Dynamical model for pion-nucleon bremsstrahlung

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A dynamical model based on effective Lagrangians is proposed to describe the bremsstrahlung reaction $\pi N \rightarrow \pi N \gamma$ at low energies. The $\Delta(1232)$ degrees of freedom are incorporated in a way consistent with both electromagnetic gauge invariance and invariance under contact transformations. The model also includes the initial- and final-state rescattering of hadrons via a T matrix with off-shell effects. The $\pi N \gamma$ differential cross sections are calculated using three different T-matrix models and the results are compared with the soft photon approximation, and with experimental data. The aim of this analysis is to test the off-shell behavior of the different T matrices under consideration.

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I. INTRODUCTION

In order to extract resonant parameters of the nucleon resonances (N^*) from the $\gamma N \rightarrow \pi N$ reaction, it is important to evaluate the background contribution to isolate the resonant peak. An important contribution to the background of this photoproduction reaction is provided by the final-state rescattering (FSI) of the πN system [1–4]. Consequently, we require knowledge of the T matrix in the off-momentum-shell regime (off-shell) to describe this rescattering process. That is, we need information about the amplitude $T[\vec{q}\,',\vec{q};z(\vec{q})]$ with $|\vec{q}| \neq |\vec{q}\,'|$, where z, which is the total energy of the πN system, is a function of the relative momentum $|\vec{q}|$ of the initial state. This particular rescattering amplitude is more properly called the half-off-shell T matrix. As is known, the T matrix can be generated by solving an integral equation of the Lippman-Schwinger or Bethe-Salpeter type by iteration of a pure phenomenological [5,6] potential, or from an effective potential based on meson-exchange models [7,8]. In all cases the so-called ''realistic'' interactions are fitted to reproduce the phase shifts in elastic πN scattering, which only depends on the on-shell ($|\vec{q}|=|\vec{q}\,'|$) values of the relative momenta. Thus elastic scattering is not useful to constrain the off-shell behavior of the T matrix, i.e., interactions which yield similar results for elastic scattering may have different behavior in the off-shell regime.

Another reaction where the πN off-shell T matrix is required is $\pi N \rightarrow \pi N \gamma$ bremsstrahlung. This process has been studied [9–11] within the soft-photon approximation (SPA). The soft-photon amplitude, defined by the first two terms of the soft-photon expansion, depends only on the electromagnetic constants of π and N and on the corresponding πN elastic-scattering amplitude (on-shell πN T matrix) [12]. It should be pointed out that Low's original amplitude fails to describe the $\pi^{\pm} N \gamma$ process near the resonance region [9]. Nevertheless the SPA for the $\pi N \gamma$ reaction, as implemented by Ref. [10], describes well the experimental data near the Δ resonance region and can provide a determination of the Δ^{++} magnetic moment [11].

Because the soft-photon amplitude depends only on the on-shell T matrix, the information on the off-shell behavior of the T matrix can be tested by adding the contributions to the radiative πN scattering within the framework of a specific dynamical model. The purpose of the present paper is to check the off-shell behavior of three different T matrices for πN rescattering in the reaction $\pi N \rightarrow \pi N \gamma$. A similar analysis of these FSI effects using different rescattering off-shell amplitudes has been done with the aim of determining the form factors of nucleons [13]. It was found that pion photoproduction reactions are very sensitive to the off-shell behavior of the πN interaction, and also that there are certain inconsistencies in fixing phenomenological form factors, to match photoproduction current and FSI effects.

For this purpose, we use a dynamical model to describe the $\pi N \rightarrow \pi N \gamma$ reaction. The gauge invariant electromagnetic current is constructed explicitly, with vertices and propagators derived from the relevant hadronic and electromagnetic Lagrangians. We also include two-body meson exchange currents, and the full energy-momentum dependence of the πN T matrix, which exhibits its off-shell behavior. Finally, we implement this model with different T matrices in order to compare their different off-shell dependences.

This paper is organized as follows. In Sec. II we will construct the gauge-invariant amplitude for radiative πN scattering. In Sec. III we give a summary of the corresponding results obtained in the SPA approximation in order to make comparisons with our dynamical model. The Lagrangians and propagators used to construct the gauge-invariant current for our process are provided in Sec. IV. Finally, the results and conclusions are given in Sec. V.

II. GAUGE-INVARIANT BREMSSTRAHLUNG AMPLITUDE

In the pion-nucleon bremsstrahlung process we deal with a problem of the scattering by two potentials [14]: the strong pion-nucleon and the electromagnetic interactions. The cross section for the $\pi N \rightarrow \pi N \gamma$ process reads

$$d\sigma=\int \frac{d\vec{k}}{\omega_\gamma}\int \frac{d\vec{q}_f}{\omega(\vec{q}_f)}\int \frac{d\vec{p}_f}{E(\vec{p}_f)}(2\pi)^4\delta^4(p_i+q_i-p_f-q_f-k) \times\frac{1}{2}\sum_{\epsilon_\lambda,ms_f,ms_i}\left|\frac{m_N^2}{2\sqrt{2}} \times M_{\pi N\gamma,\pi N}(\epsilon_\lambda,k;q_f,p_f,ms_f;q_i,p_i,ms_i)\right|^2, \qquad (1)$$

where $q=(\omega,\vec{q})$, $p=(E,\vec{p})$, and $k=(\omega_\gamma,\vec{k})$ denote pion, nucleon, and photon four-momenta, respectively; ms is the nucleon's spin projection; and ϵ_λ indicates the polarization four vector of the photon. The subindices i, f refer to initial- and final-state quantities.

The Lorentz invariant amplitude[1] $M_{\pi N\gamma,\pi N}$ explicitly reads

$$M_{\pi N\gamma,\pi N}=\langle\overline{u}(\vec{p}_f,ms_f)|\hat{M}_{\pi N\gamma,\pi N}(\epsilon_\lambda,k;q_f,p_f;q_i,p_i) \times|u(\vec{p}_i,ms_i)\rangle, \qquad (2)$$

where $u(\vec{p},ms)$ denote nucleon Dirac spinors, and the amplitude operator $\hat{M}_{\pi N\gamma,\pi N}$ is obtained from the coupled channel Bethe-Salpeter equation for the $\pi N\gamma$ system as follows (we consider electromagnetic interactions at the lowest order):

$$\hat{M}_{\pi N\gamma,\pi N}=\hat{V}_{\pi N\gamma,\pi N}+i\int\frac{dq^4}{(2\pi)^4}[\hat{V}_{\pi N\gamma,\pi N}(q)\hat{G}_{\pi N}(q) \times\hat{M}_{\pi N,\pi N}(q)+\hat{M}_{\pi N,\pi N}(q)\hat{G}_{\pi N}(q)\hat{V}_{\pi N\gamma,\pi N}(q)] +i^2\int\frac{dq^4}{(2\pi)^4}\frac{dq^4}{(2\pi)^4}[\hat{M}_{\pi N,\pi N}(q') \times\hat{G}_{\pi N}(q')\hat{V}_{\pi N\gamma,\pi N}(q',q)\hat{G}_{\pi N}(q)\hat{M}_{\pi N,\pi N}(q)]. \qquad (3)$$

The symbol $\int dq^4$ will indicate integration over intermediate four-momenta variables.

