R-matrix analysis of interference effects in $^{12}C(\alpha,\alpha)^{12}C$ and $^{12}C(\alpha,\gamma)^{16}O$

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The $\alpha+^{12}C$ d phase shift and the $^{12}C(\alpha,\gamma)^{16}O$ $E2$ S factor are analyzed in the *R*-matrix theory. A new method is proposed to link experimental and calculated parameters within the *R*-matrix formalism. We show that interference patterns in the phase shift near 3 MeV are very helpful to constrain the parameter set. Starting from this analysis, we fit the $^{12}C(\alpha,\gamma)^{16}O$ $E2$ S factor, and find low-energy values larger than currently accepted. Different interference patterns are found, but do not appear to affect the astrophysical energies. Consequently, direct measurements at low energies are necessary for astrophysical applications.

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I. INTRODUCTION

The $^{12}C(\alpha,\gamma)^{16}O$ reaction rate is the most important input in many stellar models [1]. It determines the $^{12}C/^{16}O$ ratio after helium burning, and hence the evolution of massive stars. Typically this reaction rate should be known with an uncertainty of no more than 20% to derive reliable conclusions in stellar models.

A general problem in nuclear astrophysics is the smallness of the cross sections at relevant energies. Because of the Coulomb barrier, the repulsion between charged particles makes the cross section usually too low to be measured in laboratories. However, in many cases, especially in proton-induced reactions, the measured cross sections can be extrapolated down to the stellar regime with a sufficient accuracy [2]. Quite recently, measurements within the Gamow energy region have been performed on the $^{3}He(^{3}He,2p)^{4}He$ reaction using underground facilities [3].

The situation is very different in the $^{12}C(\alpha,\gamma)^{16}O$ reaction. It has been known for a long time that the low-energy cross section is determined by the contribution of two weakly bound states (2_1^+ at 6.92 MeV and 1_1^- at 7.12 MeV) located just below the $\alpha+^{12}C$ threshold in ^{16}O. These states are responsible for an enhancement of the S factor at low energies and make the extrapolation of the data rather complicated. The situation is even more tedious since both the $E1$ and $E2$ multipolarities are expected to contribute. The cross section at stellar energies (typically 300 keV) being definitely too small to be measured in laboratories (of the order of 10^{-17} b), the challenge is to find indirect methods to access this stellar regime with a reasonable confidence [4]. It has been argued in the past [5–7] that the $E1$ contribution of the $^{12}C(\alpha,\gamma)^{16}O$ cross section could be fairly well constrained by the ^{16}N β-delayed α spectrum. Measurements have been performed [8,9], and turned out to reduce uncertainties in the $E1$ S factor [9]. Although some ambiguities still exist [10], the $E1$ component is now believed to be determined within 30%.

The current understanding of the $E2$ contribution is, however, poorly known. In spite of the numerous investigations performed so far (see references in Ref. [11]), the $E2$ term is expected to be the main uncertainty in the $^{12}C(\alpha,\gamma)^{16}O$ S factor [12]. A new indirect method has been recently suggested by Chow *et al.* [13] from a measurement of the β delayed proton spectrum of ^{17}Ne. Observations of the subsequent breakup of ^{16}O into α and ^{12}C particles might provide in a near future some useful information on the $E2$ component.

Our aim in the present work is to investigate some qualitative aspects of the $\alpha+^{12}C$ phase shifts and of the $^{12}C(\alpha,\gamma)^{16}O$ $E2$ S factor. The elastic phase shifts are experimentally known in a fairly wide energy range [14,15]. Although these data have already been used by several authors [5,6,16,12] to determine some parameters involved in the capture cross section, we want here to go into more details, and to pay attention to an energy region which could be more useful for the parameter determination. In the second part of the paper, we focus on the $^{12}C(\alpha,\gamma)^{16}O$ $E2$ S factor and especially on interference effects which could affect the high-energy region ($E \geqslant 2.5$ MeV).

The method used here is the *R*-matrix theory [17] which has been widely adopted by many authors for the $^{12}C(\alpha,\gamma)^{16}O$ reaction as well as for several other reactions (see references in Ref. [12]). In this method, the physics of the problem is determined by the properties (energy, α and γ widths) of some poles. The drawback is that, although the poles are related to physical resonances or bound states, their properties are not directly linked to experimental data. This problem is responsible for the difference between ''formal'' or ''calculated'' properties, which enter the *R*-matrix analysis, and the ''observed'' properties, which correspond to the experimental data [5]. When a single pole is included in the model, the link between ''calculated'' and ''observed'' values is rather straightforward. However, the problem becomes much more difficult when one has to deal with several poles [5,18], as in the $^{12}C(\alpha,\gamma)^{16}O$ reaction. The difficulty is to constrain the parameter set of ''calculated'' inputs with ''observed'' experimental data. In this paper, we present a new method allowing to derive ''calculated'' parameters from ''observed'' parameters in a simple way. This technique will

be used for the analysis of the $\alpha+{}^{12}$C phase shift and of the ${}^{12}C(\alpha,\gamma)^{16}O$ cross section.

