Meson exchange and nucleon polarizabilities in the quark model

F. Cano[1] and M. Traini[1,2]

[1]*Dipartimento di Fisica, Università degli Studi di Trento, I-38050 Povo, Italy*
[2]*Istituto Nazionale di Fisica Nucleare, G.C. Trento, I-38050 Povo, Italy*
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Modifications to the nucleon electric polarizability induced by pion and sigma exchange in the q-q potentials are studied by means of sum rule techniques within a nonrelativistic quark model. Contributions from meson exchange interactions are found to be small and in general reduce the quark core polarizability for a number of hybrid and one-boson-exchange q-q models. These results can be explained by the constraints that the baryonic spectrum impose on the short-range behavior of the mesonic interactions.

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I. INTRODUCTION

Electric (α) and magnetic (β) polarizabilities are fundamental observables characterizing the response of the nucleon to an external (quasi-)static electromagnetic field. In particular α controls the deformation induced by the electric field and can be experimentally determined for the proton by measuring the Compton polarizability $\bar{\alpha}$ [1]. In a nonrelativistic approach $\bar{\alpha}$ differs from α by a so-called retardation term $\Delta\alpha$:

$$\bar{\alpha} = \alpha + \Delta\alpha, \tag{1}$$

where

$$\Delta\alpha = \frac{e}{3M}\langle 0| \sum_{i=1}^{3} e_i(\vec{r}_i - \vec{R})^2 |0\rangle = \frac{e^2\langle r^2 \rangle_{ch}}{3M}, \tag{2}$$

and the (static) polarizability, obtained from the low-energy theorems [2], is

$$\alpha = 2\sum_{n\neq 0} \frac{|\langle n|D_z|0\rangle|^2}{E_n - E_0}. \tag{3}$$

In the previous expressions $|0\rangle$ is the ground state (nucleon) and $|n\rangle$ are the excited states allowed by the dipole operator

$$D_z = \sum_{i=1}^{3} e_i(\vec{r}_i - \vec{R})_z. \tag{4}$$

E_n and $E_0 \equiv M$ denote the masses of these excited states and the proton, respectively, e_i and $\vec{r}_i$ stand for the charge and position of the (constituent) quark, and $\vec{R}$ is the center-of-mass coordinate.

A recent analysis of Compton scattering experiments [3] yields

$$\bar{\alpha}^p = \alpha^p + \Delta\alpha^p = (12.1 \pm 0.8 \pm 0.5)10^{-4} \text{ fm}^3. \tag{5}$$

The static polarizability of the proton (α^p) is obtained by substracting the retardation contribution in Eq. (5), while for the neutron one has access to α^n directly [4] by means of a neutron-nucleus scattering (specifically n-Pb [5]) and the final results read [1,5]:

$$\alpha^p = (7.0 \pm 2.2 \pm 1.3)10^{-4} \text{ fm}^3, \tag{6}$$

$$\alpha^n = (12.0 \pm 1.5 \pm 2.0)10^{-4} \text{ fm}^3.$$

These observables have been calculated in a wide variety of hadronic models, including bag and soliton models, chiral perturbation theory, and dispersion relation methods (see [6] for a review). The nonrelativistic constituent quark model seems to present a serious and peculiar problem [7–10]; it cannot reproduce the spectrum and the experimental values (6) simultaneously. This can be straightforwardly illustrated within the harmonic-oscillator model [11]. In this case the sum over excited states can be performed analytically and one gets

$$\alpha^p = \alpha^n = \frac{2}{9}e^2 M\langle r_p^2 \rangle_{ch}^2 = 2\frac{e^2}{M}\frac{1}{(\hbar\omega)^2}. \tag{7}$$

Equation (7) largely underestimates the proton polarizability under the requirement of a small charge radius as suggested by spectroscopy ($\hbar\omega \approx 600$ MeV). The same conclusion holds within a large class of more realistic potentials [12]. Moreover, the (approximate) charge symmetry exhibited by nonrelativistic quark models makes it difficult to explain the experimental difference $\alpha^n - \alpha^p$. It is intuitive to argue that the meson cloud surrounding the core of quarks would substantially contribute to the total polarizability of the nucleon [6,7]. As a matter of fact, it has been shown [13] that in chiral quark models a large fraction of α comes from the coupling of the photon to the charged pion fields, i.e., from the cloud. Though the mesonic cloud seems to have little effect on the baryon mass calculation, it is essential to understand many electromagnetic properties of the baryons, the most immediate example being the charge radius of the proton.

