l-forbidden $M1$ transitions and pseudospin symmetry

P. von Neumann-Cosel[1,3] and J. N. Ginocchio[2,3]
[1]*Institut für Kernphysik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany*
[2]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
[3]*Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195*
(Received 10 March 2000; published 9 June 2000)

Pseudospin symmetry relates the l-forbidden magnetic dipole transition strengths between pseudospin partners to the magnetic moments of the involved states. These relations are tested against experimental data after correction for the fragmentation of the single particle strengths. Within typical uncertainties of the experimental spectroscopic factors good agreement is found for a wide range of shell closures $Z=20-82$, $N=20-126$, and pseudo-orbital angular momenta. Systematic deviations are observed for particle states above $N=28$ and $Z=50$ where the predicted $B(M1)$ values are about an order of magnitude too large.

PACS number(s): 21.10.Ky, 24.10.Jv, 24.80.+y

I. INTRODUCTION

A near-degeneracy of shell-model orbitals with quantum numbers $[n_r,l,j=l+1/2]$ and $[n_r-1,l+2,j'=(l+2)-1/2]$ can be observed in the vicinity of shell closures throughout the nuclear landscape. The corresponding single particle states are characterized by the radial quantum number n_r and the orbital and total angular momenta l and j. This doublet structure can be expressed in terms of a ''pseudo-orbital'' angular momentum $\tilde{l}=l+1$, where the two levels represent spin-orbit partners with a ''pseudospin'' $\tilde{s}=1/2$. While this concept of a pseudospin symmetry was empirically established 30 years ago [1], a deeper understanding has been lacking. Only recently pseudospin symmetry has been shown to be a relativistic SU(2) symmetry of the Dirac Hamiltonian which occurs when the attractive scalar and repulsive vector nuclear mean fields cancel [2].

Transitions between pseudospin partners are of so-called l-forbidden $M1$ type. The term ''l-forbidden'' refers to a selection rule of the unrenormalized one-body magnetic dipole operator which does not permit a change of the radial quantum number. The description of $M1$ and the closely related GT transitions require modifications of the bare one-body operators to describe the phenomenon of spin quenching in nuclei [3,4]. In light nuclei these have been derived from microscopic calculations of Arima *et al.* [4] and Towner and Khanna [5] which give rise to the spin and orbital correction terms as well as a tensor term ($[Y_2\otimes \mathbf{s}]^{J=1}$) in the $M1$ and GT operators. Alternatively, Brown and Wildenthal [6] deduced such corrections from an empirical fit to a large body of data. In general, both methods agree quite well with each other.

In allowed $M1$ and GT transitions, the weak tensor corrections are usually buried under the dominating spin strength. In contrast, l-forbidden transitions are mainly governed by the tensor part, thus providing experimental insight into this otherwise hardly accessible contribution [7]. The higher-order corrections to the l-forbidden transitions are theoretically expected to be dominated by Δ admixtures into the nuclear wave functions [4,5] and they are a unique observable in this respect. When scaled to the free-nucleon strength, the delta correction is expected to be essentially the same for the isovector $M1$ and GT operators.

Because of the pure single-hole character of the low-lying $1d_{3/2}$ and $2s_{1/2}$ states, l-forbidden transitions in $A=39$ nuclei have been considered a particularly interesting test case for the above predictions. With considerable experimental efforts it has been established that the $M1$ strength is about an order of magnitude larger relative to the GT strength [8–12] in contradiction to the microscopic calculations [4,5]. On the other hand, the result agrees with the empirical findings [6]. This discrepancy has recently been confirmed in the $A=32$ system [13] and is also indicated for l-forbidden $1f_{5/2}\rightarrow 2p_{3/2}$ transitions in the $A=57$ nuclei [14]. Indeed, this is one of the major problems remaining in our understanding of electromagnetic and β-decay observables in light nuclei. As was pointed out repeatedly [10–12] there is no simple way to improve the microscopic calculations of the $M1$ transitions, e.g., by varying the interaction strength, because core polarization and Δ isobar contributions to the l-forbidden $M1$ and GT matrix elements scale strictly within the nonrelativistic models.

