PHYSICAL REVIEW C, VOLUME 62, 011001(R)

Pion production in NN collisions

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(Received 27 December 1999; published 31 May 2000)

The recently proposed irreducible tensor formalism for hadron scattering and reactions is extended to pion production in NN collisions and a form akin to that of Wolfenstein for elastic NN scattering is derived together with exact partial wave expansions for the amplitudes. Explicit formulas are derived for spin observables relevant to differential as well as total cross section measurements employing a polarized beam on a polarized target which are currently of experimental interest.

PACS number(s): 13.75.Cs, 25.10.+s, 21.30.Cb, 25.40.Ve

The study of pion production in pp collisions near threshold has attracted considerable attention [1–19] thanks to advances in new technology during this decade [10]. As the reactions involve only a few lowest order final state partial waves at threshold, which in turn limit the initial partial waves through conservation of total angular momentum, and are characterized at the same time by large momentum transfers, these studies are expected to reveal rare facets of short range spin-dependent interactions involving the nucleons. On the theoretical side, the calculations following Koltun and Reitan [20] underestimated [21] the cross section by about a factor of 5, while the models of Schillaci, Silbar, and Young [22] or Lee and Matsuyama [23] were found to be inadequate to account for the data [4]. To bridge the gap between experiment and theory, several mechanisms have been invoked [23–46] such as the exchange of heavy mesons σ,ρ,ω or two pions, or the off shell extrapolation of the vertex form factor apart from effects due to Δ channel and other low lying nucleon resonances beyond Δ and final state interaction. While the CEOBEM calculations [38] with final state interactions between nucleons account well for the cross sections near threshold with substantial contributions from ω exchange, the more recent computations [46] using a chiral power counting approach led to estimates which were found to be considerably smaller than the data. Such models are likely to be tested more incisively by the rapidly increasing data on spin observables. Moreover, as a majority of the calculations deal with only the lowest partial waves, they may have to be upgraded to include higher partial waves to be able to account for the data on spin observables.

The purpose of this Rapid Communication is to present an irreducible tensor formalism for $NN\rightarrow NN\pi$, which incidentally leads to an expression akin to that for elastic NN scattering in terms of the well-known Wolfenstein amplitudes [47]. Such a phenomenological framework, based purely on invariance considerations and involving only a small number of terms, can be expected to facilitate a model independent analysis of spin observables along with the differential and total cross sections. These amplitudes *à la* Wolfenstein are also shown to have precise partial wave expansions which further enable in depth discussions at near threshold energies. We also present explicit expressions for spin observables of current experimental interest in terms of the irreducible tensor amplitudes connecting initial and final channel spin states.

Introducing the Jacobi coordinates in the final state characterized by an invariant mass W of the NN system and absorbing all the relevant phase space and other factors, the matrix $\mathbf{M}$ in spin space for $NN\rightarrow NN\pi$ may be expressed in the form

$$\mathbf{M}=\sum_{s_f,s_i=0}^{1}\sum_{\lambda=|s_f-s_i|}^{s_f+s_i}(S^{\lambda}(s_f,s_i)\cdot M^{\lambda}(s_f,s_i)), \tag{1}$$

where s_i,s_f denote the initial and final channel spins, respectively. The same notations as in [48,49] are used, where the irreducible tensor operators $S^{\lambda}_{\mu}(s_f,s_i)$ of rank λ, which effect transitions from s_i to s_f, are defined. The irreducible tensor approach was outlined in [48] for hadron scattering and reactions involving two body final states. The irreducible tensor amplitudes $M^{\lambda}_{\mu}(s_f,s_i)$, for the reaction $NN\rightarrow NN\pi$ which involves a three body final state, are now given by

$$M^{\lambda}_{\mu}(s_f,s_i)=\sum_{l_f,l,L_f,j_f,j,l_i} g_{\alpha}M^{j}_{l(l_fs_f)j_f;l_is_i}(E,W) \times((Y_{l_f}(\hat{\mathbf{p}}_f)\otimes Y_l(\hat{\mathbf{q}}))^{L_f}\otimes Y_{l_i}(\hat{\mathbf{p}}_i))^{\lambda}_{\mu}, \tag{2}$$

