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# Single boson images via an extended Holstein-Primakoff mapping

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The Holstein-Primakoff mapping for pairs of bosons is extended in order to accommodate single boson mapping. The proposed extension allows a variety of applications and especially puts the formalism at finite temperature on firm ground. The new mapping is applied to the $O(N+1)$ anharmonic oscillator with global symmetry broken down to $O(N)$. It is explicitly demonstrated that $N$-Goldstone modes appear. This result generalizes the Holstein-Primakoff mapping for interacting bosons as developed by Aouissat, Schuck, and Wambach [Nucl. Phys. **A618**, 402 (1997)].

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Boson expansion theory (BET) has played a significant role over the past decades in our understanding of the nuclear many-body problem. Starting with the pioneering work of Marumori and co-workers [1], and of Belyaev and Zelevinsky [2], the interest in this subject has culminated in the 1980's through the formulation of the interacting boson model [3]. Of particular interest, not only for the many-body problem, but also, as has become apparent, for quantum-field theory (QFT), is the perturbative boson expansion (PBE) approach. Extensive use of it has been made in nuclear physics, in order to extract anharmonicities beyond the random-phase approximation (RPA) (see Refs. [4,5] for reviews). Up until very recently its application to QFT has not attracted much attention and, therefore, has not been fully developed so far. The Holstein-Primakoff mapping for boson pairs, first introduced in [6,7], was recently applied, however, to the $O(N)$ vector model [8]. It was demonstrated that the mapping is able to systematically classify the dynamics according to the $1/N$ expansion, rendering a promising and efficient alternative to the well-known functional methods. Furthermore, considering the model in the phase of spontaneously broken symmetry, the powerful machinery of the PBE approach as developed for deformed nuclear systems could be transcribed to QFT. As a consequence the Goldstone theorem as well as the whole hierarchy of Ward identities were exactly satisfied [8].

However, the PBE in general, and the Holstein-Primakoff mapping (HPM) in particular [9], rely on the bosonization of pairs of particles. Thereby, images for particle pairs are generated in an ideal Fock space, while single particle images are absent after mapping. This problem has been appreciated for the fermionic case in the early days of the boson expansion theory. Marshalek has proposed an extension of HPM for fermions in order to allow for a perturbative boson expansion for both even and odd nuclei [10]. In the present Rapid Communication, we point out the occurrence of the same problem in the case of the PBE for purely bosonic models.

The need for an extended bosonic HPM to include single bosons clearly revealed itself in [8] where the lack of ideal single-boson states was an obstacle for defining unambiguously the two-point function for the Goldstone mode. While to leading order in the $1/N$ expansion this problem was circumvented in [8], a next-to-leading order calculation makes an extended version of the HPM mandatory.

Finite-temperature applications of the PBE approach is another issue where an extended HPM to include single bosons is definitely called for. In the following, we wish to sketch a derivation of an extended version of HPM for bosons. We will also discuss an application to the $O(N+1)$ anharmonic oscillator where it will be demonstrated explicitly that this new method is capable of including single-boson images with the correct asymptotic energy.

As a starting point let us consider a system with two types of bosonic creation and annihilation operators: $a^+$, $a$ and $b^+$, $b$. Pairing these in all possible ways leads to ten group generators of the noncompact $Sp(4)$ group. The pairs $a^+a$, $aa$, $a^+a^+$, and analogously the pairs of $b$ operators form two commuting $Sp(2)$ subgroups. The number conserving bilinears $a^+a$, $b^+b$, $a^+b$, and $ab^+$ span a closed $U(2)$ algebra. There remain the bilinears $a^+b^+$ and $ab$ which do not belong to any nontrivial subgroup of $Sp(4)$.

Our goal will be to first set up the boson images of the ten group generators, replacing in the end the $b$ operators by $c$ numbers (the condensate). This will lead us to the boson image of the semidirect product group $Sp(2)\otimes N(1)$ made up of the elements $a^+a$, $aa$, $a^+a^+$, $a$, $a^+$, and $1_d$, respectively. The latter is the desired system because it involves even and odd numbers of boson operators.

