Energy shifted level densities in rare earth region

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The energy shift between the level densities of the even-odd and even-even isotopes ^{161,162}Dy and ^{171,172}Yb is measured as a function of excitation energy. The results are compared with predictions from various semiempirical models. The energy shift procedure works well for excitation energies between 3.5 and 7 MeV in the even-even nucleus, yielding a relative energy shift close to the experimental pairing gap parameter Δ.

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The Fermi-gas model for finite nuclei has been shown to account for the nuclear level density at high excitation energy [1]. The model describes the nucleus as a gas of noninteracting fermions confined to the nuclear volume and neglects shell effects and pair correlations.

The level density of odd-odd nuclei is relatively high already around the ground state due to the coupling of the two valence particles to the core. However, for odd-mass and even-even nuclei the level density is considerably lower due to the formation of Cooper pairs. A handful of semiempirical approaches has been suggested to describe this effect of pair correlations by a simple energy shift of the level density function.

In the conventional shifted Fermi-gas model [2–4] the excitation energy is shifted using the pairing energy parameter Δ. The shifts are Δ and 2Δ for odd-mass and even-even nuclei, respectively, yielding approximately the level density found for the neighboring odd-odd system. This description turned out to be too rigid to reproduce the level densities at low and high excitation energies, simultaneously. A two-component level density formula with energy shifts was later introduced [5]. Here, the first $\sim$10 MeV of excitation energy is described by a constant temperature formula, and at higher energies the shifted Fermi-gas model is applied.

A simpler and rather well working version is the back-shifted Fermi-gas model [6,7], where the Fermi-gas formula is used for all excitation energies. The model has only two parameters: the back-shifted energy and the level density parameter a, both being free parameters in order to fit the data.

There are several unclear points in using these approaches. The main questions concern the functional form of the level density and the justification of a shift of the excitation energy to describe the level densities of neighboring nuclei. The fact that the absolute energy shifts seldom coincide with the paring gap parameter Δ (or 2Δ) indicates that one or both assumptions are not fulfilled.

The subject of this work is to extract experimental energy shifts and to investigate the quality of the energy shift procedure as function of excitation energy. Furthermore, it is interesting to compare the value of the energy shift parameter to the pairing gap parameter Δ.

In the vicinity of the ground band, levels can be counted reliably up to a certain excitation energy, typically 1.5 MeV in even rare earth nuclei. The level density can also be derived by the level spacing of neutron resonances at the neutron binding energy B_n. In between these energies few experimental results are available.

Recently, a new method (see Ref. [8], and references therein) has been developed which allows for the simultaneous extraction of the level density and the γ-strength function over a wide energy region. The experiments were carried out with 45 MeV ^{3}He projectiles at the Oslo cyclotron. The particle-γ coincidences are measured with the CACTUS array using the (^{3}He, $\alpha\gamma$) reaction on ^{162,163}Dy and ^{172,173}Yb self-supporting targets. The transferred spin is approximately $2-6\ \hbar$, and the nuclear system is believed to thermalize prior to γ emission. The charged ejectiles were detected with eight particle telescopes placed at an angle of 45° relative to the beam direction. An array of 28 NaI γ-ray detectors with a total efficiency of $\sim$15% surrounded the target and particle detectors.

The level density is deduced from γ-ray spectra recorded at a number of initial excitation energies E, determined by the measured α energy. These data are the basis for making the first generation (or primary) γ-ray matrix, which is factorized according to the Brink-Axel hypothesis [9,10] as

$$P(E,E_\gamma) \propto \rho(E-E_\gamma)\sigma(E_\gamma), \tag{1}$$

where the level density ρ and the γ-energy dependent function σ are determined by an iterative procedure.

It has been shown [8] that if one solution for ρ and σ is found, the corresponding functions $A\exp[\alpha(E-E_\gamma)]\rho$ and $B\exp(\alpha E_\gamma)\sigma$ give exactly the same fit to the $P(E,E_\gamma)$ matrix. The values of A, B, and α can be determined by additional conditions. The A and α parameters are used for absolute normalization of the level density; they are adjusted to reproduce the number of levels observed in the vicinity of the ground state and the neutron resonance spacing at the neutron binding energy B_n. In the following we will only concentrate on the level density ρ.