II. *R*-MATRIX FORMALISM

A. Elastic scattering

In the *R*-matrix approach [17,19], the configuration space is assumed to be divided into two regions. In the internal region (of radius a), the physics of the problem is described by a set of orthogonal wave functions $\chi_\lambda^{JM\pi}$, yielding for the radial part

$$\psi_{\rm int}^{JM\pi}(E,r)=\sum_\lambda f_\lambda^{J\pi}(E)\chi_\lambda^{JM\pi}(r) \quad \text{for} \quad r\leqslant a, \tag{1}$$

where r is the relative coordinate between the colliding nuclei, and the $f_\lambda^{J\pi}(E)$ coefficients are to be determined from the Schrödinger equation. Here, we restrict ourselves to reactions involving zero-spin nuclei, and to single-channel systems. In the external region, the nuclear part of the interaction is, by definition, negligible and the wave function normalized to the unit flux condition reads

$$\psi_{\rm ext}^{JM\pi}(E,r)=[I_J(kr)-U^{J\pi}(E)O_J(kr)]/krv^{1/2} \quad \text{for} \quad r\geqslant a, \tag{2}$$

where $U^{J\pi}$ is the collision matrix, I_J and O_J the Coulomb ingoing and outgoing functions, respectively, defined from the Coulomb regular and irregular functions F_J and G_J, k is the wave number, and v the relative velocity.

Since the kinetic energy is not hermitian over a finite part of the configuration space, the Bloch operator $\mathcal{L}(L)$ is introduced in the Schrödinger equation, yielding

$$[H-E+\mathcal{L}(L)]\psi_{\rm int}^{JM\pi}(E,r)=\mathcal{L}(L)\psi_{\rm ext}^{JM\pi}(E,r), \tag{3}$$

where

$$\mathcal{L}(L)=\frac{\hbar^2}{2\mu a}\delta(r-a)\left(\frac{d}{dr}-\frac{L}{r}\right)r \tag{4}$$

is a surface operator, μ being the reduced mass. The Bloch operator ensures the hermiticity of $H+\mathcal{L}(L)$ over the internal region. Constant L does not depend on r. It can be taken as (see, for example, Ref. [20])

$$\begin{aligned} L&=ka\frac{O_J'(ka)}{O_J(ka)}=S_J(ka)+iP_J(ka) \quad \text{for } E\geqslant 0\\ &=2ka\frac{W'_{-\eta,J+1/2}(2ka)}{W_{-\eta,J+1/2}(2ka)}=S_J(ka) \quad \text{for} \quad E\leqslant 0, \end{aligned} \tag{5}$$

where η is the Sommerfeld parameter and W the Whittaker function.

Solving Eq. (3) with Eqs. (1) and (2) provides the collision matrix

$$U^{J\pi}=\frac{I_J(ka)}{O_J(ka)}\frac{1-L^\star R^{J\pi}}{1-LR^{J\pi}}=\exp(2i\delta^{J\pi}), \tag{6}$$

where we have introduced the phase shift $\delta^{J\pi}$ and the R matrix defined as

$$R^{J\pi}(E)=\sum_\lambda\frac{\tilde\gamma_\lambda^2}{E_\lambda-E}. \tag{7}$$

Here and in the following, the boundary constant parameter [21] is chosen equal to zero. This parameter is usually introduced to simplify the determination of the observed values, but is not necessary here.

In the definition of the R matrix, E_λ is the eigenvalue associated to the basis function χ_λ, and the formal reduced width $\tilde\gamma_\lambda^2$ is defined from

$$\tilde\gamma_\lambda=\left(\frac{\hbar^2 a}{2\mu}\right)^{1/2}\chi_\lambda(a). \tag{8}$$

Throughout this paper, indices $J\pi$ are not written for $\tilde\gamma_\lambda^2$ and E_λ; in addition, the formal quantities are denoted by a tilde (e.g., $\tilde\gamma_\lambda^2$), whereas observed values are denoted without the tilde (e.g., γ_λ^2). The phase shift $\delta^{J\pi}$ defined by Eq. (6) involves a Coulomb term $\delta_{\rm C}^{J\pi}$, a hard-sphere phase shift $\delta_{\rm HS}^{J\pi}$, and a nuclear phase shift $\delta_N^{J\pi}$:

$$\begin{aligned} \delta^{J\pi}&=\delta_C^{J\pi}+\delta_{\rm HS}^{J\pi}+\delta_N^{J\pi},\\ \delta_C^{J\pi}&=\arg\Gamma(J+1+i\eta),\\ \delta_{\rm HS}^{J\pi}&=-\arctan F_J(ka)/G_J(ka),\\ \delta_N^{J\pi}&=\arctan\frac{P_J(ka)R^{J\pi}(E)}{1-S_J(ka)R^{J\pi}(E)}. \end{aligned} \tag{9}$$

In the literature, the *R*-matrix theory is used in two ways. In general, the pole parameters $\tilde\gamma_\lambda^2$ and E_λ are fitted to experimental data, such as resonance energy or width, or elastic phase shifts. In that case, the wave functions χ_λ are completely hidden. On the other hand, the *R*-matrix theory can also be employed in variational calculations, to correct a possibly inappropriate asymptotic behavior of the basis wave functions [22]. In that way, energies E_λ and reduced widths $\tilde\gamma_\lambda$ are calculated from the basis functions, and used to determine the collision matrix.