A possible explanation may be relativistic corrections as discussed already by Bohr and Mottelson [15]. These have been successfully applied, e.g., to interpret magnetic moments of pseudospin partners at the $N=82$ shell closure [16]. Here, we focus on an interpretation of l-forbidden $M1$ transitions based on the relativistic origin of pseudospin symmetry. For relativistic single-nucleon wave functions, unrenormalized magnetic dipole transitions between pseudospin partners are not forbidden. The transition between the upper components of the Dirac wave function vanishes for such transitions, but the transitions between upper and lower components do not. Because the radial parts of the lower component wave functions of both partners in the pseudospin doublet are the same [17], magnetic dipole transitions are related to the magnetic moments of each state in the doublet [18]. As a result, one finds a natural explanation of finite $M1$ and GT transitions probabilities within the doublet which can be expressed through the magnetic moments of the initial and final states. It is certainly interesting to what extent these predictions are confirmed by experimental data.

TABLE I. Experimental information on the magnetic properties of pseudospin partners and pseudospin symmetry predictions for neutron-odd nuclei. Given are the the pseudo-orbital angular momentum $\tilde{l}$, the involved single particle states, their magnetic moments μ and spectroscopic factors S, and the $B(M1)\downarrow$ transition strength between them. All experimental results are from [19] except where noted. The last two columns present the pseudospin symmetry prediction for the $B(M1)\downarrow$ strengths and the correction C to the spectroscopic factors needed to reproduce the data.

Nucl.	$\tilde{l}$	Conf.	E_x (MeV)	S	μ (μ_N)	Conf.	E_x (MeV)	S	μ (μ_N)	$B(M1)^{1/2}_{\mathrm{exp}}$ (μ_N)	$B(M1)^{1/2}_{\mathrm{th}}$ (μ_N)	C
$^{37}_{18}\mathrm{Ar}_{19}$	1	$2s_{1/2}$	1.409	0.44		$1d_{3/2}$	0.000	0.88	$+1.15(1)$	0.118(17)[a]	0.069	0.97/1.30
$^{39}_{20}\mathrm{Ca}_{19}$	1	$2s_{1/2}$	2.469	0.93		$1d_{3/2}$	0.000	0.99	$+1.02(0)$	0.110(9)	0.110	1
$^{53}_{24}\mathrm{Cr}_{29}$	2	$2p_{3/2}$	0.000	0.56	$-0.48(0)$	$1f_{5/2}$	1.006	0.25		0.305(16)	0.315	-0.10/0.98
$^{57}_{28}\mathrm{Ni}_{29}$	2	$2p_{3/2}$	0.000	0.90[b]	$-0.80(0)$	$1f_{5/2}$	0.769	0.90[b]		0.161(9)	0.656	0.35/0.58
$^{57}_{26}\mathrm{Fe}_{31}$	2	$2p_{3/2}$	0.014	0.42		$1f_{5/2}$	0.137	0.59	$+0.94(1)$	0.046(2)	0.081	1.07/1.25
$^{57}_{26}\mathrm{Fe}_{31}$	2	$2p_{3/2}$	0.014	0.42	$-0.15(0)$	$1f_{5/2}$	0.137	0.59		0.046(2)	0.304	0.13/0.25[c]
$^{59}_{28}\mathrm{Ni}_{31}$	2	$2p_{3/2}$	0.000	0.82		$1f_{5/2}$	0.339	0.68	$+0.43(19)$	0.122(8)	0.400	0.30/0.63
$^{91}_{40}\mathrm{Zr}_{51}$	3	$2d_{5/2}$	0.000	0.75	$-1.30(0)$	$1g_{7/2}$	2.201	0.44		0.095(26)	0.077	0.80/1.02
$^{95}_{42}\mathrm{Mo}_{53}$	3	$2d_{5/2}$	0.000	0.59	$-0.91(0)$	$1g_{7/2}$	0.762	0.18		0.141(18)	0.091	0.51/1.11
$^{97}_{44}\mathrm{Ru}_{53}$	3	$2d_{5/2}$	0.000	0.57	$-0.79(1)$	$1g_{7/2}$	0.421	0.61		0.122(4)[d]	0.241	0.58/0.86
$^{139}_{58}\mathrm{Ce}_{81}$	1	$3s_{1/2}$	0.255	0.90		$2d_{3/2}$	0.000	0.85	0.96(4)	0.078(19)	0.015	0.91/1.06
$^{141}_{60}\mathrm{Nd}_{81}$	1	$3s_{1/2}$	0.193	0.95		$2d_{3/2}$	0.000	1.00	1.01(1)	0.062(4)	0.134	0.83/0.94
$^{145}_{60}\mathrm{Nd}_{85}$	4	$2f_{7/2}$	0.000	0.52	$-0.66(0)$	$1h_{9/2}$	0.748	0.22		0.069(21)[d]	0.164	0.52/0.80
$^{207}_{82}\mathrm{Pb}_{125}$	2	$3p_{3/2}$	0.898	0.95		$2f_{5/2}$	0.570	0.80	$+0.80(3)$	0.23(2)	0.288	0.52/0.95
$^{209}_{82}\mathrm{Pb}_{127}$	5	$2g_{9/2}$	0.000	0.83	$-1.47(0)$	$1i_{11/2}$	0.779	0.86		0.100(5)[d]	0.085	0.85/1.01