Recently some quark models have emerged that consider mesonic exchanges in the q-q potential [14–16], as a phenomenological way to include chiral symmetry breaking. In addition a large debate rose on the need to invoke Goldstone boson exchange in connection with the notion of constituent

quark [17], or the possibility of reproducing the observed spectrum within the simple effective (one) gluon exchange (OGE) model [18].

The consideration of the meson exchange in the potential relates two different ''worlds'' of degrees of freedom: the low-energy nuclear physics degrees of freedom (mesons and baryons) and the fundamental description in terms of quarks and gluons. At the moment there is no clear evidence of a smooth transition between these two ''worlds'' [19] and a study of the effects induced by the q-q meson interaction can shed some light on the problem.

In particular, one can raise the question: to which extent can these mesonic degrees of freedom contribute to explain the electromagnetic properties of the baryon, i.e., relegate the missing mesonic cloud to play a marginal role. The answer to this question would be of some interest to understand if OBE-based quark models are in a better shape than OGE-based (or hybrid) models to describe the electromagnetic structure of baryons. The electric polarizability is a peculiar observable sensitive, in principle, to both quark and meson degrees of freedom and its study by means of sum rules can represent a useful tool to elucidate the comparative advantages and disadvantages of OBE models and OGE (or hybrid) models. Furthermore, the sum-rule method is sensitive to very basic properties of the models, averaging many fine details of the interaction.

II. THE THEORETICAL FRAMEWORK

Our starting point to calculate the electric static polarizability, including the effects of the meson and gluon q-q interactions, has to be simple enough to have a direct relation with the potential model and, in addition, independent of the space of states introduced by Eq. (3). In the harmonic-oscillator case the sum over excited states can be directly evaluated but in more realistic scenarios one has to resort to other techniques such as variational methods [7] or sum rules [10].[1] We make use of a sum rule technique which, washing out all the complications of the baryonic spectrum, requires the knowledge of the nucleon wave function only, and constrains the numerical result to satisfy the stringent inequality [12]:

$$2\frac{m_0^2(D_z)}{m_1(D_z)} \leq \alpha \leq 2\frac{m_0(D_z)}{E_{10}}, \tag{8}$$

where E_{10} is the energy gap between the nucleon and the first electric dipole excitation, $D_{13}(1520)$, and the moments (sum rules) of the dipole operator read

$$m_0(D_z) = \langle 0|D_z D_z|0\rangle, \tag{9}$$

$$m_1(D_z) = \frac{1}{2}\langle 0|[D_z,[H,D_z]]|0\rangle. \tag{10}$$

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[1] One could also note that variational and sum rule approaches are intimately connected [12,20].

The potential model enters in two basic ways: (i) explicitly in the Hamiltonian, Eq. (10); (ii) implicitly in the ground-state wave function $|0\rangle$ ($H|0\rangle = E_0|0\rangle$).

The general structure of the potential models we consider can be written

$$V_{qq} = V_{\text{Conf}} + V_{\text{OGE}} + V_{\text{OPE}} + V_{\text{OSE}}, \tag{11}$$

where V_{Conf} defines the confining potential, V_{OGE} embodies the one-gluon exchange interaction, and V_{OPE} (V_{OSE}) is the one-pion (one-sigma) exchange term. The general structure of these interactions is

$$V_{\text{OGE}}(r_{12}) = [V_{\text{OGE}}^C(r_{12}) + V_{\text{OGE}}^S(r_{12})\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_{\text{OGE}}^T(r_{12})\hat{S}_{12}], \tag{12}$$

$$V_{\text{OPE}}(r_{12}) = [V_{\text{OPE}}^S(r_{12})\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_{\text{OPE}}^T(r_{12})\hat{S}_{12}]\vec{\tau}_i\cdot\vec{\tau}_j, \tag{13}$$

$$V_{\text{OSE}}(r_{12}) = V_{\text{OSE}}^C(r_{12}), \tag{14}$$

with $\hat{S}_{ij} = 3(\vec{\sigma}_i\cdot\hat{r}_{ij})(\vec{\sigma}_j\cdot\hat{r}_{ij}) - (\vec{\sigma}_i\cdot\vec{\sigma}_j)$; r_{ij} is the interquark distance and $\vec{\sigma}_i(\vec{\tau}_i)$ are the spin (isospin) operators of the ith quark.

In order to illustrate the relevance of the meson exchange effects, let us first calculate the sum rules $m_1(D_z)$ and $m_0(D_z)$ neglecting, in the Hamiltonian, the contribution due to V_{OPE} and V_{OSE}, Eqs. (13),(14). One obtains

$$m_0(D_z) = \frac{1}{3}e^2\langle r_p^2\rangle_{ch} + \frac{2}{3}e^2\langle r_n^2\rangle_{ch}, \tag{15}$$

$$m_1(D_z) = \frac{e^2}{3m_q} \tag{16}$$

for both proton and neutron. The nonvanishing value of $m_1(D_z)$ in Eq. (16) is due to the conmutator of the kinetic energy and the dipole operator and no additional contribution comes from the confining and OGE potentials which commute with the dipole operator (4).