B. Capture cross section

The capture cross section of order L from an initial state with spin $J_i\pi_i$ to a final state with spin $J_f\pi_f$ reads, in the *R*-matrix formalism [19]

$$\begin{aligned} \sigma_L(E,J_i\pi_i\rightarrow J_f\pi_f)=&\frac{8\pi(L+1)}{\hbar L(2L+1)!!^2}\frac{2J_f+1}{2J_i+1}k_\gamma^{2L+1}\\ &\times|\langle\psi^{J_f\pi_f}\|\mathcal{M}_L^E\|\psi_{\rm int}^{J_i\pi_i}(E)\rangle_{\rm int}\\ &+\langle\psi^{J_f\pi_f}\|\mathcal{M}_L^E\|\psi_{\rm ext}^{J_i\pi_i}(E)\rangle_{\rm ext}|^2, \end{aligned} \tag{10}$$

where $\psi^{J_f\pi_f}$ ($\psi^{J_i\pi_i}$) is the final (initial) wave function, k_γ is the photon wave number, and $\mathcal{M}_L^E$ is the electric multipole operator of order L. In definition (10), the first term of the right-hand side represents the internal contribution, while the second term represents the external part [subscript int (ext) refers to integration performed over the internal (external) region only]. Notice that this latter contribution indirectly depends on the nuclear interaction through the collision matrix which appears in $\psi_{\rm ext}^{J_i\pi_i}$. The relative importance of both components depends on the reaction and on the energy. For weakly bound systems, such as ^{7}Be$+p$, the external contribution is strongly dominant, and Eq. (10) reduces to the extranuclear-capture approximation [23]. In the present case, the binding energy of ^{16}O with respect to the $\alpha+^{12}$C threshold in fairly large (7.16 MeV) and, up to a very good approximation, the external part can be neglected. In this approximation, the cross section reads [5]

$$\sigma_L(E,J_i\pi_i\rightarrow J_f\pi_f) = \frac{\pi}{k^2}(2J_i+1) \times \frac{\left|\sum_\lambda \varepsilon_\lambda[\tilde\Gamma_\lambda^\alpha(E)\tilde\Gamma_\lambda^\gamma(E)]^{1/2}/(E_\lambda-E)\right|^2}{|1-LR^{J_i\pi_i}(E)|^2}, \tag{11}$$

where the formal α and γ widths are defined by

$$\tilde\Gamma_\lambda^\alpha(E)=2\tilde\gamma_\lambda^2 P_J(E),$$

$$\tilde\Gamma_\lambda^\gamma(E)=\frac{8\pi(L+1)}{\hbar L(2L+1)!!^2}\frac{2J_f+1}{2J_i+1}k_\gamma^{2L+1} \times|\langle\psi^{J_f\pi_f}\|\mathcal{M}_L^E\|\chi_\lambda^{J_i\pi_i}\rangle_{\rm int}|^2 \tag{12}$$

and ε_λ is a phase coefficient, equal to $+1$ or -1. The phase coefficients are written explicitly to account for the sign of the electromagnetic matrix elements. With this definition, the square root in Eq. (11) is always positive. The energy dependence of $\tilde\Gamma_\lambda^\gamma(E)$ reads

$$\tilde\Gamma_\lambda^\gamma(E)=\tilde\Gamma_\lambda^\gamma(E_r)\left(\frac{E-E_f}{E_r-E_f}\right)^{2L+1}, \tag{13}$$

where E_f is the energy of the final state and E_r the resonance energy (see below). In the following $\tilde\Gamma_\lambda^\gamma(E_r)$ will be denoted as $\tilde\Gamma_\lambda^\gamma$. Again $\tilde\Gamma_\lambda^\gamma$ must be considered either as a parameter, or as the result of a variational calculation involving basis states $\chi_\lambda^{J_i\pi_i}$.

C. Link between "formal" and "observed" values

One of the main drawbacks of the *R*-matrix method is that the pole parameters E_λ, $\tilde\gamma_\lambda^2$, and $\tilde\Gamma_\lambda^\gamma$ are related, but not equal, to resonance or bound state physical properties. This problem has been addressed by several authors [18,12]. The resonance energy E_λ^r is defined as the solution of

$$S(E_\lambda^r)R(E_\lambda^r)=1. \tag{14}$$

Simultaneously, the observed α width Γ_λ^α is deduced from a Breit-Wigner parametrization of the phase shift near the resonance energy

$$\delta_N^{J\pi}(E)\approx\arctan\frac{\Gamma_\lambda^\alpha}{2(E_\lambda^r-E)}. \tag{15}$$

From Eq. (9), we have, for unbound states,

$$\gamma_\lambda^2=\frac{R_r}{(SR)_r'},$$

$$\Gamma_\lambda^\alpha=\frac{2P_rR_r}{(SR)_r'}=2P_r\gamma_\lambda^2, \tag{16}$$

where subscript r refers to the function calculated at $E=E_\lambda^r$, and the derivative is performed with respect to energy. For bound states, we define the reduced width in the same way; the total width vanishes. Equation (16) defines the observed reduced width γ_λ^2, which not only depends on the formal reduced width $\tilde\gamma_\lambda^2$ of pole λ, but also on the properties of the other poles.

An interesting situation is the single pole approximation, where

$$R(E)=\frac{\tilde\gamma_0^2}{E_0-E}. \tag{17}$$

In this case, Eqs. (14) and (16) reduce to the well-known expressions

$$E_0^r+\tilde\gamma_0^2S(E_0^r)-E_0=0 \tag{18}$$

and

$$\gamma_0^2=\tilde\gamma_0^2/(1+\tilde\gamma_0^2S_r'). \tag{19}$$

The problem for ^{12}C$(\alpha,\gamma)^{16}$O is more complicated. We have, in the 2^+ partial wave, several states which must be included. For these states, some properties are experimentally known and should be used to limit the parameter space. It is therefore desirable to have an efficient technique to pass from observed quantities to calculated values.