The experimental level densities for the ^{161,162}Dy and ^{171,172}Yb nuclei are shown as data points in Figs. 1 and 2. The same normalized level densities were for the first time extracted with the new technique [8] in Ref. [11]. In the extraction technique, we exclude data with γ energies below 1 MeV due to methodical problems in the first generation spectra. Therefore, the level density is generally determined

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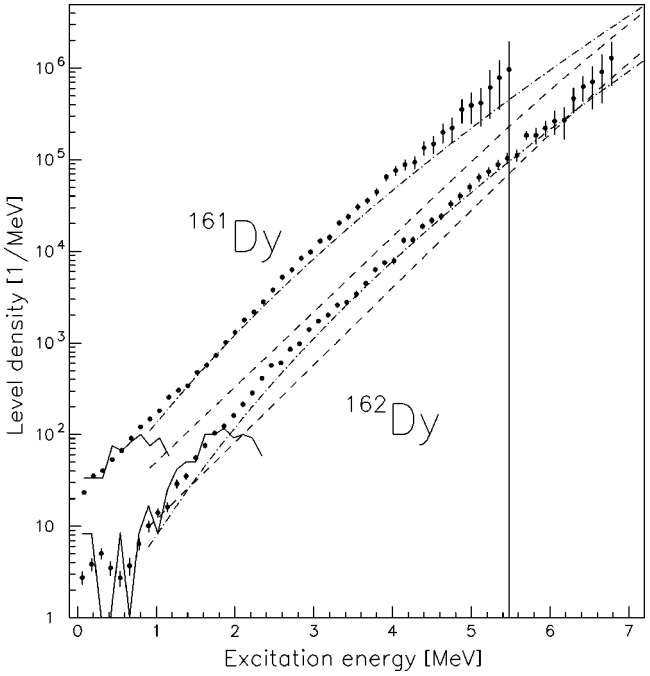

FIG. 1. Observed level densities for ^{161,162}Dy as functions of excitation energy. The experimental data points are compared to the density of known levels at low excitation energy (solid lines). The figure also includes the semiempirical level density formulas of Gilbert and Cameron [5] (dashed curves) and von Egidy *et al.* [6] (dash-dotted curves). Upper and lower points and/or curves are for ^{161}Dy and ^{162}Dy, respectively. Since Gilbert and Cameron give no parameters for ^{162}Dy, we use the ^{160}Dy parameter set.

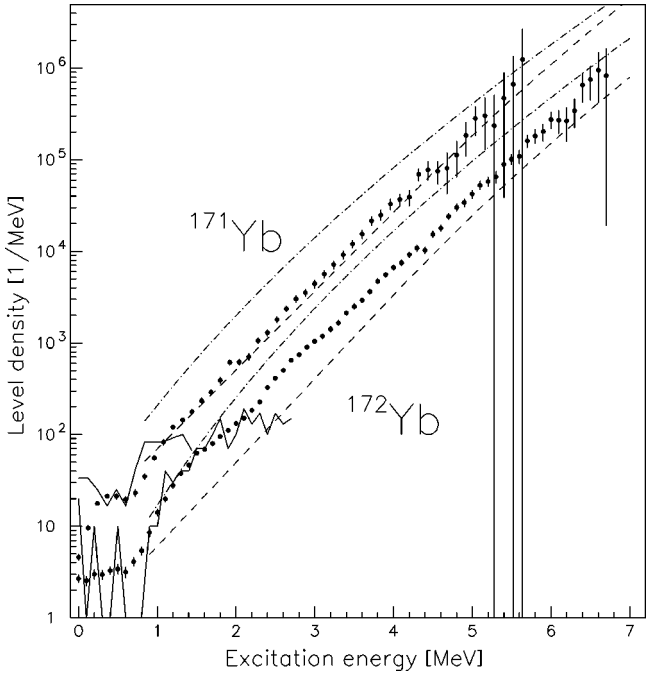

FIG. 2. Level densities for ^{171,172}Yb as functions of excitation energy (see text of Fig. 1).

only up to $E=B_n-1$ MeV. The figures also include the level densities obtained from counting known levels [12].

