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Exact treatment of the Pauli exclusion operator in nuclear matter

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We performed a nuclear matter G-matrix calculation removing the angle-average approximation from the treatment of the Pauli exclusion operator. Although the corrections to the standard angle-averaged matrix elements are very small, the dependence of the new G matrix on the projection quantum number M of the relative angular momentum J is significant on the scale of typical medium effects.

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I. INTRODUCTION

Proton-nucleus elastic and inelastic scattering can be used to test our understanding of the nuclear force in the nuclear medium. To simplify this complicated many-body problem, the effects of the nucleons surrounding the interacting pair are typically incorporated into an effective nucleon-nucleon (*NN*) interaction where differences from the original free *NN* force increase with nuclear density. These effects have important consequences, not only for the reaction cross section but also for the sensitivity to the spin projection of the interacting nucleons. Experiments completed over the last few years near 200 MeV now provide high precision polarization data for a number of discrete nuclear states. These data can be used to test models of the effective *NN* interaction since, at energies below the pion production threshold, the free *NN* interaction is well constrained by the available two-body data.

Recently we have undertaken a systematic study of proton-nucleus inelastic scattering to several of these discrete states [1–3]. In part, our analysis was motivated by the ongoing discussion in the literature about possible medium effects beyond conventional many-body mechanisms, such as density-dependent changes of meson spectral properties. These effects are suggested by dilepton production measurements in heavy-ion collisions [4,5] and have been linked to changes of the quantum chromodynamics (QCD) vacuum in the nuclear medium [6–10]. In particular, a reduction of the ρ mass in the medium has been debated extensively, and experimental evidence for such reduction has been reported from diverse places [11–16]. In view of the chief role played by the ρ meson in generating the tensor force, unnatural parity (p,p') transitions that are sensitive to the spin-dependent part of the effective interaction are a suitable way to explore these hypotheses [17,18]. Although a distorted wave Born approximation (DWBA) calculation, together with our density-dependent effective interaction, reproduces the main trend of these measurements [2,3], several notable discrepancies exist, especially for the diagonal polarization transfer coefficients [3]. Because the established medium effects contained in our model do not provide a fully satisfactory picture of these data, we have been exploring the possibility of less conventional effects. For the isovector transitions, the data support the need for a reduction of the tensor force in the medium, which would suggest a lowering of the effective ρ-meson mass and thus a link to the alleged modifications of meson spectral properties mentioned above.

On the other hand, while examining the possibility of novel medium effects, it is important to ensure that the more established ones are well under control. Prominent among the conventional medium effects at energies around 200 MeV is the Pauli blocking mechanism, which is typically treated within the so-called angle-average approximation [19]. In this approximation, the exact (nonspherical) Pauli exclusion operator is replaced by its (spherically symmetric) angle average. This approximation was examined in an earlier work and found to be satisfactory for the central and spin-orbit parts of the isoscalar interaction [20]. Recently, these calculations have received new attention [21,22]. These studies report a small but not negligible (attractive) contribution to the binding energy of nuclear matter from a consideration of the full Pauli exclusion operator.

As a result of problems with the effective tensor interaction in (p,p') transitions [3], we wish to re-examine this issue with respect to proton-nucleus inelastic scattering, paying particular attention to changes that would affect the nonspherical components in the nuclear force because these are likely to impact the calculation of polarization observables. In the next section, we describe some technical aspects of the calculation, then show examples of the numerical results (Sec. III). The exact treatment of the Pauli exclusion operator Q generates an effective *NN* interaction that has a different structure than the previous one due to the new dependence on the projection quantum number M of the total relative angular momentum J. We demonstrate that, when such an M dependence is averaged out, the remaining corrections to the standard matrix elements are negligible. However, the dependence on M is significant and would carry forward into the DWBA calculation of the (p,p') scattering matrix. While such (p,p') calculations are not yet available, we can assess the potential importance of this dependence by comparing its size to the scale of well-known medium effects typically applied to the interaction in proton-nucleus scattering. Our conclusions are summarized in Sec. IV.