In terms of the above operator amplitude, the T matrix, defined as

$$\hat{T}(q_f,p_f;q_i,p_i)=\frac{1}{(2\pi)^3}\hat{M}_{\pi N,\pi N}(q_f,p_f;q_i,p_i), \qquad (4)$$

satisfies the integral equation

$$\hat{T}=\hat{U}+i\int\frac{dq^4}{(2\pi)^4}\hat{U}(q)\hat{G}(q)\hat{T}(q),$$

$$\hat{U}=\frac{1}{(2\pi)^3}\hat{V}_{\pi N,\pi N},$$

$$\hat{G}=(2\pi)^3\hat{G}_{\pi N}. \qquad (5)$$

In the previous equations $\hat{V}_{ij}$ denote $\hat{M}$-matrix elements corresponding to the irreducible Feynman diagrams for each process, while $\hat{G}_i$ is the product of Feynman propagators of intermediate particles.

Following Thompson's prescription [15], we can represent the above integrals in a three-dimensional form as follows (we set in the center-of-mass frame of the πN system):

$$\hat{T}(\vec{q}\,',\vec{q},z)=\hat{U}(\vec{q}\,',\vec{q}) +\int d^3\vec{q}\,''\hat{U}(\vec{q}\,',\vec{q}\,'')\hat{G}_{TH}(z,\vec{q}\,'')\hat{T}(\vec{q}\,'',\vec{q},z), \qquad (6)$$

with

$$\hat{G}_{TH}(z,\vec{q}\,'')=\frac{m_N}{2\omega(\vec{q}\,'')E(-\vec{q}\,'')} \times\frac{\Sigma_{ms''}|u(-\vec{q}\,'',ms'')\rangle\langle\overline{u}(-\vec{q}\,'',ms'')|}{z-z''+i\eta}, \qquad (7)$$

where $z''=E(-\vec{q}\,'')+\omega(\vec{q}\,'')$. In the above expressions $\hat{G}_{TH}$ denotes the Thompson propagator replacing the full $\hat{G}_{\pi N}$. Feynman propagator which, as a consequence of the three-dimensional reduction, eliminates the propagation of antiparticles and puts intermediate particles on their mass shell. The kernel function $\hat{U}(\vec{q}\,',\vec{q})$ contains all the πN-interaction irreducible diagrams to be iterated in the T-matrix calculation, but usually only second-order contributions are kept.

The electromagnetic current $\hat{V}_{\pi N\gamma,\pi N}$ can be broken into two pieces,

$$\hat{V}_{\pi N\gamma,\pi N}\equiv\hat{V}^{(1)}_{\pi N\gamma,\pi N}+\hat{V}^{(2)}_{\pi N\gamma,\pi N}, \qquad (8)$$

where the upper indices denote one- and two-body contributions, respectively, which are obtained by coupling the photon to all the internal lines of $\hat{U}$. As is known, the operator $\hat{V}^{(2)}_{\pi N\gamma,\pi N}$ must be added to the electromagnetic current in order to satisfy the electromagnetic gauge invariance of the total amplitude [16], while the one-body amplitude $V^{(1)}_{\pi N\gamma,\pi N}$ vanishes for free hadrons. Both contributions to the total amplitude are illustrated in Fig. 1.

Let us now discuss some problems related to gauge invariance. The bremsstrahlung amplitude can be directly com-

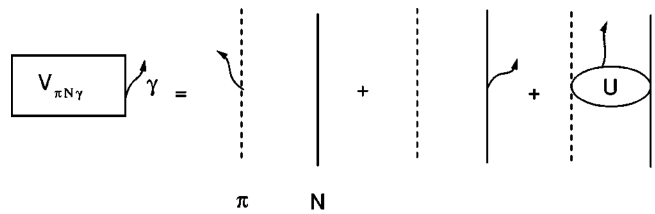

FIG. 1. One- and two-body contributions to the bremsstrahlung current amplitude.

[1]Throughout this paper, M will denote the amplitude generated by the operator $\hat{M}$, i.e., $M=\langle\overline{u}|\hat{M}|u\rangle$.

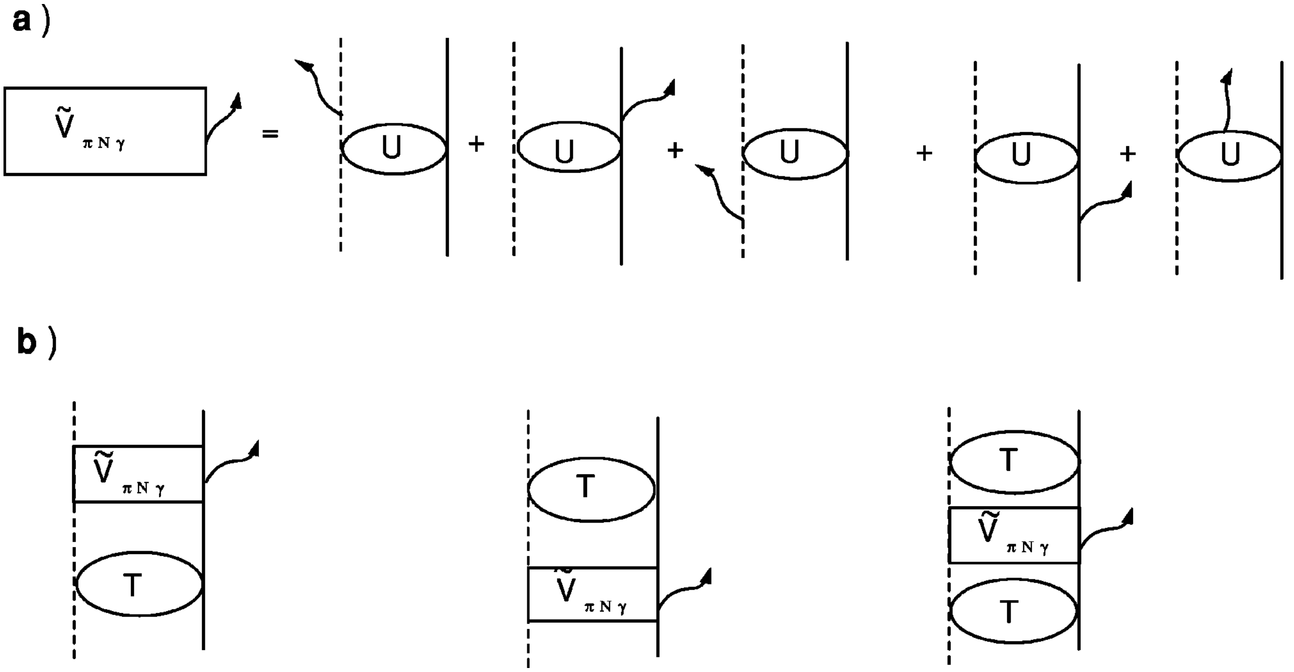

FIG. 2. (a) Gauge-invariant bremsstrahlung current amplitude. (b) Post-, pre-, and double-scattering amplitude contributions [see Eq. (9)].

