Neutral current effects in ^{16}O

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The γ-circular polarizations (P_γ) and asymmetries (A_γ) of several parity forbidden γ transitions in ^{16}O* have been investigated within an effective operator approach. Considering various strong and weak interaction models, the maximum theoretical value among the circular polarizations has been found to be 1.85×10^{-3} which corresponds to the 4.15 MeV γ ray ($2^+T=0$; $E_x=13.02$ MeV$\rightarrow2^-T=0$; $E_x=8.87$ MeV) in ^{16}O*. The isovector component of the parity nonconserving matrix element dominates the mixing of the parity doublet ($2^-T=1$; $E_x=12.9686$ MeV$-2^+T=0$; $E_x=13.02$ MeV) in ^{16}O*.

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I. INTRODUCTION

According to the standard theory of electroweak interactions, the neutral current contribution to $\Delta S=1$ and $\Delta C=1$ weak processes are strongly suppressed [1,2] and, therefore, the neutral current weak interaction between quarks can be studied best in flavor-conserving processes. This observation motivates precision studies of parity nonconservation (PNC) in low energy nuclear physics.

The present interpretation of the PNC effects at low energies, is based on a PNC nucleon-nucleon interaction, which is assumed to result from the exchange of low mass mesons (π, ρ, and ω). The PNC meson-nucleon coupling constants are calculated from the flavor-conserving weak processes.

Previous investigations of the PNC meson-nucleon vertices, for π, ρ, and ω meson exchange, within different models such as a chiral effective Lagrangian or nucleon quark model, or QCD sum rules found the weak πN coupling constant ($h_\pi^{(1)}$) to be in a range of $(2-11)\times10^{-7}$ [3–6]. The latest study of this coupling constant updated it to be $\leqslant1.3\times10^{-7}$ [7] which is significantly smaller than the above.

This value for $h_\pi^{(1)}$ falls within the range deduced by Desplanques, Donoghue, and Holstein (DDH) [8], but it is significantly smaller than the theoretical ''best value'' of 4.5×10^{-7}. The values of $h_\pi^{(1)}$ and $h_{\rho'}^{(1)}$ (see Table I) are of particular interest due to their significant sensitivity to neutral current processes at low energy. This large range of uncertainty stimulates us to investigate experiments sensitive to $h_\pi^{(1)}$ and $h_{\rho'}^{(1)}$.

Parity nonconservation in the nucleon-nucleon interaction has been observed through helicity dependent asymmetry in $\vec{p}$-p scattering [9], through nucleon-nucleus scattering induced by polarized projectiles (such as $\vec{p}$ [10] or $\vec{n}$ [11]), through spontaneous α decay [12], [13] and in the circular polarization [14–17,20] or asymmetry [8,21–23], (from polarized nuclei) of nuclear γ rays emitted in the decays. Theoretical predictions are available for new PNC experiments in induced α decay [24,25] and γ decay [26–29]. Theoretical and experimental work in nuclear PNC processes has been reviewed recently [7,8,23,30,31].

The search for parity nonconservation in complex nuclei, and especially in cases where an enhanced effect is expected from the existence of parity mixed doublets (PMD) [23,30,31] has a long history. Here PNC effects are sensitive to several factors. The most important factor is the small level spacing between states of the same spin and opposite parity in the compound nucleus. Another important factor arises from the potential increase in the ratio of parity—forbidden to parity—allowed transition matrix elements caused by the nuclear structure of the states involved. An attractive example is the case of closely spaced doublets of the same spin and opposite parity well isolated from similar levels (since it would justify a simple two-state mixing approximation) and, where the normal parity-conserving transitions are hindered by nuclear structure effects.

Detracting from the apparently ''clean'' situation for perturbation theory are the large uncertainties in the extraction

TABLE I. Weak meson-nucleon coupling constants calculated within different weak interaction models (in units of 10^{-7}). The abbreviations are KM = Kaiser and Meissner, DDH = Desplanques, Donoghue, and Holstein ''best'' values, AH = Adelberger and Haxton, and DZ = Dubovik and Zenkin.

$h_{\rm meson}^{\Delta T}$	KM	DDH	AH(fit)	DZ
h_π^1	0.19	4.54	2.09	1.30
h_ρ^0	−3.70	−11.40	−5.77	−8.30
h_ρ^1	−0.10	−0.19	−0.22	0.39
h_ρ^2	−3.30	−9.50	−7.06	−6.70
$h_{\rho'}^1$	−2.20	0.00	0.00	0.00
h_ω^0	−6.2	−1.90	−4.97	−3.90
h_ω^1	−1.00	−1.10	−2.39	−2.20

*Deceased.

of the PNC nucleon-nucleon (PNC-NN) parameters from the experimental data. As a matter of fact, the same effects responsible for enhancing PNC effects complicate a reliable determination of the nuclear matrix elements. Therefore, it is necessary to select exceptional cases, in which the nuclear structure uncertainties can be reduced by independent measurements.

Now, how does one gain experimental access to these terms? Clear and large parity mixing signals have been found.

In resonant scattering of polarized neutrons from nuclei a huge (six orders of magnitude), enhancement of the PNC effect has been predicted [32] and observed [11], however, we cannot separate specific parts of the PNC-NN interaction as in the case of low-lying PMD's. The observed properties of the PMD should help to determine the relative strengths of the different components of the PNC nucleon-nucleon interaction [3,4,8,23]. Usually, there are seven terms in the Hamiltonian [33], four of them ($\sim h_{\pi}^{(1)}$, $h_{\rho}^{(1)}$, $h_{\omega}^{(1)}$, and $h_{\rho'}^{(1)}$) are of the isovector type (dominated by the neutral currents [23]), two ($\sim h_{\rho}^{(0)}$, $h_{\omega}^{(0)}$) are of the isoscalar type (dominated by the charged currents [23]), and one ($\sim h_{\rho}^{(2)}$) is isotensor term (dominated by the charged currents [23]).

Parity mixing has been studied in simplified nucleon-nucleus interaction cases with the goal of isolating at least the characteristic strength of the nucleon-nucleus weak force. As weak interactions do not conserve the isospin, they may be approximately characterized by either the strengths of the weak proton-nucleus and neutron-nucleus potentials or equivalently by the isovector and isoscalar components of the nucleon-nucleus potentials.

The main contribution coming from the isovector part is assumed to be due to the one-pion exchange term, while the main contribution coming from the isoscalar part is assumed to be due to one-ρ-meson exchange term. Other contributions to the weak hadron-nucleus interaction potential [33] appear to be experimentally inaccessible. Therefore, two independent experiments should be sufficient, in principle, for the determination of the above simplified nucleon-nucleus weak forces. For example, theoretical analysis [23,34,35], shows the PMD in ^{19}F is dominated by the strength of the proton-nucleus weak force [36]. On the other hand, the PMD in ^{18}F, is well known to be dominated by the isovector part of this force. Experimentally observed parity mixing in ^{19}F [21,22] is accounted for by the ''best DDH values'' [8] of meson-nucleon weak coupling constants. However, parity mixing in ^{18}F is barely compatible [17–20,37], with the extremum of the theoretical uncertainties.

Another *pair* of independent experiments for the determination of the nucleon-nucleus weak forces could be the isoscalar α decay: ^{16}O ($J^{\pi}T=2^-0$,8.87 MeV)$\rightarrow\alpha+{}^{12}$C (g.s.) [12,13] coupled with an experiment involving the isovector PMD: ($J^{\pi}T=2^+0$,13.02 MeV; $J^{\pi}T=2^-1$,12.9686 MeV) in ^{16}O proposed in Refs. [24,25]. References [24,25] propose investigating isovector PMD via the resonance ^{15}N$(\vec{p},\alpha)^{12}$C reaction with two observables: the longitudinal A_L and the irregular transverse A_b analyzing powers.

In the present paper we study the PMD via parity forbidden γ transitions in ^{16}O*. Considering various strong and weak interaction models, the maximum theoretical value of the circular polarization calculated for the above parity forbidden γ transitions has been found to be that corresponding to the 4.15 MeV γ ray ($2^+T=0$; $E_x=13.02$ MeV $\rightarrow 2^-T=0$; $E_x=8.87$ MeV) in ^{16}O*. Its value is typically larger than the analogous value experimentally obtained for the ^{19}F case [21,22]. In order to compare with previous works [24,25] we examine the PNC-matrix element within different strong and weak interaction models. We conclude that the PMD is mainly affected by the isovector character of the parity mixing matrix element analogous to the ^{18}F [23] and ^{21}Ne [33] cases.