[a] $E2$ contribution estimated from shell model [20].
[b] From shell model [21].
[c] Not included in Figs. 1 and 3, see text.
[d] Pure $M1$ transition assumed.

II. APPLICATION OF PSEUDOSPIN SYMMETRY TO DATA

For a comparison to data one must be aware that the relations derived in Ref. [18] hold for pure single particle states only. The single particle strength is distributed over many states by the residual interaction and only in odd-mass nuclei near closed shells one can expect to find states which carry large fractions of the total strength. Therefore, we restrict ourselves to nuclei with one or three particles (holes) with respect to a closed shell. The calculation of $B(M1)$ values from magnetic moments described in Ref. [18] is modified to include the fractional single particle strength by correcting with spectroscopic factors from single-nucleon stripping (pickup) reactions.

This leads to the following relations:

$$\left(\frac{B(M1;j'\to j)}{gS_jS'_j}\right)^{1/2}=\sqrt{\frac{j+1}{2j+1}}\left(\frac{\mu_j^{\mathrm{exp}}}{S_j}-\mu_n\right), \tag{1}$$

$$\left(\frac{B(M1;j'\to j)}{gS_jS'_j}\right)^{1/2}=\sqrt{\frac{2j+1}{j+1}}\frac{j+2}{2j+3}\left(\frac{\mu_{j'}^{\mathrm{exp}}}{S'_j}+\frac{j+1}{j+2}\mu_n\right) \tag{2}$$

for neutron-odd nuclei and

$$\left(\frac{B(M1;j'\to j)}{gS_jS'_j}\right)^{1/2}=\frac{(j+2)(2j+1)(\mu_{j'}^{\mathrm{exp}}/S'_j)-(2j+3)(j+1)(\mu_j^{\mathrm{exp}}/S_j)+4(j+1)^2\mu_p}{2(2j+3)\sqrt{(j+1)(2j+1)}} \tag{3}$$

for proton-odd nuclei. Here, $j'=\tilde{l}+1/2, j=\tilde{l}-1/2$ are the total angular momenta of the pseudospin partners, and $\mu_{j,j'}^{\mathrm{exp}}$ are the corresponding experimental magnetic moments. Further, $\mu_p=1.79\mu_N$ and $\mu_n=-1.91\mu_N$ stand for the anomalous magnetic moments (defined as the difference to the Dirac prediction) of proton and neutron, respectively, $S_{j,j'}$ are the spectroscopic factors, and g denotes a statistical factor depending on the initial and final state. Note that the magnetic dipole operator for the neutron transitions depends on only one term proportional to the magnetic moment of the neutron, whereas because of the charge the magnetic dipole operator for proton transitions contains an additional term due to the orbital motion. For this reason two relations exist for neutron and only one for proton transitions.

TABLE II. Same as Table I, but for proton-odd nuclei.