We will follow earlier work for interacting fermions by Evans and Kraus [11], Klein, Rafelski, and Rafelski [12], and Klein, Cohen, and Li [13] in which a mapping for the ten generators of the $SO(5)$ group was derived. Use is made especially of the work of the latter groups of authors to derive, this time, the mapping of the ten generators of the noncompact $Sp(4)$ group mentioned above. Since there is no room to go into details (which will be presented elsewhere) we essentially will only give the result here.

One first realizes that the six generators of the two commuting $Sp(2)$ algebras can be mapped via the usual HPM. The difficult task lies in finding an adequate mapping for the generators $a^+b^+$ and $ab$, which allows one to close of the full $Sp(4)$ algebra. The reader is invited to consult Ref. [12] for a similar derivation. Introducing a set of three new bosonic operators $\alpha$, $A_1$, and $A_2$, one can show that the net result for the complete mapping reads

 

$$(a^+a^+)_I = A_1^+\sqrt{2+4(n_1+m)},$$
$$(aa)_I = ((a^+a^+)_I)^+, \quad (a^+a)_I = 2n_1+m,$$
$$(b^+b^+)_I = A_2^+\sqrt{2+4(n_2+m)},$$
$$(bb)_I = ((b^+b^+)_I)^+, \quad (b^+b)_I = 2n_2+m,$$
$$(a^+b^+)_I = \alpha^+\sqrt{2+4(n_1+m)}\sqrt{2+4(n_2+m)}\Phi(m) + 4\Phi(m)A_2^+A_1^+\alpha,$$
$$(ab)_I = ((a^+b^+)_I)^+,$$
$$(a^+b)_I = \frac{1}{2}[(bb)_I,(a^+b^+)_I], \quad (b^+a)_I = ((a^+b)_I)^+, \tag{1}$$

where $n_1$, $n_2$, and $m$ are occupation number operators defined by

$$m = \alpha^+\alpha, \quad n_i = A_i^+A_i, \quad (i=1,2). \tag{2}$$

The ''+'' in the Holstein-Primakoff square root indicates the noncompact character of the group at hand. Finally, the function $\Phi$ is given by

$$\Phi(m) = \left[\frac{r+m^2}{4(m+1)(2m+1)(2m-1)}\right]^{1/2}, \tag{3}$$

where $r$ is a constant which is fixed using physical conditions as will be discussed in the next section.

These results constitute only an intermediate step towards our final goal. As stated earlier, one wishes to extend the usual HPM for boson pairs, in such a way as to allow the mapping of single bosons as well. In other words, and following the original Belyaev-Zelevinsky approach, one needs to achieve a realization of the following algebra:

$$[aa, a^+a^+] = 2+4\,a^+a,$$
$$[aa, a^+a] = 2\,aa,$$
$$[a, a^+a^+] = 2\,a^+,$$
$$[a, a^+a] = a, \tag{4}$$

where all other possible commutators are assumed but not explicitly shown here. This is nothing but the algebra of the semidirect product group $Sp(2)\otimes N(1)$. The first two commutation relations in Eq. (4) remind us of the $Sp(2)$ algebra, and as such, one can propose the bosonic HPM as a second realization for it. Here again, the difficulty lies in finding an adequate mapping for the single bosons so as to close the algebra above. A way out is to notice that, by considering the limit in which the operators $b$ and $b^+$ are transformed into the identity operator, one can ultimately contract the whole $Sp(4)$ group to a nonisomorphic semidirect group $Sp(2)\otimes N(1)$. This singular transformation, which can be thought of as a contraction *à la* Inönü-Wigner or Saletan [14,15], gives a clear hint on how to proceed with the desired extension. Indeed, the single bosons can be deduced from the contraction of the generators $a^+b^+$, $ab$, $a^+b$, and $ab^+$ down to the generators $a$ and $a^+$. With this intuitive picture in mind, one can show that the following mapping for the five relevant generators constitutes a realization of the algebra in Eq. (4):

