Evidence for proton four-particle–four-hole intruder excitations in neutron deficient nuclei in the Pb region

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The recent observation of low-lying collective bands built on 0^+ intruder states in the neutron-deficient Pb region is discussed within the context of two-particle–two-hole and four-particle–four-hole proton excitations across the $Z=82$ proton shell closure.

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The recent development of recoil-decay-tagging techniques and heavy-ion-induced fusion-evaporation reactions has led to a wealth of data in nuclei that have neutron number near $N=104$ and are situated in the Pb region. The large body of experiments carried out over the last few years has shown ample evidence for unexpected new collective band structures in both the single-closed shell $Z=82$ Pb nuclei as well as in the adjacent Hg, Pt, and Po nuclei below and above the proton shell closure, respectively [1–17].

This rich variety of data points towards the presence of specific particle-hole (p-h) excitations across the closed shell at $Z=82$. It is precisely the energy gap at the $Z=82$, $N=126$ closed shell of only approximately 3.5 MeV, combined with a very large open neutron shell (filling the 82-126 orbitals) that enables the proton-neutron quadrupole-quadrupole force to lower the excitation energy of $2p$-$2h$,$4p$-$4h$, . . . , configurations as much as to approach the ground state (for the Pb and Hg nuclei) and even cross it (for the Pt and possibly the Po nuclei too) [18,19]. Because of the increased quadrupole collectivity associated with these p-h excitations, collective bands are observed on top of the low-lying 0^+ intruder excitations.

Calculations, making use of a deformed mean-field approach that studies the possible equilibrium states [20–23], have indicated the possibilities to produce rather close-lying oblate and prolate minima in the total energy surface for the Pb nuclei while approaching the neutron midshell region at $N=104$, next to the spherical ground-state configuration. In the Hg nuclei with a ground state corresponding to a slightly deformed oblate configuration, a second prolate configuration is predicted, mimimizing its energy near midshell ($N=104$), whereas for the Pt nuclei, a crossing of both minima is implied and the prolate deformed minimum becomes the lowest configuration at midshell. For the Po nuclei, the situation looks somewhat more complicated [24] with an oblate minimum, approaching the spherical ground-state configuration near $N=110$, and a prolate minimum, becoming dominant in the ground state for even lower neutron numbers.

Many-particle many-hole (mp-nh) excitations cannot easily be incorporated in full large-scale shell-model studies because of the extreme dimensions of the model spaces obtained. These mp-nh excitations, however, can be handled within an algebraic framework of the interacting boson model [25,26]. In this approach, explored in detail in a series of papers [27–30], both particle and hole shell-model configurations are handled as interacting particle and holelike s and d bosons. Quite early [31] a symmetry deriving from transforming particle into hole bosons (or the other way around) was suggested as I spin or *intruder spin*. Its presence resulted in I-spin multiplets (formally analogous to isospin multiplets in light nuclei) and some interesting realizations of this symmetry were discussed in, e.g., the Sn region [32] and in the Pb region [24,30]. In the latter paper, difficulties in bringing the Po intruder states in line with the corresponding Hg intruder states showed up, indicating that this symmetry is likely to be broken to a certain extent. Still the I-spin concept can provide a good lowest-order classification of the complicated interacting system consisting of proton particle-hole excitations and a large number of valence neutrons outside of the $Z=82$, $N=126$ shell closure. Some suggestions for breaking of the I-spin classification label in, e.g., the Hg nuclei were given before in [20].

A straightforward argument for breaking the I-spin symmetry can come from mixing $4p$-$4h$ excitations across the $Z=82$ closed shell with the lowest-lying $2p$-$2h$ excitations. The best way to study the importance of $4p$-$4h$ excitations is to use the method as discussed by Heyde *et al.* [33] in which a shell-model basis was given to describe both the low energy and the mass dependence of the lowest $2p$-$2h$ 0^+ intruder states near closed shells. We now extend this method in order to describe both the $2p$-$2h$ and $4p$-$4h$ intruder configurations. An attempt has been carried out in this respect [34] but due to the lack of data no conclusive statements could be made in one or another direction. The large amount of data that have become available at the neutron midshell region near $N=104$ in the last few years has started the present investigation. (See Fig. 1.)