Nucl.	$\tilde{l}$	Conf.	E_x (MeV)	S	μ (μ_N)	Conf.	E_x (MeV)	S	μ (μ_N)	$B(M1)^{1/2}_{\rm exp}$ (μ_N)	$B(M1)^{1/2}_{\rm th}$ (μ_N)	C
$^{67}_{31}Ga_{36}$	2	$2p_{3/2}$	0.000	0.36	+1.85(0)	$1f_{5/2}$	0.359	0.79	1.40(65)	0.130(7)	0.105	0.96/1.38
$^{121}_{51}Sb_{70}$	3	$3d_{5/2}$	0.000	0.92	+3.36(0)	$2g_{7/2}$	0.058	0.75[a]	+2.52(1)	0.134(4)	0.860	−0.016/0.27
$^{127}_{53}I_{74}$	3	$3d_{5/2}$	0.000	0.60[b]	+2.81(0)	$2g_{7/2}$	0.058	0.70[b]	+2.54(1)	0.148(7)	0.480	0.19/0.57
$^{129}_{53}I_{76}$	3	$3d_{5/2}$	0.028	0.59	+2.80(0)	$2g_{7/2}$	0.000	0.66	+2.62(3)	0.134(7)	0.608	0.14/0.45
$^{131}_{53}I_{78}$	3	$3d_{5/2}$	0.150	0.53	+2.79(50)	$2g_{7/2}$	0.000	0.64	+2.74(1)	0.10(1)	0.560	0.24/0.49
$^{203}_{81}Tl_{122}$	1	$3s_{1/2}$	0.000	0.70	+1.62(0)	$2d_{3/2}$	0.279	0.46	−0.02(17)	0.046(1)	0.084	0.80/0.94
$^{205}_{81}Tl_{122}$	1	$3s_{1/2}$	0.000	0.70	+1.64(0)	$2d_{3/2}$	0.204	0.42	−0.08(5)	0.028(1)	0.044	0.89/0.97

[a]Estimated from systematics in Sb isotopes.
[b]Estimated from systematics in I isotopes.

The experimental information [19] on *l*-forbidden *M*1 transitions fulfilling the selection criteria described above are summarized in Table I for neutron-odd and in Table II for proton-odd cases. The spectroscopic factors $S_{j,j'}$ are taken from one-nucleon transfer reactions populating the respective states. For transitions between neutron-odd states one has in principle two independent predictions from Eqs. (1) and (2). However, there is only one case (^{57}Fe) where experimental information on the magnetic moments of both pseudospin partners is available. Because of the need to know the magnetic moments of both levels, the data are much more limited for proton-odd transitions. The next-to-last columns of the Tables I and II present the predictions of Eqs. (1)–(3). Generally $B(M1)\downarrow$ values are given for the experiments. If the state with $j'=\tilde{l}+1/2$ is higher in energy, then the theoretical $B(M1)$ in Eqs. (1–3) are $B(M1)\downarrow$ and $g=1$. If the state with $j=\tilde{l}-1/2$ is higher in energy, then $g=(\tilde{l}+1)/\tilde{l}$ in order for the relations in Eqs. (1)–(3) to be correct.

III. DISCUSSION

A comparison of the experimental *l*-forbidden strengths with the pseudospin predictions is displayed in Figs. 1 and 2 for neutron and proton transitions. Nuclei with 1 and 3 particles (holes) with respect to the shell closure are distinguished as circles and triangles, respectively. In neutron-odd nuclei (Fig. 1) one finds on the average reasonable agreement with experimental $B(M1)$ strengths although a few cases scatter considerably. The predictive power is better for closed-shell ± 1 nuclei (with two marked exceptions, see below), while the pseudospin results for closed-shell ± 3 nuclei tend to be somewhat too large. For proton-odd transitions (Fig. 2) a clear separation in two groups is observed. For three out of seven experimental results the pseudospin predictions are acceptable while the other four are about an order of magnitude above the data.

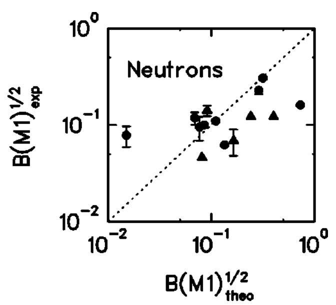

FIG. 1. Experimental *l*-forbidden *M*1 transition strengths in neutron-odd nuclei vs pseudospin symmetry predictions, Eqs. (1),(2). Circles correspond to one particle (hole), triangles to three particles (holes) with respect to closed shells.

Since the pseudospin predictions are calculated from data it is necessary to evaluate the effect of the experimental uncertainties. The largest contribution clearly comes from the spectroscopic factors. It is difficult to quantify, but comparison of *S* values deduced from different reactions [e.g., (d,p) vs (t,d), etc.] and typical errors of the measured cross sections suggest a range of $\pm 20\%$. In order to see whether agreement with the experimental strengths can be achieved by a variation of the spectroscopic factors within this limit we determine a correction factor *C* with $S^{\rm eff}_{j,j'}=C\times S_{j,j'}$ necessary to reproduce the data with Eqs. (1)–(3). For simplicity, we assume *C* to be the same for S_j and $S_{j'}$. Because the sign of the *M*1 matrix element is not determined by the experiments, always two solutions exist. These are given in the last columns of Tables I and II.