in terms of the partial wave reaction amplitudes $M^{j}_{l(l_fs_f)j_f;l_is_i}(E,W)$, which completely take care of the dependence on the c.m. energy E at which the reaction takes place, while the angular dependence is solely governed by $((Y_{l_f}(\hat{\mathbf{p}}_f)\otimes Y_l(\hat{\mathbf{q}}))^{L_f}\otimes Y_{l_i}(\hat{\mathbf{p}}_i))^{\lambda}_{\mu}$. Here $\mathbf{p}_i=p_i\hat{\mathbf{p}}_i,\mathbf{p}_f=p_f\hat{\mathbf{p}}_f$ denote, respectively, the initial and final momenta associated with the relative motion of the nucleons and $\mathbf{q}=q\hat{\mathbf{q}}$ denotes the pion momentum in c.m., while the geometric factors g_{α} are explicitly given by

$$g_{\alpha}=(4\pi)^3(i)^{l_i-l-l_f}(-1)^{l+l_f+l_i-j+s_i}[j]^2[j_f][L_f] \times[s_f]^{-1}W(l_is_iL_fs_f;j\lambda)W(s_fl_fjl;j_fL_f), \tag{3}$$

where α denotes, collectively, $\alpha=\{l,l_f,L_f,s_f,j_f,j,l_i,s_i,\lambda\}$. At energies E near threshold, the orbital angular momentum l carried by the pion is limited to low values $l=0$ or $l=1$ at the most, where the resonance Δ may enhance the partial wave amplitudes. At beam energies that yield maximum pion momentum fractions ($\eta = p_i^{max}/m_\pi c$) of $\eta = 0.22, 0.42$, and 0.50, a recent analysis [12] shows that the p-wave channel containing the Δ contributes less than one tenth of the total cross section at $\eta=0.22$ and less than one third of the total cross section at $\eta=0.42$ and 0.50, while the nonresonant p-wave channel contributes less than 1% at these energies. Conservation of parity implies that the summation over l, l_f, and l_i must be limited to only those terms which satisfy $(-1)^{l_i} = (-1)^{l_f+l+1}$, while Pauli principle demands further that $(-1)^{l_i+s_i+t_i} = -1 = (-1)^{l_f+s_f+t_f}$, where t_i, t_f denote, respectively, the initial and final isospins of the two nucleon system. In the case of $pp \to d\pi^+$, it is clear that j_f, s_f, t_f are limited to $j_f = s_f = 1, t_f = 0$ and l_f can take only two values $l_f = 0,2$ corresponding to the S and D states of the deuteron; moreover $\mathbf{p}_f$ here denotes the Fourier component with respect to which integration has to be carried out taking into account the bound state structure functions. Thus **M** for $pp \to d\pi^+$ may also be expressed in the same form as Eq. (1), but with s_f limited to $s_f = 1$ and the simpler partial wave expansion

$$M^\lambda_\mu(1,s_i) = \sum_{l,l_i,j} b_\beta M^j_{l1;l_i s_i}(E)(Y_l(\hat{\mathbf{q}}) \otimes Y_{l_i}(\hat{\mathbf{p}}_i))^\lambda_\mu, \quad (4)$$

with $\beta = \{l, l_i, s_i, j, \lambda\}$, and

$$b_\beta = \sqrt{3}[\lambda][j](-1)^{l+l_i-j+s_i} W(l1 l_i s_i; j\lambda) \quad (5)$$

for the irreducible tensor amplitudes in the spin space of the two nucleons.