Let us start with the pole energies and α widths. At a resonance energy E_λ^r, we have

$$R(E_\lambda^r)=\frac{\tilde\gamma_\lambda^2}{E_\lambda-E_\lambda^r}+R_\lambda^0, \tag{20}$$

where R_λ^0 represents the contribution of the other poles. From Eqs. (14) and (16), we deduce

$$E_\lambda=E_\lambda^r+S_r\tilde\gamma_\lambda^2/(1-S_rR_\lambda^0),$$

$$\tilde{\gamma}_\lambda^2 = \gamma_\lambda^2 \times \frac{(1-S_r R_\lambda^0)^2}{1-S_r' \gamma_\lambda^2}. \quad (21)$$

This system is solved iteratively, starting with $R_\lambda^0 = 0$. Then the calculated values E_λ and $\tilde{\gamma}_\lambda^2$ are determined in the first iteration. Coefficients R_λ^0 are recalculated, and the process is repeated until convergence. In practice, a few iterations only are needed. This method is well adapted to numerical calculations and turns out to be very efficient. Notice, however, that derivation of Eq. (21) requires the assumption that the contribution of other poles than λ is independent of energy near E_λ^r. In the $^{12}C(\alpha,\gamma)^{16}O$ reaction, the $E2$ S factor involves narrow (or bound) well separated states, and the method is applicable. It has been tested by using the formal parameters (21) in Eq. (14). Solutions of this equation and the α widths obtained with Eq. (16) are equal to the starting values.

Using the capture cross section (11), we find a similar expression to derive the γ width:

$$\sqrt{\Gamma_\lambda^\gamma} = S_r \gamma_\lambda \sum_{\lambda'} \frac{\varepsilon_{\lambda'} \tilde{\gamma}_{\lambda'} \sqrt{\tilde{\Gamma}_{\lambda'}^\gamma}}{E_{\lambda'} - E_\lambda^r}. \quad (22)$$

The calculated γ widths $\tilde{\Gamma}_\lambda^\gamma$ are therefore obtained from the resolution of a linear system. If we consider the single-pole approximation (17), Eqs. (18) and (19) are directly recovered by using Eq. (21) with $R_\lambda^0 = 0$. For the γ width we find the usual relationship

$$\Gamma_0^\gamma = \tilde{\Gamma}_0^\gamma / (1 + \tilde{\gamma}_0^2 S_r'). \quad (23)$$

With the help of Eqs. (21) and (22), we can easily obtain the R-matrix parameters from experimental (''observed'') data. Notice that the present development is limited to single-channel systems or, in other words, to one-dimensional R and U matrices. These formulas are more efficient than the methods used so far [16,12]. They will be extensively used in the following.

III. FITS OF THE $l=2$ PHASE SHIFT

In order to limit the number of parameters, we start with the study of the $\alpha + ^{12}C$ phase shift. We use the experimental data of D'Agostino Bruno *et al.* [14] and of Plaga *et al.* [15] at energies $E \leqslant 4.5$ MeV. Data at higher energies are also available [14], but are not used in the fits. In the overlapping region, a good agreement between both data sets is obtained. Since our aim here is a qualitative investigation of the phase shifts, we neglect target thickness effects, included in the analysis of Ref. [12]. Throughout the paper the channel radius is $a = 6.5$ fm, as recommended by Barker and Kajino [16].

The R-matrix fits are performed with four poles: the 2_1^+ subthreshold state ($\lambda = 1$), the 2_2^+ ($\lambda = 2$) and 2_3^+ ($\lambda = 3$) resonances at $E_{c.m.} = 2.68$ and 4.36 MeV, respectively, and a background term ($\lambda = 4$) at 10 MeV. In practice, the 2.68 MeV resonance is very narrow and does not affect the phase shift, but will be important in the S factor. We have checked the sensitivity of the results with different values of the background energy, and the conclusions remain unchanged. Table I lists the parameter set.

TABLE I. Energies and observed reduced widths [24] for poles $\lambda = 1$ to 4. γ_1^2 and γ_4^2 are fitted.

λ	E_λ^r (MeV)	γ_λ^2 (MeV)
1	-0.24	γ_1^2
2	2.68	3.68×10^{-4}
3	4.36	1.39×10^{-2}
4	10	γ_4^2

For poles 2 and 3, the γ^2 values correspond to the experimental widths (0.625 keV and 73 keV, respectively). In Fig. 1, we present the χ^2 values as a function of γ_1^2 (γ_4^2 is optimized). Let us remind the reader that for each parameter set, the experimental requirements of Table I are satisfied. The fitting procedure has been performed in two conditions. In a first step, we have limited the energy range to 3.4 MeV. This corresponds to the range adopted by Barker and Kajino [16], and does not include the 4.36 MeV resonance. Buchmann *et al.* [12] also exclude some data points but do not give further precisions. In these conditions, the χ^2 curve is rather flat, and the minimum is obtained near $\gamma_1^2 = 0$, which would exclude any contribution from the 2_1^+ subthreshold state. This conclusion has been drawn by Buchmann *et al.* On the contrary, extension of the energy range to 4.2 MeV provides a more interesting structure of the χ^2 curve. A fairly deep minimum is obtained near $\gamma_1^2 = 0.20$ MeV (and $\gamma_4^2 = 3.5$ MeV). This γ_1^2 value is very similar to the result of a microscopic multicluster calculation of the $\alpha + ^{12}C$ system [25]. Further extension of the energy range would be meaningless since it would cover a sharp resonance region and any small uncertainty on its properties would affect the fit in an inappropriate way.