The reference model that we will use throughout the paper will be the one of Valcarce *et al.* [14] that contains all the terms (12)–(14). The constituent quark mass is taken to be $m_q = M_N/3 = 313$ MeV, and the resulting charge radii of the nucleon in this model are $\langle r_p^2\rangle_{ch} = 0.252$ fm^2 and $\langle r_n^2\rangle_{ch} = -2.58\times10^{-2}$ fm^2, so that one obtains, from Eqs. (15),(16),

$$3.10\times10^{-4} \text{ fm}^3 \leq \alpha^{p,n} \leq 3.76\times10^{-4} \text{ fm}^3, \tag{17}$$

a result which can be regarded as the typical outcome of a constituent quark model including the OGE potential, since the m_q and $\langle r^2\rangle$ used above are quite common in a large class of models that reproduce the basic features of the hadronic spectrum (see [12] for an extensive study of polarizabilities in a number of quark models).

The Isgur-Karl (IK) model [21] deserves a specific comment since the inclusion of an unknown potential term to remove the harmonic-oscillator degeneracy prevents explicit calculations of the sum rules. Nevertheless, since the un-

known potential has a central character and commutes with the dipole operator, it preserves the simple form (16) for the energy-weighted sum rule and the final numerical results are still compatible with those quoted above. A more refined approach to α in the case of the IK model, as developed within a variational framework [10], also produces results which are consistent with the constraints (17) as long as the bounds (8) are considered.

In the following we will investigate boson exchange corrections to the simple result (17) as generated by the inclusion of pion and sigma exchanges and to this end it is convenient to discuss lower and upper bounds separately. In fact they involve different dynamical effects and ingredients.

III. LOWER BOUND: DIPOLE SUM RULE

For the sake of clarity let us begin the investigation of the lower bound discussing the analogous nuclear limit, which somehow motivates our interest in the mesonic exchanges in the interaction.

When photon-nucleus interactions are considered, it is well known (see, e.g., Ref. [22]) that the first moment of the the dipole strength can be written

$$m_1(D_z) = e^2 \frac{NZ}{2MA}(1+K), \qquad (18)$$

where N and Z represent the number of neutrons and protons, respectively, and $A = N + Z$. The first term in Eq. (18) arises from the kinetic energy conmutator and K is an enhancement factor due to the (isospin-dependent) N-N interaction arising from boson exchange. Considering an OPE potential model the values for K range from 0.5 to 1, depending on the importance of the tensor correlations [23].

The nuclear example makes clear the modifications to $m_1(D_z)$ one should expect in the case of nucleon. An interesting feature, indeed, distinguishes the nucleon from the nuclear wave function: due to the color degrees of freedom, the spin-isospin wave function for the (dominant) SU(6) symmetric component of the three-quark system must be symmetric, whereas it results antisymmetric in the nuclear case. As a consequence we expect corrections to the energy-weighted sum rule (16) consistent with

$$m_1(D_z) = \frac{e^2}{3m_q}(1+\kappa), \qquad (19)$$

where κ embodies the additional contribution coming from V_{OPE} [cf. Eq. (13)] and constrained by $-1 < \kappa \leqslant 0$ because of the symmetry properties of the quark wave function.

An explicit calculation of this factor κ gives

$$\kappa = -\frac{3}{2}m_q\langle 0|(\vec{\tau}_1 \cdot \vec{\tau}_2 - \tau_1^z \tau_2^z)r_{12}^2[V_{\mathrm{OPE}}^S(r_{12})\vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$+ V_{\mathrm{OPE}}^T(r_{12})\hat{S}_{12}]|0\rangle, \qquad (20)$$

By making use of the q-q potential model proposed by Valcarce *et al.* [14] and its numerical solution [24][2] for the wave function one gets, for both proton and neutron, the surprising *positive* value $\kappa = 0.088$ which produces a *reduction* of the lower bound of (roughly) 10%. In spite of the change in the spin-isospin wave function symmetry and contrary to our expectations κ keeps the sign of the nuclear enhancement factor K.

In order to understand this nonintuitive result it is very instructive to investigate more closely the corresponding moment $m_1(D_z)$ in nuclei. In addition, if we want to compare the results obtained in the two systems, the framework must be as consistent as possible.