puted from Eqs. (2)–(4), by making a reduction that replaces $\hat{G}_{\pi N}$ by $\hat{G}_{TH}$ in Eq. (3) [17]. This procedure, however, introduces some inconveniences. First, $V_{\pi N\gamma,\pi N}$ is not gauge invariant by itself because it involves the addition of zero- and second-order terms in the hadronic vertices, and thus $M_{\pi N\gamma,\pi N}$ is not manifestly gauge invariant. Second, the three-dimensional reduction destroys any possibility of obtaining gauge invariance, because we lose the propagation of antiparticles.[2]

We can follow an alternative procedure that makes the bremsstrahlung amplitude manifestly gauge invariant and where the Thompson reduction does not introduce the problems mentioned above. If we substitute Eqs. (4) and (5) into Eq. (3), only in the one-body component of the amplitude $M^{(1)}_{\pi N\gamma,\pi N}\equiv M_{\pi N\gamma,\pi N}(\hat{V}^{(1)}_{\pi N\gamma,\pi N})$, we can isolate the lowest order nonzero contribution of the one-body current. After the three-dimensional reduction, the total amplitude can be rewritten as

$$M_{\pi N\gamma,\pi N}\equiv[\tilde{V}_{\pi N\gamma,\pi N}+\tilde{M}^{\rm pre}_{\pi N\gamma,\pi N}+\tilde{M}^{\rm post}_{\pi N\gamma,\pi N}+\tilde{M}^{\rm double}_{\pi N\gamma,\pi N}], \tag{9}$$

with

$$\tilde{V}_{\pi N\gamma,\pi N}=\langle\overline{u}(-\vec{q}\,'-\vec{k}/2,ms_f)|\times\hat{\tilde{V}}_{\pi N\gamma,\pi N}(\epsilon_\lambda,\vec{k},\vec{q}\,',\vec{q})|u(-\vec{q},ms_i)\rangle,$$

$$\tilde{M}^{\rm pre}_{\pi N\gamma,\pi N}=\int dq''^3\langle\overline{u}(-\vec{q}\,',ms_f)|\times\hat{T}^{(-)+}(\vec{q}\,',\vec{q}\,'',z')\hat{G}_{TH}(z',\vec{q}\,'')\times\hat{\tilde{V}}_{\pi N\gamma,\pi N}(\epsilon_\lambda,\vec{k},\vec{q}\,'',\vec{q})|u(-\vec{q}+\vec{k}/2),ms_i\rangle,$$

$$\tilde{M}^{\rm post}_{\pi N\gamma,\pi N}=\int dq''^3\langle\overline{u}(-\vec{q}\,'-\vec{k}/2,ms_f)|\times\hat{\tilde{V}}_{\pi N\gamma,\pi N}(\epsilon_\lambda,\vec{k},\vec{q}\,',\vec{q}\,'')\hat{G}_{TH}(z,\vec{q}\,'')\times\hat{T}(\vec{q}\,'',\vec{q},z)|u(-\vec{q},ms_i)\rangle,$$

$$\tilde{M}^{\rm double}_{\pi N\gamma,\pi N}=\int dq''^3\int dq'''^3\langle\overline{u}(-\vec{q}\,'-\vec{k}/2,ms_f)|\times\hat{T}^{(-)\dagger}(-\vec{q}\,'-\vec{k}/2,\vec{q}\,'',z')\hat{G}_{TH}(z',\vec{q}\,'')\times\hat{\tilde{V}}_{\pi N\gamma,\pi N}(\epsilon_\lambda,\vec{k},\vec{q}\,'',\vec{q}\,''')\hat{G}_{TH}(z,\vec{q}\,''')\times\hat{T}(\vec{q}\,''',\vec{q},z)|u(-\vec{q},ms_i)\rangle, \tag{10}$$

where the current,

$$\hat{\tilde{V}}_{\pi N\gamma,\pi N}=i\hat{V}^{(1)}_{\pi N\gamma,\pi N}\hat{G}\hat{U}+i\hat{U}^\dagger\hat{G}\hat{V}^{(1)}_{\pi N\gamma,\pi N}+\hat{V}^{(2)}_{\pi N\gamma,\pi N}, \tag{11}$$

generates a gauge-invariant Born amplitude $\tilde{V}_{\pi N\gamma,\pi N}$, that involves all possible ways of attaching a photon to the

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[2]As an example let us consider the amplitude for photon emission off a charged pion. This contribution involves the product of the electromagnetic vertex Γ_π and the pion propagator Δ_π in the following form: $\hat{\Gamma}_\pi(q\pm k,q)\epsilon^*_\lambda\Delta_\pi(q\pm k)=\pm e(2q\pm k)\epsilon^*_\lambda[1/(q\pm k)^2-m^2_\pi]$. When we replace $\epsilon_\lambda\rightarrow k$ to verify gauge invariance we obtain, $\hat{\Gamma}_\pi(q\pm k,q)\epsilon^*_\lambda\Delta_\pi(q\pm k)=\pm e$. The three-dimensional reduction replaces the full propagator $\Delta_\pi(q)$ by $\Delta^+_\pi(q)=[1/2\omega(\vec{q})][1/[q^0-\omega(\vec{q})]$ and such a relation is no longer fulfilled.

πN-scattering amplitude U, and contains the full propagator $G \sim \hat{G}_{\pi N}$. The operator $\hat{T}^{(-)}(z')$, where $z' = z + \omega_\gamma$, obeys Eq. (6) if we change $\eta \rightarrow -\eta$ in Eq. (7).

The superscript pre (post) in Eq. (9) indicates that the photon is emitted before (after) the action of the T matrix, while the superscript double refers to a double-scattering term where the photon is emitted from internal lines between two T matrices. In the above equations, the Born, pre, and double amplitudes were evaluated in the initial center-of-mass frame ($\vec{q}_f = \vec{q}' - \vec{k}/2$, $\vec{p}_f = -\vec{q}' - \vec{k}/2$, $\vec{q}_i = -\vec{p}_i = \vec{q}$), while the other (post) amplitude was evaluated in the corresponding final frame ($\vec{q}_f = -\vec{p}_f = \vec{q}'$, $\vec{q}_i = \vec{q} + \vec{k}/2$, $\vec{p}_i = -\vec{q} + \vec{k}/2$). The different terms in Eq. (9) are illustrated in Fig. 2(a) ($\tilde{V}_{\pi N\gamma,\pi N}$) and in Fig. 2(b) (remaining terms).

III. SOFT PHOTON APPROXIMATION

In this section we present a brief review of the soft-photon approximation (SPA) to the radiative πN scattering process. This will be helpful for the introduction of the notation and for later comparison with our dynamical model. Within the SPA [9], the T matrices can be represented as an expansion in powers of the photon energy ω_γ. Using the two energy–two angle kinematics [10] we can write this expansion as follows:

$$\hat{T}(s,t,\Delta) = \hat{T}(s,t,m^2) + \frac{\partial \hat{T}}{\partial \Delta}\frac{\partial \Delta}{\partial \omega}\omega_\gamma + O(\omega_\gamma^2), \tag{12}$$

where (s,t,Δ) indicates one of the most convenient sets of variables among (s_i, t_p, Δ_{q_i}), (s_i, t_q, Δ_{p_i}), (s_f, t_p, Δ_{q_f}), and (s_f, t_q, Δ_{p_f}) to describe the process. These Lorentz-invariant variables are defined as

$$s_i = (q_i + p_i)^2, \quad s_f = (q_f + p_f)^2,$$
$$t_p = (p_i - p_f)^2, \quad t_q = (q_i - q_f)^2,$$
$$\Delta_{q_i} = (q_i - k)^2, \quad \Delta_{q_f} = (q_f + k)^2,$$
$$\Delta_{p_i} = (p_i - k)^2, \quad \Delta_{p_f} = (p_f + k)^2. \tag{13}$$

If we set $k=0$ in the previous equations, we get $\Delta_{q_i} = \Delta_{q_f} = m_\pi^2$ and $\Delta_{p_i} = \Delta_{p_f} = m_N^2$, i.e., they reduce to the particles' masses in every case. Therefore, when $k \neq 0$, the Δ variables provide a convenient set to measure the off-shell character of the intermediate particles, and the derivatives in Eq. (12) account for off-shell effects in the T matrix.