The level densities for the ^{161}Dy and ^{171}Yb isotopes are about five times higher than for the neighboring ^{162}Dy and ^{172}Yb isotopes. The latter isotopes seem to exhibit the same slope at high excitation energy. However, the presence of bumps modifies this simple picture, in particular at low excitation energies.

The energy region up to $5-10$ MeV has been described by the constant temperature formula [5,6] given by

$$\rho = C\exp(E/T), \tag{2}$$

where the normalization factor C and the temperature T are constants. Also level densities based on the Fermi-gas model are frequently adopted in this energy region [5–7]

$$\rho = \frac{\exp[2\sqrt{aU}]}{12\sqrt{2}a^{1/4}U^{5/4}\sigma}, \tag{3}$$

where σ is the spin cutoff parameter and U is the shifted energy. As examples of such approaches Figs. 1 and 2 also include level densities from Gilbert and Cameron [5] and from von Egidy *et al.* [6]. Full details on the formulas, parametrizations, and choice of parameters are given in Refs. [5,6].

The level densities of Gilbert and Cameron are described by Eq. (2) in the excitation region below ~ 5 MeV, and at higher energies they use Eq. (3). The description is rather poor, except for ^{171}Yb. Gilbert and Cameron give temperatures $[T=(d\ln\rho/dE)^{-1}]$ which are lower in the even-even systems, contrary to the tendency of our data. It seems that these authors have, for the even-even nuclei, anchored their constant temperature level density curves to the ground-state band, rather than to levels at ~ 1.7 MeV, where the two quasiparticle regime appears. Also scarce data at the time their compilation was made (1965) could be the reason for the poor agreement. Even so, we think the two-component level density is a reasonable approach. In the first MeV's of excitation energy, nucleon pairs (Cooper pairs) are broken and thus prevent the temperature to rise as fast as predicted by the Fermi-gas formula. This mechanism is discussed in Ref. [11] and references therein. For excitation energies around and above the neutron binding energy, the Fermi-gas conditions are probably fulfilled. Here, the pairing correlations are quenched and a high density of single particle levels is present.

In Ref. [6] von Egidy *et al.* have tested both the constant temperature and the back-shifted Fermi-gas formulas. They find that both approaches give similar χ^2 fits to experimental data. The suggested temperatures are close to 0.6 MeV for all four nuclei, almost 0.1 MeV higher than our data indicate. In Figs. 1 and 2 we show only the Fermi-gas results. Within this model both the level density parameter a and the correction to the backshift C_1 $(U=E-\Delta-C_1)$ are based on global parametrizations as function of the mass number A. With this restriction, one may say that the level density curves describe our data points rather well. However, a clear shortcoming is that these expressions increase too slowly as function of excitation energy. The experimental level densities, in particular for the Yb isotopes, show a functional form closer to the constant temperature formula.

Global fits for all mass numbers can give deviations of up to a factor of 10 from known average neutron resonance spacings [6]. Of course, better local fits to experimental data could be achieved, both with Eq. (2) and/or Eq. (3). However, a common approach is difficult to construct since all four nuclei exhibit different functional forms. These variations are probably connected with details in the quenching of the pair correlations in the individual nuclei.

It is commonly believed that neighboring odd-odd, odd-even, even-odd, and even-even isotopes reveal the same level density if a proper shift is applied to the excitation energy. With the present experimental data, we have the opportunity to test how well the energy-shift procedure works for the level densities of even-odd (eo) and even-even (ee) systems.

Neglecting collective excitations and residual interactions, the Fermi-gas model can describe the level density of the odd-odd (oo) nucleus rather successfully. The level density of the other neighboring nuclei can then be estimated by

$$\rho_{\mathrm{oe}}(E)=\rho_{\mathrm{oo}}(E-\Delta_n), \tag{4}$$

$$\rho_{\mathrm{eo}}(E)=\rho_{\mathrm{oo}}(E-\Delta_p), \tag{5}$$

$$\rho_{\mathrm{ee}}(E)=\rho_{\mathrm{oo}}(E-\Delta_n-\Delta_p), \tag{6}$$

where E is the excitation energy.