The paper is organized as follows. An effective PNC operator ($\mathrm{H}_{\mathrm{PNC}}^{\mathrm{eff}}$) is deduced in Sec. II. In Sec. III the formulas of the circular polarization (P_{γ}) and γ asymmetry (A_{γ}) in the ^{16}O case is presented. Discussions concerning the weak interaction models and the corresponding PNC matrix elements are given in Sec. IV. Section V is devoted to the analysis of the particular behavior of the circular polarizations and γ asymmetries of the ^{16}O decay from its $J^{\pi}T=2^+0$, 13.02 MeV state. Section VI will contain the conclusion.

II. EFFECTIVE PNC INTERACTION HAMILTONIAN

Starting from the standard model there are two major layers for obtaining the effective PNC interaction Hamiltonian. The bare PNC interaction Hamiltonian ($\mathrm{H}_{\mathrm{PNC}}$) is given by the exchange of heavy $W^{\pm}$ and Z^0 bosons between the quarks of the nucleons and mesons [8]. In principle one would then obtain an effective QCD $\mathrm{H}_{\mathrm{PNC}}$ ($\mathrm{H}_{\mathrm{PNC}}^{\mathrm{QCD}}$), for example, by generalyzing the method given in Ref. [38], and references cited therein. This would produce the effective weak coupling constants in analogy to the procedure valid for the quark masses and QCD strong coupling constants. From this $\mathrm{H}_{\mathrm{PNC}}^{\mathrm{QCD}}$ interaction we would then obtain an effective nuclear PNC interaction Hamiltonian ($\mathrm{H}_{\mathrm{PNC}}^{\mathrm{eff}}$). In the following we shall develop this second layer starting from $\mathrm{H}_{\mathrm{PNC}}$ [8] mentioned above—we recognize that the full development including $H_{\mathrm{PNC}}^{\mathrm{QCD}}$ remains a challenging issue. The microscopic theory of effective operators [39–44] has been derived in several ways based on the ideas introduced by Bloch and Horowitz [45] and by Morita [46].

Recent years have seen an upsurge of interest in relating the effective parameters of the established nuclear models to the properties of the nucleons. This is generally referred to as the microscopic approach to nuclear structure. Its fundamental aims are to derive the model-space dependence and nucleus dependence of the microscopic models, to define the limitations of these models, and to indicate the nature of the corrections they require, starting from the observed properties and mutual free-space interactions of the nucleons.

We follow well-established theory and, for completeness, develop our framework showing the cross relationships between our different approaches. We also establish our notation in this development.

The Hamiltonian of the system is $H=T+v$, where T is the total kinetic energy operator and v is the sum of the bare two-body interactions. We rewrite H in the form $H=H_0+V$, where H_0 is a completely solved reference Hamiltonian with known eigenenergies and eigenvectors

$$H_0|\Phi_n\rangle=\mathcal{E}_n|\Phi_n\rangle, \tag{1}$$

and with the unperturbed ground state $|\Phi_0\rangle$. In practice, this requires $H_0=T+U$, where U is the single-particle potential. Consequently $V=v-U$ with both two-body and one-body parts. The full Schrödinger equation ($H|\Psi_n\rangle=E|\Psi_n\rangle$) is, of course, impossible to solve exactly for a many particle system with more than three or four nucleons. A blend of nonperturbative and perturbative methods may be used to relate the exact solution to the known solutions of the unperturbed Hamiltonian H_0, taken to be parity conserving.

This theoretical analysis is usually performed for the parity conserving (PC) part of the Hamiltonian (H_{PC}). To be specific for our applications, we will attach the labels PC and PNC where appropriate. Thus, $H=H_{\mathrm{PC}}+H_{\mathrm{PNC}}=H_0+V$, i.e., $H_{\mathrm{PC}}=H_0+V_{\mathrm{PC}}$, $H_0=T+U$, $v_{\mathrm{PC}}=U+V_{\mathrm{PC}}$, and $V=V_{\mathrm{PC}}+H_{\mathrm{PNC}}$, where H_0 has the known eigenenergies $\mathcal{E}_n$ and PC eigenvectors $|\Phi_n\rangle$.

The Hilbert space spanned by $|\Phi_n\rangle$ is separated into two subspaces by the projection operators P and Q, where $P+Q=1$ and

$$P=\sum_{n\in D}|\Phi_n\rangle\langle\Phi_n|. \tag{2}$$

The notation $n\in D$ means that the states $|\Phi_n\rangle$ included in the summation span a specified and finite subspace (the valence space) D of the Hilbert space. Since P and Q are defined in terms of $|\Phi_n\rangle$ eigenstates of H_0, they commute with H_0, and we obtain

$$(E-H_0-PV)P|\Psi\rangle=PVQ|\Psi\rangle, \tag{3}$$

$$(E-H_0-QV)Q|\Psi\rangle=QVP|\Psi\rangle. \tag{4}$$

The second equation is formally solved for $Q|\Psi\rangle$, which is then substituted in the first equation, by using

$$Q|\Psi\rangle=\mathcal{G}_Q(E)QVP|\Psi\rangle, \tag{5}$$

where

$$\mathcal{G}_Q(E)=\frac{Q}{(E-H_0-QVQ)}. \tag{6}$$

Then using the definition

$$(E-H^{\mathrm{eff}})P|\Psi\rangle=0, \tag{7}$$

we obtain

$$H^{\mathrm{eff}}=H_0+V\mathcal{G}_Q(E)V. \tag{8}$$

Equation (7) should be solved in the subspace D only, and the eigenenergies should be the exact ones.

Now we use the separation $V=V_{\mathrm{PC}}+H_{\mathrm{PNC}}$ of the exact residual interaction and expand to first order in $QH_{\mathrm{PNC}}Q$ to obtain

$$\mathcal{G}_Q(E)=\mathcal{G}_Q^{\mathrm{PC}}(E)+\mathcal{G}_Q^{\mathrm{PC}}(E)H_{\mathrm{PNC}}\mathcal{G}_Q^{\mathrm{PC}}(E), \tag{9}$$

where

$$\mathcal{G}_Q^{\mathrm{PC}}(E)=\frac{Q}{(E-H_0-QV^{\mathrm{PC}}Q)}. \tag{10}$$

The effective Hamiltonian becomes

$$H^{\mathrm{eff}}=H_{\mathrm{PC}}^{\mathrm{eff}}+H_{\mathrm{PNC}}^{\mathrm{eff}}+\mathcal{O}(H_{\mathrm{PNC}}^2), \tag{11}$$

in which

$$H_{\mathrm{PC}}^{\mathrm{eff}}=H_0+V_{\mathrm{PC}}^{\mathrm{eff}}, \tag{12}$$

where

$$V_{\mathrm{PC}}^{\mathrm{eff}}=V_{\mathrm{PC}}+V_{\mathrm{PC}}\mathcal{G}_Q^{\mathrm{PC}}(E)V_{\mathrm{PC}}, \tag{13}$$

and

$$H_{\mathrm{PNC}}^{\mathrm{eff}}=[1+H_{\mathrm{PC}}\mathcal{G}_Q^{\mathrm{PC}}(E)]H_{\mathrm{PNC}}[\mathcal{G}_Q^{\mathrm{PC}}(E)H_{\mathrm{PC}}+1]. \tag{14}$$

Now it is straightforward to see that the matrix elements of $H_{\mathrm{PNC}}^{\mathrm{eff}}$ between the unperturbed states $|\Phi_n\rangle$ are equal to the matrix elements of the bare H_{PNC} interaction [8] between the correlated states $|\Psi_n\rangle$. For our initial investigation, we will circumvent the usual full development of the $H_{\mathrm{PC}}^{\mathrm{eff}}$, in favor of using semirealistic two-body correlations. We implement correlations in the following manner. We can write the correlated states in terms of unperturbed states:

$$|\Psi_n\rangle=\Omega_{\mathrm{PC}}|\Phi_n\rangle\approx\Pi_{i<j}f_{ij}^{\mathrm{Jastrow}}|\Phi_n\rangle, \tag{15}$$

where

$$\Omega_{\mathrm{PC}}=1+\mathcal{G}_Q^{\mathrm{PC}}(E_n)H_{\mathrm{PC}}, \tag{16}$$

is a Moeller-like operator and $f_{ij}^{\mathrm{Jastrow}}$ is the two-body Jastrow correlation operator, assumed to be state independent [41].