Defining 2×2 matrices $\sigma^\lambda_\mu(n)$ in the spin space of the two nucleons with $n=1,2$ through

$$\sigma^0_0(n) = 1, \quad \sigma^1_0(n) = \sigma_z(n),$$

$$\sigma^1_{\pm 1}(n) = \mp \frac{1}{\sqrt{2}}(\sigma_x(n) \pm i\sigma_y(n)), \quad (6)$$

where $\sigma_x, \sigma_y, \sigma_z$ denote the Pauli spin matrices, and noting that

$$S^\lambda_\mu(s_f,s_i) = \sum_{\lambda_1,\lambda_2=0}^{1} G(s_f,s_i;\lambda_1,\lambda_2)(\sigma^{\lambda_1}(1) \otimes \sigma^{\lambda_2}(2))^\lambda_\mu, \quad (7)$$

where the geometrical factors are explicitly given by

$$G(s_f,s_i;\lambda_1,\lambda_2) = \tfrac{1}{2}[s_f]^2[s_i][\lambda_1][\lambda_2]\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s_f \\ \frac{1}{2} & \frac{1}{2} & s_i \\ \lambda_1 & \lambda_2 & \lambda \end{Bmatrix}, \quad (8)$$

we may express **M**, given by Eq. (1), in a form akin to that of Wolfenstein [47] for elastic NN scattering. We have

$$\mathbf{M} = A + B\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + (\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{U} + (\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{V} + (\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{W} + ((\boldsymbol{\sigma}_1\otimes\boldsymbol{\sigma}_2)^2\cdot\mathcal{T}^2(1,1)), \quad (9)$$

where

$$A = \mathcal{T}^0_0(0,0), \quad B = -\frac{1}{\sqrt{3}}\mathcal{T}^0_0(1,1), \quad (10)$$

and the spherical components of **U**, **V**, and **W** are given by

$$\mathrm{U}^1_\mu = \tfrac{1}{2}[\mathcal{T}^1_\mu(1,0) + \mathcal{T}^1_\mu(0,1)], \quad \mathrm{V}^1_\mu = \tfrac{1}{2}[\mathcal{T}^1_\mu(1,0) - \mathcal{T}^1_\mu(0,1)], \quad (11)$$

$$\mathrm{W}^1_\mu = \frac{i}{\sqrt{2}}\mathcal{T}^1_\mu(1,1),$$

in terms of the irreducible tensor amplitudes

$$\mathcal{T}^\lambda_\mu(\lambda_1,\lambda_2) = \sum_{s_f,s_i} G(s_f,s_i;\lambda_1,\lambda_2)M^\lambda_\mu(s_f,s_i), \quad (12)$$

which readily provide, through the use of Eqs. (2), (3), and (8), explicit partial wave expansions for the amplitudes for $NN \to NN\pi$ which are akin to the Wolfenstein amplitudes in the case of elastic NN scattering. Terms containing $\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2$ and $\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2$, which are absent in the case of NN scattering, are present here in Eq. (9) since channel spin is not conserved in $NN \to NN\pi$. We may also note that appropriate combinations of these amplitudes, viz.,

$$M^\lambda_\mu(s_f,s_i) = 4[s_f]^{-2}\sum_{\lambda_1,\lambda_2=0}^{1} G(s_f,s_i;\lambda_1,\lambda_2)\mathcal{T}^\lambda_\mu(\lambda_1,\lambda_2), \quad (13)$$

do in turn constitute the irreducible tensor amplitudes $M^\lambda_\mu(s_f,s_i)$, which directly yield physically interesting information on the initial singlet and triplet spin state contributions to $NN \to NN\pi$ in polarized beam and polarized target experiments [14–18].