Let us consider, therefore, the N-N potential generated by the q-q interaction we used to calculate the lower bound to α, namely the nuclear potential of Ref. [14]: the interaction contains a central part and an isospin-dependent term and it provides a good description of phase shifts and deuteron properties. By assuming a simple Gaussian wave function for a three nucleon system with a harmonic-oscillator parameter $\alpha_{\mathrm{ho}}^2 = 0.5 \; \mathrm{fm}^{-2}$, the enhancement factor in Eq. (18) becomes $K \approx 0.1$. The spin-isospin matrix elements contain the expected opposite sign (with respect to the nucleon case) as already anticipated, but the spatial integral contributes with an additional opposite sign and both κ and K are positive. The underlying reason is that the nucleon and the nuclear wave functions are sensitive to very separate regions of the OPE potential. Let us look at the OPE potential model of Ref. [14] in more detail. It can be written

$$V_{\mathrm{OPE}}^S(r_{ij}) = \frac{1}{3}\alpha_{ch}\frac{\Lambda_{\mathrm{CSB}}^2}{\Lambda_{\mathrm{CSB}}^2 - m_\pi^2}m_\pi$$

$$\times \left[Y(m_\pi r_{ij}) - \frac{\Lambda_{\mathrm{CSB}}^3}{m_\pi^3}Y(\Lambda_{\mathrm{CSB}}r_{ij}) \right], \quad (21)$$

$$V_{\mathrm{OPE}}^T(r_{ij}) = \frac{1}{3}\alpha_{ch}\frac{\Lambda_{\mathrm{CSB}}^2}{\Lambda_{\mathrm{CSB}}^2 - m_\pi^2}m_\pi$$

$$\times \left[H(m_\pi r_{ij}) - \frac{\Lambda_{\mathrm{CSB}}^3}{m_\pi^3}H(\Lambda_{\mathrm{CSB}}r_{ij}) \right], \quad (22)$$

where

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[2]Quite recently the potential model of Ref. [14] has been criticized by Glozman *et al.* [25] who consider the agreement with the empirical spectrum is only apparent and obtained because of the truncation in the hypercentral components. However, it has been argued [26] that the authors of Ref. [25] did not take into account the dependence of the regularization of the OGE interaction on the model space used in the calculation and the agreement can be restored by considering consistent regularizations. The discussion is still ongoing [27], but, as it will become clear in the following, our numerical conclusions are quite independent of the specific debate and the results obtained making use of the criticized potential are quite similar to other potential models.

TABLE I. Comparison between different values of κ, as defined in Eq. (20), predicted by three potential models. Values in parentheses correspond to a Gaussian wave-function approximation.

	Valcarce *et al.* [14]	Dziembowski *et al.* [16]	Glozman *et al.* [25]
κ	0.088	0.164 (0.143)	0.139 (0.138)

$$Y(x) = \frac{e^{-x}}{x}, \quad H(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right)Y(x), \quad (23)$$

and m_π, m_σ, and m_q are the pion, sigma, and quark masses, respectively, α_{Ch} is the chiral coupling constant. The cutoff parameter Λ_{CSB} controls the size of the pion-quark interaction region.

From Eqs. (21) and (22) and taking into account the rather large value for the cutoff Λ_{CSB} (4.2 fm^{-1}), one can check that for distances $\lesssim 1.5$ fm the regularized part of the interaction dominates. The nucleon wave function is mostly concentrated within distances $\lesssim 0.5$ fm and is sensitive to the short-range (or regularized) part, while the nuclear wave function is mainly sensitive to the long-range tail of the OPE. This simple argument justifies the change of sign previously emphasized and we can conclude that, despite of the use of the same terminology, the only common feature of the OPE in nucleons and in nuclei is its spin-isospin structure. The spatial structure of the OPE interaction looks very different moving from nucleons to nuclei: the quarks in the nucleon are sensitive to the short-range part only, which survives in the chiral limit [28] but is poorly known, whereas the nucleons in the nuclei are mostly influenced by the long-range tail.

At this point one might wonder whether our conclusion for κ can be considered as general or it is a particular feature of the employed model. To check this point, and taking advantage of the flexibility of the sum-rule techniques, we have repeated the calculation of κ for other interquark potential models, namely, the hybrid model proposed by Dziembowski, Fabre de la Ripelle, and Miller in Ref. [16] and the version of the OBE model due to Glozman, Papp, Plessas, Varga, and Wagengrunn presented in [25]. For those potentials we used a simple Coulomb-type [$\Psi \propto \exp(-\xi/\beta)$] approximation for the wave functions whose size parameter is fixed to reproduce the proton root-mean-square radius predicted by those potential models. The sum rule value of $m_0(D_z)$ is fixed by the size of the nucleon and therefore is well reproduced also within such approximate procedure. The other ingredient of the lower bound, namely $m_1(D_z)$, is quite insensitive to the details of the wave function behavior once the size of the system is reproduced [23]. For example, for the potential of Valcarce *et al.* the Coulomb approximation of the wave function gives $\kappa = 0.075$ against $\kappa = 0.088$ of the calculation with the full wave function.