The Bethe-Goldstone equation may be obtained from Eq. (15) when retaining only the two-nucleon correlations [47,48]. For the Jastrow operator we will use the Miller and Spencer [49] expression. Recognizing that H_{PNC} involves short-range operators we may involve higher order terms in $QH_{\mathrm{PNC}}Q$ in addition to the terms occuring in Eq. (14). Then the PNC effective interaction becomes

$$H_{\mathrm{PNC}}^{\mathrm{eff}}=[1+H_{\mathrm{PC}}\mathcal{G}_Q^{\mathrm{PC}}(E)]t_{\mathrm{PNC}}[\mathcal{G}_Q^{\mathrm{PC}}(E)H_{\mathrm{PC}}+1], \tag{17}$$

where

$$t_{\mathrm{PNC}}=H_{\mathrm{PNC}}+H_{\mathrm{PNC}}\mathcal{G}_Q^{\mathrm{PC}}(E)H_{\mathrm{PNC}}, \tag{18}$$

which acts as a PNC reaction matrix by analogy with

$$t_{\rm PC}=V_{\rm PC}+V_{\rm PC}\mathcal{G}_Q^{\rm PC}(E)V_{\rm PC}, \tag{19}$$

the well-known parity conserving reaction matrix [42]. There are of course fundamental differences between $t_{\rm PC}$ and $t_{\rm PNC}$. For $t_{\rm PC}$ the strong short-range repulsion in $V_{\rm PC}$ makes it difficult to solve. This is one reason for us to simply use the Jastrow correlated basis states. However, for $t_{\rm PNC}$ it should be sufficient to work to leading order in $H_{\rm PNC}$. Thus, a PNC matrix element may be calculated according to

$$M_{\rm PNC}=\langle\Phi_n|H_{\rm PNC}^{\rm eff}|\Phi_{n'}\rangle=\langle\Psi_n|t_{\rm PNC}|\Psi_{n'}\rangle=\langle\Psi_n|H_{\rm PNC}|\tilde{\Psi}_{n'}\rangle, \tag{20}$$

where

$$|\tilde{\Psi}_n\rangle=|\Psi_n\rangle+\mathcal{G}_Q^{\rm PC}(E)H_{\rm PNC}|\Psi_n\rangle. \tag{21}$$

This last equation can be solved by complete analogy with the usual Bethe-Goldstone equation [48] when retaining only the two-nucleon correlations due to the PNC nucleon-nucleon interactions.

Our approximation scheme proceeds as follows. According to Kallio and Day [48] we take the H_0 to be

$$H_0=H_{\rm osc}+\sum_m c_m|m\rangle\langle m|, \tag{22}$$

where $H_{\rm osc}$ is the oscillator Hamiltonian. The parameters c_m are to be determined from the conditions

$$c_m+\left\langle m\left|\frac{1}{2}m\omega^2r^2\right|m\right\rangle=\langle m|U|m\rangle=\sum_{n\leq F}[\langle mn|t_{\rm PC}|mn\rangle -\langle mn|t_{\rm PC}|nm\rangle], \tag{23}$$

where the sum is over the occupied or ''core'' states. For $m>F$, $c_m=0$.

By complete analogy with the Kallio and Day approach for the ordinary Bethe-Goldstone equation [48], we may write an analogous equation for the two-body $|\tilde{\Psi}\rangle$ solution of Eq. (21)

$$[E_0+\Delta-H_{\rm osc}(1,2)]|\tilde{\Psi}(1,2)\rangle=\Delta|\Psi^{\rm BG}(1,2)\rangle +QH_{\rm PNC}(1,2)|\tilde{\Psi}(1,2)\rangle, \tag{24}$$

with

$$[E_0+\Delta-H_{\rm osc}(1,2)]|\Psi^{\rm BG}(1,2)\rangle =\Delta|\Phi^{\rm osc}(1,2)\rangle+QH_{\rm PC}(1,2)|\Psi(1,2)\rangle. \tag{25}$$

Here $|\Psi^{\rm BG}(1,2)\rangle$ is the two-nucleon solution of the Bethe-Goldstone equation [48], Q is the Pauli operator, $H_{\rm PNC}$ is the bare PNC weak two-nucleon interaction [33], $H_{\rm PC}(1,2)$ is the two-nucleon bare PC Hamiltonian [$T+v_{\rm PC}(1,2)$, where $v_{\rm PC}(1,2)$, e.g., is the Reid soft-core [50] potential]. Also, $E_0=E_n+E_m$, $E_{n,m}$ are the single-particle eigenenergies of $H_{\rm osc}(1,2)$ where the indices m,n stand for the oscillator single-particle quantum numbers and

$$\Delta=c_n+c_m. \tag{26}$$

The above equation for $|\tilde{\Psi}(1,2)\rangle$ is analogous to the two-nucleon Bethe-Goldstone equation [see Eqs. (25) and (19a) of Ref. [48]], the only change is that the PC NN potential v should be replaced by $H_{\rm PNC}(1,2)$ (e.g., from Ref. [33]). The technique to solve the above equation for $|\tilde{\Psi}(1,2)\rangle$ is analogous to that proposed by Kallio and Day except for the boundary conditions at the origin for which we choose

$$\langle r|\psi\rangle=0 \quad \text{for} \quad r=0. \tag{27}$$

III. THE γ ASYMMETRIES

The degree of circular polarization of the emitted γ rays is given [see Ref. [51], Chap. 9, Sec. III, Eq. (9.38)] by a sum of PNC and PC contributions

$$P_\gamma(\cos\theta)\equiv\frac{W_{\rm right}(\theta)-W_{\rm left}(\theta)}{W_{\rm right}(\theta)+W_{\rm left}(\theta)}=(P_\gamma)_0R_\gamma^{\rm PNC}(\cos\theta) +R_\gamma^{\rm PC}(\cos\theta), \tag{28}$$

where θ represents the angle between the emitted photon and the axis of polarization and W represents the angular distribution of the circular polarized radiation

$$(P_\gamma)_0=2\frac{M_{\rm PNC}}{\Delta E}\sqrt{\frac{b_+\tau_-}{b_-\tau_+}\left(\frac{E_\gamma^-}{E_\gamma^+}\right)^3} \tag{29}$$

and

$$\begin{aligned}R_\gamma^{\rm PNC}(\cos\theta)=\sqrt{\frac{1+\delta_-^2}{1+\delta_+^2}}\Bigg(&\sum_{\nu=0,2,4}P_\nu(\cos\theta)B_\nu(2)\\&\times[F_\nu(1122)+F_\nu(2222)\delta_+\delta_+\\&+F_\nu(1222)(\delta_-+\delta_+)]\Bigg)\\&\times\Bigg(\sum_{\nu=0,2,4}P_\nu(\cos\theta)B_\nu(2)[F_\nu(1122)\\&+F_\nu(2222)\delta_-^2+2F_\nu(1222)\delta_-]\Bigg)^{-1},\end{aligned} \tag{30}$$

is a multiplier coming from the orientation of the nucleus in the initial excited state when the mixing ratios do not vanish.

The F_ν coefficients are defined by

$$F_\nu(LL'I'I)=(-1)^{I'+3I-1}[(2I+1)(2L+1)(2L'+1)]^{1/2}$$

$$C(LL'\nu;1-10)W(LL'II;\nu I'), \quad (31)$$

where C is the Clebsch-Gordan coefficient $C(J_1J_2J_3;M_1M_2M_3)$ and W is the Racah coefficient. The parity conserving (PC) quantity is given by [51]

$$R_\gamma^{\mathrm{PC}}(\cos\theta)=\left\{\sum_{\nu=1,3}P_\nu(\cos\theta)B_\nu(2)[F_\nu(1122)+F_\nu(2222)\delta_-^2+2F_\nu(1222)\delta_-]\right\}\times\left\{\sum_{\nu=0,2,4}P_\nu(\cos\theta)B_\nu(2)\times[F_\nu(1122)+F_\nu(2222)\delta_-^2+2F_\nu(1222)\delta_-]\right\}^{-1}, \quad (32)$$

where P_ν are Legendre polynomials, and

$$B_\nu(2)=\sum_M(2\nu+1)^{1/2}C(2\nu2;M0M)p(M). \quad (33)$$

Here $p(M)$ is the polarization fraction of the M state, which determines the degree of the orientation of the nucleus.

In the above equations, δ_- is the $M2/E1$ mixing ratio, δ_+ is the $E2/M1$ mixing ratio, and τ_+,τ_-, $b_+,b_-,E_\gamma^-,E_\gamma^+$ are the lifetime, branching ratios, and transition energies for the respective parity states from the doublet.

In order to measure a PNC effect one must find situations for which the R_γ^{PC} part vanishes. Two particular cases have this property. In the first case, an initially unpolarized nucleus has $B_0(2)=1$, $B_{\nu\neq0}(2)=0$, and $F_0(LL'22)=\delta_{LL'}$ [50]. In this particularly simple case, the circular polarization reduces to the well-known expression

$$(P_\gamma)_{un}=(P_\gamma)_0\sqrt{\frac{1+\delta_-{}^2}{1+\delta_+{}^2}}\frac{(1+\delta_-\delta_+)}{1+\delta_-{}^2}.$$

In the second case, one may prepare a polarized state by choosing $p(M)=\delta_{M0}$ for which, $B_{\nu=1,3}(2)=0$ and the R_γ^{PC} part vanishes. Another observable which reflects a PNC effect is the forward-backward asymmetry (FBA) of the emitted γ rays by polarized nuclei

$$\mathrm{FBA}_\gamma(\theta)\equiv\frac{W(\theta)-W(\pi-\theta)}{W(\theta)+W(\pi-\theta)}. \quad (34)$$

This observable has been successfully used in the ^{19}F case [21,22] in order to avoid the small efficiency of the Compton polarimeters when one measures the degree of circular polarization. If the mixing ratios are small ($\delta_+,\delta_-\ll1$) one can show that [29]

$$\mathrm{FBA}_\gamma(\theta)\simeq(P_\gamma)_0R_\gamma^{\mathrm{PC}}(\cos\theta). \quad (35)$$

The angular distribution described by this formula has a maximum for $\theta=0°$ [29]. It has the advantage that the parity conserving (PC) circular polarization $R_\gamma^{\mathrm{PC}}(\cos\theta)$ can be measured experimentally. For all these cases the $(P_\gamma)_0$ quantities essentially describe the PNC effect.