Within the SPA the total bremsstrahlung amplitude can be split into external (E) and internal (I) contributions:

$$M_{\pi N\gamma,\pi N} \equiv M^E_{\pi N\gamma,\pi N} + M^I_{\pi N\gamma,\pi N}, \tag{14}$$

where we can identify

$$M^E_{\pi N\gamma,\pi N} \equiv \tilde{M}^{\text{pre}}_{\pi N\gamma,\pi N}(\hat{\tilde{V}}_{\pi N\gamma,\pi N} \rightarrow \hat{V}^{(1)}_{\pi N\gamma,\pi N}) + \tilde{M}^{\text{post}}_{\pi N\gamma,\pi N}(\hat{\tilde{V}}_{\pi N\gamma,\pi N} \rightarrow \hat{V}^{(1)}_{\pi N\gamma,\pi N}),$$

and the internal contribution $M^I_{\pi N\gamma,\pi N}$ can be obtained ''by imposing'' the gauge-invariance condition

$$M_{\pi N\gamma,\pi N}(\epsilon^\mu_\lambda = k^\mu) = 0. \tag{15}$$

In this approach, the total amplitude depends only on the static electromagnetic properties of the external particles and on the elastic πN-scattering amplitude [see Eq. (16) below] [12]. Therefore, the total amplitude at this order eliminates any dependence on off-shell effects and model-dependent contributions.

Up to terms of $O(\omega_\gamma^0)$, the total amplitude is given by

$$M_{\pi N\gamma,\pi N} \approx \left\langle \overline{u}(\vec{p}'_+, m_{s_f}) \middle| \left\{ \hat{e}_\pi \left[\frac{q'_- \cdot \epsilon_\lambda}{q'_- \cdot k} - \frac{(q'_- + p'_+)\cdot \epsilon_\lambda}{k \cdot (q'_- + p'_+)} \right] \hat{T}(s_i, t_p, m_\pi^2) + \left[\frac{[\hat{e}_N p'_+ - \hat{R}(p'_+)]\cdot \epsilon_\lambda}{p'_+ \cdot k} - \frac{[\hat{e}_N (q'_- + p'_+) - \hat{R}(p'_+)]\cdot \epsilon_\lambda}{k \cdot (q'_- + p'_+)} \right] \hat{T}(s_i, t_q, m_N^2) \right\} \middle| u(\vec{p}, m_{s_i}) \right\rangle - \left\langle \overline{u}(\vec{p}', m_{s_f}) \middle| \left\{ \hat{e}_\pi \hat{T}(s_f, t'_p, m_\pi^2) \left[\frac{q_+ \cdot \epsilon_\lambda}{q_+ \cdot k} - \frac{(q_+ + p_-)\cdot \epsilon_\lambda}{k \cdot (q_+ + p_-)} \right] + \hat{T}(s_f, t'_q, m_N^2) \left[\frac{[\hat{e}_N p_- - \hat{R}(p_-)]\cdot \epsilon_\lambda}{p_- \cdot k} - \frac{[\hat{e}_N (q_+ + p_-) - \hat{R}(p_-)]\cdot \epsilon_\lambda}{k \cdot (q_+ + p_-)} \right] \right\} \middle| u(\vec{p}_-, m_{s_i}) \right\rangle, \tag{16}$$

where

$$q \equiv [\omega(\vec{q}), \vec{q}], \quad p \equiv [E(-\vec{q}), -\vec{q}], \quad q' \equiv [\omega(\vec{q}'), \vec{q}'], \quad p' \equiv [E(-\vec{q}'), -\vec{q}'],$$
$$q'_- \equiv [\omega(\vec{q}'_-), \vec{q}'_-], \quad p'_+ \equiv [E(\vec{q}'_+), -\vec{q}'_+], \quad q_+ \equiv [\omega(\vec{q}_+), \vec{q}_+], \quad p_- \equiv [E(\vec{q}_-), -\vec{q}_-],$$
$$t_p = (p'_+ - p)^2, \quad t_q = (q'_+ - q)^2, \quad t'_p = (p_- - p')^2, \quad t'_q = (q_+ - q')^2,$$
$$\hat{R}_\mu(x) \equiv \frac{1}{4}\hat{e}_N[\not{k}, \gamma_\mu] + \frac{\hat{\kappa}_N}{8 m_N}\{[\not{k}, \gamma_\mu], \not{x}\}, \quad \vec{q}_\pm = \vec{q} \pm \vec{k}/2, \quad \vec{q}'_\pm = \vec{q}' \pm \vec{k}/2, \tag{17}$$

and $\hat{e}_\pi = e\mathcal{T}_z$, $\hat{e}_N = e(1+\tau_z)/2$, and $\hat{\kappa}_N = \kappa_p(1+\tau_z)/2 + \kappa_n(1-\tau_z)/2$ denote the charge and anomalous magnetic moment operators of pions and nucleons. As anticipated, Eq. (16) shows that within the SPA the $M_{\pi N\gamma,\pi N}$ amplitude depends only on the elastic T matrix, because derivative terms of $\hat{T}$ cancel in the addition of internal and external contributions. Let us emphasize that any dependence on the structure of internal contributions (in particular, the dependence of off-shell effects) are of higher order in ω_γ and must be included explicitly in the amplitude in a gauge-invariant way. This is the purpose of the forthcoming section.

IV. DYNAMICAL MODEL

In this section we compute the bremsstrahlung amplitude along the lines developed in Eqs. (9)–(11), using a potential $\hat{U}$ obtained from effective Lagrangians [18], and three specific models for the T matrix to describe the πN rescattering.

The three models for the T matrix will be called OBQA (one boson quasipotential approach), SEP (separable), and NEW (new one boson exchange model), respectively. The OBQA version for the T-matrix interaction [19], is based on a model that includes π and ρ mesons exchange through a correlated 2π exchange potential. The SEP model for πN rescattering is generated [20] by a phenomenological separable potential. Finally, the NEW model [21] is obtained from the exchange of π and (sharp) ρ mesons.

