Degrees of deformation at scission and correlated fission properties of atomic nuclei

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The degrees of the deformation of atomic nuclei at scission configurations of the mass-symmetric and mass-asymmetric fission modes are studied. The β shape elongation of the fissioning nucleus, associated with the asymmetric fission is found to be constant for $A_f = 213-260$, while that associated with the symmetric fission is larger but also constant for a wide range of fissioning nuclei ($A_f = 205-260$) except for the low-energy induced and spontaneous fission of heavy nuclei with $A_f = 245-262$ where a change of the nucleon number by one unit is found to cause a very rapid change in β. A systematic correlation between the scission deformation and the fission properties such as the mass-yield distribution and the TKE are found. The constancy of the β gives rise to new formulas of $\mathrm{TKE}_{\mathrm{sym}} = 0.1173\times(Z_f^2/A_f^{1/3}) + 7.5$ MeV for symmetric fission, and of $\mathrm{TKE}_{\mathrm{asym}} = 0.1217\times(Z_f^2/A_f^{1/3}) + 3.5$ MeV for asymmetric fission. The physical origin of the well-known simple linear function for the TKE released in the fission process is found to be the invariance of the degrees of the elongation of scissioning nuclei. The mass dispersion of fission products was found to have a direct correlation with the β value; as β becomes larger, the width of the mass yield distribution becomes wider. Our understanding of the spontaneous fission properties of heavy nuclides in the ^{258}Fm region is presented. In this region, two different scission configurations, one giving $\beta \sim 1.53$ which is the characteristic value for the asymmetric deformation and the other giving $\beta \sim 1.33$ which is for the symmetric deformation, are found.

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I. INTRODUCTION

Fission study is of significant physical interest because it represents a large scale rearrangement process of numerous nucleons. The most fundamental observable fission properties are the mass yield distribution and the total kinetic energy distribution of fission products. For the mass yield distribution, a systematic change in the shape from the symmetric one in the lead-bismuth region to a triple-humped shape in the actinium-radium region, and then to a double-humped one in the thorium and heavier elements is known [1,2]. For the average total kinetic energy (TKE), a simple dependence on the nuclear parameter, $Z_f^2/A_f^{1/3}$, is empirically known [3]. The formation mechanism of those observed mass yield curves, and the physical origin of the simple linear TKE function in such a complicated process as fission still remain as unsolved puzzles today, more than 60 years after the discovery of fission.

The experimental observations related with the entrance channels of fission such as fission isomers [4] and intermediate structure in fission resonances [5] have been theoretically elucidated by the existence of a double humped potential barrier [6]. The observed fission barrier heights were also theoretically understood by lowering of the saddle height with an inclusion of the reflection asymmetric degree of freedom in the potential energy calculation [7] which also helped understand the mass-asymmetric fission. Nevertheless, we are still far from a satisfactory understanding of the exit-channel properties of fission which are closely related with the observed mass yield and TKE distributions.

In the present work, the deformation properties of the scissioning nucleus in mass-symmetric and mass-asymmetric processes have been investigated by a systematic analysis of observed TKE values for 47 fissioning systems. As a result of such a systematic study, invariance of the scission deformation is found and presented in Sec. II. The empirical formulas for the total kinetic energy that will replace the TKE systematics of Viola *et al.* [8] have been derived from the constant β-value (scission deformation), which are given in Sec. III. Section IV presents the results for the correlation between the β and the FWHM (full width at the half maximum) of the mass yield curve. These findings help understand not only the formation mechanism of the fission mass-yield distribution but also the drastic changes in fission properties of very heavy element nuclides observed in nuclei around ^{258}Fm [9–14], as discussed in Sec. V. It is to be noted that some results in Sec. II have been partly published in Ref. [15].

II. DEGREES OF DEFORMATION AT SCISSION

A. Definition of shape elongation

As a first order approximation, the distance between the charge centers of two touching nuclei (Z_1, A_1) and (Z_2, A_2) at scission can be estimated from the total kinetic energies, $\mathrm{TKE}(A_1, A_2)$, released in a mass split producing A_1 and A_2 fragments. The $\mathrm{TKE}(A_1, A_2)$ value is considered to be the sum of the Coulomb interaction energy between the two separating fragments at scission, the nuclear attractive force present between the touching fragments, and the pre-scission kinetic energy arising from the collective motion from saddle to scission. It has been pointed out that the nuclear attractive force and pre-scission kinetic energies are nearly canceled near scission [16], and, thus, the measurable value of the $\mathrm{TKE}(A_1, A_2)$ is approximately equal to the Coulomb repul-

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sion energy. Recently, a dynamical calculation using the two-dimensional Langevin equation under the assumption of one-body dissipation, showed that the pre-scission kinetic energy was about 7 MeV irrespective of the mass of the fissioning nuclide and excitation energy [17]. It amounts to approximately 3–5 % of the total kinetic energy for fissioning systems studied in this work. Accordingly, the distance between the two charge centers of two fragment nuclei at scission can be estimated from the following equation:

$$D(A_1,A_2)=Z_1\times Z_2\times e^2/\mathrm{TKE}(A_1,A_2), \qquad (2.1)$$

where $D(A_1,A_2)$ is the distance between the two charge centers of the nascent fragments. In order to allow a comparison of the degree of scission deformation among various fissioning nuclei, a shape elongation, β, is defined as

$$\beta=D(A_1,A_2)/D_0(A_1,A_2), \qquad (2.2)$$

where $D_0(A_1,A_2)$ is the distance between the charge centers of two touching spherical fragments for which the radius is given by $R=1.17\times A_i^{1/3}$ $(i=1,2)$ fm. The quantity β, hence, is a measure of the degree of deformation for the fissioning nucleus at scission, namely, a parameter that shows the amount of deviation from the two spheres.