Our results for κ are summarized in Table I, where we can see that in spite of the differences in the absolutes values (due essentially to the different strength α_{Ch} and cutoff Λ parameters) the sign of κ is always positive. Therefore, our previous conclusions are quite general and rely on the com-
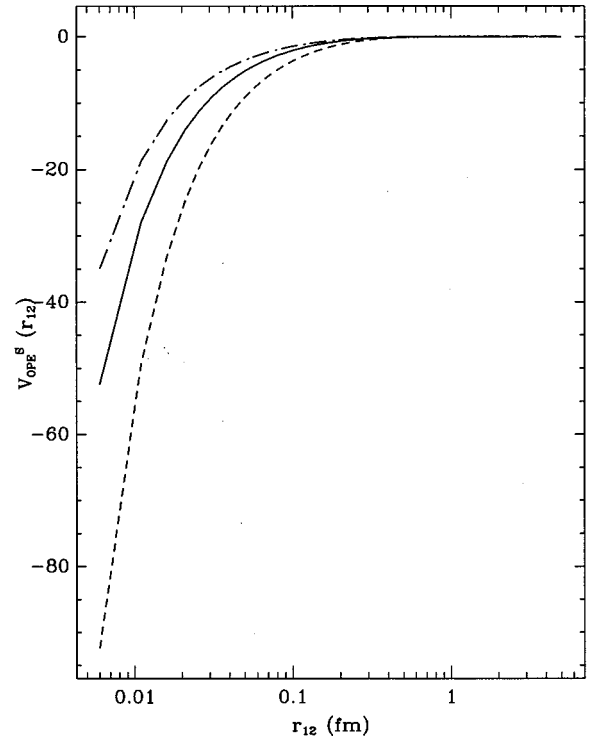

FIG. 1. Behavior of the $V_{OPE}^S(r_{12})$ (in fm^{-1}) for different models (solid line: Valcarce *et al.* [14]; dashed line: Dziembowski *et al.* [16]; dot-dashed line: Glozman *et al.* [25]).

mon short-range behavior of the interquark potentials, as it is confirmed in Fig. 1 where they are compared in the region $r_{12} \lesssim 5$ fm. In fact these similarities are a direct consequences of the rather large values of Λ employed in the models. In conclusion, the introduction of meson exchange contributions lead to a smaller value of the lower bound and their inclusion does not represent any improvement.

IV. UPPER BOUND: TWO-BODY CHARGE DENSITIES

The discussion of the previous section leads to the conclusion that the lower bound to the nucleon polarizability is rather small even including meson exchange effects in the q-q potential. Does such a conclusion mean that the range of the allowed values for α is simply enlarged by the presence of virtual mesons? The question opens the need for a reinvestigation of the upper bound. In fact, it is evident from Eq. (8) that a shift in the allowed values of α is achieved more easily by increasing $m_0(D_z)$ rather than by lowering $m_1(D_z)$. In addition one can note that $m_0(D_z)$ is extremely transparent: it depends on the definition of the dipole operator (and the nucleon ground state) only. The expression for D_z used in the previous section was obtained by assuming a nonrelativistic charge density:

$$\rho_{NR}(\vec{q}) = \sum_{i=1}^{3} e_i e^{i\vec{q}(\vec{r}_i - \vec{R})}, \quad (24)$$

the dipole operator being defined as

TABLE II. Contribution of the OPE two-body charge to the m_0 sum rule (in 10^{-4} fm^2 units) for different models.

		Valcarce *et al.* [14]	Dziembowski *et al.* [16]	Glozman *et al.* [25]
$m_0(D_z^{\mathrm{NR}})$	p,n	4.88	5.15	2.25
$m_0(D_z^{\mathrm{NR}}+D_z^{\mathrm{OPE}})$	p	4.55	5.59	2.19
	n	4.38	4.98	1.94

$$D_z = -i \left. \frac{\partial \rho(\vec{q})}{\partial q_z} \right|_{\vec{q} \to 0}. \tag{25}$$

Corrections to the charge density [29] will translate into modifications to the dipole operator, i.e., to $m_0(D_z)$. However, there are modifications of the charge density which leave the dipole operator unchanged.