The operator $\hat{U}$ is constructed from a Lagrangian density that includes the nucleon (N), the Δ isobar, and the π, ρ, and σ mesons in the following form:

$$\hat{\mathcal{L}}_{\text{hadr}} = \hat{\mathcal{L}}_{\pi NN} + \hat{\mathcal{L}}_{N\Delta\pi} + \hat{\mathcal{L}}_{\rho NN} + \hat{\mathcal{L}}_{\rho\pi\pi} + \hat{\mathcal{L}}_{\sigma NN} + \hat{\mathcal{L}}_{\sigma\pi\pi}, \tag{18}$$

where the individual terms are given by

$$\hat{\mathcal{L}}_{\pi NN}(x) = -\left(\frac{f_{\pi NN}}{m_\pi}\right)\overline{\psi}(x)\gamma_5\vec{\tau}(\not\partial\vec{\pi}(x))\psi(x),$$

$$\hat{\mathcal{L}}_{N\Delta\pi}(x) = \left(\frac{f_{N\Delta\pi}}{m_\pi}\right)\overline{\psi}^{\mu}_{\Delta}(x)\vec{T}^{\dagger}(\partial_\mu\vec{\pi}(x))\psi(x) + \text{H.c.},$$

$$\hat{\mathcal{L}}_{\rho NN}(x) = -\frac{1}{2}g_\rho\overline{\psi}(x)\left[\gamma_\mu - \frac{\kappa_\rho}{2m_N}\sigma_{\mu\nu}\partial^\nu\right] \times\vec{\tau}\cdot\vec{\rho}^{\,\mu}(x)\psi(x),$$

$$\hat{\mathcal{L}}_{\rho\pi\pi}(x) = -g_{\rho\pi\pi}\vec{\rho}_\mu(x)[\vec{\pi}(x)\wedge\partial^\mu\vec{\pi}(x)],$$

$$\hat{\mathcal{L}}_{\sigma NN}(x) = g_\sigma\overline{\psi}(x)\psi(x)\sigma(x),$$

$$\hat{\mathcal{L}}_{\sigma\pi\pi}(x) = \left(\frac{g_{\sigma\pi\pi}}{2m_\pi}\right)\sigma(x)(\partial_\mu\vec{\pi}(x))(\partial^\mu\vec{\pi}(x)). \tag{19}$$

The isotopic fields $\psi(x)$ and $\psi^{\mu}_{\Delta}(x)$ denote the N and Δ baryons, respectively, while $\vec{\pi}(x)$, $\vec{\rho}^{\,\mu}(x)$, and $\sigma(x)$ denote the pion, ρ-meson, and σ-meson fields. The arrow over the meson fields refers to the isospin space. $\vec{T}^{\dagger}$ stands for the isospin 1/2 to 3/2 transition operator, and $f_{\pi NN}$, $g_\rho(\kappa_p)$, g_σ, $f_{N\Delta\pi}$, $g_{\rho\pi\pi}$, and $g_{\sigma\pi\pi}$ are the corresponding coupling constants.

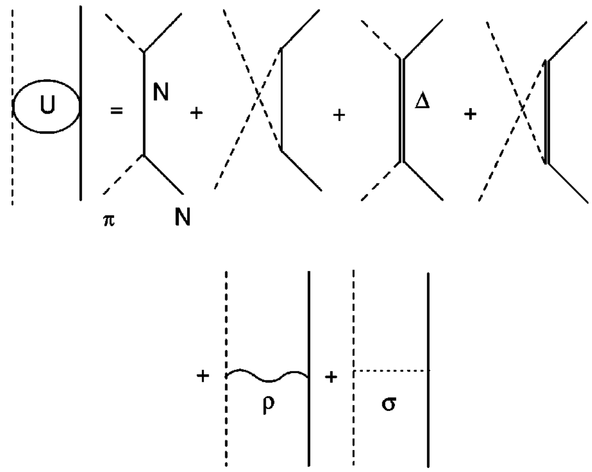

FIG. 3. Born amplitude corresponding to the πN potential operator $\hat{U}$. The first diagram denotes the nucleon-pole, and the Δ pole corresponds to the third diagram. The fifth and sixth diagrams correspond to ρ and σ mesons exchange.

The electromagnetic currents can be obtained from the Lagrangian density

$$\hat{\mathcal{L}}_{\text{elec}} = \hat{\mathcal{L}}_{\gamma NN} + \hat{\mathcal{L}}_{\gamma\pi NN} + \hat{\mathcal{L}}_{\gamma\pi\pi} + \hat{\mathcal{L}}_{\gamma\Delta\Delta} + \hat{\mathcal{L}}_{\gamma\pi N\Delta} + \hat{\mathcal{L}}_{\gamma\rho\pi\pi} + \hat{\mathcal{L}}_{\gamma\sigma\pi\pi}, \tag{20}$$

with

$$\hat{\mathcal{L}}_{\gamma NN}(x) = -e\overline{\psi}(x)\left[\hat{e}_N\gamma_\mu - \frac{\hat{\kappa}}{2m_N}\sigma_{\mu\nu}\partial^\nu\right]A^\mu(x)\psi(x),$$

$$\hat{\mathcal{L}}_{\gamma\pi NN}(x) = -e\left(\frac{f_{\pi NN}}{m_\pi}\right)\overline{\psi}(x)\gamma_5\gamma_\mu[\vec{\tau}\times\vec{\pi}(x)]_3\psi(x)A^\mu(x),$$

$$\hat{\mathcal{L}}_{\gamma\pi\pi}(x) = -e[\vec{\pi}(x)\partial_\mu\vec{\pi}(x)]_3A^\mu(x),$$

$$\hat{\mathcal{L}}_{\gamma\Delta\Delta}(x) = -e\overline{\psi}^{\nu}_{\Delta}(x)\hat{\Gamma}_{\nu\mu\alpha}\psi^{\mu}_{\Delta}(x)A^\alpha(x),$$

$$\vec{\mathcal{L}}_{\gamma\pi N\Delta}(x) = e\left(\frac{f_{N\Delta\pi}}{m_\pi}\right)\overline{\psi}^{\mu}_{\Delta}(x)[\vec{\pi}(x)\times\vec{T}^{\dagger}]_3\psi(x)A_\mu(x) + \text{H.c.},$$

$$\hat{\mathcal{L}}_{\gamma\rho\pi\pi}(x) = eg_\rho\{[\vec{\pi}(x)\cdot\vec{\pi}(x) - \pi_3(x)\pi_3(x)]\rho^{\nu}_3(x) - \pi_3(x)[\vec{\pi}(x)\cdot\vec{\rho}^{\,\nu}(x) - \pi_3(x)\rho^{\nu}_3]\}A_\nu(x),$$

$$\hat{\mathcal{L}}_{\gamma\sigma\pi\pi}(x) = -2e\left(\frac{g_{\sigma\pi\pi}}{2m_\pi}\right)\sigma(x)[\vec{\pi}(x)\times\partial_\mu\vec{\pi}(x)]_3A^\mu(x). \tag{21}$$

The electromagnetic vertex operator of the Δ isobar is given by [22,23]

$$\hat{\Gamma}_{\nu\mu\alpha}=\hat{e}_{\Delta}\left[\left(\gamma_{\alpha}g_{\nu\mu}-\frac{1}{3}\gamma_{\alpha}\gamma_{\nu}\gamma_{\mu}-\frac{1}{3}\gamma_{\nu}g_{\mu\alpha}+\frac{1}{3}\gamma_{\mu}g_{\nu\alpha}\right)-\frac{\hat{\kappa}_{\Delta}}{2m_{\Delta}}\sigma_{\alpha\beta}k_{\beta}g_{\nu\mu}\right], \tag{22}$$

where $F_{\mu\nu}=\partial_{\mu}A_{\nu}(x)-\partial_{\nu}A_{\rho}(x)$, $A_{\mu}(x)$ being the electromagnetic four potential. $\hat{\kappa}_{\Delta}$ and $\hat{e}_{\Delta}$ are the anomalous magnetic moment[3] and charge operators whose action upon the Rarita-Schwinger field give as eigenvalues the corresponding values of these properties of the Δ isobar.

