyssymmetry coefficients are slightly greater than the experimental ones. This circumstance shows that dyssymmetry coefficients are highly sensitive to some peculiarities of the MED shape. Thus, Figs. 2 and 3, except in the above-mentioned case, demonstrate very good agreement between

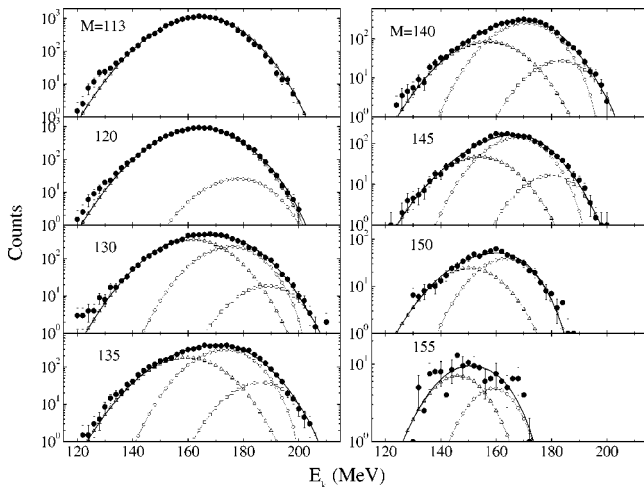


FIG. 3. Examples of decomposition of the experimental (filled symbols) differential distributions of E_k into separate components (open symbols) for the fixed fission fragment masses (solid curves). Designation of modes is the same as in Fig. 2.

TABLE II. Characteristics of independent fission modes found from the analysis of experimental MED for ^{226}Th .

	Strasbourg			Catania		
	$S1+S3$	$S2$	S	$S1+S3$	$S2$	S
$Y(\%)$	2.1	21.4	76.5	2.8	21.9	75.3
$\bar{M}_H(u)$	136.9	136.9	113.0	138.6	137.6	113.0
$\sigma_M^2(u^2)$	21.6	44.5	139.5	44.3	50.6	150.6
$\bar{E}_k(\text{MeV})$	184.2	171.6	162.5	184.7	172.0	162.7
$\sigma_E^2(\text{MeV}^2)$	71.6	82.1	118.3	72.4	82.8	110.6

the description and experiment, and this is a very important argument in favor of the principle assumptions of this approach.

From the comparison of these figures it can be also seen that in the cases when we succeed to describe within a unified approach the dependences $Y_{\text{exp}}(M)$, $\bar{E}_{k,\text{exp}}(M)$, $\sigma_{E,\text{exp}}^2(M)$, and $\gamma_1(M)$, the description of the total matrix also turns out to be good. Thus, relatively smooth matrices of fission fragments are quite fully characterized by the above mentioned properties and in most cases it is possible to confine the data analysis to the description of only these four dependences.

In Fig. 2 the two sets of data on the $Y_i(M)$, $\bar{E}_{k,i}(M)$, and $\sigma_{E,i}^2$ values of the independent modes agree well and they also agree with dependences expected for ^{226}Th from the analysis of fission modes in related works. It is well seen from Table II which shows mass and energy characteristics obtained in this work for the three fission modes.

Table II shows that for ^{226}Th as well as for other nuclei from related works the kinetic energies E_k and kinetic energy variances σ_E^2 of these modes are in the following relationship: $E_k(S1) > E_k(S2) > E_k(\text{sym})$ and $\sigma_E^2(S1) < \sigma_E^2(S2) < \sigma_E^2(\text{sym})$. The mean value of the standard-two mode mass is also close to the expected value $M \approx 138$ for light actinides.