For evaluating β from the experimentally observed TKE for the mass split producing fragments A_1 and A_2, the average charges Z_1 and Z_2 of the two fragments are needed. It is known from the past experience that the ECD (equal charge displacement) model works well for estimating the most probable charge in low energy fission, but for higher energy fission the UCD (unchanged charge distribution) model works better [2]. For low-energy, light-particles, such as neutron and proton induced fission, there are a number of experimental data [18–21] and also some elaborate empirical equations proposed [22] for the most probable charge. It has been indicated that the difference in β values derived by the UCD and the experimentally measured charges is at the largest ~1% of the absolute β values [15]. In the following, shape elongations for scissioning nuclei of 47 fission systems with the excitation energy from 0 to more than 100 MeV are hence evaluated using the Z values expected from the UCD model.

B. Shape elongations as a function of the fragment mass

The β values for the low-energy (~15 MeV) proton induced fission of the ^{226}Ra, ^{232}Th, ^{238}U, ^{244}Pu, and ^{248}Cm as a function of the primary fragment mass number have been given in the previous paper [15]. The results indicated that the β values for scission configurations leading to the products in the symmetric mass division region remain constant with the mass split. The β values for the asymmetric mass division are slightly fragment mass dependent going from $\beta=1.55$ at $A=160$ to the minimum of 1.52 at $A=134$. Those observations are for nuclei fissioning both symmetrically and asymmetrically with comparable occurrence, and producing a fragment mass-yield curve with a double-humped shape. The results for nuclei fissioning symmetrically and producing a broad symmetric mass-yield curve with a single peak centered at $A_f/2$ are given in Fig. 1. These β values obtained from the analysis of the experimental TKE(A_1,A_2) data in literature [23–25,27,28] are for the high-energy p, ^{3}He, ^{4}He, and heavy-ion induced fissions. They are plotted as a function of the mass ratio (volume ratio) of the pair fragments, A_1/A_2. The constancy of the β value for the scission configuration leading to the symmetric mass division mode is better seen. The values indeed stay constant at 1.60 for a wide range of A_1/A_2 in the symmetric fission of nuclei with $Z<84$. In the heavy-ion induced fission of heavier nuclei, it is 1.65 at symmetry and slightly decreases to 1.60 at $A_1/A_2>1.5$. The difference of 0.05 amounts to only 3% of the β value of 1.65.

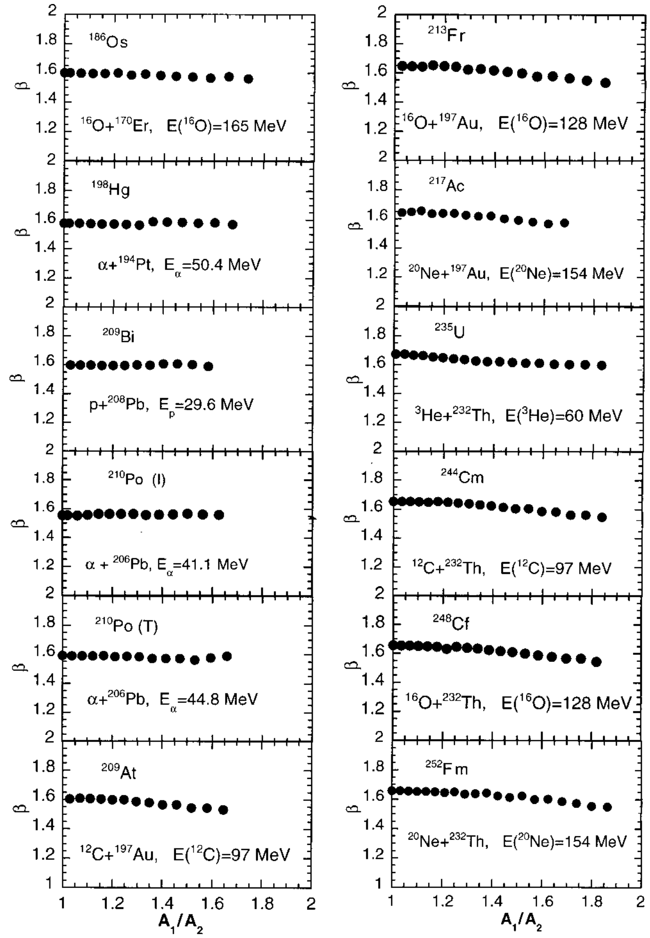

FIG. 1. The shape elongation of the scissioning nucleus leading to each mass split is plotted as a function of the ratio of nuclear volumes of pair fragments, A_1/A_2. The results are for the high-energy p, ^{3}He, ^{4}He and heavy-ion induced fission of various nuclei producing broad symmetric mass-yield curves of a single peak.

C. Elongations of scission shapes for asymmetric and symmetric fission

The β values for the scission configurations leading to the asymmetric mass division that produce the heavier fragment mass of $A_H=140$ are plotted in Fig. 2 *versus* the mass number of the fissioning nucleus A_f (hereafter referred to as β_{asym}). The reason for choosing this mass as the representa-