The propagators for the $\pi(\sigma)$, ρ, N, and Δ hadrons obtained from the above Lagrangians are given, respectively, by

$$\Delta_{\pi,\sigma}(q)=\frac{1}{q^{2}=m_{\pi,\sigma}^{2}+i\eta},$$

$$D_{\rho}^{\mu\nu}(q)=\frac{-g^{\mu\nu}+q^{\mu}q^{\nu}/m_{\rho}^{2}}{q^{2}-m_{\rho}^{2}+i\eta},$$

$$S(q)=\frac{\not{q}+m_{N}}{q^{2}-m_{N}^{2}+i\eta},$$

$$G^{\mu\nu}(q)=\frac{\not{q}+m_{\Delta}}{q^{2}-m_{\Delta}^{2}+i\eta}\left[-g^{\mu\nu}+\frac{1}{3}\gamma^{\mu}\gamma^{\nu}+\frac{2}{3}\frac{q^{\mu}q^{\nu}}{m_{\Delta}^{2}}-\frac{1}{3}\frac{q^{\mu}\gamma^{\nu}-q^{\nu}\gamma^{\mu}}{m_{\Delta}}\right]-\frac{2}{3m_{\Delta}^{2}}(q^{2}-m_{\Delta}^{2})[(\gamma^{\mu}q^{\nu}-\gamma^{\nu}q^{\mu})+(\not{q}+m_{\Delta})\gamma^{\mu}\gamma^{\nu}]. \tag{23}$$

Observe that we have kept the off-shell part of $G^{\mu\nu}(q)$ in Eqs. (23); without this term it is impossible to get gauge invariance consistently simultaneously using the vertex given in Eq. (22) [24]. As was mentioned previously, the potential operator $\hat{U}$ can be computed by using these Feynman rules. The scattering amplitude U is depicted in Fig. 3, while the amplitude $\tilde{V}_{\pi N\gamma,\pi N}$ can be obtained by coupling the photon to all diagrams in U as shown in Fig. 4.

Good convergence properties of the scattering equations given in Eqs. (10) can be obtained by introducing hadronic form factors, which are supposed to describe the composite nature of hadrons. It is a common practice to use different parametrizations of the form factors for different T matrices. For example, the OBQA model uses monopole and dipole forms with cutoff parameters ranging from $\Lambda=1200-1600$ MeV [19]. In the case of the SEP interaction different form factors are introduced for each partial wave component [20], while in the NEW model form factors usually advocated are of monopolar form with $\Lambda=1300-2300$ MeV [21]. However, the introduction of form factors replacing point vertices in $\tilde{V}_{\pi N\gamma,\pi N}$ spoils the gauge invariance of the total amplitude. Fortunately, the gauge invariance of the amplitude can be recovered by using the method of Gross and Riska [25] which, however, does not yield unique electromagnetic couplings to hadrons. Therefore, we follow the more simple prescription of using a common form factor [13,20] of the monopole type

$$f(\vec{q}'')=\frac{\Lambda^{2}}{\Lambda^{2}+\vec{q}''^{2}}, \tag{24}$$

where the scale Λ can be adjusted at a given incident energy for each T-matrix model.

[3]We restrict ourselves to the Δ^{++} contribution, the only one for which we have experimental information on κ_{Δ} [11].

V. NUMERICAL RESULTS AND CONCLUSIONS

The differential cross section $d\sigma/d\Omega_{\pi}d\Omega_{\gamma}d\omega_{\gamma}$ to be compared with experimental data can be obtained from Eq. (1), where the amplitude $M_{\pi N\gamma,\pi N}$ is calculated from Eqs. (9)–(11). The dynamical model approximation (DMA) advocated in the present paper contains the following steps. The current operator $\hat{\tilde{V}}_{\pi N\gamma,\pi N}$ is computed from the effective Lagrangians given in Eqs. (18) and (20), with the propagators obtained in Eqs. (23), and the monopole form factor given in Eq. (24) to have good convergence of the intermediate momentum integrals.

As is known, double scattering terms have a significant contribution in the case of *proton-proton* bremsstrahlung, mainly in the endpoint region of ω_{γ} [26]. In the present work we will neglect $\tilde{M}^{\text{double}}_{\pi N\gamma,\pi N}$ in Eq. (10), because the numerical calculation of the three-dimensional integrals requires an enormous computational effort. Nevertheless, we keep double-scattering-like contributions in post- and preamplitudes coming from current components $iU^{\dagger}\hat{G}\hat{V}^{(1)}_{\pi N\gamma,\pi N}$ and $i\hat{V}^{(1)}_{\pi N\gamma,\pi N}\hat{G}\hat{U}$, respectively. In order to compare the approaches provided by the DMA and SPA, we fix $M_{\pi N\gamma,\pi N}$ to coincide quantitatively at low photon energies.

For illustration purposes, we will implement the DMA approach with the OBQA, SEP, and NEW T matrices for the specific example of $\pi^{+}p\rightarrow\pi^{+}p\gamma$. In this case, we have contributions coming from the diagrams depicted in Figs. 4(a) (with intermediate Δ), 4(b) (with intermediate N), and 4(c). The different coupling constants and masses needed to evaluate $\hat{V}_{\pi N\gamma,\pi N}$ were taken from the model II of Ref. [19], from [20], and from [21], and are displayed in Table I. For direct pole diagrams we use bare masses and coupling constants since they get dynamically dressed in the T-matrix scattering (5). In the OBQA case, we replace the 2π correlated exchange potential by π and ρ sharp mass exchange terms, since they do not lead to sizable differences as shown in Ref. [19]. In the SEP case we use the same coupling constants and masses, because the scattering potential is not generated from a dynamical model.