In the same limit as above ($\delta_+,\delta_-\ll1$), the expression for the circular polarization for the γ rays emitted from one member of the PMD lying at 13 MeV excitation energy in an unpolarized ^{16}O* nucleus reads

$$(P_\gamma)_{\mathrm{un}}=2\frac{M_{\mathrm{PNC}}}{\Delta E}\frac{m_1}{e_1}. \quad (36)$$

It contains the enhancement factor

$$\frac{m_1}{e_1}=\sqrt{\frac{\Gamma_{m_1}}{\Gamma_{e_1}}}, \quad (37)$$

where m_1 (e_1) represents the matrix element corresponding the magnetic (electric) transition of the PMD.

Analyzing the following five γ transitions:

$2^+T=0$, 13.02 MeV $\rightarrow 2^-T=0$, 8.8719 MeV,
$2^+T=0$, 13.02 MeV $\rightarrow 2^-T=0$, 12.53 MeV,
$2^-T=1$, 12.9686 MeV $\rightarrow 2^+T=0$, 11.52 MeV,

TABLE II. Experimental data [52] for the γ transitions used in the calculations of the circular polarization [see Eqs. (29) and (36)].

E_i^* (MeV)	$I_i^{\pi_i}T_i$	E_f^* (MeV)	$I_f^{\pi_f}T_f$	Branching ratio (%)	Γ_γ (eV)
12.97	2^-1	0.00	0^+0		$(3.4\pm0.9)\times10^{-2}$
		6.13	3^-0	63 ± 6	(2.3 ± 0.2)
		7.12	1^-0	12 ± 3	(0.44 ± 0.1)
		8.87	2^-0	25 ± 3	(0.9 ± 0.1)
8.87	2^-0	0.00	0^+0	7.2 ± 0.2	$(2.6\pm0.4)\times10^{-4}$
		6.05	0^+0	0.122 ± 0.033	$(3.1\pm1.0)\times10^{-6}$
		6.13	3^-0	77.7 ± 16.0	$(2.8\pm0.3)\times10^{-3}$
		6.92	2^+0	3.6 ± 0.5	$(1.5\pm0.3)\times10^{-4}$
		7.12	1^-0	11.4 ± 0.5	$(4.2\pm0.8)\times10^{-4}$

TABLE III. The calculated (with OXBASH) $B(E1)$ ratios necessary for evaluating the coefficient κ^2 [see Eq. (39)].

Interaction	$\sqrt{\frac{B(E1)_{\mathrm{OXB}}(13.02\to 8.87)}{B(E1)_{\mathrm{OXB}}(8.87\to 6.92)}}$
ZBM I	0.2055
ZWM	0.2811
ZBM II	0.3134
REWIL	0.1226
ZBMO	0.2055

$2^- T=1$, 12.9686 MeV $\to 2^+ T=0$, 9.8445 MeV,
$2^- T=1$, 12.9686 MeV$\to 2^+ T=0$, 6.9171 MeV;

the maximum enhancement factor is obtained for the first transition (see Table II and Ref. [52]). The calculations read

$$\Gamma_{m_1}(2^-1;E_x=12.97 \ \mathrm{MeV}\to 2^-0;\ \ E_x=8.87 \ \mathrm{MeV}) =(0.9\pm 0.1) \ \ \mathrm{eV}$$

and Γ_{e_1} is estimated as

$$\Gamma_{e_1}=\kappa^2\left(\frac{4.1 \ \mathrm{MeV}}{1.95 \ \mathrm{MeV}}\right)^3\Gamma_{e_1}(2^-0;E_x=8.87 \ \mathrm{MeV}\to 2^+0;\ \ E_x =6.92 \ \mathrm{MeV}), \tag{38}$$

where

$$\Gamma_{e_1}(2^-0;E_x=8.87 \ \mathrm{MeV}\to 2^+0;\ \ E_x=6.92 \ \mathrm{MeV}) =(1.5\pm 0.3)\times 10^{-4} \ \ \mathrm{eV}.$$

The above formulas contain two theoretical quantities: the PNC matrix element and the ratio κ^2, where the last one can be estimated theoretically as follows:

$$\kappa^2=\frac{B(E1)_{\mathrm{OXB}}(13.02\to 8.87)}{B(E1)_{\mathrm{OXB}}(8.87\to 6.92)}. \tag{39}$$

The relation of the circular polarization $(P_\gamma)_0$, to the PNC matrix element $\langle 2^+,T=0(13.02 \ \mathrm{MeV})|H^{\mathrm{eff}}_{\mathrm{PNC}}|2^-,T=1(12.9686 \ \mathrm{MeV})\rangle$, where the $H^{\mathrm{eff}}_{\mathrm{PNC}}$ is the effective parity nonconserving interaction, is given by

$$|(P_\gamma)_0|=0.99\times 10^{-3} \ \ \mathrm{eV}^{-1}\frac{1}{|\kappa|}|\langle 2^+,T=0(13.02 \ \ \mathrm{MeV})|H^{\mathrm{eff}}_{\mathrm{PNC}}|2^-,T=1(12.9686 \ \ \mathrm{MeV})\rangle|. \tag{40}$$

Taking the average value of $|\kappa|=0.2$ (see Table III), and the averaged value $M_{\mathrm{PNC}}\approx 0.36$ eV (see next section and Table IV for the isovector contribution only) we obtain for the circular polarization the value $(P_\gamma)_0\approx 1.85\times 10^{-3}$, which is above the experimental value obtained for the ^{19}F case [21,22]. Other values for this quantity can be obtained by a multiplication by the factor M_{PNC}(in eV)/0.36 eV, where the values for the PNC matrix element M_{PNC} can be found in the Table IV (see Sec. V).

IV. PARITY NONCONSERVING MATRIX ELEMENT

The calculation of the weak matrix element has been performed with the standard PNC potential, arising from the exchange of π, ρ, and ω mesons [Eqs. (8)–(21) from Ref. [33]], together with various descriptions of the effective NN interaction. In the present case we consider the mixing of 2^-, $T=1$, $E_x=12.9686$ with 2^-, $T=0$, $E_x=12.530$ MeV in ^{16}O.

A sample of the values for the weak coupling constants are given in Table I, while the values for the $F_{k,s}$ constants are given in Table V. The abbreviations DDH, KM, AH, and DZ stand for the models developed in Refs. [3,4,8,23], respectively.

Corresponding to the above definitions, we may define the following matrix element:

$$M_{k,s}=\langle \Psi^f_{n'}(I^{-\pi}T')|f_{k,s}|\tilde{\Psi}^i_n(I^\pi T)\rangle, \tag{41}$$

where $|\Psi\rangle$ and $|\tilde{\Psi}\rangle$ are the eigenvectors as defined in Eqs. (15) and (21), respectively. According to Ref. [24] the matrix element of the PNC interaction reads

$$\begin{aligned}\langle 2^+,T=0(13.02 \ \ \mathrm{MeV})|H_{\mathrm{PNC}}|2^-(12.9686 \ \ \mathrm{MeV})\rangle &=0.88\langle 2^+,T=0(13.02 \ \ \mathrm{MeV})|H_{\mathrm{PNC}}|2^-,T=1(12.9686 \ \ \mathrm{MeV})\rangle \\ &\quad -0.47\langle 2^+,T=0(13.02 \ \ \mathrm{MeV})|H_{\mathrm{PNC}}|2^-,T=0(12.53 \ \ \mathrm{MeV})\rangle \\ &=0.88\sum_{k,s=\pi,\rho,\omega}F^{\Delta T=1}_{k,s}M_{k,s}-0.47\sum_{k,s=\rho,\omega}F^{\Delta T=0}_{k,s}M_{k,s}.\end{aligned} \tag{42}$$

TABLE IV. Values of different parts of the PNC matrix element for different descriptions of the nucleus (in units of eV). In the first column M_{π}^{PNC} represents the pion part of the M^{PNC}, M_{IV}^{PNC} signifies the isovector part of the M^{PNC}, $M_{4\rho}^{\mathrm{PNC}}$ stands for the isoscalar ρ-meson part of the M^{PNC}, $M_{\mathrm{IS}}^{\mathrm{PNC}}$ represents the the full isoscalar part of the M^{PNC}, M_{-}^{PNC} represents the M^{PNC}, obtained from Eq. (42) (where the contribution of the $|2^-,T=0\rangle$ state due to the isospin mixing into the $|2^-,12.9686\ \mathrm{MeV}\rangle$ state has a negative sign), M_{+}^{PNC} represents the M^{PNC}, obtained from Eq. (42) (where the contribution of the $|2^-,T=0\rangle$ state due to the isospin mixing into the $|2^-,12.9686\ \mathrm{MeV}\rangle$ state has a positive sign). The next columns contain results (the matrix elements $M_{k,s}$ from Table VII multiplied with the corresponding coefficients from Table V) corresponding to models whose description is given in the text. For each component the value obtained within the DDH weak interaction model is given in the first row, the second row incorporates the value obtained within the DZ weak interaction model while the third row incorporates the value obtained within the KM weak interaction model.