However, in the case of the standard-one mode, we obtained in both cases a broad, asymmetric and two-humped distribution with $M \approx 137$ instead of the narrow asymmetric distribution $Y_1(M)$ with the mean mass $M \approx 133$. It is very well seen in a large-scale Fig. 4 showing comparison results of the decomposition of $Y_{\text{exp}}(M)$ from both experiments. This behavior of the standard-one mode is caused by the peculiarities of the ^{226}Th fission rather than by the experimental errors. The fact that the dependences extracted from the analysis of the experimental data obtained by means of different methods are similar is in favor of such assumption (a slight discrepancy in masses is most probably explained by incomplete accounting of the losses in the start detector). This at first sight strange behavior of $Y_1(M)$ can be understood when taking into account the results of the recent work [38]. The authors managed to show convincingly that the standard-three mode exists as predicted by Brosa. In addition, it was established there that in contrast to ($S1$) and ($S2$), the standard-three mode is caused by the shell effects in the close-to-sphere neutron shell in the light fission frag-

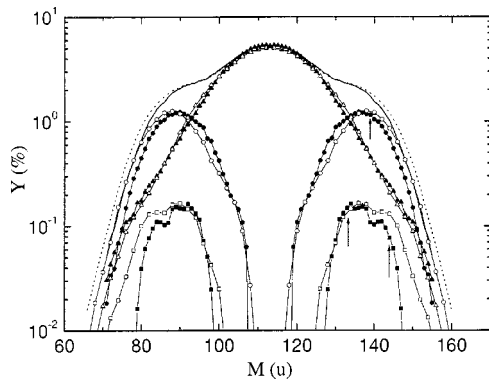


FIG. 4. Comparison of the experimental mass yields $Y(M)$ measured at Strasbourg (the thick solid line) with those measured at Catania (the dashed line). The yields of independent fission modes are also compared (the filled symbols correspond to the Strasbourg measurements and the open ones to those made at Catania).

ment with $N \approx 50$. This is the reason why the high kinetic energies, which are close in values to E_k for mode $S1$, correspond to this mode. Proceeding from this it becomes clear that in the three-component analysis mode $S3$ will manifest itself as a distortion of the $S1$ mode which is seen in Figs. 1 and 3.

Figure 5 clearly demonstrates the way mode $S3$ can mani-

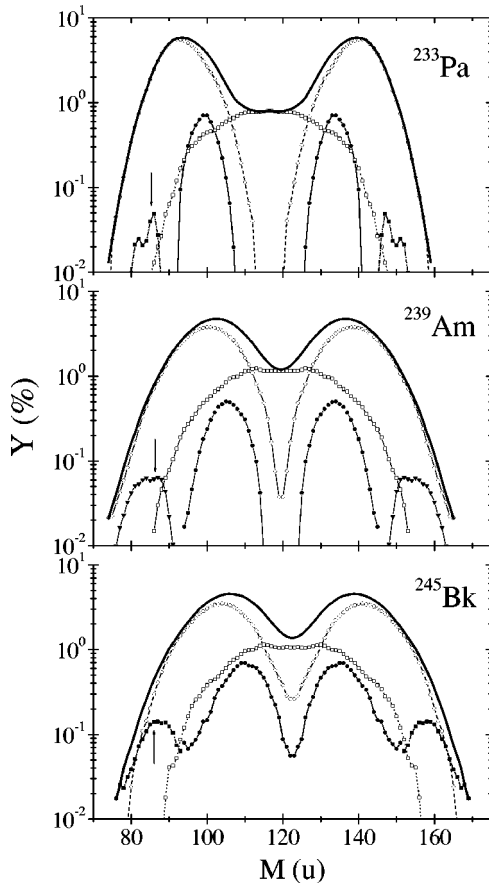


FIG. 5. Results of the MED decomposition for compound nuclei ^{233}Pa , ^{239}Am , and ^{245}Bk formed in the reaction (p,f) , taken from Ref. [38] (see the text).