As an example let us consider the rather common inclusion of a photon-quark form factor which is supposed to take into account the structure of the constituent quarks:

$$e_q \to e_q(q^2) = \frac{e_q}{1 + \frac{1}{6}\vec{q}^{\,2} r_{\gamma q}^2}. \tag{26}$$

The replacement (26) increases the charge radius [15] of the baryon but has no effect on the dipole operator. The same conclusion holds for any relativistic correction to Eq. (24), such as the Darwin-Foldy term, which does not modify the spherical symmetry of the quark charge distribution.

A trivial solution, sometimes invoked, would be the introduction of *ad hoc* intrinsic polarizabilities of the quark [or equivalently, intrinsic dipole form factors, analogous to Eq. (26)], but the predictive power of the model is lost unless it is extended to the study of other observables (for example, generalized polarizabilities in virtual Compton scattering). A more ambitious approach, beyond the scope of the present work, would be to explain how this intrinsic polarizabilities (or intrinsic form factors) are built from more fundamental physical mechanisms, such as pion loop fluctuations (see, for instance, [30]).

Since we are interested in those mesonic effects directly related to the spectral Hamiltonian we will investigate in detail a source of corrections to the charge density [and consequently to the dipole operator of Eq. (25)] which is directly related to the form of the potentials we are studying: the two-body exchange terms which appear as a consequence of current conservation when isospin or velocity-dependent interactions are included. The q-q potentials considered in the present investigation contain, in fact, terms of this kind: the OPE interaction (13). In addition to the pion exchange current required by current conservation

$$\vec{q} \cdot \vec{J}_{\mathrm{OPE}} = [V_{\mathrm{OPE}}, \rho_{\mathrm{NR}}], \tag{27}$$

a two-body charge density $\rho^{\mathrm{OPE}}(\vec{q}) = \rho_{\pi q \bar{q}}(\vec{q})$ can be associated (see [31] and references therein) and the corresponding modification to the dipole operator considered. Let us emphasize that, in fact, the pion exchange current contributions

(27) have been already included in the calculation of the energy-weighted sum rule $m_1(D_z)$ of Eq. (19). They are embodied in the additional term κ which does not vanish precisely because the conmutator (27) is different from zero.[3] Self-consistency would, therefore, require us to consider also the additional mentioned two-body densities in $m_0(D_z)$. The order by order procedure can stop here (as to the first-order pion terms) and we do not need to reconsider two-body densities contribution to $m_1(D_z)$.

Let us explicitly show the corrections to the dipole operator coming from the pion exchange interaction (21). One has

$$D_z^{\mathrm{OPE}} = -i \left. \frac{\partial \rho^{\mathrm{OPE}}(\vec{q})}{\partial q_z} \right|_{\vec{q} \to 0}$$

$$= -e \frac{\alpha_{Ch}}{m_q} \frac{\Lambda_{\mathrm{CSB}}^2}{\Lambda_{\mathrm{CSB}}^2 - m_\pi^2} \sum_{i<j} \left[\left(\frac{1}{6} \vec{\tau}_i \cdot \vec{\tau}_j + \frac{\tau_{jz}}{2} \right) \sigma_{iz} \cdot \vec{\sigma}_j \hat{r}_{ij} \right.$$

$$\left. + (i \leftrightarrow j) \right] \left(G(m_\pi r_{ij}) - \frac{\Lambda_{\mathrm{CSB}}^2}{m_\pi^2} G(\Lambda_{\mathrm{CSB}} r_{ij}) \right), \tag{28}$$

with

$$G(x) = \left(1 + \frac{1}{x} \right) Y(x). \tag{29}$$

From the numerical point of view the pion contribution in the model of Valcarce *et al.* lowers the non-energy-weighted sum rule from $m_0(D_z) = 4.88 \times 10^{-4}$ fm^2 to $m_0(D_z) = 4.55 \times 10^{-4}$ fm^2 for the proton and $m_0(D_z) = 4.38 \times 10^{-4}$ fm^2 for the neutron [the neutron-proton difference originates not only from the SU(6)-breaking components in the nucleon wave function, but also from the structure of the operator (28) which is sensitive to the total isospin of the system even in the SU(6)-symmetric limit].

The modifications in $m_0(D_z)$ renormalize slightly both upper and lower bounds to the polarizability which becomes

$$2.48 \times 10^{-4} \text{ fm}^3 \leq \alpha^p \leq 3.51 \times 10^{-4} \text{ fm}^3,$$

$$2.29 \times 10^{-4} \text{ fm}^3 \leq \alpha^n \leq 3.38 \times 10^{-4} \text{ fm}^3. \tag{30}$$

A similar trend is observed for other potentials (see Table II). The inclusion of $\rho_{\pi q \bar{q}}$ has a small effect on m_0 and in general it reduces the initial one-body contribution. As a result the upper bounds to α^p with the other two potentials

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[3]In photonuclear physics this is known as the Siegert theorem.