Interact.	ZBMI	ZBMII	REWIL	ZWM	ZWMO
M_{π}^{PNC}	0.5942	0.1892	0.6914	1.6271	0.4320
	0.1678	0.0534	0.1953	0.4595	0.1226
	0.0248	0.0079	0.0288	0.0678	0.0182
M_{IV}^{PNC}	0.6201	0.1968	0.7191	1.6759	0.4536
	0.2232	0.0706	0.2598	0.6111	0.1631
	0.0532	0.0170	0.0679	0.1458	0.0387
$M_{4\rho}^{\mathrm{PNC}}$	−0.7320	−0.2850	−0.0975	0.3270	0.0962
	−0.5309	−0.2067	−0.0707	0.2372	0.0697
	−0.2367	−0.0922	−0.0375	0.1057	0.0310
$M_{\mathrm{IS}}^{\mathrm{PNC}}$	−0.7921	−0.3078	−0.1053	0.3401	0.1018
	−0.6404	−0.2401	−0.0883	0.2751	0.0836
	−0.2764	−0.1008	−0.0441	0.1163	0.0360
M_{+}^{PNC}	0.9181	0.3144	0.8433	0.4312	0.3532
	0.4011	0.1615	0.2602	0.4023	0.1011
	0.1792	0.1228	0.0804	0.0632	0.0172
M_{-}^{PNC}	0.1734	0.0322	0.7491	0.7504	0.4472
	−0.1015	−0.0377	0.1812	0.6609	0.1806
	−0.0855	0.0328	0.0382	0.1815	0.0517

The advantage of the M quantities is that their ratios in the case of the single-particle approximation, without short-range correlations and for a zero range force, are quite simple rational numbers. Since H_{PNC} is a short-range operator, its matrix elements are expected to be sensitive to short-range correlations (SRC) induced by the operator $\mathcal{G}_Q^{PC}(E_n)H_{PC}$ from Eq. (16).

To examine this sensitivity issue, we treat the SRC by two approximation methods. The first one follows Refs. [16,47,53] and neglects three and more particle correlations, i.e.,

$$|\Psi\rangle\approx e^{S_2^{^{16}\mathrm{O}}}|\Phi^{^{16}\mathrm{O}}\rangle, \tag{43}$$

where

$$S_2^{^{16}\mathrm{O}}=\frac{1}{4}\sum_{\rho_1\rho_2}\sum_{\nu_1\nu_2}a^{\dagger}_{\rho_1}a^{\dagger}_{\rho_2}a_{\nu_1}a_{\nu_2}\langle\rho_1\rho_2|S_2^{^{16}\mathrm{O}}|\nu_1\nu_2\rangle, \tag{44}$$

TABLE V. The expressions for the coefficients $F_{k,s}$ multiplying matrix elements $M_{k,s}$ given in Tables VI and VII is tabulated in the first column. Numerical values (in units of 10^{-6}) are given for the ''best'' values of the PNC meson-nucleon couplings in the DDH approach [8], as well as for the values obtained by Kaiser and Meissner [3], and Dubovik and Zenkin [4].

$F_{k,s}^{\Delta T}$	KM	DDH	DZ
$F_{0,\pi}^{1}=\frac{1}{2\sqrt{2}}g_{\pi}h_{\pi}^{1}$	0.090	2.160	0.61
$F_{1,\rho}^{1}=-\frac{1}{2}g_{\rho}h_{\rho}^{1}$	0.014	0.027	0.06
$F_{2,\rho}^{1}=-\frac{1}{2}g_{\rho}h_{\rho}^{1}(1+\mu_v)$	0.066	0.127	0.34
$F_{3,\rho}^{1}=\frac{1}{2}g_{\rho}h_{\rho}^{1}$	−0.014	−0.027	−0.06
$F_{1,\omega}^{1}=-\frac{1}{2}g_{\omega}h_{\omega}^{1}$	0.437	0.480	0.96
$F_{2,\omega}^{1}=-\frac{1}{2}g_{\omega}h_{\omega}^{1}(1+\mu_s)$	0.384	0.423	0.85
$F_{3,\omega}^{1}=-\frac{1}{2}g_{\omega}h_{\omega}^{1}$	0.437	0.480	0.96
$F_{4,\rho}^{0}=-g_{\rho}h_{\rho}^{0}(1+\mu_v)$	4.850	14.940	10.88
$F_{5,\rho}^{0}=-g_{\rho}h_{\rho}^{0}$	1.032	3.180	2.32
$F_{6,\omega}^{0}=-g_{\omega}h_{\omega}^{0}(1+\mu_s)$	1.038	1.408	2.89
$F_{7,\omega}^{0}=-g_{\omega}h_{\omega}^{0}$	1.179	1.600	3.28
$F_{0,\rho}^{1}=-\frac{1}{2}g_{\rho}h_{\rho'}^{1}$	0.307	0.000	0.00
$F_{8,\rho}^{2}=-\frac{1}{2\sqrt{6}}g_{\rho}h_{\rho}^{2}(1+\mu_v)$	0.886	2.542	1.79
$F_{9,\rho}^{2}=-\frac{1}{2\sqrt{6}}g_{\rho}h_{\rho}^{2}$	0.189	0.541	0.38

in which ν_i and ρ_i are occupied and unoccupied, respectively, single-particle states entering the structure of $|\Phi^{^{16}\mathrm{O}}\rangle$. The corresponding two-nucleon correlation function has the form

$$\langle x_1x_2|\psi_2^{^{16}\mathrm{O}}|\nu_1\nu_2\rangle=\mathcal{A}[\phi_{\nu_1}(x_1)\phi_{\nu_2}(x_2)]+\langle x_1x_2|S_2^{^{16}\mathrm{O}}|\nu_1\nu_2\rangle, \tag{45}$$

where $\phi_{\nu_1}(x_1)$ and $\phi_{\nu_2}(x_2)$ are single-particle bound states obtained with an oscillator potential with $\hbar\omega=14$ MeV. The spatial part ($|\Psi(1,2)\rangle$) of the above two-nucleon correlated wave function is the solution of Eq. (25).

In the present calculations we use the Reid soft-core potential [50] as the origin of the effective interaction. The $|\Phi\rangle$ are configuration mixed states arising from effective interactions of the REWIL type [54] within the valence ZBM [55] model space. This effective interaction describes quite well a series of low-energy observables such as the position of the low-lying excited levels in ^{16}O*, the electromagnetic transitions between these levels, α and β decay, and several nuclear reaction rates. The $|\Phi\rangle$ states, however, do not describe directly the short-range correlations due to the very

TABLE VI. Values (in units of MeV) for the 2^+ –2^- mixing matrix elements ($M_{k,s}$) for different descriptions of the nucleus. In the first column, as suggested by Eq. (42), are the symbols of the $M_{k,s}$ quantities. The next three columns contain results corresponding to models of short-range correlations in the PNC matrix elements using different approaches: MS stands for the Miller and Spencer approach, REID stands for the results obtained by solving the Bethe-Goldstone equation, while ''Uncorrelated'' represents the results with no SRC included. All the results are obtained by using the OXBASH shell model code with REWIL interaction (see the text). For each component the contribution corresponding to the ^{12}C core is given in the first row, the second row specifies the contribution of the valence nucleons, while the third row is their sum.