To illustrate the low sensitivity of the fit with respect to the total width of the 4.36 MeV level, we have repeated the procedure with the α width measured by Bilaniuk *et al.* [26] in a $^{14}N(^3He,p)^{16}O$ experiment (64 ± 5 keV). The corresponding fit is presented in Fig. 1 as a dotted line for $E_{max} = 4.2$ MeV (the curves for 3.4 MeV are indistinguishable).

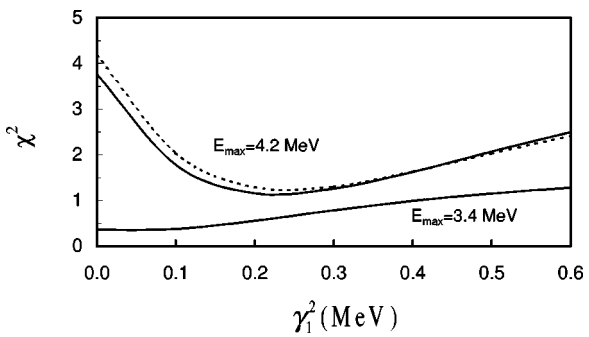

FIG. 1. χ^2 fits of the d phase shifts for different maximum energies. The dotted line shows the sensitivity with respect to the α width of pole 3 (see text).

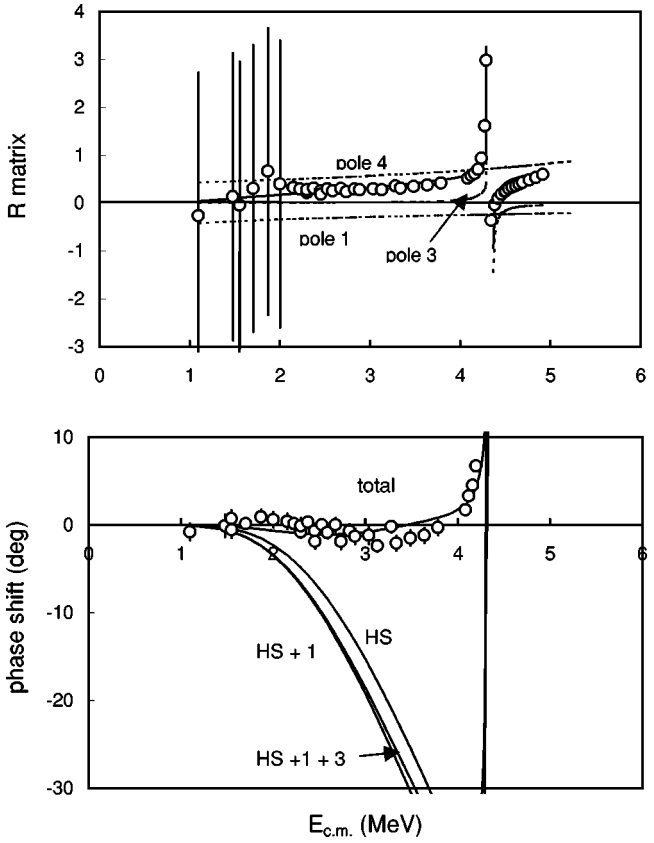

FIG. 2. Upper panel: *R* matrix derived from the experimental data of Ref. [15] (circles), with the present fit (full line), and individual contributions of poles 1, 3, and 4 (dashed lines). Lower panel: experimental phase shifts [15] (without the Coulomb term) with the *R*-matrix fit; the hard-sphere phase shift is denoted as HS; calculations limited to some poles are also shown.

To go further in the understanding of the fit, we give in Fig. 2 an analysis of the phase shift, and of the corresponding *R* matrix. From the experimental data of Plaga *et al.* [15], we have extracted ''experimental'' *R* matrices at 6.5 fm by inverting Eq. (6). We also show the individual contribution of the poles included in the present fit (pole 2 is not shown). Different interesting conclusions can be drawn from the figure. (i) Below 2 MeV, the error bars on the phase shift (typically 1°) are converted as infinite error bars on the *R* matrix. In other words, for these very small phase shifts, any *R* matrix gives a phase shift within the experimental error bar. This means that these energies cannot be considered as valuable constraints on the fit. (ii) Except around the 4.36 MeV resonance, the *R* matrix is always positive. However, the *R*-matrix expansion (7) limited to the first pole gives a negative contribution since $\tilde{\gamma}_0^2$ is positive and E_0 is negative. This is illustrated in Fig. 2, where we show this contribution obtained in our best fit. In other words, a simple analysis of the experimental *R* matrix suggests that the background contribution must be important to compensate the effect of the subthreshold state. (iii) Near the 4.36 MeV resonance, we find a singularity of the *R* matrix which is expected to appear at the poles. The parameters of this resonance are well known experimentally and yield a contribution almost negligible except near 4.36 MeV (see curve labeled ''pole 3'' in Fig. 2). Hence, the shape of the *R* matrix near the resonance is strongly affected by the pole 1 and background contributions, and this energy region turns out to be very sensitive to the conditions of the calculation.