$$(aa)_I = \sqrt{2+4(n_1+m)}A_1, \quad (a^+a^+)_I = (aa)_I^+,$$
$$(a^+a)_I = 2\,n_1+m,$$
$$(a)_I = \sqrt{2+4(n_1+m)}\Gamma_1(m)\alpha + 2\alpha^+A_1\Gamma_1(m),$$
$$(a^+)_I = (a)_I^+, \tag{5}$$

where the occupation number operators are, as before, given by $n_1 = A_1^+A_1$, $m = \alpha^+\alpha$, while the function $\Gamma_1$ reads

$$\Gamma_1(m) = \left[\frac{z_1+m^2}{2(m+1)(2m+1)(2m-1)}\right]^{1/2}. \tag{6}$$

Here, too, $z_1$ is a constant which will be fixed by using physical conditions as will be explained. It is straightforward to verify, through a direct evaluation of the commutators in Eq. (4), that this is indeed a proper realization. This completes our considerations concerning the mapping. In the following, the formalism will be applied to the interesting case of $N$ oscillators and used to develop the $1/N$ expansion.

As an application, let us consider the anharmonic oscillator with an $O(N+1)$ symmetry broken down to $O(N)$. The properly scaled Hamiltonian of the system is given by

$$H = \frac{\vec{P}_\pi^2}{2} + \frac{P_\sigma^2}{2} + \frac{\omega^2}{2}[\vec{X}_\pi^2 + X_\sigma^2] + \frac{g}{N}[\vec{X}_\pi^2 + X_\sigma^2]^2 - \sqrt{N}\eta X_\sigma. \tag{7}$$

Here we have considered an explicit ($\eta \neq 0$) and a spontaneous ($\langle X_\sigma\rangle \neq 0$) symmetry breaking along the $X_\sigma$ mode. The variables $\vec{X}_\pi, X_\sigma$ and their conjugate momenta $\vec{P}_\pi, P_\sigma$ are expressed in second quantization as

$$\vec{X}_\pi = \frac{1}{\sqrt{2\omega}}(\vec{a}+\vec{a}^+), \quad \vec{P}_\pi = i\sqrt{\frac{\omega}{2}}(\vec{a}^+ - \vec{a}),$$

$$X_\sigma = \frac{1}{\sqrt{2\mathcal{E}_\sigma}}(b+b^+), \quad P_\sigma = i\sqrt{\frac{\mathcal{E}_\sigma}{2}}(b^+ - b). \tag{8}$$

The frequency, $\mathcal{E}_\sigma$, of the mode $X_\sigma$ will be fixed later. The subscripts $\pi$ and $\sigma$ are used in analogy with the linear $\sigma$ model in QFT, where these modes represent the pion and sigma fields, respectively.

To sort out the dynamics according to the $1/N$ expansion, one needs to adapt the mapping derived previously to the situation of $N$ oscillators. This can be done in a straightforward way. It can be shown that the mapping in this case takes the form

$$(\vec{a}\vec{a})_I=\sqrt{2N+4(n_1+m)}A_1,$$
$$(\vec{a}^+\vec{a})_I=2n_1+m, \quad (\vec{a}^+\vec{a}^+)_I=(\vec{a}\vec{a})_I^+,$$
$$(a_i)_I=\sqrt{2N+4(n_1+m)}\Gamma_N(m)\alpha_i+2\alpha_i^+A_1\Gamma_N(m),$$
$$(a_i^+)_I=(a_i)_I^+, \tag{9}$$

where $N$ is an integer, $n_1=A_1^+A_1$, and $m=\Sigma_i\alpha_i^+\alpha_i$, while $\Gamma_N$ is a generalization of the $\Gamma_1$ function, of the last section to the case of $N$ oscillators. It reads

$$\Gamma_N(m)=\left[\frac{z_N+m^2+m(N-1)}{2(m+1)(2m+N)(2m+N-2)}\right]^{1/2}. \tag{10}$$

The constant $z_N$ will be fixed below. One can also easily verify that this mapping leads to a realization of the following algebra:

$$[(\vec{a}\vec{a}),(\vec{a}^+\vec{a}^+)]=2\,N+4(\vec{a}^+\vec{a}),$$
$$[(\vec{a}\vec{a}),(\vec{a}^+\vec{a})]=2(\vec{a}\vec{a}),$$
$$[a_i,(\vec{a}^+\vec{a}^+)]=2\,a_i^+,$$
$$[a_i,(\vec{a}^+\vec{a})]=a_i. \tag{11}$$

For a finite $N$, the $O(N+1)$ anharmonic oscillator is purely quantum mechanical. For an infinite number of degrees of freedom $N\to\infty$, on the other hand, it can be used to mimic the quantum-field situation of the breaking and restoration of a continuous symmetry.