Interact.	MS	REID	Uncorrelated
$M_{0\pi}$	0.3131	0.3000	0.3945
	−0.0070	−0.0088	−0.0088
	0.3061	0.2912	0.3857
$M_{1\rho}$	0.0170	0.0151	0.0527
	0.0012	0.0007	0.0031
	0.0182	0.0158	0.0558
$M_{2\rho}$	0.0198	0.0213	0.0602
	0.0007	0.0008	0.0022
	0.0205	0.0221	0.0624
$M_{3\rho}$	0.0160	0.0132	0.0451
	0.0030	0.0022	0.0092
	0.0190	0.0154	0.0543
$M_{4\rho}$	−0.0068	−0.0066	−0.0221
	−0.0021	−0.0020	−0.0074
	−0.0089	−0.0086	−0.0295
$M_{5\rho}$	0.0003	0.0003	0.0011
	0.0020	0.0019	0.0052
	0.0023	0.0022	0.0063
$M_{1\omega}$	0.0159	0.0155	0.0511
	0.0011	0.0009	0.0037
	0.0170	0.0164	0.0548
$M_{2\omega}$	0.0188	0.0185	0.0611
	0.0007	0.0008	0.0022
	0.0195	0.0193	0.0633
$M_{3\omega}$	0.0150	0.0146	0.0488
	0.0028	0.0023	0.0092
	0.0178	0.0169	0.0580
$M_{6\omega}$	−0.0021	−0.0022	−0.0072
	0.0002	0.0003	0.0008
	−0.0019	−0.0019	−0.0064
$M_{7\omega}$	−0.0035	−0.0033	−0.0121
	−0.0011	−0.0010	−0.0037
	−0.0046	−0.0043	−0.0158
$M_{0\rho}$	0.0198	0.0193	0.0611
	0.0019	0.0011	0.0063
	0.0217	0.0204	0.0674

small model space. The results are reported in Table VI column ''REID.''

In our second approximation scheme we treat the SRC directly by following Refs. [24,25] and utilizing the Jastrow correlations, i.e., Eq. (15). We take the Miller and Spencer [49] correlation function, where the two-particle correlation in coordinate space has a state independent form

$$f(r)=1-\exp(-ar^2)(1-br^2); \quad a=1.1 \ \ \mathrm{fm}^{-2};$$

$$b=0.68 \ \ \mathrm{fm}^{-2}. \tag{46}$$

The results are reported in Table VI column ''MS,'' where again the $|\Phi\rangle$ are configuration mixed wave functions arising from effective interactions of the REWIL type [54] within the valence ZBM [55] model space. In the Table VI column ''Uncorrelated,'' the calculations have been performed under the approximation $|\Psi\rangle \approx |\Phi\rangle$, i.e., with configuration mixing but no SRC.

The Miller-Spencer parametrization is semirealistic and matches rather well with the short-range correlations generated in the solution of the Bethe-Goldstone equation for the Reid soft-core model in the $^1S_0$ and $^3P_0$ channels. Consequently, the results in Table VI do not show large differences between our two treatments of correlations.

Our results in Table VI include only S-wave and P-wave contributions in accord with the approximations of previous efforts [24,25]. Neglecting the tensor-admixed D wave admits the possibility of significant corrections whose sign depends on the specific transition. For the π exchange contribution in the $^3P_1-{}^3S_1(+{}^3D_1)$ transition, the D wave compensates a large part of the short-range repulsion [56]. On the other hand, for the isoscalar ρ exchange contribution in the $^1P_1-{}^3S_1(+{}^3D_1)$ transition, the D wave provides further suppression. Our future efforts will explore this quantitatively.

The nuclear structure calculations depend drastically on the valence model space [57] in some cases. We present in Table VII the detail on the contributions of the different components of the PNC potential to the PNC matrix element

$$\langle 2^+, T=0(13.02 \ \ \mathrm{MeV})|H_{\mathrm{PNC}}|2^-, T=1(12.9686 \ \ \mathrm{MeV})\rangle.$$

To facilitate the comparison, we present the basic matrix elements

$$M_{k,s}=\langle 2^+, T=0(13.02 \ \ \mathrm{MeV})|f_{k,s}|2^-, T'\rangle, \tag{47}$$

where the operators $f_{k,s}$ are defined by Eqs. (10)–(19) of Ref. [33]. In each case we present in Table VII the contribution of the ^{12}C core and the valence components, along with the total contribution. For comparison, we present a simple case where the PNC is due solely to the mixing of the single-nucleon orbits $1p_{1/2}$ and $2s_{1/2}$.

In order to manage the uncertainty in the sign of the PNC interaction, we have adopted the procedure of Ref. [33]. That is, we set the overall phase of the largest contributions to be the same with all interactions and with both our treatments of

TABLE VII. Values of the matrix elements $M_{k,s}$ for different descriptions of the nucleus (in units of MeV). In the first column, are the symbols of the $M_{k,s}$ quantities (see the text). The next columns contain results corresponding to models whose description is presented in the text. The results corresponding to the oversimplified model, where the PNC mixing is due to a single-particle matrix element between the nucleon in the $2s_{1/2}$ and $1p_{1/2}$ orbits, are given in the seventh column (valence). For each component the contribution corresponding to the ^{12}C core is given in the first row, the second row specifies the contribution of the valence nucleons, while the third row represents their sum.

Interact.	ZBMI	ZBMII	REWIL	ZWM	ZWMO	Valence
$M_{0\pi}$	0.2680	0.0571	0.3131	0.6438	0.1975	0.7832
	0.0071	0.0305	−0.0070	0.0095	0.0031	
	0.2751	0.0876	0.3061	0.6533	0.2006	
$M_{1\rho}$	0.0145	0.0031	0.0170	0.0349	0.0107	0.0430
	0.0032	−0.0028	0.0012	0.0055	0.0002	
	0.0177	0.0003	0.0182	0.0404	0.0109	
$M_{2\rho}$	0.0170	0.0036	0.0198	0.0408	0.0125	0.0491
	0.0042	−0.0050	0.0007	0.0066	−0.0004	
	0.0212	−0.0014	0.0205	0.0474	0.0121	
$M_{3\rho}$	0.0137	0.0029	0.0160	0.0329	0.0101	0.0398
	0.0019	0.0023	0.0030	0.0050	0.0024	
	0.0156	0.0052	0.0190	0.0379	0.0125	
$M_{4\rho}$	−0.0425	−0.0152	−0.0068	0.0191	0.0045	−0.0776
	−0.0063	−0.0038	−0.0021	0.0027	0.0019	
	−0.0488	−0.0190	−0.0089	0.0218	0.0064	
$M_{5\rho}$	−0.0021	−0.0007	0.0003	−0.0009	0.0002	0.0002
	−0.0012	0.0041	0.0020	−0.0048	−0.0012	
	−0.0033	0.0034	0.0023	−0.0057	−0.0010	
$M_{1\omega}$	0.0136	0.0029	0.0159	0.0327	0.0100	0.0388
	0.0030	−0.0026	0.0011	0.0052	0.0002	
	0.0166	0.0003	0.0170	0.0379	0.0102	
$M_{2\omega}$	0.0161	0.0034	0.0188	0.0387	0.0119	0.0517
	0.0040	−0.0047	0.0007	0.0063	−0.0004	
	0.0201	−0.0013	0.0195	0.0450	0.0115	
$M_{3\omega}$	0.0128	0.0027	0.0150	0.0308	0.0095	0.0412
	0.0018	0.0021	0.0028	0.0046	0.0022	
	0.0146	0.0048	0.0178	0.0354	0.0117	
$M_{6\omega}$	−0.0134	−0.0048	−0.0021	0.0060	0.0014	−0.0283
	−0.0038	0.0003	0.0002	−0.0027	0.0005	
	−0.0172	−0.0045	−0.0019	0.0033	0.0019	
$M_{7\omega}$	−0.0221	−0.0079	−0.0035	0.0099	0.0023	−0.0382
	−0.0026	−0.0013	−0.0011	0.0014	0.0009	
	−0.0247	−0.0092	−0.0046	0.0113	0.0032	
$M_{0\rho}$	0.0170	0.0036	0.0198	0.0408	0.0125	0.0441
	0.0015	0.0024	0.0019	0.0035	0.0017	
	0.0185	0.0060	0.0217	0.0443	0.0142	

correlations. The sign of the final matrix element may, nevertheless, be different after summation of all terms. This is seen for some of the results in Table VII.

The calculations of the nuclear structure levels where performed using the OXBASH program [58], which includes options for different model spaces and different effective two-nucleon interactions. In the present paper we used the ZBM model space where ZBM I and ZBM II are the interactions I and II from Zuker, Buck, and McGrory [55]. The ZBM model space contains $1s_{1/2}$ and $1p_{3/2}$ orbits filled and the active (valence) particles were restricted to the $1p_{1/2}$, $2s_{1/2}$, and $1d_{5/2}$ orbits. The single-particle energies were fitted as in Ref. [54], and the two-body matrix elements (TBME) were taken to be the Kuo and Brown G-matrix elements [39,40]. This approach is analogous to the approaches developed in Refs. [59,60], however, the valence space is smaller here.