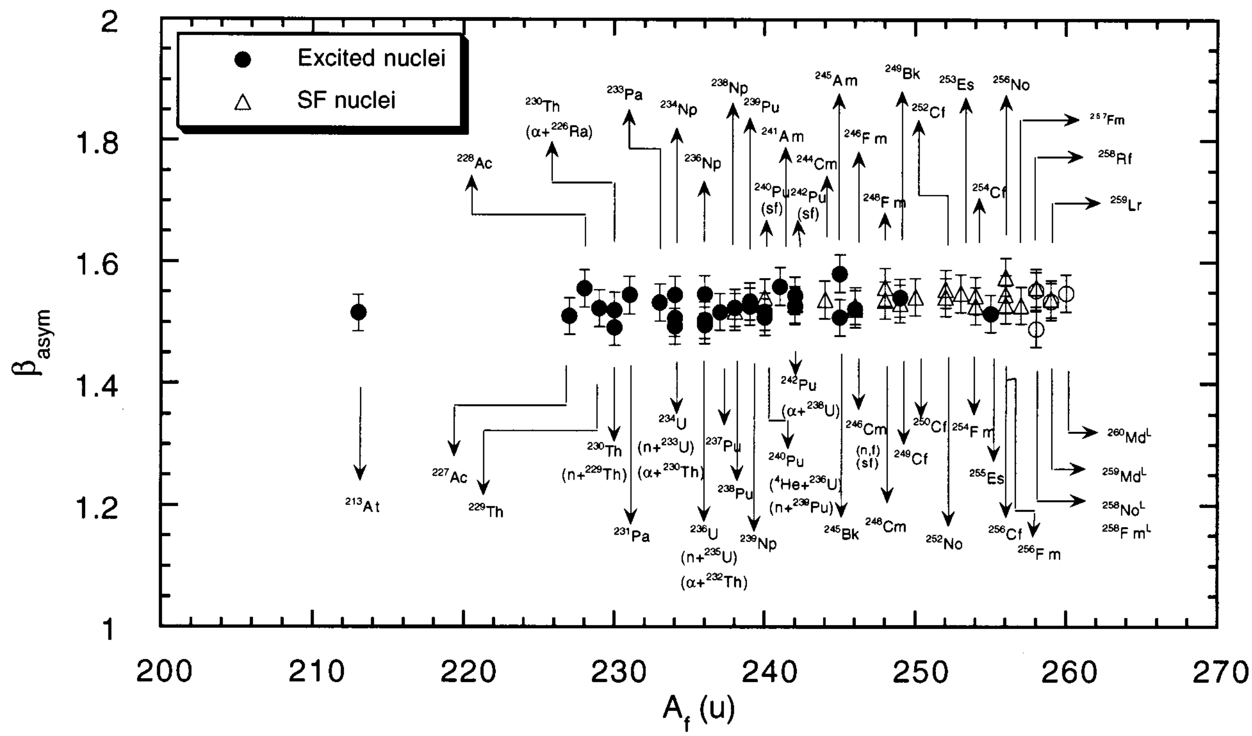

FIG. 2. Shape elongations of scissioning nuclei for the asymmetric mass split are plotted as a function of the fissioning mass A_f. Each data point is indicated by the name of the fissioning nucleus. Solid circles are results corresponding to the induced fission while open triangles for spontaneous fission. Open circles are those corresponding to the low-TKE component observed in the *bimodal fission* process of heavy actinides.

tive one for the asymmetric fission is that (1) $A_H=140$ is mostly at the peak of the heavier asymmetric mass yield distribution, (2) it is the mean mass of the heavier asymmetric mass yield curve in most cases, and (3) the results in Ref. [15] indicate that a choice of other mass numbers among asymmetric fission products does not essentially alter the β_{asym} in Fig. 2. The data of filled points are for low-energy particle-induced fission while open triangles are for spontaneous fission. They were obtained via the analysis of the experimental TKE(A_1,A_2) data in Refs. [1,25,26,29–50]. Open circles are those for the low-TKE components reported for the *bimodal fission* in the ^{258}Fm region [12]. As seen from the figure, the β_{asym} values are surprisingly similar for both particle-induced and spontaneous fission of all fissioning nuclei, indicating that the degree of deformation at scission for the asymmetric mass division is independent of the excitation energy of the fissioning nucleus. The standard deviation of all data points in Fig. 2 is 0.02 which is comparable to the range of the variation of β_{asym} values for different mass splits from $A=132$ to 160 in Ref. [15]. The average β_{asym} value for the asymmetric mode can be hence summarized as 1.53 ± 0.02. It is also interesting to note that the lower TKE component reported for the *bimodal fission* gives the β value of about 1.53 (open circles). This agreement of the β value strongly suggests that the ordinary asymmetric fission is competing with the strong shell-affected symmetric fission in the ^{258}Fm region, as suggested in the theoretical work by Ćwiok *et al.* [51]. Further discussion on this point is given in Sec. V.

In Fig. 3, the β values evaluated for the symmetric fission process in 47 fissioning nuclei from ^{205}Bi to ^{262}Rf are plotted as a function of the mass number of fissioning nuclides, A_f (hereafter referred to as β_{sym}). It was evaluated from the TKE(A_1,A_2) of the pure symmetric mass split producing fragments of $A_1=A_2=A_f/2$. Solid points are for low-energy fission ($E_X<30$ MeV), open circles for high-energy fission ($E_X>65$ MeV), open triangles for spontaneous fission and open squares are for the high-TKE component reported for *bimodal fission*. The following observations can be summarized from the figure. (1) The β_{sym} value is constant and about 1.65 throughout the whole region of the fissioning nuclei from $A_f=205$ to 245 beyond which it gradually becomes smaller in the spontaneous fission toward the value of 1.33 that corresponds to the abnormally high TKE release observed in the spontaneous fission of very heavy nuclides in the ^{258}Fm region. Therefore, two extreme types of the shape elongation exist in the mass-symmetric fission mode. The one is for the low-energy fission in the region from preactinide up to the actinide until $A_f\sim245$ (shown by solid circles) and for high-energy fission (shown by open circles) in a wide range of A_f, where the β_{sym} is nearly a constant of ~1.65. The other is for the spontaneous fission in the region of $A_f\sim260$, where the β_{sym} again becomes nearly a constant of a very smaller value of ~1.33. The smaller elongation of the fissioning nucleus in the very heavy A_f region (A_f ~260) is probably related to the effects of fragment shells of $N=82$ and/or $Z=50$ on the mass-symmetric deformation