The expression for the energy of a proton $4p$-$4h$ intruder configuration can be given as

$$E_{intr.}(4p\text{-}4h)\simeq 4(\epsilon_p-\epsilon_h)+\Delta E_M(4p\text{-}4h) -\Delta E_{pair}+\Delta E_Q(4p\text{-}4h), \qquad (1)$$

where the various terms describe the unperturbed energy to create the $4p$-$4h$ configuration, a monopole correction due to a change in proton single-particle energy while changing the neutron number, the pairing-energy correction because essentially 0^+-coupled pairs are formed, and the quadrupole

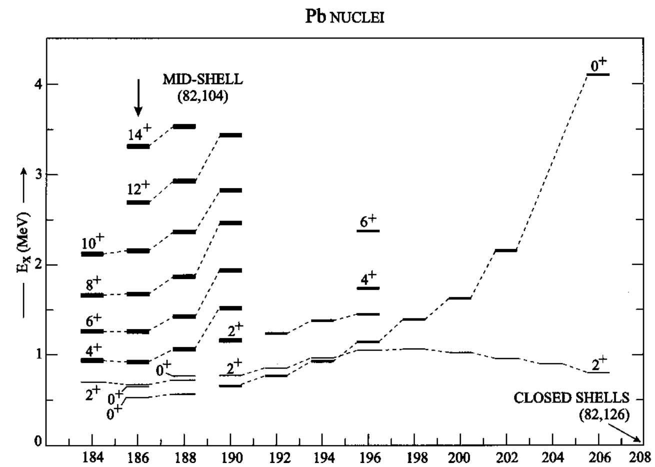

FIG. 1. Systematics of the lowest 0^+ states in the even-even Pb nuclei. The first excited 2^+ state is also given for reference. The band members of the yrast structure are given in the mass region $184 \leqslant A \leqslant 190$. The references are denoted in the introductory part of the present paper.

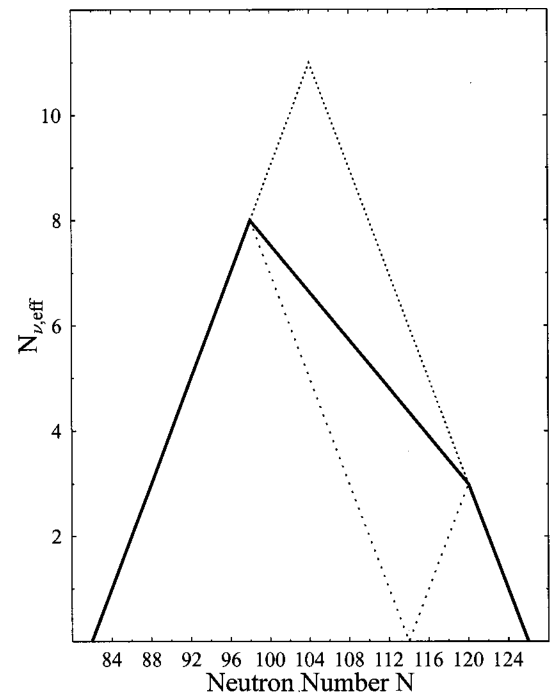

FIG. 2. The variation of the effective neutron number $N_{\nu,eff.}$ for the situation that the full shell for neutrons between 82 and 126 can be reduced to a single shell [approximation (a) in the text]. We also show the case [approximation (b)] in which a subshell closure at $N=114$ is taken into account.

binding energy originating in the proton-neutron force, respectively. Starting from the expressions of the unperturbed energy, the monopole and the quadrupole energy corrections, a scaling with the number of particles and holes is obvious, i.e.,

$$\Delta E_M(4p\text{-}4h) = 2\Delta E_M(2p\text{-}2h), \tag{2}$$

$$\Delta E_Q(4p\text{-}4h) = 2\Delta E_Q(2p\text{-}2h). \tag{3}$$

For the pairing correction though, the Pauli effect causes a systematic lowering when increasing the number of pairs and the ratio of the pairing energy gain for $n/2$ pairs over one pair becomes

$$\frac{\Delta_{pair}(n/2)}{\Delta_{pair}(1)} = \frac{n}{2}\left(1 - \frac{n}{2\Omega} + \frac{1}{\Omega}\right), \tag{4}$$

where Ω denotes the shell-model degeneracy.

We apply the above method to study the mass dependence of the $2p$-$2h$ and $4p$-$4h$ intruder configurations from the neutron shell closure at $N=126$ into the midshell region near $N=104$. We thereby start from values for the unperturbed particle-hole excitation energy at the $Z=82$ closed shell, the pairing strength, and the monopole corrections as derived in detail before [33,34]. This results in values for $4(\epsilon_p - \epsilon_h) = 13.3$ MeV and a pairing energy gain of $\Delta_{pair} = 3.75$ MeV when taking the proton degeneracy to be $\Omega_\pi = 16$. The monopole shift was deduced from the results in the variation in the $1p$-$1h$ energy difference with the hole moving in the $3s_{1/2}$ or the $2d_{3/2}$ orbitals and the particle promoted into the $1h_{9/2}$ orbital.