V. DISCUSSION OF THE RESULTS

The comparison of the calculated nuclear structure parts ($M_{k,s}$) within different model spaces and different effective interactions (including the results of Ref. [25]) with the predictions of the PNC single-particle model (column labeled ''valence'' in Table VII) shows that the core contribution is suppressed by a factor 1.2–4 for the isovector part, when one excludes the results given within the ZBMII model.[1] This suppresion is due to two facts, one is that the $\frac{1}{2}^+$ states and $\frac{1}{2}^-$ states are not described by pure configurations with a neutron in $2s_{1/2}$ and $1p_{1/2}$ orbits respectively. The other fact is a pairing effect contained in the effective interaction. Due to pairing, the dominant PNC contribution for the transition $2s_{1/2}-1p_{1/2}$ is cancelled in the range of $\simeq 20-30\,\%$ by the similar, but time reversed, transition $1\overline{p_{1/2}}-2\overline{s_{1/2}}$.

For the isoscalar contribution, we have some similarities, but the pairing effect is much more pronounced, since the contribution of the second transition, $1\overline{p_{1/2}}-2\overline{s_{1/2}}$, becomes comparable to the first one, and even larger in some cases. The overall result, as seen in Table VII, is almost a complete cancellation (REWIL) or even a change in sign in other cases (ZWM,ZWMO). The relative weight of the isoscalar and isovector contributions can be seen more easily in the different models in Table VII, since those on the left favor a neutron transition, while those on the right favor a proton transition. The isovector contribution is relatively stable and varies by a factor of 1.5 at most if one excludes the ZBMII results. There is a possibility of a total absence of the isoscalar contribution (REWIL).

Analyzing the contribution of the valence nucleons $(1p_{1/2},1d_{5/2},2s_{1/2})$ also proves instructive. Due to the fact that core nucleons generally contribute coherently to the single-particle PNC interaction, one might *a priori* expect that they would increase the core contribution. Looking at Table VII shows that this is true in many cases, for the relative state transition $^3S_1-{}^3P_1$ (dominant in $M_{0\pi}$, $M_{0\rho}$, $M_{3\rho}$, and $M_{3\omega}$) as well as for the transition $^3S_1-{}^1P_1$ (dominant in $M_{4\rho}$, $M_{5\rho}$, $M_{6\omega}$, and $M_{7\omega}$, after appropriately separating in this case the contributions arising from the transition $^1S_0-{}^3P_0$). This does not apply for the isovector $^1S_0-{}^3P_0$ transition (dominant in $M_{1\rho}$, $M_{1\omega}$, and $M_{2\omega}$), whose contribution is quite small (ZWMO) or even destructive (ZBMII). For the isoscalar $^1S_0-{}^3P_0$ transition, the situation is much more complex (decrease for ZWM and increase for ZBMII for absolute values reminiscent of the $^3S_1-{}^3P_1$). Clearly, the results are very sensitive to strong interactions in $^1S_0$ and $^3S_1$ states, whose relative strength in nuclei is not well determined (see discussions in Ref. [61], and some other references therein). With this significant uncertainty in the quantitative contributions of the S waves, we are not impelled to retain the contributions from partial waves higher than P waves at the present time.

As for the core contribution, the dependence of the behavior of the results on the transition can be traced back to specific ''pairing'' effects and to a more or less destructive interference of the contributions of the single-particle transitions $2s_{1/2}$–$1p_{1/2}$ and the time reversed one $1\overline{p_{1/2}}$–$2\overline{s_{1/2}}$.

Considering various strong and weak interaction models the average value of PNC matrix element is

$$M_{\rm PNC}=\langle 2^+,T=0(13.02\ \ \mathrm{MeV})|H_{\rm PNC}|2^-(12.9686\ \ \mathrm{MeV})\rangle \approx 0.36\ \ \mathrm{eV}. \qquad (48)$$

This can be considered the main result of our current effort. This leads to our prediction of 1.85×10^{-3} for the circular polarization of the 4.15 MeV γ ray $(2^+T=0;\ E_x=13.02\ \ \mathrm{MeV}\rightarrow 2^-T=0;\ E_x=8.87\ \ \mathrm{MeV})$ in ^{16}O*.

VI. CONCLUSION

In the present work an effective Hamiltonian approach for the PNC weak interaction has been investigated. It has been shown that the circular polarization or γ asymmetry of the γ ray emitted in the following transition $(2^+T=0;\ E_x=13.02\ \ \mathrm{MeV}\rightarrow 2^-T=0;\ E_x=8.87\ \ \mathrm{MeV})$ in ^{16}O*, can provide a measurable and sensitive test to determine the PNC matrix element ($M_{\rm PNC}$), connecting the parity mixed doublet $(2^-,T=1;\ 2^+,T=0)$ in ^{16}O. This $M_{\rm PNC}$ has been calculated within the OXBASH shell model code, using five strong (PC) and four weak (PNC) interaction models. A theoretical value of 0.36 eV has been obtained as an average to predict the above mentioned experimentally relevant observables.

This new ^{16}O case enlarges the number of two-level systems sensitive to the isovector parity nonconservation. It is accessible experimentally with four independent polarization observables. Two of them are the longitudinal and irregular transverse analyzing powers of the $^{15}\mathrm{N}(\vec{p},\alpha_0)^{12}\mathrm{C}$ resonance reaction (see Refs. [24,25] if we make the pair of independent experiments), and another two are the circular polarization and γ-asymmetry of the γ-ray emitted in the transition: $2^+T=0;\ E_x=13.02\ \ \mathrm{MeV}\rightarrow 2^-T=0;\ E_x=8.87\ \ \mathrm{MeV}$ in ^{16}O*. The $M_{\rm PNC}$ is sensitive to the $\Delta T=1$ components of the weak interaction Hamiltonian responsible for parity nonconservation ($\mathrm{H_{PNC}}$) and especially to the part described by the weak pion exchange, which itself is related to the neutral current contributions. Current experiments provide only an upper limit for the weak pion constant. New experiments are necessary in order to fix this weak pion constant and therefore determine the neutral current contributions in the $\mathrm{H_{PNC}}$.

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[1]As a matter of fact, in Ref. [25] the third 2^+ eigenvalue given by the OXBASH code (see Fig. 2 of Ref. [25]) has been adopted for comparison with the experimental 2^+ state at 13.02 MeV. Here, as well as in Ref. [24], we have adopted the fourth 2^+ eigenvalue given by the OXBASH code.