If both the beam and the target are polarized, the initial spin state of the NN system is characterized by the density matrix

$$\rho^i = \tfrac{1}{4}(1 + \boldsymbol{\sigma}_1\cdot\mathbf{P}_1)(1+\boldsymbol{\sigma}_2\cdot\mathbf{P}_2), \quad (14)$$

in terms of the beam and target polarizations $\mathbf{P}_1$ and $\mathbf{P}_2$. Introducing the notations

$$P^0_0(n) = 1, \quad P^1_0(n) = P_{nz}, \quad P^1_{\pm 1}(n) = \mp\frac{1}{\sqrt{2}}(P_{nx} \pm iP_{ny}), \quad (15)$$

with $n=1,2$, and using already known properties [48] of the operators given by Eq. (7), we obtain the differential cross section for $NN \to NN\pi$ as

$$\frac{d^2\sigma}{d^3p_f d\Omega}=\frac{1}{4}\sum_{k_1,k_2=0}^{1}\sum_{k=|k_1-k_2|}^{k_1+k_2}\times((P^{k_1}(1)\otimes P^{k_2}(2))^k\cdot B^k(k_1,k_2)), \quad (16)$$

in terms of the irreducible tensors

$$B^k_\nu(k_1,k_2)=\sum_{s_f=0}^{1}(2s_f+1)\times\sum_{s_i,s_i'=0}^{1}\sum_{\lambda,\lambda'}\mathcal{G}(s_f;\lambda\lambda';s_is_i';k_1k_2k)\times(M^\lambda(s_f,s_i)\otimes M^{\dagger\lambda'}(s_f,s_i'))^k_\nu, \quad (17)$$

which are bilinear in the channel spin irreducible tensor amplitudes $M^\lambda_\mu(s_f,s_i)$, whose complex conjugates $M^\lambda_\mu(s_f,s_i)^*$ define

$$M^{\dagger\lambda}_\mu(s_f,s_i)=(-1)^\mu M^\lambda_{-\mu}(s_f,s_i)^*. \quad (18)$$

The geometrical factors in Eq. (17) are given by

$$\mathcal{G}(s_f;\lambda\lambda';s_is_i';k_1k_2k)=2(-1)^{\lambda+\lambda'+s_i+s_f}(-1)^{k_1+k_2}[s_i][s_i'][\lambda][\lambda'][k_1]\times[k_2]W(s_i's_i\lambda'\lambda;ks_f)\begin{Bmatrix}\frac{1}{2}&\frac{1}{2}&s_i\\ \frac{1}{2}&\frac{1}{2}&s_i'\\ k_1&k_2&k\end{Bmatrix}, \quad (19)$$

where the 9-j symbol ensures that contributions from terms with $s_i\neq s_i'$ can arise only for $k=1$ when $k_1=k_2$. Clearly, the term with $k_1=k_2=k=0$ in Eq. (16) represents the unpolarized differential cross section

$$\frac{d^2\sigma_0}{d^3p_f d\Omega}=\frac{1}{4}B^0_0(0,0)=\frac{1}{4}\sum_{s_i=0}^{1}\sum_{s_f=0}^{1}(2s_f+1)\sum_\lambda\mathcal{G}(s_f;\lambda\lambda;s_is_i;000)\times(-1)^{-\lambda}[\lambda]^{-1}\sum_\mu|M^\lambda_\mu(s_f,s_i)|^2, \quad (20)$$

where the two terms with $s_i=0,1$ represent the contribution of the two initial channel spin states, respectively. The coefficients of $(P^{k_1}(1)\otimes P^{k_2}(2))^k_\nu$ when either of k_1,k_2, or both are nonzero define, respectively, the beam and target analyzing powers $\mathbf{A}(1,0),\mathbf{A}(0,1)$ and the initial spin correlations $A^k_\nu(1,1)$. Thus

$$\frac{d^2\sigma}{d^3p_f d\Omega}=\frac{d^2\sigma_0}{d^3p_f d\Omega}\left[1+\mathbf{P}_1\cdot\mathbf{A}(1,0)+\mathbf{P}_2\cdot\mathbf{A}(0,1)+\sum_{k=0}^{2}(\mathbf{P}_1\otimes\mathbf{P}_2)^k\cdot A^k(1,1)\right], \quad (21)$$

where the asymmetries $\mathbf{A}(1,0)$, $\mathbf{A}(0,1)$ and spin correlations $A^k_\nu(1,1)$ are given by

$$A^k_\nu(k_1,k_2)=B^k_\nu(k_1,k_2)/B^0_0(0,0), \quad (22)$$

in terms of their spherical components.