In calculating the neutron number dependence of the $2p$-$2h$ and $4p$-$4h$ intruder 0^+ configurations we furthermore determine the quadrupole energy contribution using the SU(3) expression given in [33], i.e.,

$$\Delta E_Q \simeq 2\kappa \Delta N_\pi N_\nu, \tag{5}$$

in which ΔN_π denotes the number of pairs excited out of the closed shell configuration at $Z=82$, i.e., $\Delta N_\pi = 2$ for a $2p$-$2h$ excitation and 4 for a $4p$-$4h$ excitation, and with

$$\kappa = \kappa_0 \sqrt{(\Omega_\pi - N_\pi)(\Omega_\nu - N_\nu)}, \tag{6}$$

the SU(3) quadrupole strength. To determine the latter, we use both (a) the full 82-126 neutron shell as a single shell with degeneracy $\Omega_\nu = 22$ and (b) a subshell closure at $N=114$. Indications for a possible subshell effect have been studied by [35]. In this case we have to consider the intervals (i) $N_\nu = (N-82)/2$ and $\Omega_\nu = 16$ for $N<98$, (ii) $N_\nu = (126-N)/2$ and $\Omega_\nu = 22$ for $N>120$, and (iii) a linear variation for N_ν from $8 \rightarrow 3$ corresponding to a linear variation of Ω_ν from $16 \rightarrow 22$ for the interval $98 \leqslant N \leqslant 120$ (see also Fig. 2). Using the approximation (a), a value of the quadrupole coupling strength $\kappa_0 = -6$ to -8 keV is needed to obtain the right energy range for the $2p$-$2h$ intruder excitations. However, the predicted lowering of the excitation energy towards midshell is much faster than what is experimentally observed. Including the subshell effect at $N=114$, values for $\kappa_0 = -11$ to -12 keV result when fitting to the experimental $2p$-$2h$ and $4p$-$4h$ 0^+ intruder excitation energies in the Pb nuclei. The quadrupole strength derived in this way conforms with independent IBM model calculations [36,37].

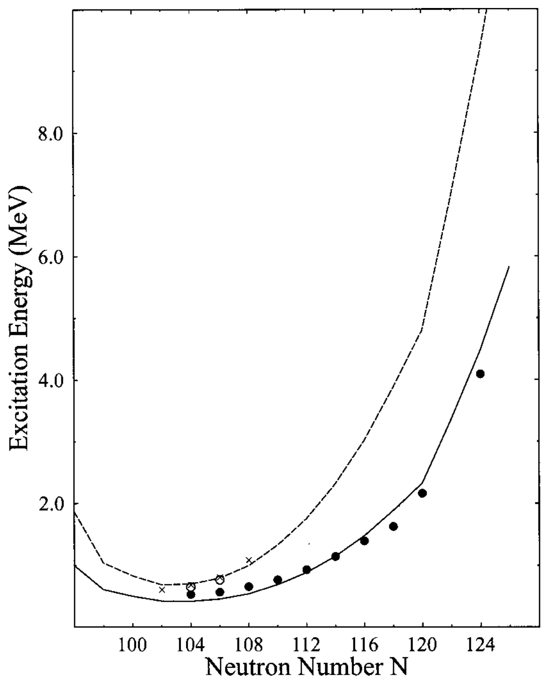

FIG. 3. The mass dependence for the $2p$-$2h$ (full line) and the $4p$-$4h$ (dashed lines) shell-model intruder configurations. The precise parameters describing these curves are discussed in the text. The filled (open) circles denote the experimentally detected 0_2^+ (0_3^+) states in the Pb isotopes [2,6,9], and the crosses are deduced via rotational band extrapolation $E_x = E_0 + AI(I+1) + BI^2(I+1)^2$ from the band structure observed in the midshell region Pb nuclei.