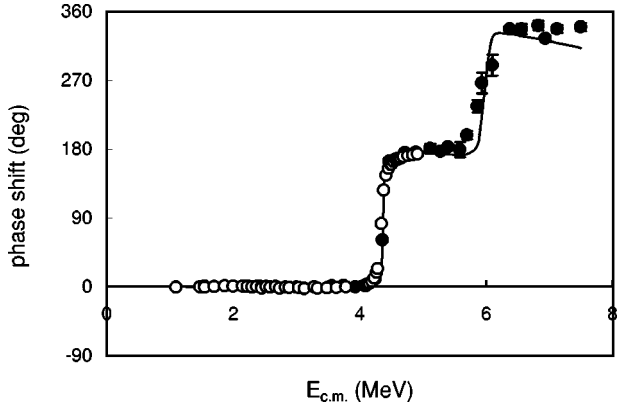

FIG. 3. Fit of the experimental phase shifts of Ref. [14] (filled circles) and of Ref. [15] (open circles). The Coulomb phase shift has been subtracted.

In the lower panel of Fig. 2, we present the phase shift expanded in its different terms [see Eq. (9)]. The hard-sphere phase shifts are much lower than experiment. Furthermore, the inclusion of the subthreshold state alone still lowers the theoretical phase shift. This feature led Buchmann *et al.* [12] to the conclusion that γ_1^2 should be very close to zero, and that a better agreement needs a smaller channel radius (4.5 fm). The present analysis shows that a non-negligible γ_1^2, as well as a large channel radius necessary to account for the observed cascade transitions [16], are still compatible with the experimental phase shifts, provided that an important background contribution is taken into account.

From this analysis, it turns out that the interference pattern in the *d* phase shift between 3 and 4.2 MeV provides a rather strong constraint on the parameter set. Since resonance energies are known as well as the α width of the 4.36 MeV resonance, we are left with only two parameters: γ_1^2 and γ_4^2, and the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ analysis is greatly simplified.

For the sake of completeness, we present in Fig. 3 the phase shifts calculated up to 7.5 MeV, and compare them with the high-energy data of D'Agostino Bruno *et al.* [14]. The parameters are those of Table I with our best values for γ_1^2 and γ_4^2 (0.20 and 3.5 MeV, respectively). To account for the 5.86 MeV resonance, we have added a fifth pole ($E_5^r = 5.86$ MeV, $\Gamma_5^\alpha = 0.52$ MeV taken from Ref. [14]) to the *R*-matrix expansion. The agreement with the data is quite acceptable if one keeps in mind that no fitting procedure has been used beyond 4.2 MeV.

IV. ANALYSIS OF THE *E*2 *S* FACTOR

In this section, we use the parameters fitted on the elastic phase shifts to investigate the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ *E*2 *S* factor. In this way, the number of free parameters is strongly reduced. In fact, since the γ widths of poles 1, 2, and 3 are experimentally known, we have one free parameter only (the γ

TABLE II. *R*-matrix parameters for capture data [24]. Γ_4^γ is fitted.

λ	E_λ^r (MeV)	γ_λ^2 (MeV)	Γ_λ^γ (eV)	ε_λ
1	-0.24	0.20 [a]	9.7×10^{-2}	$+1$
2	2.68	3.68×10^{-4}	5.7×10^{-3}	ε_2
3	4.36	1.39×10^{-2}	0.61	ε_3
4	10	3.5 [a]	Γ_4^γ	ε_4

[a]From the phase-shift fit.

width of pole 4, corresponding to the background) and different combinations of three phase coefficients [ε_2, ε_3, and ε_4 in Eq. (11), assuming $\varepsilon_1=+1$]. We use the data of Refs. [27–29,11], and the parameters are summarized in Table II.

First we consider the χ^2 values as a function of Γ_4^γ, and for different (ε_2,ε_3) choices. For $\varepsilon_4=-1$, all χ^2 values are unphysically large, and accordingly, this phase is fixed as $\varepsilon_4=+1$ in the following. The results are presented in Fig. 4, where we find out that the minima are obtained at different Γ_4^λ values when the interference signs ε_2 and ε_3 are changed. The χ^2 values at the minima are rather similar (between 3.6 and 5.4), and significantly larger than unity.

The best fits are illustrated in Fig. 5, and compared to the experimental data. Contrary to the phase shifts, the high-energy region ($E\approx3-4$ MeV) does not give strong constraint on the *S* factor at low energies. Important differences are observed beyond 3 MeV whereas the 300 keV *S* factors are nearly identical for each set of interference signs (from 190 to 220 keV b). From the current experimental situation, it is not possible to draw definite conclusions about these signs.

The present $S_{E2}(300\ \text{keV})$ value is larger than in recent works [16,12,29] which provide $S_{E2}(300)=50-180$ keV b, $S_{E2}(300)\leqslant140$ keV b, and $S_{E2}(300)=7-95$ keV b, respectively. With respect to the *R*-matrix studies of Refs. [16] and [12], we include the 2.68 and 4.36 MeV states which were neglected in those references (the potential model used in Ref. [29] does not describe these states). On the other hand, instead of fitting simultaneously the phase shift and the *S* factor, we try to reduce the number of free parameters as much as possible. To this end, we first consider the phase shift, and then analyze the *S* factor. Using experimentally known properties of the 2^+ states, we have

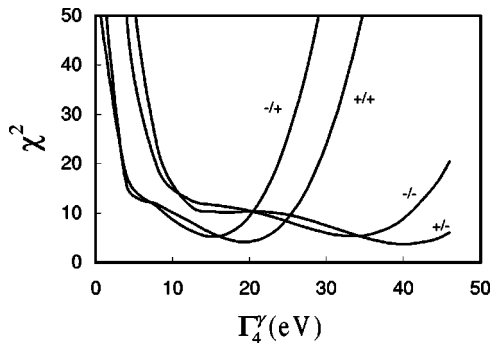

FIG. 4. χ^2 values of the $E2$ *S* factor for different $\varepsilon_2/\varepsilon_3$ sets (see text).