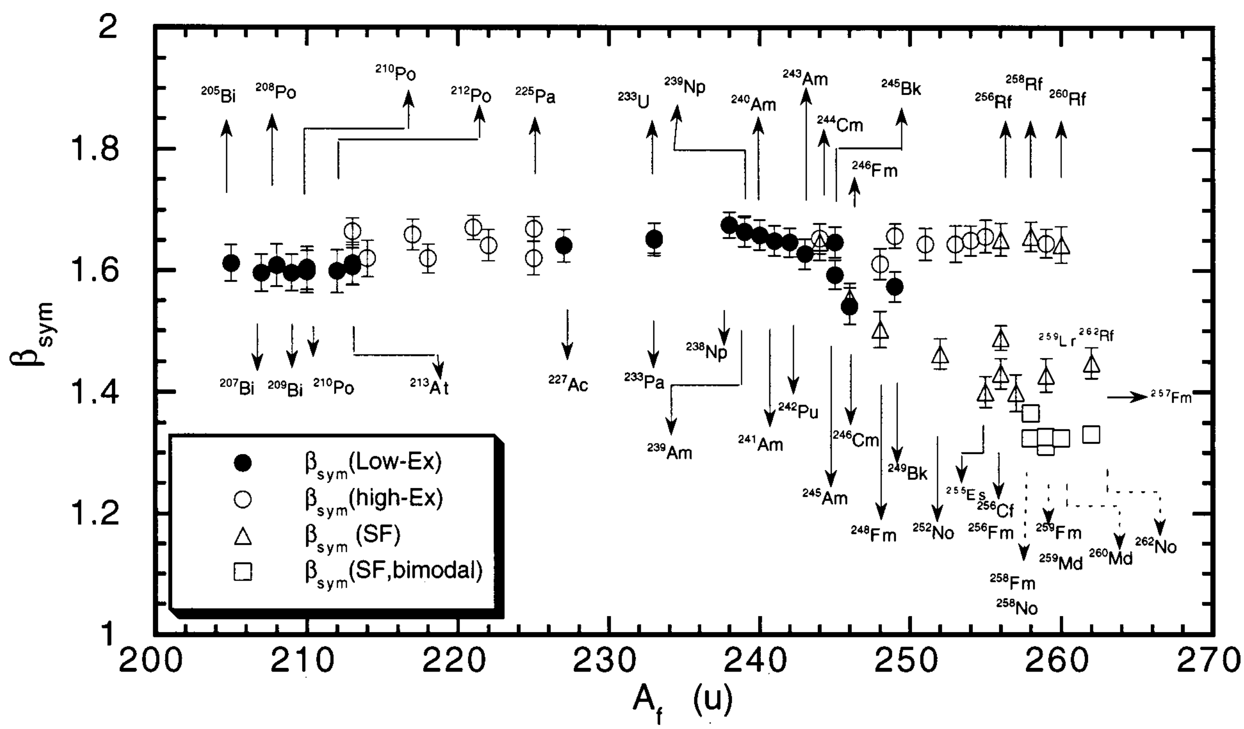

FIG. 3. Shape elongations of scissioning nuclei that experienced mass-symmetric deformation are plotted as a function of the fissioning mass A_f. Data points are labeled with the name of the fissioning nucleus. Solid circles are the results for the low energy nuclei ($E_X < 30$ MeV), open triangles are for spontaneous fission, open circles are those of the high excitation energy nuclei ($E_X > 65$ MeV). Open squares show the results of the high-TKE component observed in the *bimodal fission* of very heavy nuclei in the ^{258}Fm region.

motion. (2) In the region of $A_f > 245$, the β_{sym} values (open circles) becomes $\sim$1.65 in the high energy fission observed in heavy-ion induced reactions, probably due to washing-out of the shell structures in hot nuclei. (3) In the same A_f region, at low energy, evident difference in β values (shown by solid circles, open triangles, and open squares) exists for nuclides in the actinide and heavy actinide regions. This may indicate that the nuclear properties of very heavy nuclei cannot be simply or directly extrapolated by the knowledge from light elements, as recently demonstrated by some related theoretical findings using dynamical calculations by the Polish groups [52,53].

III. NEW FORMULAS FOR PREDICTING TKE RELEASE IN FISSION PROCESS

In the present systematic study, it is demonstrated that if the shell-affected symmetric fission observed in the low-energy fission of heavy nuclides with $A_f > 245$ are excluded, there are only two kinds of β values for tearing off a big nucleus into two smaller portions: one is $\beta = 1.65$ for nuclei in the symmetric fission mode, and the other is $\beta = 1.53$ for those in the asymmetric fission mode.