The pairing gap parameters Δ_p and Δ_n can be determined from empirical masses of a sequence of isotones or isotopes where [13]

$$\Delta_p=\frac{1}{4}\left|S_p(N,Z+1)-2S_p(N,Z)+S_p(N,Z-1)\right|, \tag{7}$$

$$\Delta_n=\frac{1}{4}\left|S_n(N+1,Z)-2S_n(N,Z)+S_n(N-1,Z)\right|, \tag{8}$$

and S_p and S_n are proton and neutron separation energies [12], respectively. The pairing gap parameter can alternatively be calculated by the empirical formula [13]

$$\Delta=12A^{-1/2}\mathrm{MeV}, \tag{9}$$

which is valid for both neutrons and protons.

Equations (7) and (8) depend on the proton (Z) and neutron (N) numbers and should, in principle, give the best estimate. However, Eq. (9) gives a smooth function which neglects local shell effects, and is probably more correct if Δ is interpreted as a pure pairing parameter.

From the extracted level densities for the ^{161,162}Dy and ^{171,172}Yb nuclei, we can investigate the energy shift necessary to apply in order to simulate the level density in neighboring even-odd and even-even isotopes. The energy shift $\delta(E)$ is defined as the necessary shift of the even-odd nucleus level density in order to describe the level density in the neighboring even-even nucleus

$$\rho_{\mathrm{ee}}(E)=\rho_{\mathrm{eo}}[E-\delta(E)]. \tag{10}$$

In Fig. 3 the resulting $\delta(E)$ curves are plotted as function of the excitation energy E measured in the even-even nucleus.

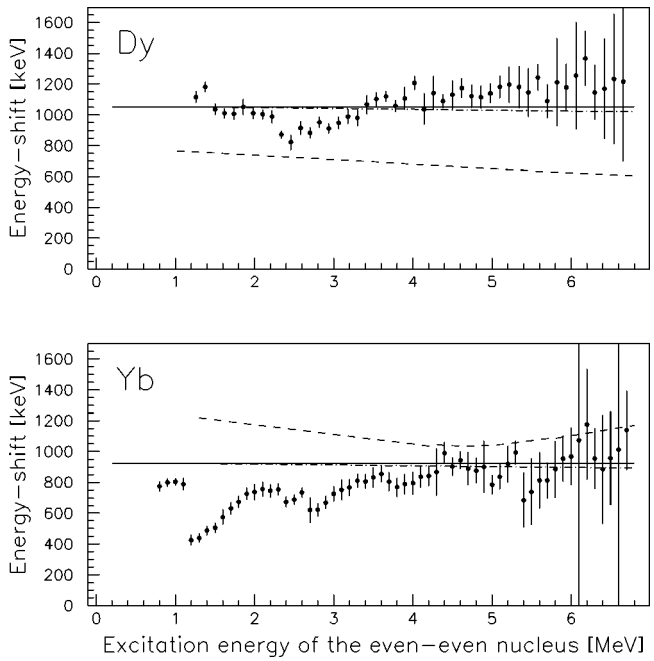

FIG. 3. The observed energy shift δ between ^{161}Dy and ^{162}Dy (upper part) and ^{171}Yb and ^{172}Yb (lower part). The pairing gap parameters Δ^{eff} (solid lines) and energy shifts from Gilbert and Cameron [5] (dashed curves) and von Egidy *et al.* [6] (dash-dotted curves) are also displayed for comparison.

In the excitation energy region between 3.5 and 7 MeV the energy shift is rather constant giving $\delta=1.13(18)$ and 0.84(19) MeV for ^{161,162}Dy and ^{171,172}Yb, respectively.