II. DESCRIPTION OF THE CALCULATION

The Brueckner-Bethe-Goldstone approach [23–26] is based on the idea that nucleons in nuclear matter move in a mean field arising from the interaction with all the other nucleons. For practical reasons, infinite nuclear matter systems are typically used in studies of the nuclear many-body problem as a working approximation to actual finite systems. Consider a nucleon with momentum $\mathbf{k}_1$ colliding with another of momentum $\mathbf{k}_2$ embedded in infinite nuclear matter. The Fermi sea is defined by the Fermi momentum k_F. If $\mathbf{k}_1$ and $\mathbf{k}_2$ are the momenta of two nucleons in the nuclear matter rest frame, it is convenient to introduce the relative momentum

$$\mathbf{k}=\tfrac{1}{2}(\mathbf{k}_1-\mathbf{k}_2) \tag{1}$$

and one-half of the center-of-mass momentum

$$\mathbf{P}=\tfrac{1}{2}(\mathbf{k}_1+\mathbf{k}_2). \tag{2}$$

Conversely, we have

$$\mathbf{k}_{1,2}=\mathbf{P}\pm\mathbf{k}. \tag{3}$$

The effective two-nucleon interaction in nuclear matter, or G matrix, is the solution of the Bethe-Goldstone equation, which is given schematically by

$$G=V+V\frac{Q}{e}G, \tag{4}$$

with V the two-body interaction, Q the Pauli exclusion operator, and $1/e$ the two-particle propagator in nuclear matter. One of the sources of density dependence in Eq. (4) is the operator Q, defined by

$$Q(\mathbf{k},\mathbf{P},k_F)=\begin{cases}1 & \text{if } k_{1,2}>k_F\\ 0 & \text{otherwise,}\end{cases} \tag{5}$$

with $k_{1,2}$ being the magnitudes of the momenta in Eq. (3). That is, Q prevents scattering into occupied intermediate states.

The exclusion operator depends not only on the magnitude of the total and relative momentum of the two nucleons but also on their directions, as it can be seen from Eq. (3). This angular dependence leads to couplings between partial waves which do not conserve angular momentum and which make numerical computation rather involved. To avoid these difficulties, it has become customary to abolish its angular dependence by replacing Q with the so-called angle-averaged Pauli projector

$$\bar{Q}(k,P,k_F)=\frac{\int d\Omega\, Q(\mathbf{k},\mathbf{P},k_F)}{\int d\Omega}, \tag{6}$$

with $d\Omega$ the solid angle element associated with the vector $\mathbf{k}$.

Employing a partial wave expansion for the two-particle states, the matrix elements for the (exact) Pauli projector can be written as

$$\begin{aligned}&\langle (l'S)J'M|Q(k,P,k_F)|(lS)JM\rangle\\ &\quad=\sum_{m_l,m_S}\langle l'm_lSm_S|J'M\rangle\langle JM|lm_lSm_S\rangle\\ &\qquad\times\langle l'm_l|Q(k,P,k_F)|lm_l\rangle,\end{aligned} \tag{7}$$

where

$$\begin{aligned}\langle l'm_l|Q(k,P,k_F)|lm_l\rangle=&\int d\Omega\, Y^*_{l'm_l}(\Omega)Y_{lm_l}(\Omega)\\ &\times\Theta(|\mathbf{P}+\mathbf{k}|-k_F)\Theta(|\mathbf{P}-\mathbf{k}|-k_F).\end{aligned} \tag{8}$$

Due to the presence of the step function which limits the integration domain, the Pauli operator in general will not be diagonal in l and J while maintaining parity conservation. The condition

$$|\mathbf{P}\pm\mathbf{k}|>k_F \tag{9}$$

implies the restriction on the angle θ between $\mathbf{k}$ and $\mathbf{P}$

$$\frac{k_F^2-P^2-k^2}{2Pk}\leqslant\cos\theta\leqslant\frac{P^2+k^2-k_F^2}{2Pk}, \tag{10}$$

thus breaking the orthogonality between l and l' that would otherwise be present in Eq. (8). In a frame where the total momentum $\mathbf{P}$ points in the direction of the z axis, the azimuthal integration with respect to the angle ϕ makes the matrix elements of Eq. (7) diagonal with respect to the projection quantum number m_l. Spin conservation makes this true also for M in Eq. (7).