As the total kinetic energy can be approximated by the Coulomb repulsion between the nascent fragments at scission, the TKE for the symmetric mass division is hence given by the following equation:

$$\mathrm{TKE}_{\mathrm{sym}} = e^2 \times \rho^2 \times (A_f/2)^2 / D_{\mathrm{sym}}, \tag{3.1}$$

where ρ is the charge density of fission fragments given by the UCD assumption, $\rho = Z_f/A_f$. The D_{sym} is the distance between the two charge centers of the nascent fragments at scission configuration, $D_{\mathrm{sym}} = \beta_{\mathrm{sym}} \times D_0$. Then, one obtains

$$\mathrm{TKE}_{\mathrm{sym}} = e^2 \times (8 \times r_0 \times (1/2)^{1/3} \times \beta_{\mathrm{sym}})^{-1} \times (Z_f^2/A_f^{1/3}). \tag{3.2}$$

With the value of $r_0 = 1.17$ fm, and $\beta_{\mathrm{sym}} = 1.65$, and with an addition of a constant term b to this equation, the above equation is fitted to the presently determined TKE data [56] for determining the value of b. The TKE formula for the symmetric fission thus derived is

$$\mathrm{TKE}_{\mathrm{sym}} = 0.1173 \times (Z_f^2/A_f^{1/3}) + 7.5\,(\mathrm{MeV}). \tag{3.3}$$

A comparison of the present TKE formula (indicated by a bold line) with the available experimental data (open circles) is given in Fig. 4, where the present formula reproduces very well the experimental data. The TKE formula by Viola *et al.* [8] and the theoretical prediction for the symmetric fission by the dynamical calculations based on the liquid drop model by Davies, Sierk, and Nix [54,55] are also shown via a broken and a dotted line, respectively. The results in Fig. 4 indicated that the dynamical models taking into account the surface plus window dissipation for the time dependence of the distribution function in phase space of collective coordinates and momenta predicted the TKE values which were approximately in agreement with those given by Eq. (3.3) (in the

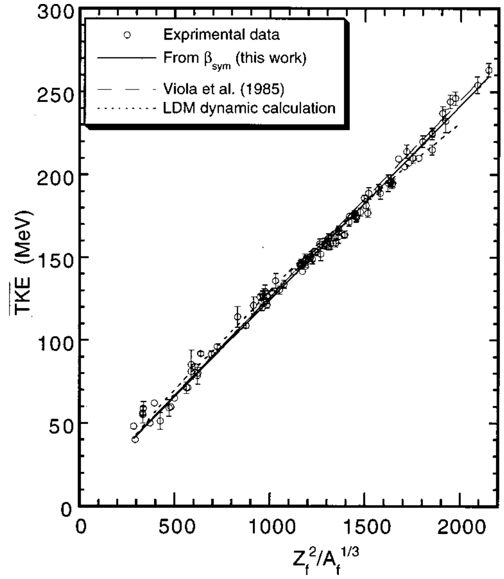

FIG. 4. The solid line shows the results of the new TKE formula for the symmetric fission mode. Circles are experimental TKE data. The broken line is from the systematics of Viola *et al.* Theoretical results of the dynamical calculation based on liquid drop model by taking into account the surface-plus-window dissipation by Nix, Davies, and Sierk are indicated by the dotted line.

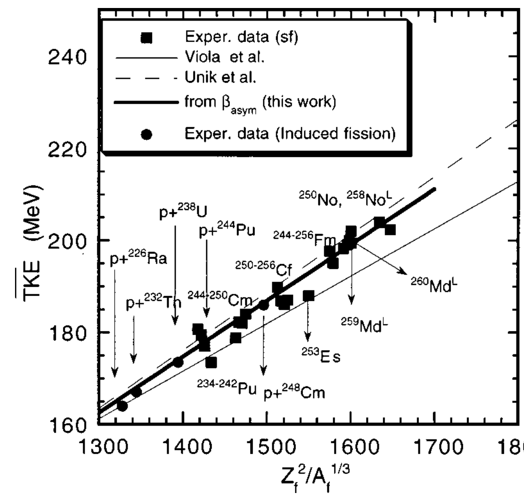

FIG. 5. The bold line shows the results of the TKE formula for the asymmetric fission. Solid squares are the experimental TKE data which are taken from Ref. [11] but modified to study the results of the asymmetric fission process. Solid circles are results from the low-energy proton-induced fission as indicated in the figure. The result of Viola *et al.* is indicated by a thin line and that of Unik *et al.* by a broken line.

region of $Z_f^2/A_f^{1/3}<1800$). In this theoretical calculation, the pre-scission kinetic energy varies as a function of the $Z_f^2/A_f^{1/3}$ and amounts to about 10 MeV, e.g., in the region of $Z_f^2/A_f^{1/3}=1400$, this is somewhat in contradiction to the present assumption of the pre-scission kinetic energy being negligible. Nevertheless, this approximate agreement between the solid and the dotted lines implies that the invariant value of $\beta=1.65$ is mainly a consequence of the liquid-drop-like property of the atomic nucleus. It is worthy, however, to note that as demonstrated in Fig. 3, the symmetric fission observed in the spontaneous fission of heavy actinides and transactinides is strongly affected by the fragment shells, and more experimental data are needed for further systematic understanding.

For the total kinetic energy released in the asymmetric fission process, the TKE expression corresponding to Eq. (3.2) becomes more complicated,

$$\mathrm{TKE}=(e^2\times 140\times(A_f-140)\times(Z_f/A_f)^2)\times(\beta_{\mathrm{asym}}\times r_0\times[140^{1/3}+(A_f-140)^{1/3}])^{-1}. \tag{3.4}$$

With two approximations of $(140^{1/3}+(A_f-140)^{1/3})\simeq 1.587\times A_f^{1/3}$ and $(140\times(A_f-140)/A_f^2)\simeq 0.24$ for the range of $A_f=225-260$ (error introduced by this approximation is at most $\sim$4%), it becomes a simple formula as

$$\mathrm{TKE}_{\mathrm{asym}}=0.1861\times\beta_{\mathrm{asym}}^{-1}\times(Z_f^2/A_f^{1/3}). \tag{3.5}$$

With the asymmetric shape elongation β_{asym} of 1.53 the coefficient becomes 0.1217 for this function. Then, with an addition of a constant term the linear function is fitted to the experimental TKE data reported in literature and to those recently determined using a double velocity TOF method [56] and the following TKE function for the asymmetric fission mode is derived:

$$\mathrm{TKE}_{\mathrm{asym}}=0.1217\times(Z_f^2/A_f^{1/3})+3.5\,(\mathrm{MeV}). \tag{3.6}$$

The experimental data for the asymmetric fission are plotted in Fig. 5 and compared with the new TKE formula. The data shown by solid circles are from our experiment [56], while the data shown by solid square are taken from Ref. [11]. The dashed line is the one proposed by Unik *et al.* [48], the thin line represents the TKE formula of Viola *et al.* The new TKE formula is indicated by a bold line which reproduces the experimental data well.