Using the mapping in Eq. (9), one can expand the Hamiltonian of the system in powers of the operators $A$, $\alpha$, $b$ and their Hermitian conjugates. One then arrives at an expansion of the form $H=\mathcal{H}^{(0)}+\mathcal{H}^{(1)}+\mathcal{H}^{(2)}+\mathcal{H}^{(3)}+\mathcal{H}^{(4)}+,\cdots,$ where the superscripts indicate powers of operators without normal ordering. This expansion is in fact not unique and therefore the preservation of the symmetries is not necessarily guaranteed. A more useful approach consists in organizing the expansion in powers of the parameter $N$, such that $H=NH_0+\sqrt{N}H_1+H_2+(1/\sqrt{N})H_3+(1/N)H_4+,\cdots$.

This is possible if one chooses a coherent state as the variational ground state for the model

$$|\psi\rangle=\exp[\langle A_1\rangle A_1^++\langle b\rangle\, b^+]|0\rangle. \tag{12}$$

This trial vacuum state must accommodate two condensates, respectively, for the $X_\sigma$ mode and the newly introduced boson $A_1$ (see Ref. [8] for details). The mode $\alpha$, on the other hand, is not allowed to condense. The ground-state energy, $NH_0=(\langle\psi|H|\psi\rangle)/\langle\psi|\psi\rangle$, calculated on the coherent state, takes the following form:

$$H_0=\frac{\omega}{2}(2d^2+1)+\frac{gs^2}{\omega}(d+\sqrt{1+d^2})^2$$
$$+\frac{g}{4\omega^2}(d+\sqrt{1+d^2})^4+\frac{\omega^2 s}{2}+gs^4-\eta s, \tag{13}$$

where we have introduced for convenience the rescaled condensates $s=(1/\sqrt{N})\langle X_\sigma\rangle$, $d=\sqrt{2/N}\langle A\rangle$.

The coherent ground state is fully determined by requiring that the values taken by the two condensates above lead to the minimum of $H_0$. The minimization procedure with respect to $s$ and $d$ gives the following two coupled BCS equations:

$$\omega^2+4gs^2+\frac{2g}{\omega}(d+\sqrt{1+d^2})^2=\frac{\eta}{s},$$
$$2\omega d\sqrt{1+d^2}+(d+\sqrt{1+d^2})^2\Delta=0, \tag{14}$$

where $\Delta=(2gs^2/\omega)+g/\omega^2(d+\sqrt{1+d^2})^2$ is the gap parameter .

To gather the full dynamics of the leading order in the $1/N$ expansion one needs to generate the terms $H_1$ and $H_2$ of the Hamiltonian. This can be done by using the parameter differentiation techniques (see Ref. [8] for details). The net result for both $H_1$ and $H_2$ then reads

$$H_1=\frac{1}{\sqrt{2}}\left[2\omega d+\frac{(d+\sqrt{1+d^2})^2}{\sqrt{1+d^2}}\Delta\right](\tilde{A}_1+\tilde{A}_1^+)$$
$$+\left[\frac{2gs}{\omega}(d+\sqrt{1+d^2})^2+\omega^2 s+4gs^3-\eta\right]\frac{(\beta^++\beta)}{\sqrt{2\mathcal{E}_\sigma}},$$

$$H_2=\mathcal{H}_0+\mathcal{E}_\sigma\beta^+\beta+\left[\omega+\Delta+\frac{\Delta d}{\sqrt{1+d^2}}\right]m$$
$$+\left[2\omega+2\Delta+\frac{\Delta d}{\sqrt{1+d^2}}\right]\tilde{n}_1+(\tilde{A}_1+\tilde{A}_1^+)^2$$
$$\times\left[\frac{\Delta d(2+d^2)}{4\sqrt{(1+d^2)^3}}+\frac{g}{2\omega^2}\,\frac{(d+\sqrt{1+d^2})^4}{1+d^2}\right]$$
$$+2gs\frac{(\beta^++\beta)(\tilde{A}_1+\tilde{A}_1^+)}{\omega\sqrt{\mathcal{E}_\sigma}}\,\frac{(d+\sqrt{1+d^2})^2}{\sqrt{1+d^2}}. \tag{15}$$