We performed the exact calculation for angular momentum states up to $J=6$, after which we used the Born approximation, which is independent of the Pauli operator. To solve the Bethe-Goldstone equation within the new scheme, we first regrouped the partial waves by ST channels. For example, when $S=0$ and $T=1$, the states which can potentially couple (up to $J=6$) are $^1S_0$, $^1D_2$, $^1G_4$, and $^1I_6$. The value of M determines the actual number of allowed couplings. For $M=0$, all of these states will couple. Thus the size of the matrix to be inverted increases considerably with respect to the angle-averaged calculation. Due to the presence of new matrix elements from Eq. (7) appearing in the partial wave expansion of Eq. (4), the solution of the Bethe-Goldstone equation now has the general structure (suppressing for simplicity momentum and energy dependence)

$$G=\langle lJ|G_M^{ST}|l'J'\rangle. \tag{11}$$

III. RESULTS AND DISCUSSION

Two kinds of effects result from the above treatment, as is evident from the expression in Eq. (11), the dependence of the G-matrix elements on M, the existence of otherwise for-

TABLE I. The $J=J'=2$, $l=1$, $l'=3$ matrix element (in units of GeV^{-2}). The calculations are performed at nuclear matter density, or a Fermi momentum equal to 1.35 fm^{-1}.

Standard result	M	Exact Q calculation
	0	$0.114+0.010i$
	± 1	$0.112+0.013i$
	± 2	$0.104+0.021i$
$0.1093+0.0155i$	M averaged	$0.1092+0.0154i$

bidden transitions. We will next examine the size of the two items above.

Table I shows on-shell matrix elements at 200 MeV and nuclear matter density, corresponding to a Fermi momentum of 1.35 fm^{-1}. Because our emphasis here is on the Pauli exclusion operator Q (Dirac effects are not relevant for the present discussion), calculations are done within conventional Brueckner theory (BHF) [23–26]. In Table I we show to what degree the new G-matrix elements depend on M. The transition we have chosen as an example is $^3P_2 \rightarrow {}^3F_2$ (associated with ε_2) where $J=J'=2$, $l=1$, and $l'=3$. On the left-hand side is the value of the standard matrix element calculated with the spherically averaged Q. On the right-hand side are the new G-matrix elements for each of the M values allowed in this transition. The dependence on M is noticeable, and actually quite large in the imaginary part. The last number in the right-hand column of the table shows the result when the average over M is taken for the matrix elements in that column. Clearly, the difference with the corresponding standard matrix element is very small. This is not surprising, since the difference amounts to angle averaging the solution of the integral equation instead of the kernel.

Table II shows the M dependence for some of the most important matrix elements in NN scattering. The close similarity between the standard result and the M-averaged result exemplified in Table I remains valid for all matrix elements and is not shown again. To facilitate the connection with the familiar NN states, we use the usual spectroscopic notation for partial waves. It should be kept in mind, though, that only the average over M can be directly related to the standard $^{2s+1}L_J$ partial wave indicated on the left for each case.

TABLE II. The M dependence of some $\langle lJ|G_M^{ST}|lJ\rangle$ matrix elements (units of GeV^{-2}). The calculations are performed at nuclear matter density, or a Fermi momentum equal to 1.35 fm^{-1}.

Partial wave	M	Value
$^3P_1$	0	$0.791-0.196i$
	± 1	$0.851-0.145i$
$^3D_1$	0	$0.587-0.049i$
	± 1	$0.609-0.161i$
$^3D_2$	0	$-0.891-0.140i$
	± 1	$-0.851-0.150i$
	± 2	$-0.872-0.225i$
$^3F_3$	0	$0.0881-0.0009i$
	± 1	$0.0879-0.0015i$
	± 2	$0.0879-0.0017i$
	± 3	$0.0877-0.0024i$
$^3F_4$	0	$-0.0474-0.0006i$
	± 1	$-0.0472-0.0007i$
	± 2	$-0.0478-0.0006i$
	± 3	$-0.0486-0.0011i$
	± 4	$-0.0488-0.0014i$

TABLE III. Some higher-order tensor transitions (in units of GeV^{-2}). The calculations are performed at nuclear matter density, namely a Fermi momentum equal to 1.35 fm^{-1}.

J,J',l,l'	Standard result	Result from exact Q calculation
0,2,0,2	0	$-0.0184+0.0052i$
0,4,0,4	0	$(-0.119-0.050i)\times 10^{-3}$

By sampling cases with J from 1 to 4, we want to see to what degree the M dependence is related to the absolute size of the matrix element, which decreases at large J. In general, a stronger M dependence is seen in the imaginary parts. For the (much larger) real parts, the M dependence is stronger for the lower J's where the amplitudes themselves are larger. The change is close to 10% for $^3P_1$ and drops to about 3% for $^3F_4$.