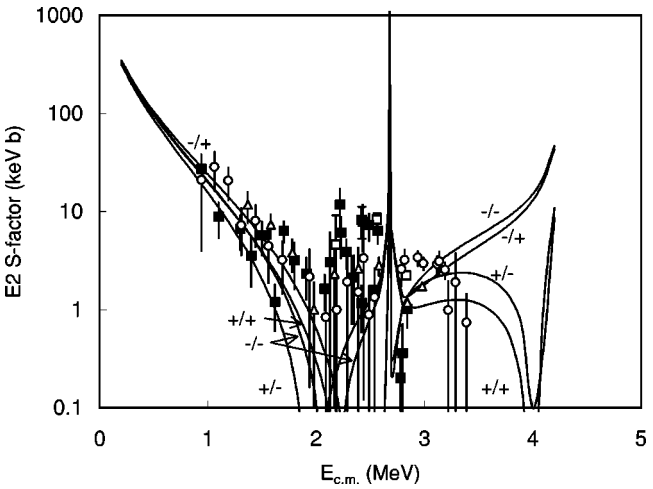

FIG. 5. $E2$ *S* factors for the best χ^2 values of Fig. 4 (see caption to Fig. 4). Experimental data are taken from Refs. [27] (open squares), [28] (filled squares), [29] (triangles), and [11] (open circles). For the σ_{E2}/σ_{E1} data of Redder *et al.*, the $E1$ fit of Ref. [9] has been used.

one free parameter only (Γ_4^γ) for the *S* factor. The energy dependence of the fit below 2 MeV is more consistent with experiment than the fits of Refs. [16,12,29], and yields a higher $S_{E2}(300)$ value. Using the empirical criteria of Ref. [12] for the error determination, we find for each fit about 15% error.

In Fig. 6, we analyze the contribution of the different poles for $\varepsilon_2=-1$, $\varepsilon_3=+1$. With the subthreshold state alone, the *S* factor decreases smoothly and is larger than experiments. Introducing the background (pole 4) provides a fairly good description of the data below 2 MeV. Consequently, the role of the background contribution extends even to very low energies.

V. CONCLUSION

The goal of this paper is to provide some qualitative properties of the $\alpha+{}^{12}\text{C}$ $l=2$ phase shifts and the ${}^{12}\text{C}(\alpha,\gamma){}^{16}\text{O}$

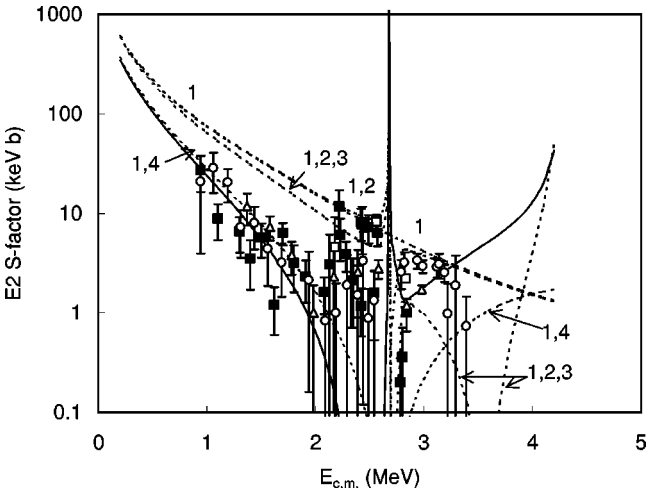

FIG. 6. *R*-matrix results with four poles (full line) and selected poles (indicated as labels: dashed lines). See caption to Fig. 5.

$E2$ S factor in the *R*-matrix formalism. We do not aim at providing a ''new'' S factor but we want to point out some interesting features which could greatly help in reducing the uncertainties.

First, we have developed a new method to link observed and calculated values in the *R*-matrix theory. This method, based on an iterative procedure, is quite simple and efficient for the $^{12}C(\alpha,\gamma)^{16}O$ reaction. It allows us to solve one of the main drawbacks of the *R*-matrix theory. We can easily constrain the parameter set with experimentally known data. This starting point is crucial for $^{12}C(\alpha,\gamma)^{16}O$ where many parameters are involved.

From the analysis of the d phase shift, we find that the interference pattern between the different contributions near 4 MeV provides a fairly strong constraint on the parameter set. This energy region is easily covered by modern experiments and the counting rate is high enough to provide accurate phase shifts. On the contrary, we have shown that, below 2 MeV, the errors on the current data are too large to be used in a fit. The experimental d phase shifts strongly deviate from the hard-sphere phase shift. Since the 2_1^+ contribution enlarges the deviation, and since the role of the 2.68 and 4.36 MeV resonances is limited to a very narrow energy region, the difference must arise from the background contribution. This contribution turns out to be of the same amplitude as the 2_1^+ subthreshold state. New data on elastic scattering would be welcome to improve the accuracy of the parameters.