It was pointed out already in early 1960s [3] that the average total kinetic energies observed in various fission systems approximately fall on a straight line if they were plotted as a function of $(Z_f^2/A_f^{1/3})$. The equation of the straight line that fits best to the existing TKE data has been changed several times as the dimension of nuclear fission studies developed in the directions of A_f, excitation energy and angular momentum. The most recent TKE formula presented by Viola *et al.* [8] is

$$\mathrm{TKE}=0.1189\times(Z_f^2/A_f^{1/3})+7.3\,(\mathrm{MeV}). \tag{3.7}$$

From a comparison with the presently derived TKE formulas (3.3) and (3.6), one sees that the TKE function proposed by Viola *et al.* is in between the present two formulas for the symmetric and asymmetric fission, but closer to the one for the symmetric. It is known that the TKE formula proposed by Viola *et al.* works very well and has been used by both theorists and experimentalists for several decades. But it was obtained from a free fit to the experimental data, which led to some necessary modification of the formula when more experimental data became available. The difference between the previously existing TKE formula and the present new TKE formulas is that, the latter are derived from a certain physical quantity (β), hence not modifiable unless some new type of the scission deformation is observed. Finally, it is worth pointing out again that the simple linear expression of the average TKE as a function of $Z_f^2/A_f^{1/3}$ essentially reflects the invariance of the shape elongation of the fissioning nuclei at scission.

Since the values of the deformation parameters fixed in Eqs. (3.6) and (3.3) were obtained in Sec. II under the assumption of a negligible pre-scission kinetic energy, one expects a near zero value of the constant term from the fits. As shown in Figs. 2 and 3, the error of the β value is about 2%, which can be a reason for the appearance of this constant term. For the TKE_{asym} of Eq. (3.6), the constant term of 3.5 MeV is within the magnitude of the error. For TKE_{sym} of Eq. (3.3), it becomes a bit larger. It is caused in part by the following reasons. The value of $\beta_{sym}=1.65$ was obtained for the fission of atomic nuclei with the mass number heavier than 205 (see Fig. 3). For the fission of the nucleus lighter than 200 (until $A_f\sim 100$), the average β_{sym} value becomes a constant of 1.60 (the reason for this slight decrease is not clear). Using the $\beta_{sym}=1.65$ to fit all available fission TKE data (including the lighter A_f region of Fig. 4) hence leads to a larger additional term for Eq. (3.3). For this reason, the solid line in the heavier A_f region of Fig. 4 becomes slightly larger than the observed data points.

IV. CORRELATION BETWEEN SCISSION DEFORMATION AND WIDTH OF MASS-YIELD DISTRIBUTION

It is conceivable that the width of the mass-yield distribution is related with the total kinetic energy as pointed out by Brosa *et al.* [57]. The lower total kinetic energy means a larger elongation of the nuclear shape at scission and thus allows the larger chance of fluctuation of mass between the two separating fragments.

The results of the present work have demonstrated that the shape elongation is an effective parameter for comparing the degrees of deformation at scission of various fissioning nuclei. Thus, a further attempt was made to study the correlation between the FWHMs (full width at half maximum) of the mass yield curves and the β values of fissioning nuclei. The results are shown in Fig. 6 for the FWHM observed for the symmetric mass distribution plotted as a function of the β_{sym} value. The solid circles are for spontaneous fissions reported for heavy nuclides in the ^{258}Fm region and open triangles are for the low-energy ($E_p\sim 13$ MeV) proton induced fission reported for the Ra-Ac nuclides [26] and those from the present work [56,58]. In the figure, the range of the FWHM for the asymmetric mass yield peaks observed in the low-energy fission of light actinides is also indicated by a shaded bar at $\beta=1.53$. The figure explicitly shows the presence of a strong correlation between the FWHMs and the degree of the deformation of the scissioning nuclei. When β is $\sim$1.33, the FWHM of the fragment mass-yield distribution

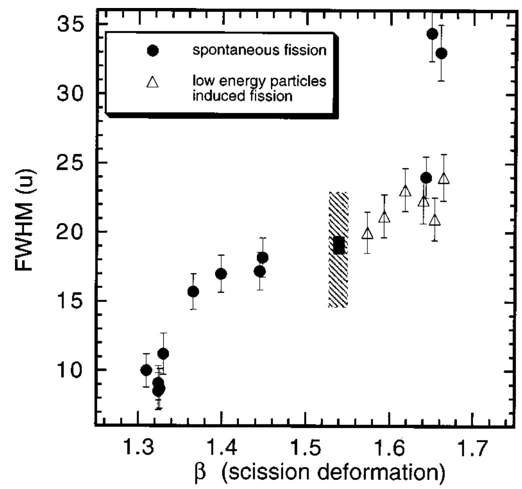

FIG. 6. Correlation between the β_{sym} values and the FWHMs of the symmetric mass-yield distributions of fission fragments. Solid circles are for spontaneous fission and open triangles for the low-energy proton-induced fission. The shaded bar indicates the variation of the asymmetric shape elongations and corresponding asymmetric FWHMs the average value of the asymmetric FWHMs for most of the asymmetric fission is indicated by a solid square in the shaded bar.

becomes $\sim$10 u, but as the scissioning nucleus is more elongated to $\beta\sim 1.65$, the FWHM, is broadened to $\sim$25 u. The two data points with FWHMs $\sim$35 u are for the fission of ^{256}Rf and ^{258}Rf, whose mass-yield distributions are somewhat asymmetrically double humped [59], whereas others are obtained from the apparently symmetric mass-yield distributions.