In the near threshold energy region, we may use the nonrelativistic form

$$E=2M+m+\frac{p_f^2}{M}+\frac{q^2}{4M}+\frac{q^2}{2m} \quad (23)$$

for energy E, to relate p_f^2 with q^2 where M and m denote, respectively, the masses of the nucleon and the pion. Moreover, the energy $\omega=(q^2+m^2)^{1/2}$ of the pion is related to the invariant mass W of the two nucleon system through the relativistic relation

$$\omega=\frac{E^2-W^2+m^2}{2E}, \quad (24)$$

so that we may express

$$p_f^2dp_f=\frac{2M+m}{4m}\cdot\frac{p_f\omega}{E}\cdot WdW. \quad (25)$$

We may now integrate Eq. (16), with respect to $d^3p_f=p_f^2dp_fd\Omega_f$. The angular integration may readily be carried out using standard properties of the spherical harmonics to yield

$$\frac{d\sigma}{d\Omega}=\mathrm{b}^0_0(0,0)+\mathbf{P}_1\cdot\mathbf{b}(1,0)+\mathbf{P}_2\cdot\mathbf{b}(0,1)-\frac{1}{\sqrt{3}}(\mathbf{P}_1\cdot\mathbf{P}_2)\mathrm{b}^0_0(1,1)+\frac{i}{\sqrt{2}}((\mathbf{P}_1\times\mathbf{P}_2)\cdot\mathbf{b}(1,1))+((\mathbf{P}_1\otimes\mathbf{P}_2)^2\cdot\mathrm{b}^2(1,1)), \quad (26)$$

where $\mathrm{b}^0_0(0,0)$ denotes the unpolarized differential cross section $(d\sigma_0)/(d\Omega)$ and

$$\mathrm{b}^k_\nu(k_1,k_2)=\frac{1}{16\pi}\sum_{l'',l_i''}F(l'',l_i'';k_1,k_2,k)\times[l_i'']C(l''l_i''k;\nu0\nu)Y_{l''\nu}(\hat{\mathbf{q}}), \quad (27)$$

if we choose the z axis along $\mathbf{p}_i$. Further,

$$F(l'',l_i'';k_1,k_2,k)=\frac{1}{4\pi}\sum_{\alpha,\alpha'}\mathcal{F}_\alpha\mathcal{F}^\star_{\alpha'}I_{\alpha\alpha'}\times\mathcal{G}(s_f;\lambda\lambda';s_is_i';k_1k_2k)\times(-1)^{l'+l_i'-\lambda'}C(ll'l'';000)\times C(l_il_i'l_i'';000)W(L_f'l_fl''l;l'L_f)\times\begin{Bmatrix}L_f&l_i&\lambda\\ L_f'&l_i'&\lambda'\\ l''&l_i''&k\end{Bmatrix}, \quad (28)$$

with

$$\mathcal{F}_\alpha = g_\alpha [s_f][l][L_f][l_i][\lambda], \tag{29}$$

involves also the integrals

$$I_{\alpha\alpha'} = \delta_{s_f s_f'} \int p_f^2 dp_f M^j_{l(l_f s_f) j_f; l_i s_i}(E,W) \times M^{j'}_{l'(l_f' s_f') j_f'; l_i' s_i'}(E,W)^*, \tag{30}$$

whose estimation is facilitated through the use of Eq. (25).