[1] S.L. Glashow, G. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).
[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[3] N. Kaiser and U.G. Meissner, Nucl. Phys. **A489**, 671 (1988); **A499**, 699 (1989); **A510**, 759 (1990); Mod. Phys. Lett. A **5**, 1703 (1990).
[4] V.M. Dubovik and S.V. Zenkin, Ann. Phys. (N.Y.) **172**, 100 (1986); V.M. Dubovik, S.V. Zenkin, I.T. Obluchovskii, and L.A. Tosunyan, Fiz. Elem. Chastitz At. Yadra **18**, 575 (1987) [Sov. J. Part. Nucl. **18**, 244 (1987)].
[5] E.M. Henley, W-Y.P. Hwang, and L.S. Kisslinger, Phys. Lett. B **367**, 21 (1996).
[6] E.M. Henley and J. Pasupathy, Nucl. Phys. **A556**, 467 (1993); E.M. Henley, W-Y.P. Hwang, and L.S. Kisslinger, Phys. Rev. D **46**, 431 (1992); T. Hatsuda *et al.*, Phys. Rev. C **49**, 452 (1993); L.J. Reinders, H. Rubinstein, and S. Yazaki, Nucl. Phys. **B213**, 109 (1983).
[7] B. Desplanques, Phys. Rep. **297**, 1 (1998).
[8] B. Desplanques, J.F. Donoghue, and B.R. Holstein, Ann. Phys. (N.Y.) **124**, 449 (1980).
[9] S. Kistryn, J. Lang, J. Liechti, Th. Maier, R. Mueller, F. Nessi-Tedaldi, M. Simoniu, J. Smyrski, S. Jaccard, W. Haerbeli, and J. Sromicki, Phys. Rev. Lett. **58**, 1616 (1987).
[10] V.J. Zeps, Ph.D. thesis, University of Washington, 1989; V.J. Zeps, E.G. Adelberger, A. Garcia, C.A. Gossett, H.E. Swanson, W. Haeberli, P.A. Quin, and J. Sromicki, in Intersections Between Particles and Nuclear Physics, Rockport, Maine, 1988, AIP Conf. Proc. No. 176 (AIP, New York, 1989); H.E. Swanson, V.J. Zeps, E.G. Adelberger, C.A. Gossett, J. Sromicki, W. Haerberli, and P. Quin, Heidelberg Conference Proceedings No. 648 (unpublished).
[11] C.M. Frankle, J.D. Bowman, J.E. Bush, P.P.J. Delheij, C.R. Gould, D.G. Haase, J.N. Knudson, G.E. Mitchell, S. Pentila, H. Postma, N.R. Robertson, S.J. Seestrom, J.J. Szymanski, S.H. Yoo, V.W. Yuan, and X. Zhu, Phys. Rev. C **46**, 778 (1992).
[12] K. Neubeck, H. Schober, and H. Waeffler, Phys. Rev. C **10**, 320 (1974).
[13] F. Carstoiu, O. Dumitrescu, G. Stratan, and M. Braic, Nucl. Phys. **A441**, 221 (1985).
[14] V.M. Lobashev, V.A. Nazarenko, L.F. Saenko, L.F. Smotritzkii, and O.I. Kharkevitch, JETP Lett. **5**, 59 (1967); Phys. Lett. **25**, 104 (1967).
[15] O. Dumitrescu, M. Gari, H. Kuemmel, and J.G. Zabolitzky, Phys. Lett. **35B**, 19 (1971).
[16] M. Gari, Phys. Rep., Phys. Lett. **6C**, 317 (1973).
[17] C.A. Barnes, M.M. Lowry, J.H. Davidson, R.E. Mars, F.B. Morinigo, B. Chang, E.G. Adelberger, and H.E. Swanson, Phys. Rev. Lett. **40**, 840 (1979).
[18] H.B. Mak *et al.*, Reports on Research in Nuclear Physics at Queens University, Kingston, Ontario, 1981 (unpublished), p. 19.
[19] M. Bini, T.F. Fazzini, G. Poggi, and N. Taccetti, Phys. Rev. C **38**, 1195 (1988); Phys. Rev. Lett. **55**, 795 (1985).
[20] P.G. Bizzeti, T.F. Fazzini, P.R. Maurenzig, A. Perego, G. Poggi, P. Sona, and N. Taccetti, Lett. Nuovo Cimento Soc. Ital. Fis. **29**, 167 (1980); P.R. Maurenzig, M. Bini, P.G. Bizzeti, T.F. Fazzini, A. Perego, G. Poggi, P. Sona, and N. Taccetti, in Proceedings of the 1979 International Conference on Neutrinos, Weak Interactions and Cosmology, Bergen, 1979, edited by A. Haatuft and C. Jarlskog (unpublished), p. 97; M. Bini, P.G. Bizzeti, and P. Sona, Phys. Rev. C **23**, 1265 (1981); Lett. Nuovo Cimento Soc. Ital. Fis. **41**, 191 (1984).
[21] E.G. Adelberger, M.M. Hindi, C.D. Hoyle, H.E. Swanson, R.D. Von Lintig, and W.C. Haxton, Phys. Rev. C **27**, 2833 (1983).
[22] K. Elsener, W. Gruebler, V. Koenig, C. Schweitzer, P.A. Schmeltzbach, J. Ulbricht, F. Sperisen, and M. Merdzan, Phys. Lett. **117B**, 167 (1982); Phys. Rev. Lett. **52**, 1476 (1984).
[23] E.G. Adelberger and W.C. Haxton, Annu. Rev. Nucl. Part. Sci. **35**, 501 (1985).
[24] O. Dumitrescu, Nucl. Phys. **A535**, 94 (1991).
[25] N. Kniest, M. Horoi, O. Dumitrescu, and G. Clausnitzer, Phys. Rev. C **44**, 491 (1991).
[26] O. Dumitrescu and G. Stratan, Nuovo Cimento A **105**, 901 (1991).
[27] O. Dumitrescu and G. Clausnitzer, Nucl. Phys. **A552**, 306 (1993).
[28] M. Horoi, Phys. Rev. C **50**, 2392 (1994).
[29] M. Horoi and G. Clausnitzer, in *Proceedings of the 4th International Seminar on Nuclear Structure, Amalfi, 1992*, edited by A. Covello and A. Gargano (World Scientific, Singapore, 1993), p. 143.
[30] P.G. Bizzeti, Riv. Nuovo Cimento **6**, 1 (1983).
[31] O. Dumitrescu, ICTP Trieste Report No. IC/95/139, 1995.
[32] V.E. Bunakov, Fiz. Elem. Chastitz At. Yadra **26**, 285 (1995) [Phys. Part. Nuclei **26**, 115 (1995)].
[33] B. Desplanques and O. Dumitrescu, Nucl. Phys. **A565**, 818 (1993).
[34] W.C. Haxton, B.F. Gibson, and E.M. Henley, Phys. Rev. Lett. **45**, 1677 (1980).
[35] B.A. Brown, W.A. Richter, and N.S. Godwin, Phys. Rev. Lett. **45**, 1681 (1980).
[36] B. Desplanques in *Proceedings of the VIII International Workshop on Weak Interactions and Neutrinos, Javea, 1982*, edited by A. Morales (World Scientific, Singapore, 1983), p. 515; J. Phys. C **3**, 55 (1984).
[37] G. Ahrens, W. Harfst, J.R. Kass, E.V. Mason, H. Schober, G. Stefens, and H. Waeffler, Nucl. Phys. **A390**, 486 (1982).
[38] J.R. Spence and J.P. Vary, Phys. Rev. C **52**, 1668 (1995); **59**, 1762 (1999).
[39] T.T.S. Kuo and G.E. Brown, Nucl. Phys. **85**, 40 (1966).
[40] T.T.S. Kuo, Annu. Rev. Nucl. Part. Sci. **24**, 101 (1974); Nucl. Phys. **A103**, 71 (1967).
[41] B.H. Brandow, Rev. Mod. Phys. **39**, 771 (1967).
[42] B.R. Barrett and M.W. Kirson, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1973), Vol. 7, p. 219.
[43] M.W. Kirson, in *Nuclear Shell Models*, edited by M. Vallieres and B.H. Wildenthal (World Scientific, Singapore, 1985), p. 290.
[44] M.B. Johnson and M. Baranger, Ann. Phys. (N.Y.) **62**, 172 (1971).
[45] C. Bloch and J. Horowitz, Nucl. Phys. **8**, 1023 (1958).
[46] T. Morita, Prog. Theor. Phys. **29**, 351 (1963).
[47] H. Kuemmel, Nucl. Phys. **A176**, 205 (1971); H. Kuemmel, K.H. Lurhmann, and J.G. Zabolitzky, Phys. Rep. **36**, 1 (1978).

[48] A. Kallio and B.D. Day, Phys. Lett. **25B**, 72 (1967); Nucl. Phys. **A124**, 177 (1968).
[49] G.A. Miller and J.E. Spencer, Ann. Phys. (N.Y.) **100**, 562 (1976).
[50] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975).
[51] R.J. Blin-Stoyle, *Fundamental Interactions and the Nucleus* (North-Holland, Amsterdam, 1973).
[52] D.R. Tilley, H.R. Weller, and C.M. Cheves, Nucl. Phys. **A564**, 1 (1993); F. Ajzenberg-Selove, *ibid.* **A460**, 1 (1986); **A281**, 1 (1977).
[53] M. Gari, H. Kuemmel, and J.G. Zabolitzky, Nucl. Phys. **A161**, 625 (1971); **A155**, 256 (1970).
[54] B.S. Reehal and B.H. Wildenthal, Part. Nuclei **6**, 137 (1973).
[55] A.P. Zuker, B. Buck, and J.B. McGrory, Phys. Rev. Lett. **21**, 39 (1968).
[56] B. Desplanques and J. Missimer, Nucl. Phys. **A300**, 286 (1978); B. Desplanques, *ibid.* **A242**, 423 (1975).
[57] M. Horoi, G. Clausnitzer, B.A. Brown, and E.K. Warburton, Phys. Rev. C **50**, 775 (1994); M. Horoi, G. Clausnitzer, B.A. Brown, and E.K. Warburton, *NATO-ASI Series B: Physics 334*, edited by W. Scheid and A. Sandulescu (Plenum, New York, 1994); M. Horoi and B.A. Brown, Phys. Rev. Lett. **74**, 231 (1995).
[58] B.A. Brown, A. Etchegoyen, and W.D.M. Rae, MSU-NSCL Report No. 524, 1985 (unpublished); B.A. Brown, W.E. Ormand, J.S. Winfield, L. Zhao, A. Etchegoyen, W.M. Rae, N.S. Godwin, W.A. Richter, and C.D. Zimmerman, MSU-NSCL Report No. 524, 1988 (unpublished); B.A. Brown and B.H. Wildenthal, Annu. Rev. Nucl. Part. Sci. **38**, 29 (1988).
[59] D.C. Zheng, B.R. Barrett, J.P. Vary, and R.J. McCarthy, Phys. Rev. C **49**, 1999 (1994).
[60] D.C. Zheng, J.P. Vary, and B.R. Barrett, Phys. Rev. C **50**, 2841 (1994).
[61] B. Desplanques *et al.*, Z. Phys. C **51**, 499 (1991); J. Bernabeu *et al.*, *ibid.* **46**, 323 (1990).