V. SCISSION SHAPE ELONGATION IN THE FISSION OF HEAVY NUCLIDES

As pointed out in Sec. II C and depicted in Fig. 3, the degree of the scission shape elongation β_{sym} starts decreasing as the fissioning mass becomes larger than $A_f=245$ from the value of about 1.65 observed in the symmetric fission of the lighter nuclides and in the fission of highly excited nuclides. Such deviations are more systematically plotted in Fig. 7 as a function of the neutron and proton number of the fissioning nuclide. The figure shows that the deviations are observed even for Am nuclides ($Z=95$) and it becomes conspicuous for $Z\sim 99$ and $N\sim 156$. It is to be noted that such deviations are the same in the fission of ^{256}Fm and ^{258}Fm regardless of whether it is induced by neutron absorption or spontaneous decay as learned from the results of the fission of ^{256}Fm [48,49] and ^{258}Fm [10,30].

These systematic variations of β_{sym} as a function of Z and N of the fissioning nuclide indicate that the extremely high TKE observed in the so-called *bimodal fission*, e.g., of ^{258}Fm [12–14], and the sudden variation of the mass yield curve to the symmetric narrow shape as summarized by Hoffman *et al.* and depicted in Fig. 8 are certainly related to the shell effects of either the fragment spherical shells and/or the neutron and proton shell effects of the fissioning nuclide.

As pointed out in Sec. II C, the lower average TKE ob-

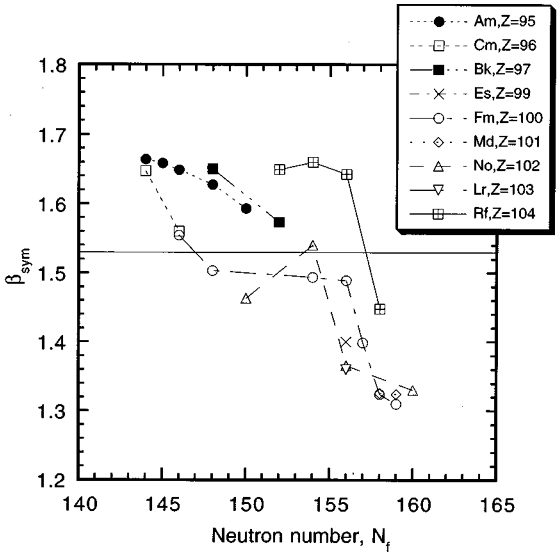

FIG. 7. The degrees of the scission deformation for symmetric fission β_{sym} of heavy nuclei ($Z>95$) plotted as a function of the number of neutrons and protons of the fissioning system. The different symbol is used for the different fissioning nucleus as shown in the figure. The symmetric shape elongation β_{sym} studied here are for spontaneous fission except for $Z=95$ and 96 which are for low-energy proton induced fission. The lines connecting the data points are drawn for guiding the eyes, while a solid line is drawn at $\beta=1.53$ which represents the scission deformation for the asymmetric fission path.

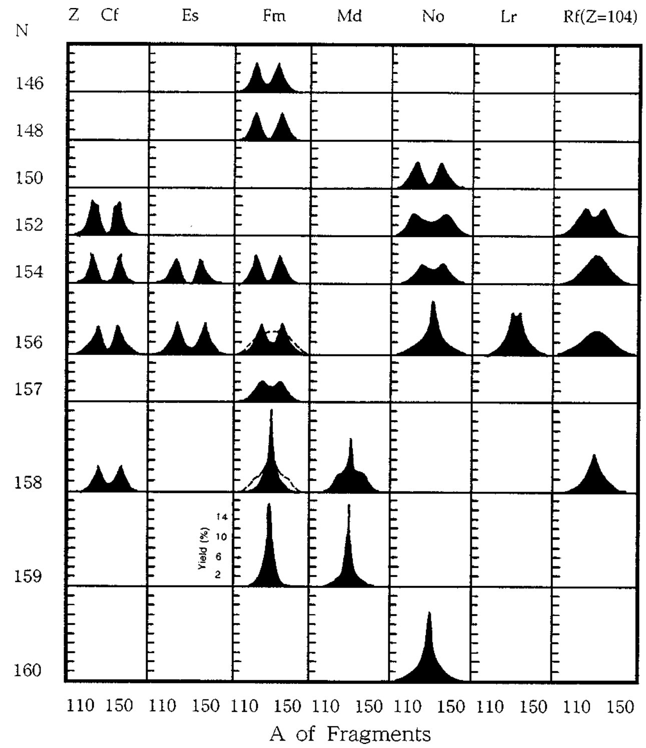

FIG. 8. Schematic representation of all known mass-yield distributions for spontaneous fission of trans-Bk isotopes. It is from Ref. [11] except for the dashed curves added in this work for neutron-induced fission.