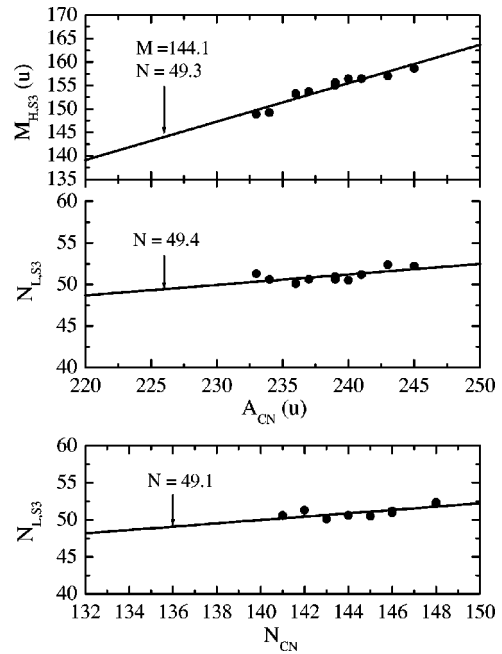


FIG. 6. Extrapolation of the mean mass value and the neutron number of mode $S3$ (shown by the arrow) for the region of actinide nuclei up to ^{226}Th (the filled symbols correspond to the results of Ref. [38]) in dependence on the mass number A_{CN} of the fissioning nucleus and the number of neutrons in the compound nucleus N_{CN} .

fest itself in the fission of ^{226}Th . The figure shows the results of Ref. [38] devoted to the three-component MED for nuclei from ^{233}Pa to ^{245}Bk . It is seen from the figure that for these nuclei modes $S1$ and $S2$ are clearly distinguished. However, considering the heavy group of fission fragments, the peak position of mode $S3$ rapidly moves in the direction from heavy to lighter masses of fission fragments with a decrease in the compound nucleus mass, in contrast to modes $S1$ and $S2$ whose peak positions do not practically change. To understand where the peak of $S3$ for ^{226}Th turns out to be, we built three empirical dependences using the data from Ref. [38]. The results are presented in Fig. 6. It shows the data for ten nuclei (from ^{233}Pa to ^{245}Bk) on the heavy fission fragment mass and the number of neutrons in the light fragment of mode $S3$ as a function of the mass and the neutron number of the compound nucleus. As is seen, these dependences are easily described by linear functions. In the case of ^{226}Th , the extrapolation of these dependences to $A=226$ and $N=136$ yields the values of the heavy fission fragment mass $M \approx 144$ and the number of neutrons in the light fission fragment $N_L \approx 49.3$. Using a hypothesis on the unchanged charge density, one can easily obtain mass $M \approx 144$ of the complementary heavy fission fragment using N_L which is in agreement with the previous estimation.

The positions of peaks for modes $S1$ (133), $S2$ (139), and $S3$ (144) expected from the above discussion are shown in Fig. 4 by arrows. As is seen from the figure, these values are close to the peak positions of yields of the corresponding modes obtained in this analysis. This fact allows us to make a conclusion about the significant role of mode $S3$ in the formation of the ^{226}Th fission fragment MED.

From the comparison of the decomposition of the ^{226}Th fission fragment MED with that of heavier nuclei (see Fig. 5) it follows that with increasing the mass of the fissioning nucleus the relative contribution of mode $S1$ into the asymmetric fission rapidly decreases. It agrees well with the results of Refs. [9,10,28,30] in which this mode was not called up for analyzing the MED's of fission fragments in the region of ^{227}Ac – ^{233}Pa , and its contribution if any was very small. At the same time, works [1,2,8] showed that in the fission of preactinide nuclei the contribution of the high-energy and $S2$ modes were comparable. One possible explanation of the irregular behavior of the high-energy mode can be obtained from the systematics presented in Fig. 6. Indeed, if the far extrapolation is justified, in the vicinity of lead the location of the mode $S3$ peak will correspond to the heavy fission fragment mass $M \approx 130$ and it will lead to an increase in the yield of the high-energy mode.

Another peculiarity of the ^{226}Th MED decomposition is the behavior of the symmetric fission mode yields. In both cases the yields noticeably differ from the Gaussian distribution. A similar deviation of the mass yields in the symmetric fission at low excitation energies was studied in much detail in Refs. [38,61–63]. Using the results of analysis of the preactinide nucleus mass distribution, a noticeable influence of strongly deformed neutron shells with $N=52$ and 68 on the mass yields was established. It is not excluded that in the fission of ^{226}Th we encounter similar effects. Especially as for ^{226}Th , the number of neutrons in every fission fragment is close to $N \approx 68$ and this value is the same for the strongly deformed shell, the position of which in this case exactly coincides with $A/2$.