TABLE III. Contributions to m_0 (D_z) (in 10^{-4} fm^2 units) when two-body OPE and OSE terms are considered in the dipole operator for the potential model of Valcarce et al. [14]. Each column shows the contribution of different pieces of the dipole operator according to Eq. (32) with the notation $\langle \mathcal{O} \rangle = \langle 0|\mathcal{O}|0 \rangle$.

	$\langle D_z^{NR} D_z^{NR} \rangle$	$\langle D_z^{NR} D_z^{OPE} + D_z^{OPE} D_z^{NR} \rangle$	$\langle D_z^{NR} D_z^{OSE} + D_z^{OSE} D_z^{NR} \rangle$	$\langle D_z^{OSE} D_z^{OPE} + D_z^{OPE} D_z^{OSE} \rangle$	$\langle D_z^{OPE} D_z^{OPE} \rangle$	$\langle D_z^{OSE} D_z^{OSE} \rangle$
Proton	4.88	−1.16	−0.105	−0.398	0.831	0.620
Neutron	4.88	−1.12	−0.050	−0.398	0.591	0.617

shown in Table II are 4.2×10^{-4} fm^3 for Dziembowsky et al. and 1.49×10^{-4} fm^3 for Glozman et al.

A comparison with Eq. (15) shows that the present results for m_0 are consistent with a shrinking of the size of the nucleon by OPE, an effect already discussed by the authors of Refs. [15,31,32]. Two-body mesonic currents have been proved to be crucial to explain some observables such as $N - \Delta$ electric multipoles and neutron charge radius [31]. However, in these cases the leading one-body contribution is strongly suppressed by symmetry reasons. The argument is not valid in general and cannot be invoked for the electric polarizability (neither for the charge radius of the proton) where these two-body currents are not sufficient to parametrize all the nonvalence degrees of freedom seen by electromagnetic probes. Nevertheless before drawing more definite conclusions on the effects of meson exchange on the nucleon electric polarizability, it is worth mentioning that two-body charge density modifications come also from the other terms of the q-q interaction. Their origin is a little more subtle than the pion contributions and has to do basically with the relativistic corrections to the one-body charge density of Eq. (24). As a result the charge density can be written as a sum of a nonrelativistic contribution plus the ones coming from OPE, OSE, OGE, and the confinement potential; namely,

$$\rho(\vec{q}) = \rho_{NR}(\vec{q}) + \rho_{\pi q\bar{q}}(\vec{q}) + \rho_{\sigma q\bar{q}}(\vec{q}) + \rho_{g q\bar{q}}(\vec{q}) + \rho_{conf}(\vec{q}).$$
(31)

On the same basis the dipole operator acquires the corresponding components:

$$D_z = D_z^{NR} + D_z^{OPE} + D_z^{OSE} + D_z^{OGE} + D_z^{conf}.$$
(32)

Since we want to focus on mesonic contributions, we will not consider here the OGE and confinement component, and the only additional component we will calculate explicitly is the one coming form the OSE potential term, obtaining

$$D_z^{OSE} = -\alpha_{Ch} \frac{1}{2m_q} \left(\frac{m_\sigma}{m_\pi} \right)^2 \frac{\Lambda_{CSB}^2}{\Lambda_{CSB}^2 - m_\sigma^2} \sum_{i<j} \left[\left(\frac{1}{2} e_i r_{iz} m_\sigma \right. \right.$$

$$\left. + (i \leftrightarrow j) \right) \left(G(m_\sigma r_{ij}) - \frac{\Lambda_{CSB}^2}{m_\sigma^2} G(\Lambda_{CSB} r_{ij}) \right)$$

$$\left. + [e_i \hat{r}_{ij} + (i \leftrightarrow j)] \left(Y(m_\sigma r_{ij}) - \frac{\Lambda_{CSB}^3}{m_\sigma^3} Y(\Lambda_{CSB} r_{ij}) \right) \right].$$
(33)

The detailed contributions to $m_0(D_z)$ from the different terms of the dipole operator $D_z = D_z^{NR} + D_z^{OPE} + D_z^{OSE}$ are summarized in Table III. The full result, sum of all the entries in Table III, is $m_0(D_z) = 4.66 \times 10^{-4}$ fm^2 for the proton and $m_0(D_z) = 4.51 \times 10^{-4}$ fm^2 for the neutron, values slightly larger than the corresponding results (30) obtained including OPE only, but still smaller than nonrelativistic impulse approximation obtained considering the one-body contribution D_z^{NR} only. Therefore, the final bounds on the polarizability remain

$$2.60 \times 10^{-4} \text{ fm}^3 \leq \alpha^p \leq 3.59 \times 10^{-4} \text{ fm}^3,$$

$$2.43 \times 10^{-4} \text{ fm}^3 \leq \alpha^n \leq 3.48 \times 10^{-4} \text{ fm}^3. \quad (34)$$