Here, $\mathcal{H}_0$ is a constant. Since one is not particularly interested in the ground-state energy, the latter will not be further specified. The shifted operators $\tilde{A}_1=A_1-\langle A_1\rangle$, $\beta=b-\langle b\rangle$, and $\tilde{n}_1=\tilde{A}_1^+\tilde{A}_1$ annihilate the coherent state $|\Psi\rangle$. The frequency $\mathcal{E}_\sigma$ of the $X_\sigma$ mode is fixed such that the bilinear part of $H_2$ in the $\beta$ operators is diagonal. It is purely of perturbative character, and the frequency is explicitly given by

$$\mathcal{E}_\sigma^2=\omega^2+12gs^2+\frac{2g}{\omega}(d+\sqrt{1+d^2})^2. \tag{16}$$

Using the gap equations (14) and the easily verifiable identities

$$\Delta = -2d\sqrt{1+d^2}[\omega(d-\sqrt{1+d^2})^2],$$

$$\omega+\Delta = (1+2d^2)[\omega(d-\sqrt{1+d^2})^2], \qquad (17)$$

one can establish that $H_1$ vanishes at the minimum. From $H_2$, and more precisely from the coefficient of $m = \Sigma_{i=1}^{N}\alpha_i^+\alpha_i$, one can deduce the existence of $N$ uncoupled modes. The common frequency of these modes is denoted by $\mathcal{E}_\pi$ and is given by

$$\mathcal{E}_\pi^2 = \omega^2(d-\sqrt{1+d^2})^4 = \omega^2+4gs^2+\frac{2g}{\mathcal{E}_\pi} = \frac{\eta}{s}. \qquad (18)$$

These $N$ modes are our first asymptotic states. Furthermore, it can be easily verified that they have Goldstone character. In other words, their frequency vanishes in the exact symmetry limit ($\eta=0$), and for a finite condensate ($s\neq 0$).

It is evident from the ansatz above that the model suffers from infrared divergences. However, since it is used for demonstration purposes only, we choose to disregard this difficulty here. Clearly the new and important result that has been obtained shows up in the fact that the proposed mapping provides asymptotic states in the ideal Fock space which correspond to the images of the single bosons. It should be stressed that this is a nontrivial finding which, as shown above, is a direct consequence of the extended HPM. It reproduces the result anticipated earlier in this paper, in clear departure from the HPM for boson pairs [8] and in accordance with the Goldstone theorem.

So far, only the mapping of the bilinears in Eq. (9) was involved in expanding the Hamiltonian. The single-boson part of the mapping, on the other hand, was not directly used. The latter enters, however, in the definition of the two-point function $\langle\Psi|TX_{\pi,i}(t)X_{\pi,i}(t')|\Psi\rangle$, where $|\Psi\rangle$ is the coherent ground state. To leading order in $1/N$ and after a Fourier transform one obtains

$$D_{\pi,\,ij}(s) = \int dt\, e^{i\sqrt{s}(t-t')}\langle\Psi|TX_{\pi,i}(t)X_{\pi,j}(t')|\Psi\rangle$$

$$= \delta_{ij}\frac{2\,N\,\Gamma_N^2(0)}{s-\mathcal{E}_\pi^2+i\,\eta}. \qquad (19)$$

The fact that the residue at the pole has to be $2\,N\,\Gamma_N^2(0) = 1$, leads to $z_N = N-2$.