It is interesting to note that the differential cross section for $NN \to d\pi$ is also expressible in the same form as Eq. (26) with the simpler

$$F(l'' l_i''; k_1 k_2 k) = \frac{3}{4\pi} \sum_{\beta,\beta'} b_\beta b_{\beta'} M^j_{l1;l_i s_i}(E) \times M^{j'}_{l'1;l_i' s_i'}(E)^* C(l l' l''; 000) \times C(l_i l_i' l_i''; 000) \mathcal{G}(1;\lambda\lambda'; s_i s_i'; k_1 k_2 k) \tag{31}$$

instead of that given by Eq. (28). The simpler partial wave amplitudes $M^j_{l1;l_i s_i}(E)$ in Eq. (31) may themselves be considered as

$$M^j_{l1;l_i s_i}(E) = \int d^3 p_f \sum_{l_f = 0,2} \mathcal{S}_{l_f}(p_f^2) M^j_{l(l_f 1)1; l_i s_i}(E,W), \tag{32}$$

where $\mathcal{S}_{l_f}(p_f^2)$, $l_f = 0,2$ denote the deuteron structure functions for the S and D states, respectively, and the $M^j_{l(l_f 1)1; l_i s_i}(E,W)$ on the right hand side of Eq. (32) denote off-shell partial wave amplitudes for $NN \to NN\pi$ extended down to the invariant mass $W < 2M$ corresponding to the mass of the deuteron.

Integrating Eq. (26) with respect to $d\Omega$ readily yields the total cross section

$$\sigma = \sigma(0,0,0) - \frac{1}{\sqrt{3}} \sigma(1,1,0) \mathbf{P}_1 \cdot \mathbf{P}_2 + \sqrt{\frac{15}{2}} \sigma(1,1,2) \times ((\mathbf{P}_1 \otimes \mathbf{P}_2)^2 \cdot (\hat{\mathbf{p}}_i \otimes \hat{\mathbf{p}}_i)^2), \tag{33}$$

with $\sigma(k_1, k_2, k)$ being given by

$$\sigma(k_1, k_2, k) = \frac{1}{4\sqrt{4\pi}} F(0,k; k_1 k_2 k). \tag{34}$$

Clearly Eq. (33) is equivalent to Eq. (1) of Bilenky and Ryndin [50]. However, their further analysis is of limited validity [51], applicable only when $\mathbf{P}_1$ and $\mathbf{P}_2$ are collinear with the axis of quantization $\hat{\mathbf{q}}$ which has been chosen along $\hat{\mathbf{p}}_i$. This is particularly to be noted in view of the current capability [14,17] of carrying out experiments employing 12 possible combinations of a polarized beam (up, down) with a polarized target (up, down, left, right, forward, backward). Clearly, $\sigma(0,0,0) = \sigma_s + \sigma_t$ denotes the unpolarized cross section, while $\sigma(1,1,0) = -\sigma_s + 1/3(\sigma_t)$, where σ_s, σ_t denote, respectively, the singlet and triplet contributions.

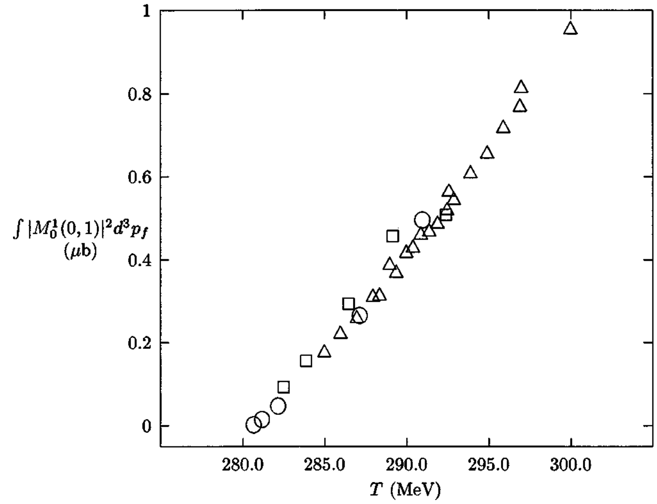

FIG. 1. The integrated $|M_0^1(0,1)|^2$ as a function of the bombarding energy from existing data of Ref. [1] (denoted by squares), Ref. [4] (denoted by triangles), and Ref. [6] (denoted by circles).