A final comment on the terms neglected in Eq. (32) is in order. Previous analysis of the contribution of ρ_{conf} to the nucleon electric form factors show a quite large sensitivity to the potential model: in [31] the confinement two-body charge produces a reduction of the charge radius of the proton whereas in [32] an increment is found. Furthermore, there is some sensitivity to the details of the employed wave function (see [15] for a comparison). While the ρ_{conf} term is present in both OBE-based and hybrid models, in the latter we can find an additional two-body density $\rho_{g q\bar{q}}(\vec{q})$ that gives a positive contribution to the size of the proton [15,31]. One can estimate that this increment of the square charge radius of the proton, that in our reference model is of 0.063 fm^2, would presumably induce an increment (through m_0) of the allowed values for α^p of, roughly, $\lesssim 1 \times 10^{-4}$ fm^3. Nonetheless, in some models [31] such a large effect on the size of the nucleon is strongly suppressed by the confinement two-body current. Since the emphasis of our work is on meson exchange effects we will not discuss this effect further.

Additional relativistic corrections to the current could certainly be considered [8,29,33], and they would also receive contributions from pion and sigma exchange. In Ref. [8] the effects of relativistic corrections to $\rho_{NR}(\vec{q})$ were calculated [up to order $(1/m^2)$] and found to be small ($\lesssim 5\%$) and also reduced the value of α. On the same grounds, potential-dependent corrections to the static polarizability have been proposed [34] and discussed [35]. However, we believe that the comparison between Eqs. (34) and (17) gives a good idea of the importance of the meson exchange for the nucleon polarizability. In addition the validity of our conclusions are largely independent on the details of q-q interaction model

and rely on their basic features. In particular, the aspects which play a relevant role for the present investigation are the small core size for the nucleon wave function (which basically determines m_0) and the sign and short-range behavior of the mesonic interaction (that enters in m_1), largely constrained by the $N-\Delta$ splitting in the spectrum.

V. SUMMARY

We have shown that the nucleon polarizability is quite a transparent observable to elucidate the role (or better, the lack of role) of meson-exchange effects between quarks in a nonrelativistic approach. The robustness of the employed method, the sum rule technique, resides on the fact that the sum over infinite excitations is avoided and transformed into conmutators that depend on very basic features of the q-q interaction, so that it is possible to compare, in a simple and direct way, a large class of q-q potential models. It has been shown that, unlike in nuclei, meson exchange currents are not relevant to explain experimental evidences.

The comparison between the nuclear and the nucleon case reflects the rather different scales involved in the problem. While the former is sensitive to the long-distance tail of the OPE potential the latter is mostly affected by the regularized short-range part. A different choice of the OPE interaction does not change substantially the behavior of κ in Sec. III. This is basically a consequence of the constraints that hadronic spectroscopy (and in particular the $N-\Delta$ splittings) imposes on the short-range behavior of the pseudoscalar potential. Furthermore, the small size of the nucleon seems to be also a common requirement of the baryonic spectroscopy.

Another type of corrections to the polarizability are due to the two-body charge density generated by meson exchanges (π and σ considered here in more detail). Their contribution tends to reduce the allowed values of α. The same trend is observed in other calculations for the charge radius of the proton [31,32]. These facts are pointing out the limitations of mesonic two-body terms to parametrize the effects of nonvalence degrees of freedom in the nucleon. The gluonic two-body current present in OGE-based and hybrid models might contribute to some extent to increase the allowed values for the polarizability.

The picture of the nucleon as a core plus a meson cloud, that emerges not only from constituent quark models but also from Nambu–Jona-Lasinio and Skyrmion models [36], have no analogy in nuclei. Meson exchange currents, or equivalently the OPE and OSE terms, still leave much room to the cloud in the description of the electromagnetic properties of the nucleon. The inclusion of the internal polarizability of pions and/or more refined treatment of the $|qqq\ q\bar{q}\rangle$ component in the wave function seems to be unavoidable and it should be further investigated. In other words, it means that the efforts in the description of the electromagnetic baryonic transitions should be concentrated on the construction of the effective photon-quark (and/or photon-quark-antiquark, etc.) operator since the hadronic wave functions cannot account for all the physical ingredients required to explain the electromagnetic structure of the baryons. This conclusion is reinforced by some calculations of other electromagnetic observables, for which the use of hadronic wave functions based on OBE potentials [15,37] does not represent much improvement with respect to the calculations based on OGE interactions [38].

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