We will compare the theoretical predictions to the experimental cross sections measured by Nefkens [27] (EXP I) and Myer [28] (EXP II), which have been reported for different

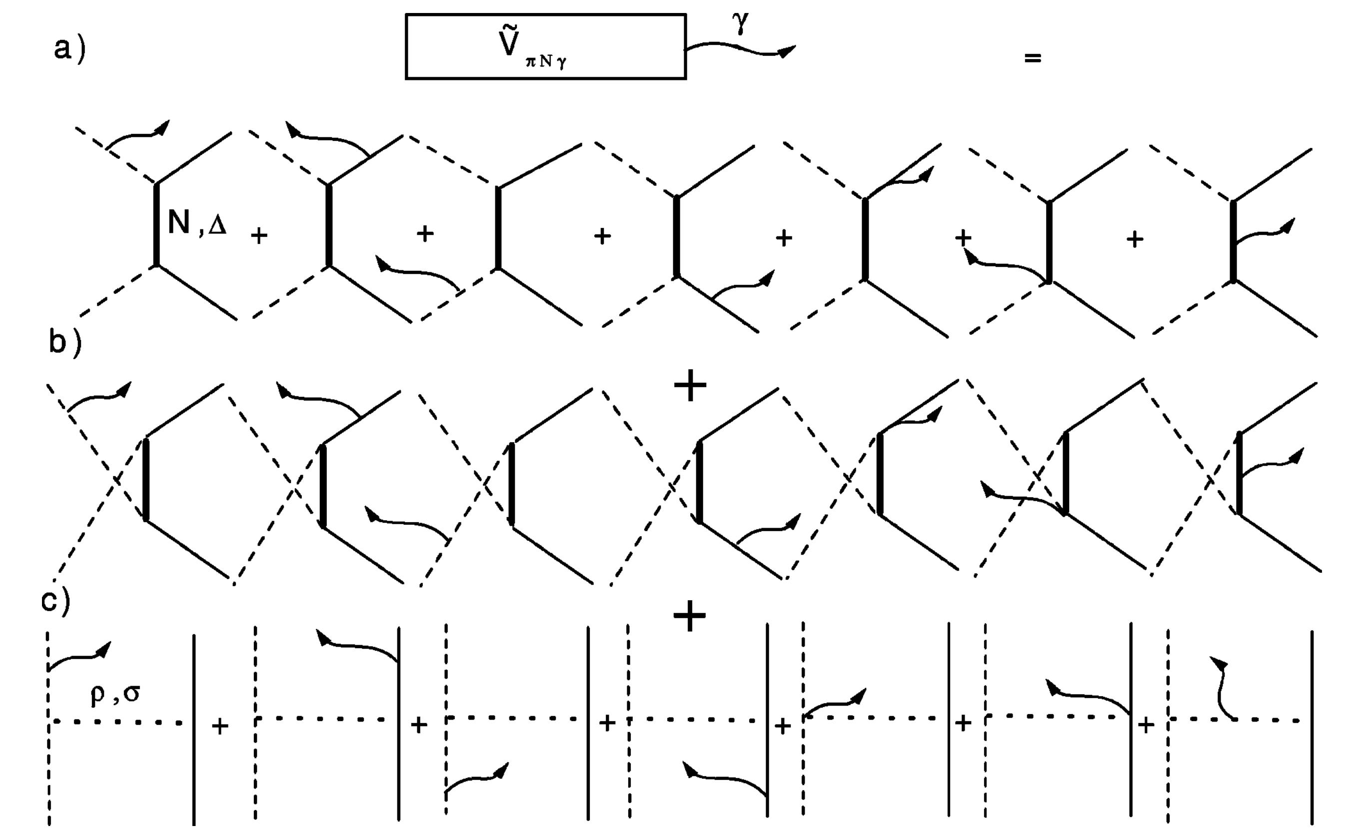

FIG. 4. The gauge-invariant amplitude obtained by coupling a photon to the Born terms in Fig. 3, together with the two-meson exchange currents (fifth, sixth, and seventh graphs in each line): (a) contributions obtained from the *N* and Δ direct-pole diagrams in Fig. 3; (b) terms generated by the cross-pole diagrams in Fig. 3; and (c) diagrams obtained from ρ and σ exchange contributions in Fig. 3.

kinematical configurations. In EXP I the pions were detected at fixed angles $\theta_\pi = 50.5°$, $\phi_\pi = 180°$, for three different energies of incident pions (269, 298, and 324 MeV), and the photons were detected at various θ_γ, ϕ_γ angles in the range of energies $\omega_\gamma = 0-150$ MeV. In EXP II θ_π ranges from 55° to 95°, the incident energy of the pion is fixed at 299 MeV and the photons were detected at angles $\theta_\gamma = 120°$, $\phi_\gamma = 0°$ with energies in the range $\omega_\gamma = 0-140$ MeV. In EXP II the resulting cross section measurements were averaged over θ_π.

Our results for the cross sections in the DMA approach, for the respective ansatz of the *T* matrix, are shown in Figs. 5 and 6. The SPA and experimental values are also plotted for comparison. The predictions of the DMA approach shown in Fig. 5 using the three models of the *T* matrix, are compared to the results of EXP I for photon angles given by $G_{14} \equiv (\theta_\gamma = 103°, \phi_\gamma = 180°)$. Since the parameters entering *T* matrices are usually quoted to reproduce the elastic phase shifts, the cutoff parameters Λ were fixed in order to have good coincidence between the predictions of the different interactions and the SPA at low ω_γ values. We get (in MeV units) $\Lambda_{\rm OBQA} = 750,700,600$, $\Lambda_{\rm SEP} = 700,600,500$, and $\Lambda_{\rm NEW} = 550,500,450$, for incident pion energies $T_{\rm lab} = 269,298,324$, respectively. Observe that the value of Λ found in the case of the SEP interaction for $T_{\rm lab} = 298$ MeV is consistent with the one previously found for pion photoproduction at $T_{\rm lab} = 300$ MeV [20], while the OBQA values are roughly consistent with the form factors used in Ref. [19], which correspond to a monopole form factor with $\Lambda \approx 800$ MeV [13]. On the other hand, the resulting cross sections for the conditions of EXP II are shown in Fig. 6. In this case, the differential cross section is averaged over $\theta_\pi = 55°-75°$, and $\theta_\pi = 75°-95°$, for $T_{\rm lab} = 299$ MeV. However, we keep for consistency the values of Λ obtained at $T_{\rm lab} = 298$ MeV in EXP I.

As we can observe, the SPA reproduces very well the experimental cross section for EXP I in the whole range of

TABLE I. Relevant masses (in MeV) and coupling constants used in the computation of $\tilde{V}_{\pi N\gamma,\pi N}$.

	OBQA	SEP	NEW
m_N	938.926	938.926	938.926
$m_{0\Delta}$	1515	1450	1405
m_ρ	769	769	769
m_σ	650	650	650
$f^2_{NN\pi}/4\pi$	0.0778	0.0778	0.778
$f^2_{0N\Delta\pi}/4\pi$	0.21	0.64	0.18
k_Δ	1.6	1.6	1.6
$g^2_{NN\rho}/4\pi$	5.05	0.563	0.90
k_ρ	2.69	3.7	6.1
$g^2_{NN\sigma}/4\pi$	8.94	8.94	13.
$g^2_{\pi\pi\rho}/4\pi$	5.05	0.563	2.9
$g^2_{\pi\pi\sigma}/4\pi$	0.6	0.6	0.25