At threshold for $NN \to NN\pi$, it is clear that $W = 2M$ and we may also set $l = l_f = 0$. Considering, in particular $pp \to pp\pi^\circ$ for which $t_i = t_f = 1$, which in turn implies that $s_f = j_f = L_f = 0$ and $l_i = s_i = \lambda = 1$, so that

$$M_0^1(0,1) = \frac{i}{\sqrt{3}} (4\pi)^{3/2} M^0_{0(00)0;11}(E,W) \tag{35}$$

is the only irreducible tensor amplitude which contributes to the reaction, if we choose the z axis along the beam. This can be expected [14] to be valid for bombarding energies up to about 300 MeV. Therefore one can study empirically the energy dependence of this amplitude from the existing measurements of the total cross section [4] as shown in Fig. 1. Without loss of generality, one may choose $M_0^1(0,1)$ to be real and positive and determine the other irreducible tensor amplitudes in magnitude as well as in relative phase with respect to $M_0^1(0,1)$. As the bombarding energy increases, irreducible tensor amplitudes

$$M^1_\mu(1,0) = \frac{i(4\pi)^2}{3} [M^0_{0(11)0;00}(E,W) Y_{1\mu}(\hat{\mathbf{p}}_f) - M^2_{0(11)2;20}(E,W) (Y_1(\hat{\mathbf{p}}_f) \otimes Y_2(\hat{\mathbf{p}}_i))^1_\mu], \tag{36}$$

with $l = 0, l_f = 1$, start contributing in addition to Eq. (35). The $l = 1, l_f = 0$ partial wave amplitude is absent since it cannot simultaneously satisfy the requirements of the Pauli principle and parity conservation. At energies up to 400 MeV the only other irreducible tensor amplitudes that are expected to contribute to $pp \to pp\pi^\circ$ are

$$M_\mu^\lambda(1,1)=\frac{(4\pi)^3 i}{\sqrt{3}}\sum_{L_f,j_f,j}(-1)^j[j]^2[j_f][L_f]W(11j1;j_fL_f)$$
$$\times[-W(11L_f1;j\lambda)M^j_{1(11)j_f;11}(E,W)$$
$$\times((Y_1(\hat{\mathbf{p}}_f)\otimes Y_1(\hat{\mathbf{q}}))^{L_f}\otimes Y_1(\hat{\mathbf{p}}_i))^\lambda_\mu$$
$$+W(31L_f1;j\lambda)M^j_{1(11)j_f;31}(E,W)$$
$$\times((Y_1(\hat{\mathbf{p}}_f)\otimes Y_1(\hat{\mathbf{q}}))^{L_f}\otimes Y_3(\hat{\mathbf{p}}_i)]^\lambda_\mu), \quad (37)$$

with $l=l_f=1$ and $\lambda=0,1,2$. However, one would require data at the double differential level for the cross section as well as the spin observables to be able to determine empirically the bilinears $(M^\lambda(s_f,s_i)\otimes M^{\dagger\lambda'}(s_f',s_i'))^k_\nu$ and hence the irreducible tensor amplitudes $M^1_\mu(1,0)$ and $M^\lambda_\mu(1,1)$, individually.

Even at still higher energies where additional partial wave amplitudes have to be included in Eqs. (35), (36), and (37) it is possible to set l_f as zero if we select events with $W=2M$ (i.e., events with maximum pion c.m. momentum η). The cross section measured as a function of the pion angle θ would then provide a clear insight into the relative contribution of higher partial waves $l\geqslant 2$ as E increases. It would be interesting to study this experimentally.

Finally, it may also be mentioned that in contrast to Eqs. (35) and (36), the irreducible tensor amplitudes

$$M^\lambda_\mu(s_f,s_i)=\delta_{s_f,s}\delta_{s_i,s}M^\lambda_\mu(s) \quad (38)$$

for elastic NN scattering [52], where channel spin is conserved. Consequently, it is not possible to detect the singlet-triplet entanglement present in the initial state through elastic scattering, while it is possible [51] to do so employing $NN\rightarrow NN\pi$.

Two of us (G.R. and P.N.D.) acknowledge, with thanks, the support of C.S.I.R. (India).

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