The situation is different for the $^{12}C(\alpha,\gamma)^{16}O$ $E2$ S factor. Above 3 MeV, where the cross sections are reasonably large and therefore easily accessible to experiments, several interference patterns can be obtained with similar χ^2 values. The structure of the S factor beyond 3 MeV does not have a direct implication on the 300 keV value and, therefore, direct measurements at energies as low as possible are necessary to derive a reliable S factor for astrophysical applications.

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[1] D. D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis* (Chicago Press, Chicago, 1983).
[2] C. Angulo *et al.*, Nucl. Phys. **A656**, 3 (1999).
[3] R. Bonetti, C. Broggini, L. Campajola, P. Corvisiero, A. D'Alessandro, M. Dessalvi, A. D'Onofrio, A. Fubini, G. Gervino, L. Gialanella, U. Greife, A. Guglielmetti, C. Gustavino, G. Imbriani, M. Junker, P. Prati, V. Roca, C. Rolfs, M. Romano, F. Schuemann, F. Strieder, F. Terrasi, H. P. Trautvetter, and S. Zavatarelli, Phys. Rev. Lett. **82**, 5205 (1999).
[4] C. Brune, W. Geist, R. Kavanagh, and K. Veal, in *Proceedings of the International Symposium Nuclei in the Cosmos V, Volos, 1998*, edited by N. Prantzos and S. Harissopoulos (Editions Frontières, Paris, 1998), p. 107.
[5] F. C. Barker, Aust. J. Phys. **24**, 777 (1971).
[6] F. C. Barker, Aust. J. Phys. **40**, 25 (1987).
[7] D. Baye and P. Descouvemont, Phys. Rev. C **38**, 2463 (1988).
[8] Z. Zhao, R. H. France III, K. S. Lai, M. Gai, E. L. Wildes, R. A. Kryger, J. A. Winger, and K. B. Beard, Phys. Rev. C **48**, 429 (1993).
[9] R. E. Azuma, L. Buchmann, F. C. Barker, C. A. Barnes, J. M. D'Auria, M. Dombsky, U. Giesen, K. P. Jackson, J. D. King, R. G. Korteling, P. McNeely, J. Powell, G. Roy, J. Vincent, T. R. Wang, S. S. M. Wong, and P. R. Wrean, Phys. Rev. C **50**, 1194 (1994).
[10] R. H. France III and M. Gai, in *Proceedings of the International Symposium Nuclei in the Cosmos V* [4], p. 111.
[11] H.-P. Trautvetter, G. Roters, C. Rolfs, S. Schmidt, and P. Descouvemont, Nucl. Phys. **A621**, 161c (1997).
[12] L. Buchmann, R. E. Azuma, C. A. Barnes, J. Humblet, and K. Langanke, Phys. Rev. C **54**, 393 (1996).
[13] J. C. Chow, A. C. Morton, R. E. Azuma, N. Bateman, R. N. Boyd, L. Buchmann, J. M. D'Auria, T. Davinson, M. Dombsky, W. Galster, E. Gete, U. Giesen, C. Iliadis, K. P. Jackson, J. D. King, G. Roy, T. Shoppa, and A. Shotter, Phys. Rev. C **57**, R475 (1998).
[14] M. D'Agostino Bruno, I. Massa, A. Uguzzoni, G. Vannini, E. Verondini, and A. Vitale, Nuovo Cimento A **27**, 1 (1975).
[15] R. Plaga, H. W. Becker, A. Redder, C. Rolfs, H.-P. Trautvetter, and K. Langanke, Nucl. Phys. **A564**, 291 (1987).
[16] F. C. Barker and T. Kajino, Aust. J. Phys. **44**, 369 (1991).
[17] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).
[18] F. C. Barker, H. J. Hay, and P. B. Treacy, Aust. J. Phys. **21**, 239 (1968).
[19] R. J. Holt, H. E. Jackson, R. M. Laszewski, J. E. Monaha, and J. R. Specht, Phys. Rev. C **18**, 1962 (1978).
[20] R. F. Barret, B. A. Robson, and W. Tobocman, Rev. Mod. Phys. **55**, 155 (1983).
[21] F. C. Barker, Aust. J. Phys. **25**, 341 (1972).
[22] P. Descouvemont, Nucl. Phys. **A470**, 309 (1987).
[23] R. F. Christy and I. Duck, Nucl. Phys. **24**, 89 (1961).
[24] D. R. Tilley, H. R. Weller, and C. M. Cheves, Nucl. Phys. **A564**, 1 (1993).
[25] P. Descouvemont, Phys. Rev. C **47**, 210 (1993).
[26] O. M. Bilaniuk, H. T. Fortune, and R. Middleton, Nucl. Phys. **A305**, 63 (1978).
[27] P. Dyer and C. A. Barnes, Nucl. Phys. **A233**, 495 (1974).
[28] A. Redder, H. W. Becker, C. Rolfs, H. P. Trautvetter, T. R. Donoghue, T. C. Rinckel, J. W. Hammer, and K. Langanke, Nucl. Phys. **A462**, 385 (1987).
[29] J. M. L. Ouellet, M. N. Butler, H. C. Evans, H. W. Lee, J. R. Leslie, J. D. MacArthur, W. McLatchie, H.-B. Mak, P. Skensved, J. L. Whitton, and X. Zhao, Phys. Rev. C **54**, 1982 (1996).