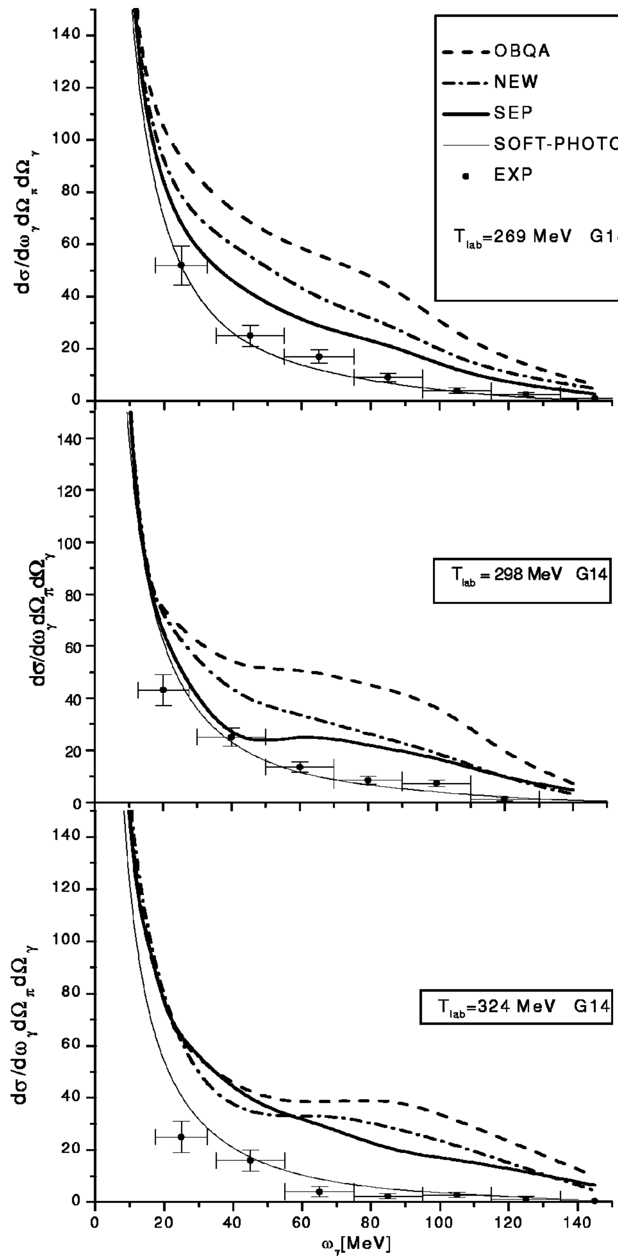

FIG. 5. $\pi N\gamma$ cross section for $T_{\mathrm{lab}}=269$, 298, and 324 MeV and $G_{14}\equiv(\theta_\gamma=103°,\phi_\gamma=180°)$ in EXP I, calculated in the DMA for the different T matrices. We also include the SPA cross section and the measured values.

measured photon energies, while it gives values somewhat below those obtained in EXP II. According to Ref. [11], the agreement with EXP II might be improved if the emission of the photon from the Δ^{++} [seventh graph in Fig. 3(a)] is included explicitly as a piece of the internal amplitude.

In almost all the cases, the predictions of the DMA lie above the experimental cross section and the SPA for energies $\omega_\gamma > 20$ MeV. One of the reasons for this may be the use of an overall form factor to cure the gauge invariance problems. The total bremsstrahlung amplitude is built up, as can be seen from Eqs. (9) and (10), by adding different components. It is not expected that the common form factor works as satisfactorily upon adding up these components as it does

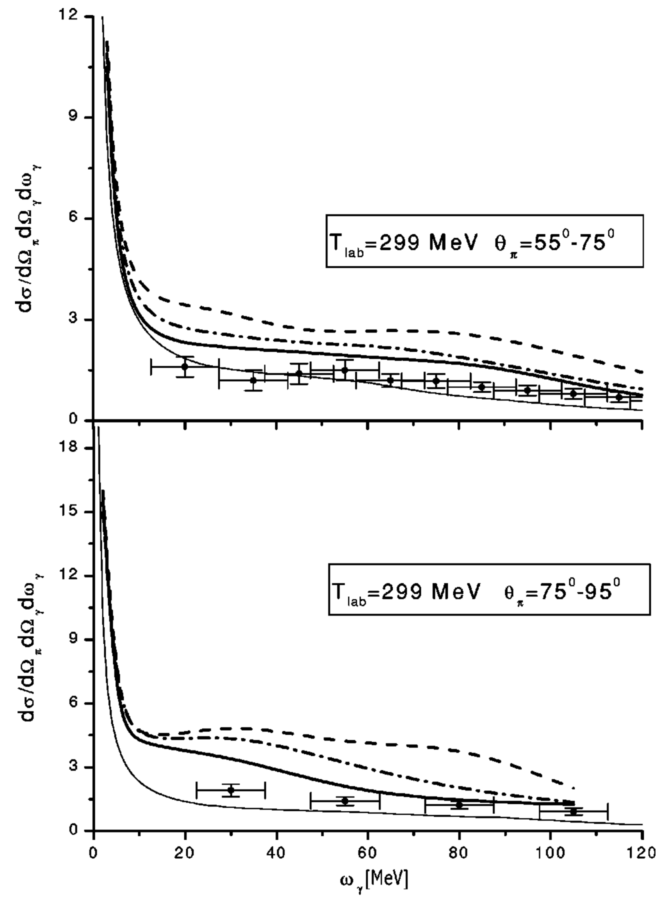

FIG. 6. Same as Fig. 5 for $T_{\mathrm{lab}}=299$ MeV. The cross sections are averaged over the angles $\theta_\pi=55°-75°$ (upper plot) and $\theta_\pi=75°-95°$ (lower plot) in EXP II.

the one used to generate the individual T matrices, which change their values from vertex to vertex. The comparison between the results of the SPA and DMA schemes shows that the additional off-shell effects, added coherently to the lowest order contributions, may have important contributions since they do not cancel exactly as the derivative terms of the T matrix appearing from the soft-photon expansion.

From Fig. 5 we can check that the SEP interaction provides the closest results to the experimental cross section with deviations starting for $\omega_\gamma > 40$ MeV. This indicates that the dynamical model involved in the SEP interaction gives the smallest off-shell effects. On the other hand, the strongest off-shell effects appear in the OBQA model. This conclusion agrees with a previous study [13] on observables in pion photoproduction experiments. Similar conclusions can be drawn from Fig. 6. Note, however, that the SPA lies somewhat below the experimental results in this case, which seems to indicate the necessity of additional dynamical degrees of freedom.

As was discussed in Sec. III, the off-shell contributions to the external and internal amplitudes within the SPA cancel each other, thus we cannot study these effects within this approximation. In addition since we get gauge invariance in the SPA by adjusting the internal amplitude, the gauge-invariant electromagnetic currents remain hidden. In the

DMA approach, these cancellations must occur explicitly between the different components of the amplitude (the so-called Born, pre, post contributions). The departure of the different T matrices from the SPA can be used to estimate the size of unbalanced off-shell terms and provide a test of their off-shell behaviors. Also, since the gauge-invariant electromagnetic current is constructed explicitly from effective Lagrangians, we can use the radiative πN reaction to study the relevance of the degrees of freedom and the parameters involved in this dynamical model.

Finally, one may wonder about the relevance of additional contributions not included in the present formulation of effective Lagrangians, for example the a_1,ω mesons. A preliminary estimation of these effects shows that, in the region of photon energies considered in this work $\omega_\gamma = 0-150\,\text{MeV}$, their contributions to the amplitude are suppressed by an order of magnitude with respect to the degrees of freedom considered here. In addition it may be required to include the double-scattering terms, in order to provide a better fit of experimental data. These considerations, together with a partial wave analysis of the studied T matrices and a more detailed analysis of each particular contribution to the electromagnetic current are beyond the scope of the present paper.

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