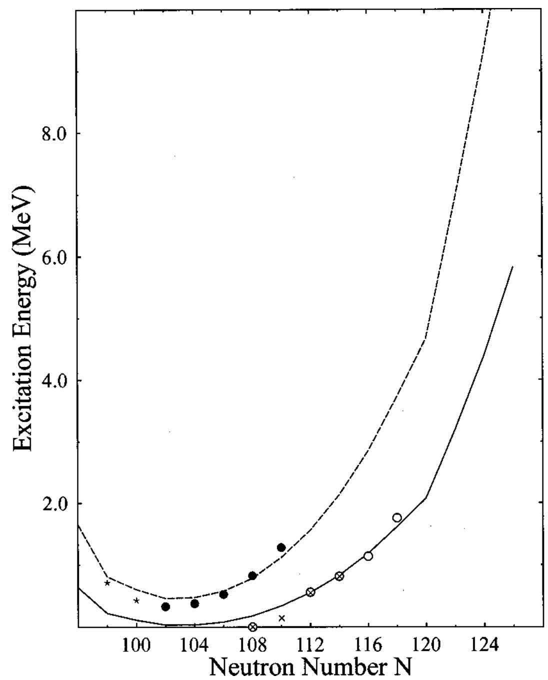

FIG. 4. The mass dependence of the $2p$-$2h$ (full line) and the $4p$-$4h$ (dashed line) shell-model intruder configurations. The quadrupole strength is fixed by the value of $\kappa_0 = -12$ keV. The open circles denote the observed lowest excited 0^+ state in the even-even Po nuclei, possibly becoming the ground state in ^{192}Po, the crosses indicate the values of the unperturbed intruder energy as obtained in a two-state mixing calculation [24]. The filled circles give the excitation energy for the first excited intruder 0^+ state in the even-even Hg nuclei, the stars indicate values obtained via rotational extrapolation $E_x = E_0 + AI(I+1) + BI^2(I+1)^2$.

In Figs. 3 and 4, we compare the results for both $2p$-$2h$ and $4p$-$4h$ 0^+ intruder configurations as derived for the Pb and for the Po and Hg nuclei. Using the value of $\kappa_0 = -11$ keV, a very good reproduction is obtained for the mass dependence of the lowest 0^+ state; in order to reproduce the second group of 0^+ states as $4p$-$4h$ intruder configurations a slightly larger value of -11.7 keV is needed. So, in Fig. 3, the results for these two slightly different κ_0 values are incorporated. We do not have a good understanding of the need to have slight variations (6%): it may possibly come from the fact that the above shell-model approach and especially the SU(3) form of the quadrupole proton-neutron interaction is still rather schematic, or from the fact that the two different types of intruder configurations are derived in an independent way, whereas mixing will appear when they approach each other in the midshell region. We have now also applied the method to the Po nuclei (see Fig. 4 where a single $\kappa_0 = -12$ keV value has been used) and the results seem to support a $2p$-$2h$ interpretation for the behavior of the lowest excited 0^+ state in these nuclei. There has been quite some debate over the character of this state [3,13,24,38–43] but the most recent experimental results seem to support the idea of an intruder $2p$-$2h$ character.

We have also displayed the results for the Hg 0^+ intruder band for which the p-h character is not firm on the basis of present-day experiments. In case of a $2p$-$2h$ interpretation this Hg band would be a member of an $I = 3/2$ multiplet, together with the ground-band for the Os nuclei and the intruder band in the Po nuclei. However, one can apply the I-spin concept here with only limited success [30]. Nazarewicz has also indicated the possibility of mixing with more complicated $4p$-$4h$ excitations as a possible explanation for deviations from a pure multiplet structure [20]. In this figure, the systematics clearly favor a $4p$-$4h$ interpretation as the major component.

The above comparison sits within a schematic approach to study the mass dependence of $2p$-$2h$,$4p$-$4h$, . . . , intruder excitations across closed shells. It is clear that the results presented here are to be interpreted within this context. Still, the specific mass dependence for different types of excitations, i.e., $2p$-$2h$ vs $4p$-$4h$ with the latter rising very steeply in energy when moving away from midshell, conforms with total potential energy studies in this mass region [20,24,44]. We furthermore obtain the clear and robust result that in the midshell region near $N = 104$ one can find both types of excitations, i.e., $2p$-$2h$ and $4p$-$4h$, coming quite close in energy. It is precisely the close presence of the intruders that will modify the ideal intruder I-spin symmetry concept.

Because large-scale shell-model calculations do not easily allow us to incorporate these mp-nh excitations in a straightforward way, a better understanding should come from appropriate but realistic truncations to such a shell-model study. Work is in progress in which mp-nh configurations are treated in an algebraic way, starting from particle and hole pairs (considered as s and d bosons) in a noncompact U(6,6) framework [45].

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