served in the *bimodal fission* of nuclei around ^{258}Fm gives the β value of 1.53 which is the value observed in all the asymmetrically divided fission events. This agreement of the β value strongly suggests the presence of the asymmetric fission in the bimodal process. This inference is supported by the theoretical study by Ćwiok *et al.* [51]. According to their dynamical calculation of fission paths in multidimensional potential surface for ^{258}Fm, two main paths are present after passing through a single barrier (no second hump of the barrier); one proceeds to a symmetric valley with a more compact shape of the scissioning nucleus and the other to a more reflection-asymmetric valley with a more elongated shape at scission. Their calculation beautifully explains the observed data in the spontaneous fission of ^{258}Fm in which two types of fission events are present with comparable yields: the one taking the trajectory through the symmetric fission valley and ending with a compact scission configuration (a small β_{sym}), and the other going through the asymmetric fission valley and ending with a more elongated scission shape of $\beta_{sym}=1.53$. They performed similar calculation for the fission of ^{254}Fm and also found two paths with different thickness of the second barrier; the path leading to the symmetric fission valley has a thicker barrier to penetrate through and reaches the scission with less compact shape compared to the one in ^{258}Fm, and the other leading to a reflection-asymmetric fission valley has a thinner barrier and ends with an elongated scission shape. Their result for ^{254}Fm is in conformity with the experimental observations which give a double-humped asymmetric mass yield curve for the spontaneous fission of ^{254}Fm. In their calculations, deformation parameters for the multipolarities of $\lambda=3,5,6$ were fixed for the minimum potential energies, but recent calculations by Möller and Iwamoto [60] show the presence of a different saddle for each deformation path to the reflection-symmetric and to the asymmetric fission valley with similar predictions for elongation of the scission shapes. It is interesting to note that, according to their theoretical calculation, the relative height of the symmetric and the asymmetric barrier is reversed if the neutron number is increased by two units: namely, the saddle leading to the asymmetric valley is lower in ^{256}Fm whereas it becomes higher than the one leading to the symmetric valley in ^{258}Fm. Thus, their calculations also qualitatively explain the mass yield diagram in Fig. 8 [11]. These theoretical calculations are also in conformity with the experimental results observed for neutron-induced fission of ^{255}Fm(n,f) [49] and ^{257}Fm(n,f) [10]. In the former, a broad symmetric mass yield curve is observed while in the latter, the contribution of the asymmetric mass yields becomes more conspicuous as shown by dotted lines in Fig. 8 (the dotted lines for the neutron-induced fissions are inserted in the present work to the original figure reported in Ref. [11]).

From comparison between the two figures, Fig. 7 for the β_{sym} and Fig. 8 for mass yield curves, the following remark can be made. As the barrier penetration phenomena in spontaneous fission are so sensitively dependent on the height and shape of the fission barriers, the effects of nuclear shells are drastically demonstrated in the observed mass yield curves; they are changed from the ordinary double humped one to a single narrow one by an addition of one neutron to the fissioning nucleus. On the other hand, the shell effects on the scission shape in the symmetric fission valley are gradually manifested as a function of N and Z of the fissioning nucleus. It is interesting to note, however, that such shell effects are not manifested in the fission events that have gone through the deformation path in the asymmetric fission valley.

It is also worthy to point out that in the fission of $A_f \sim 280$, the two fission valleys, the symmetric and asymmetric, may merge into one, due to the mass symmetry leading to fragments $A \sim 140$ which is affected by the deformed shells as observed in the ordinary asymmetric fission [16]. Consequently, fission events resulting from the symmetric and asymmetric valleys may not allow to be disentangled, and the concept of the fission path in this region may need to be modified.

VI. SUMMARY

Degrees of the deformation of the atomic nucleus at scission were estimated from the experimental total kinetic energies in terms of shape elongation β for a wide range of fissioning nuclides of $A_f = 205-262$ with $Z_f = 83-104$.

The results indicate that the β value is nearly constant throughout the whole region of the fissioning nuclides within each fission mode of the mass asymmetry and the mass symmetry except for the low-energy symmetric fission in the region of the nuclides $A_f \geqslant 245$ where the symmetric fission process may be affected by the shell structures of the product nuclei. We may define the asymmetric and symmetric fission process as the fission deformation process that proceeds through the reflection-asymmetric and the reflection-symmetric fission valley, respectively. The β values are 1.53 ± 0.02 for the asymmetric deformation and 1.65 ± 0.03 for the ordinary symmetric deformation. But for the low-energy symmetric fission in heavy nuclides with $A_f \geqslant 245$, the β values are critically dependent on the nucleonic structures of the fissioning system.

Based on the constant β value, the new empirical equations were derived for the TKE release in fission process. They are $\mathrm{TKE}_{\mathrm{sym}} = 0.1173 \times Z_f^2/A_f^{1/3} + 7.5\,\mathrm{MeV}$, for the symmetric fission and $\mathrm{TKE}_{\mathrm{asym}} = 0.1217 \times Z_f^2/A_f^{1/3} + 3.5\,\mathrm{MeV}$ for the asymmetric fission. The physical origin of the simple linear expression for the TKE release is found to be the invariance of the shape elongation of fissioning nuclei at scission.

Correlations between the degree of scission deformation and the mass width (FWHM) of fission products were found; the large shape elongation of the fissioning nucleus reflects the large variation in the shape of the mass yield curve for the symmetric fission. This will help us understand the mass division phenomena of the nucleus and the formation mechanism of the mass-yield distribution.

Some knowledge that can help us understand the sudden changes in SF properties of heavy elements are learned from the present work. The lower TKE component observed in SF of heavy nuclei around Fm was found to exhibit the scission configuration of $\beta = 1.53$ which is the elongation observed for the asymmetric deformation path. This may imply that a fission path with a reflection-asymmetric fission valley exists in the *bimodal fission* of heavy nuclides. The shape elongation of the symmetric scission configuration of heavy nuclides is dependent on the Z and N of the fissioning nuclide, and it gradually becomes smaller beyond $A_f \sim 245$ and reaches the minimum value of $\beta = 1.33$ at $N \sim 160$. These observations are supported by the recent theoretical calculations [51,60–62].

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