Assuming in each case the more favorable value (i.e., closer to 1), the correction factors are plotted in Figs. 3 and 4 as a function of neutron and proton number, respectively. One finds that agreement can be achieved for almost all neutron transitions within the estimated uncertainty for the spectroscopic factors indicated by the short-dashed lines in Fig. 3. There are two exceptions which would require values around 0.6. In cases where both spectroscopic factors are close to unity, the variations of *C* around 1 are even smaller ($<10\%$). For the proton transitions (Fig. 4) the findings from Fig. 2 are reflected with a group of four cases which would require correction factors of about $0.3-0.5$ well outside the acceptable range.

As pointed out above, Eqs. (1) and (2) can provide independent results for the same transition. For the data selected here, there is only one case (^{57}Fe) where experimental data on magnetic moments of both states are available. In addition to the result given in Table I, a value $\mu=-0.15\ \ \mu_N$ has been measured for the $E_x=0.014$ MeV, $J^\pi=3/2^-$ state

[19]. The prediction of Eq. (1) would be $[B(M1\downarrow)]^{1/2} = 0.304\ \mu_N$ in poor agreement with experiment. The spectroscopic information for this level suggests a more complex structure beyond the single particle picture. This is also reflected in shell-model calculations [22] which provide a successful description of the $M1$ transitions strengths in ^{57}Fe. While these calculations are capable to account for the magnetic moments of $1/2^-$ g.s. and the lowest $5/2^-$ state used in Table I, they completely fail for the $3/2^-$ state.

Concerning the discrepancies of the tensor corrections to the magnetic dipole and GT operators in doubly closed shell ± 1 nuclei, the present results are inconclusive. The two cases where data are sufficient for a description in pseudospin symmetry are ^{39}Ca and ^{57}Ni. A perfect description is achieved for the transition in ^{39}Ca, but the prediction for the l-forbidden $M1$ transition strength in ^{57}Ni is an order of magnitude too high. For ^{57}Ni, spectroscopic factors were recently deduced from a (d,p) experiment in reverse kinematics using a radioactive ^{56}Ni beam, but exhibit large uncertainties [21]. Instead, the S values were taken from the shell-model results described in Ref. [21]. However, it was demonstrated recently by large-scale Monte Carlo shell-model calculations that the Fermi surfaces are considerably weakened in ^{56}Ni with a probability of 50% only for a doubly closed configuration in the ground state [23]. Thus, the calculations of Ref. [21] might severely overestimate the purity of the single particle states in ^{57}Ni although the reduction to $S_{j,j'} \approx 0.6$ necessary to reproduce the experimental $B(M1)$ strength is probably too drastic to be explained by this finding.

The systematics of $2s_{1/2} \rightarrow 1d_{3/2}$ transitions in the sd shell and proton $1g_{7/2} \rightarrow 2d_{5/2}$ transitions above $Z=50$ have been discussed by Andrejtscheff *et al.* in terms of the interplay of first order and second order core polarization for the former [24] and the role of quadrupole deformation for the latter [25]. Because of the present restriction to near-closed shell nuclei a direct comparison is not possible. The data summarized in Tables I and II cover all shell closures $Z=20-82$ and $N=20-126$ and a large variety of pseudoangular momenta $\tilde{l}=1-5$. No systematic dependencies are visible except for the marked deviations of nuclei with 1 or 3 particles relative to $N=28$ (^{57,59}Ni) and $Z=50$ (^{121}Sb, ^{127,129,131}I). Their origin is at present not understood. One should keep in mind, however, that the magnetic moments themselves are quenched (see, e.g., Refs. [26,27]) and it is unclear to what extent these effects would be included in the present approach. Rather, the reproduction of a large body of data for such a weak and elusive type of transition provides strong support for the interpretation of pseudospin symmetry developed in Refs. [2,17,18].