V. THEORETICAL CALCULATIONS

Earlier in theoretical calculations [47–49] it was shown that for ^{226}Th and ^{224}Th at the surface of the potential energy there existed three valleys of fission, i.e., S , $S1$, and $S2$. For the theoretical evaluation of the results obtained by us in relation to mode $S3$, calculations for pre-scission shapes were made for the mentioned nuclei at the bottom of the valleys $S1$ and $S2$. The surface of the fissioning nucleus was parametrized by the Cassini ovaloids [32]. In this approach the elongation of the nucleus was characterized by the parameter α and the asymmetry of the shape, by the parameter α_3 . Minimization was performed for the deformations of the higher order, i.e., α_4 , α_5 , α_6 , and α_7 .

Figure 7 shows the nuclear shapes for ^{226}Th (left-hand part) near the scission point at $\alpha=0.97$ (geometrical scission of the nucleus takes place in this parametrization at $\alpha=1$ when the radius of the neck becomes equal to zero). The top of the figure shows the case of the valley $S1$, the bottom, that of $S2$. On the assumption that the nucleus undergoes scission when the thickness of the neck is minimal, the volumes (masses) of future fission fragments were calculated and their Z and N determined. It is well seen that the heavy fragments in both valleys are near spherical keeping in mind the nucleons in the neck. At the same time the deformation of the light fission fragment in those cases is different. In the valley $S2$ (bottom), the light fission fragment is obviously more elon-

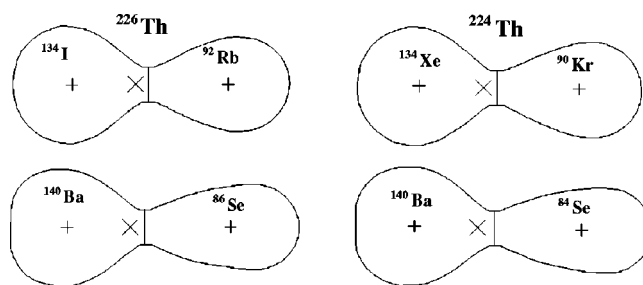


FIG. 7. Theoretical calculations of pre-scission shapes of the ^{226}Th (left-hand part) and ^{224}Th (right-hand part) nuclei: at the bottom of the $S1$ valley (upper panel) and at the bottom of the $S2$ valley (lower panel).

gated than in the valley $S1$. As was shown in Sec. IV B, in experiment we see the combined effect of modes $S1$ and $S3$; similarly, it is very difficult to separate them in theoretical calculations, that is why the influence of mode $S3$ will make itself evident in a more compact shape of the light fission fragment. The latter can evidently be near-magic by neutrons ($N \approx 50$) taking into account the nucleons in the neck. A similar idea which seems to be rather fruitful was expressed in Ref. [68] devoted to the study of spontaneous fission of ^{252}Cf . Note that our calculations of the light fission fragment deformation in both valleys for the case of ^{226}Th strongly differ from those made by Brosa *et al.* for heavier nuclei [34]. In Ref. [34], the deformation of the light fission fragment is approximately the same in both valleys.

Let us recall that at the scission point the fissioning nucleus (in the studied reaction) effectively has 224.5 nucleons instead of 226 and much less excitation energy. This is the reason why we made calculations of pre-scission shapes also for ^{224}Th , the results are presented in the right-hand part of Fig. 7. As is seen, the shapes of the nuclei under discussion look quite similar. Thus, the foregoing in principle does not depend on the fact whether the nucleus emits pre-scission neutrons or not, and thus the theoretical explanation of the appearance of mode $S3$ with the high E_k due to the influence of the spherical shell of the light fission fragment agrees well with our analysis of the experimental MED.

VI. CONCLUSION

In the present work we used a modernized method of multicomponent analysis and applied it to the experimental data for ^{226}Th formed in the reaction $^{18}\text{O} + ^{208}\text{Pb}$. In the investigated MED a noticeable presence of the high-energy mode $S3$ was found. This mode appeared due to the influence of the close-to-sphere neutron shell with $N \approx 50$ in the light fission fragment. Theoretical calculations of the pre-scission shapes of the fissioning $^{224,226}\text{Th}$ nuclei confirm this conclusion.

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