In Table III, we show some of the tensor transitions generated by the new coupling mechanism, namely those that are nondiagonal with respect to J. Although they tend to be small compared to the standard matrix elements, they allow scatterings that are not described by the matrix elements of the usual nuclear force operators. The impact on (p,p') transitions needs further consideration.

In order to perform a scattering calculation within the theoretical base available at this time, it is necessary to use a G matrix that is compatible with the standard framework for constructing the effective interaction, namely, the usual angular momentum conserving partial waves. This implies averaging out the M dependence, as well as neglecting the explicit contribution of transitions such as those in Table III. Some trace of the new coupling mechanism will still be present, but only to a very limited extent. Under these conditions, we have done a DWBA calculation of (p,p') inelastic scattering and observed it to be practically insensitive to the small corrections to the standard interaction such as those shown in the last row of Table I.

Even though at this time we are not able to perform a scattering calculation with the full G matrix of Eq. (11), we can estimate the probable impact on proton-nucleus scattering by comparing the size of the M dependence, as shown in Table II, with the size of the medium effects which are typically applied to the interaction. These changes from conventional medium effects are known [2,3]. We can then indirectly set the scale for these new effects relative to the experimental errors and the typical discrepancies between theory and experiment.

To make this comparison, we display in Table IV the typical size of medium effects on some of the matrix elements shown in Table II. The two in-medium calculations being compared include effects from conventional Brueckner theory [23–26] and from the Dirac-Brueckner approach

TABLE IV. Medium modifications on some of the matrix elements shown in Table II. The in-medium calculations (BHF and DBHF) are performed at nuclear matter density (Fermi momentum equal to 1.35 fm^{-1}) and with the spherically averaged Pauli operator.

Partial wave	Type of calculation	Matrix element (GeV^{-2})
$^3P_1$	Free space	$0.695-0.262i$
	BHF	$0.830-0.162i$
	DBHF	$1.066-0.269i$
$^3D_1$	Free space	$0.653-0.232i$
	BHF	$0.609-0.125i$
	DBHF	$0.574-0.117i$
$^3D_2$	Free space	$-0.816-0.388i$
	BHF	$-0.875-0.181i$
	DBHF	$-0.712-0.116i$
$^3F_3$	Free space	$0.0875-0.0036i$
	BHF	$0.0879-0.0018i$
	DBHF	$0.0929-0.0019i$

(DBHF) [27–30], respectively. Although the changes associated with the inclusion of conventional medium effects illustrated in Table IV appear to be uncorrelated with those associated to the treatment of the Pauli exclusion operator shown in Table II, we observe that the size of the M dependence arising from the Pauli exclusion operator is about half the size of the largest (DBHF) conventional medium effects and comparable to the BHF effects. We conclude that the neglect of this M dependence in (p,p') calculations may represent a significant theoretical uncertainty that could be important with respect to a reliable detection of new medium effects.

Finally, we point out that the energy denominator e in Eq. (4) carries an angular dependence [through the energies of two particles with momenta as in Eq. (3)] which is also handled by angle averaging. It should be noted, though, that when nonrelativistic kinetic energies are used, the angular dependence disappears entirely. [This is also a consequence of the effective mass approximation. See, for instance, Eqs. (3.8)–(3.11) of Ref. [19] or Ref. [31]]. This cancellation does not occur when relativistic kinematics are employed, as is the case in this work. The effect of removing the angle average from the energy denominator of Eq. (4) remains to be explored, although we expect it to be smaller than relativistic kinematical corrections.

IV. CONCLUSIONS

We have solved the G-matrix equation in nuclear matter at 200 MeV keeping the full, angular-dependent expression for the Pauli exclusion operator. The corrections to the standard matrix elements after averaging over M are very small and do not produce detectable differences in the scattering observables.

While the role of the M dependence and the existence of the otherwise forbidden transitions in (p,p') calculations are not assessed directly, we provide estimates for the size of the M dependence of the G-matrix elements. We observe that the differences between various components of the new G matrix are comparable in size to standard medium effects brought about by BHF or DBHF calculations. We conclude that this new degree of freedom in the G matrix may be important, especially when establishing an accurate baseline for a reliable observation and characterization of new medium effects [1].

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