The corresponding δ values should be compared with the effective pairing gap parameter defined here for even N and Z as

$$\Delta^{\mathrm{eff}}(N,Z)=\Delta_p(N,Z)+\Delta_n(N,Z)-\Delta_p(N-1,Z), \tag{11}$$

where we apply Eqs. (7) and (8). In Table I these values [and the pairing gap parameters calculated from Eq. (9)] are compared to the experimental δ values. These values coincide rather well within less than 0.2 MeV, and Fig. 3 shows that Δ^{eff} is in good agreement with the observed energy shift in

TABLE I. Energy shift δ extracted between the even-odd and even-even isotopes. (The energy shifts from semiempirical level densitiy formulas are constants based on parameters used in Refs. [5–7]. The energy shifts of von Egidy *et al.* [6] are not listed since they coincide with Δ^{eff}.)

Parameter δ or Δ (MeV)	^{162}Dy	^{172}Yb
δ from present data	1.13(18)	0.84(19)
Δ^{eff} from separation energies, Eq. (11)	1.05	0.93
Δ from empirical formula, Eq. (9)	0.94	0.91
δ from back-shifted Fermi gas [7]	0.88(50)	1.15(50)
δ from two-component level density [5]:		
Energies below $\sim$5 MeV, Eq. (2)	0.81(30) [a]	1.30(30)
Energies above $\sim$5 MeV, Eq. (3)	0.70(20)	0.69(20)

[a]The shift is calculated from the ^{160,161}Dy parameter sets.

the 3.5 – 7 MeV excitation energy region. The correspondence is less impressive when comparing experiment with energy shifts obtained from Eq. (9) and with semiempirical level density functions. The shifts from Gilbert and Cameron [5] deviate strongly from the experimental data, as also indicated from the δ values[1] of Table I. The shifts from von Egidy *et al.* [6] are determined by the experimental pairing gap Δ and the slow varying back-shift correction C_1. Therefore, these shifts coincide almost excactly with the Δ^{eff} values. The small deviations seen in Fig. 3 are due to the 0.1 MeV^{-1} increase in the level density parameters for the even-even systems.

Both the two-component formula of Gilbert and Cameron [5] and the Fermi-gas formula of Ref. [7] give δ values deviating with about 0.3 – 0.5 MeV (see Table I). This is probably due to the free adjustment of δ and other parameters, and indeed the shifts have been associated with large uncertainties by these authors. The role of δ in this type of approach is not a pure energy shift, but may also include a compensation for the unphysical form of the adopted analytical level density function. The same conclusion is evident from the compilations of Refs. [5,7], where the extracted energy shifts scatter typically within ± 0.5 MeV in this mass region.

In conclusion, the shifting of excitation energy in order to simulate the level density of neighboring isotopes works well using realistic level density functions. The experimental level densities follow each other rather closely as function of excitation energy in the 3.5 – 7 MeV region, and the energy shifts coincide within 0.2 MeV with the pairing gap parameter Δ. In fact, any reasonable mathematical form used to interpolate between the discrete levels and the resonance data will obtain relative energy shifts nearly equal to the pairing energy. Of the approaches studied here, only the formalism of von Egidy *et al.* gives a relative energy shift equal to the pairing energy. This feature is built automatically into the back-shift correction. However, the absolute level density of these authors often fails to reproduce the densities based on discrete levels and/or the resonance data. Below 3.5 MeV of excitation energy, nuclear structures assigned the various nuclei prevent the use of a simple energy shift procedure. In particular, the even-even isotopes reveal bumps in the level density function due to the breaking of Cooper pairs.

Probably, no simple level density formula can describe simultaneously the four nuclei investigated here. The Yb isotopes exhibit a constant temperaturelike behavior, while the Dy isotopes are closer to the back-shifted Fermi-gas prediction. Nevertheless, we find that the parameters used for the semiempirical formulas should undergo a revision. Here, all new low-lying levels should be included together with recent information on resonance level spacings. This effort, combined with a refined two-component formula, like the one of Gilbert and Cameron, could probably give better analytical formulas for future use. The formulas should have the ability to give a constant energy shift between the level densities of neighboring isotopes, as observed in this work for the ^{161,162}Dy and ^{171,172}Yb nuclei.

[1]Since these authors apply different temperatures for even-odd and even-even nuclei (see Figs. 1 and 2), the actual shifts are only approximately given by the δ values of Table I.

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