Besides the Goldstone modes there also exist other excitations. They can be made explicit in diagonalizing the remaining part of $H_2$. This is a straightforward procedure which can be found in [8]. In short, since the nondiagonal part of $H_2$ is at most bilinear in the operators $\tilde{A}_1, \tilde{A}_1^+, \beta, \beta^+$, a generalized Bogoliubov rotation of the type

$$Q_\nu^+ = X_\nu\beta^+ - Y_\nu\beta + U_\nu\tilde{A}_1^+ - V_\nu\tilde{A}_1, \qquad (20)$$

can be performed and leads to uncoupled modes at the minimum of the action. The diagonalization is done by recalling the usual Rowe equations of motion [4]

$$\langle RPA|[\delta Q_\nu,[H_2,Q_\nu^+]]|RPA\rangle$$

$$= \Omega_\nu\langle RPA|[\delta Q_\nu, Q_\nu^+]|RPA\rangle, \qquad (21)$$

where $|RPA\rangle$, the full ground state of the theory at this order, is a random-phase approximation (RPA) ground state, defined by $Q_\nu|RPA\rangle = 0$. The Hamiltonian can then be written in the RPA phonon basis, $|\nu\rangle = Q_\nu^+|RPA\rangle$, as follows:

$$H = NH_0 + E_{RPA} + \mathcal{E}_\pi\sum_{i=1}^{N}\alpha_i^+\alpha_i$$

$$+\sum_{\nu=\pm1,\pm2}\Omega_\nu Q_\nu^+Q_\nu + \mathcal{O}(N^{-1/2}), \qquad (22)$$

and contains three terms of order $(\sqrt{N})^2, (\sqrt{N})^1, (\sqrt{N})^0$, respectively. The coefficient of the $\sqrt{N}$ term vanishes. The contribution $E_{RPA} = \langle RPA|H_2|RPA\rangle$ is the RPA correction to the ground-state energy and will not be given explicitly here. The frequencies $\Omega_\nu$ are solutions of the characteristic equation of the RPA eigenvalues problem and given by

$$\Omega_\nu^2 = \frac{\eta}{s} + \frac{8gs^2}{1-\dfrac{4g}{\mathcal{E}_\pi}\dfrac{1}{\Omega_\nu^2-4\mathcal{E}_\pi^2}}. \qquad (23)$$

In the exact symmetry limit ($\eta=0$), there exist a pair of zero-energy solutions among the four RPA eigenvalues which correspond to two uncorrelated Goldstone modes.[1] This point is not the main purpose of the present Rapid Communication and therefore will not be discussed further. The reader may consider looking into Ref. [8] for a complete treatment of this question.

We therefore see that the Hamiltonian in Eq. (22) is the same as in [8], however, augmented by the ‘‘single-pion’’ term $\Sigma_{i=1}^{N}\alpha_i^+\alpha_i$. This extra term arises necessarily in our approach where single bosons and pairs of bosons are treated on the same footing. In [8] the single boson state has been treated on a heuristic level by neglecting exchange contributions to the self-energy. So implicitly, this amounts to the same as using Eq. (22) at the order considered. The present systematic scheme puts the treatment of Ref. [8] on a firm theoretical ground.

In this Rapid Communication we have extended previous work on the Holstein-Primakoff boson expansion for boson pairs applied to a relativistic field theory of interacting bosons [8]. The aim was to treat simultaneously single bosons and pairs of bosons which is necessary to unambiguously define the two-point function for the Goldstone mode and to extend the formalism to finite temperature.

The mapping was applied to the anharmonic oscillator with broken $O(N+1)$ symmetry. It was explicitly shown

[1]Here again, we disregard the infrared problem since the model is only used for demonstration purposes. The reader is referred to [8] for a thorough study of these questions in four space-time dimensions.

that the extension to accommodate single bosons indeed renders, to leading order of the $1/N$ expansion, $N$ uncoupled Goldstone as well as RPA phonon modes. This result is novel and inaccessible to the bosonic Holstein-Primakoff mapping for boson pairs. The latter is only able to provide RPA phonon modes as previously shown in Ref. [8]. The full power of the formalism will reveal itself in working out the next-to-leading order of the $1/N$ expansion by providing an unambiguous computation of all $n$-point functions. It also allows for a natural and straightforward extension to finite temperature. These two points will be discussed in a forthcoming publication.

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