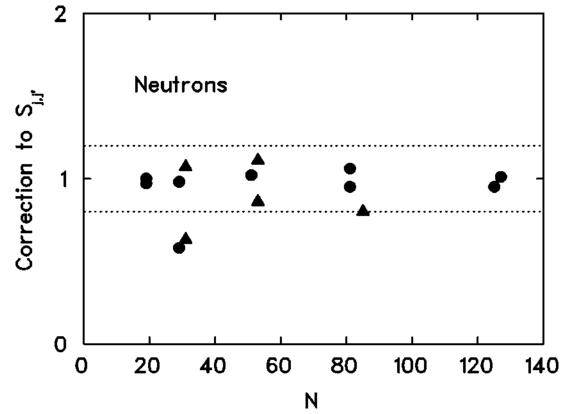

FIG. 3. l-forbidden $M1$ transitions in neutron-odd nuclei: Corrections to the spectroscopic factors necessary to bring the pseudospin symmetry predictions, Eqs. (1),(2), in agreement with the experimental data as a function of neutron number. For symbols see Fig. 1.

IV. CONCLUSIONS

The relativistic SU(2) pseudospin symmetry has been tested through experimental magnetic properties of the involved doublet of single particle states in near-closed shell nuclei. After correction for the fragmentation of the single particle strength in the wave functions, predictions connecting the l-forbidden $M1$ transitions between pseudospin partners with their magnetic moments [18] are capable to describe the data throughout the nuclear landscape. Considering the simplicity of the approach neglecting any explicit account for the well-established quenching of magnetic moments, this success seems remarkable. However,

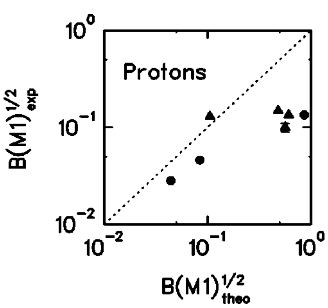

FIG. 2. Experimental l-forbidden $M1$ transition strengths in proton-odd nuclei vs pseudospin symmetry predictions, Eq. (3). For symbols see Fig. 1.

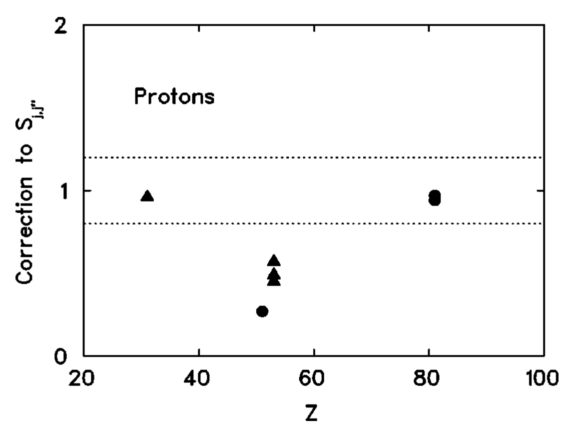

FIG. 4. l-forbidden $M1$ transitions in proton-odd nuclei: Corrections to the spectroscopic factors necessary to bring the pseudospin symmetry predictions, Eq. (3), in agreement with the experimental data as a function of atomic charge number. For symbols see Fig. 1.

pronounced deviations occur at the $N=28$ and $Z=50$ shell closures whose origin is at present not clear.

Particular interest has focused on *l*-forbidden GT and $M1$ transitions in doubly magic ±1 nuclei as a test of tensor corrections to the respective one-body operators. Microscopic predictions [3–5] account for the GT strengths, but fail for the $M1$ strengths. For the two cases where data are sufficient for a description in pseudospin symmetry we find perfect agreement for one (^{39}Ca) and disagreement for the other (^{57}Ni). However, the mass-57 system needs further investigation because of the unexpected softness of the ^{56}Ni closed core. Clearly, a measurement of the magnetic moments of the $2s_{1/2}^{-1}$ excited states in ^{39}K and ^{39}Ca and the $1f_{5/2}$ states in ^{57}Ni and ^{57}Cu would be of considerable interest to test the predictive power of the approach.

ACKNOWLEDGMENTS

The authors thank the Institute for Nuclear Theory, University of Washington, Seattle for its kind hospitality during a visit which initiated the present work. Discussions with A. Richter and H. Toki are gratefully acknowledged. This work was supported by the DFG under Contract No. Ri 242/12-2 and by the U. S. Department of Energy under Contract No. W-7405-ENG-36.

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