Coulomb excitation of vibrational states in ^{232}Th with ^{16}O projectiles

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Excited states in ^{232}Th were populated by Coulomb excitation with 80 MeV ^{16}O ions. Gamma-gamma coincidences were detected with the detector array YRAST Ball. The one-phonon quadrupole and octupole vibrations are most strongly excited. These vibrations are in ^{232}Th, particularly favorable cases for a study of their couplings, due to the small energy spacings of the β and γ bands and the $K^\pi = 1^-$ and 2^- bands, respectively. The need for additional experimental information on the γ-ray intensities and the location of members of the octupole-vibrational bands is emphasized. The higher-lying $K^\pi = 0^+$ bands with bandheads at 1079 and 1352 keV proposed previously are observed and some conflicting properties of these bands are discussed. The interpretation of the 1352 keV 0^+ band as two-octupole-phonon vibration is not supported by our data. A level at 1054 keV, previously assigned as 2^+ or 2^- level, is tentatively proposed to be the bandhead of a second-excited $K^\pi = 2^+$ band. Our $\gamma\gamma$ coincidence data establish that the unusually enhanced $E2$ transitions from a 2^+ level at 1554 keV to members of the β and γ bands proposed previously do not exist.

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I. INTRODUCTION

The lowest nonrotational excitations in deformed even-even nuclei are commonly assumed to correspond to small-amplitude quadrupole and octupole shape oscillations, giving rise to vibrational bands with $K^\pi = 0^+$ and 2^+ (denoted as β and γ vibrations, respectively) and $K^\pi = 0^-$, 1^-, 2^-, and 3^- (octupole vibrations). At higher excitation energies one expects two-quasiparticle excitations which arise from the breaking of a coupled pair of nucleons and occur above the pairing gap (~ 1.4 MeV in actinides). Finally, in a harmonic picture, two-phonon excitations are expected at approximately twice the energy of the one-phonon vibrational states ($\geqslant 1.5$ MeV in actinides). However, in a number of deformed nuclei, in particular, in the actinide region, additional intrinsic excitations have been observed with $K^\pi = 0^+$ and 2^+, and with energies well below the threshold for two-quasiparticle or two-phonon excitations. As was emphasized more then two decades ago by Bohr and Mottelson, in connection with the discussion of the low-energy excitation spectrum of ^{234}U, the interpretation of these additional excitations is crucial to the understanding of the β and γ vibrations [1].

More recently, the rotational bands observed in even-even actinides were compiled, and the existing theoretical approaches for their description summarized by Sood *et al.* [2]. These authors note that both the experimental and theoretical investigations reveal incomplete data on the low-lying excitations, and emphasize in particular that it is expected that many missing 0^+ states remain to be located.

It is clear that a complete set of experimental data for several neighboring nuclei is required for an understanding of the various excitation modes. In this respect the isotopic chain of the even thorium nuclei is particularly favorable. The isotopes accessible experimentally range from ^{216}Th with a magic neutron number across the region of octupole deformation around $A = 224$ to the quadrupole-deformed nuclei ^{232}Th and ^{234}Th. While the experimental knowledge for the lighter isotopes (up to ^{226}Th) is essentially restricted as of now to the ground and lowest $K^\pi = 0^-$ bands, there are already extensive data available on the higher-lying intrinsic excitations for the heavier isotopes, notably for ^{228}Th [3] and ^{232}Th [4,5].

The ideal nucleus for an investigation of the one- and two-phonon vibrational states, and the additional low-lying intrinsic excitations with $K^\pi = 0^+$ and 2^+, appears to be ^{232}Th. The position and $B(E\lambda)$ strength of these levels can be located in this nucleus in Coulomb excitation, and the theoretical interpretation of these data is facilitated by the well-deformed character of ^{232}Th. Surprisingly, however, there are as yet several uncertainties even for the lowest excitations, despite a large number of experimental investigations, principally by Coulomb excitation: (i) McGowan and Milner [5] report 2^+ states at 1054 and 1122 keV, whereas the Frankfurt/GSI group interprets levels at these energies as 2^- members of the $K^\pi = 2^-$ and 1^- band, respectively [6,7]. (ii) Gerl *et al.* [8] propose a two-phonon $K^\pi = 0^+$ octupole-vibrational band with its bandhead at 1352 keV and the 2^+ rotational member at 1386.6 keV. On the other hand, McGowan and Milner report a 2^+ level at 1387.2 keV with $B(E2, 0^+ \rightarrow 2^+) = 0.25$ s.p.u., a rather high value for a two-phonon band, raising the question whether this level is the 2^+ member of the band proposed in Ref. [8]. (iii) The assignment of a band with its bandhead at 1414 keV as $K^\pi = 4^+$ two-phonon $\gamma\gamma$ band [9] is questioned by the Frankfurt/GSI group, who suggest that this band might be the $K^\pi = 3^-$ octupole-vibrational band [7,8]. (iv) McGowan and Milner suggest a 2^+ level at 1554 keV as a possible candidate for a two-phonon state. This level decays according to these authors by $E2$ transitions to the β and γ vibrational bands with $B(E2)$ values which are an order of magnitude larger than the $B(E2)$ between one-phonon and zero-phonon states. As already noted by McGowan and Milner such large $B(E2)$ values ''disagree with our present understanding of collectivity in nuclei.''

In this paper we present the results from a study of $\gamma\gamma$ coincidences observed in the Coulomb excitation of ^{232}Th by ^{16}O projectiles. The oxygen beam was chosen to selectively populate the low-spin levels observed in the Coulomb excitation with α particles [5], and possibly the lowest members of the associated rotational bands. γ-γ coincidences were measured to check the various double assignments of γ rays proposed by McGowan and Milner, which were derived indirectly from a comparison of measured and calculated angular distributions of the γ ray singles with respect to the beam direction.

II. EXPERIMENTAL METHODS AND RESULTS

States in ^{232}Th were populated through Coulomb excitation by a 80 MeV ^{16}O beam at the ESTU Tandem accelerator at Yale University. The target consisted of two 10 mg/cm^2 thick ^{232}Th metal foils. The thickness was chosen to avoid the Doppler broadening of γ rays by stopping the thorium nuclei recoiling after the Coulomb excitation in the target itself. γ rays were detected with the detector array YRAST Ball [10] which consisted of 4 four-element Clover detectors and 17 single-crystal HPGe detectors, most of which were equipped with Compton suppression shields. The data were sorted off line into a γ-γ matrix. In this sorting the time spectra were corrected with the help of a 33×33 matrix containing the centroids of the time spectra, in order to correct for imperfect alignment of the timing signals of the 33 individual detectors. This enabled a reduction of the time window by a factor of 2 without appreciable loss in the coincidence counts. Background subtracted coincident spectra were produced and examined with the RADWARE interactive package [11].

A total of 3×10^8 prompt coincidence events were collected. Unfortunately, the data contained an appreciable amount of background events, most of which could be assigned to isotopes in two mass regions: (i) nuclei between ^{23}Na and ^{30}P, presumably resulting from reactions of the ^{16}O beam with the oxygen impurities in the target; (ii) nuclei between ^{38}Ar and ^{53}Mn, possibly resulting from the reaction of the ^{16}O beam with impurities of sodium and calcium in the target or with the material in the beam line (the ^{16}O projectiles were not stopped in the target, but left it with approximately 45 MeV). As an illustration a section of the total projection of the γ-γ coincidences is shown in the upper part of Fig. 1. For comparison, the spectrum in coincidence with the Th K_α x rays is shown in the lower part of the figure.

As is apparent from the spectra shown in Fig. 1 there is a strong true coincidence of the x rays with transitions in

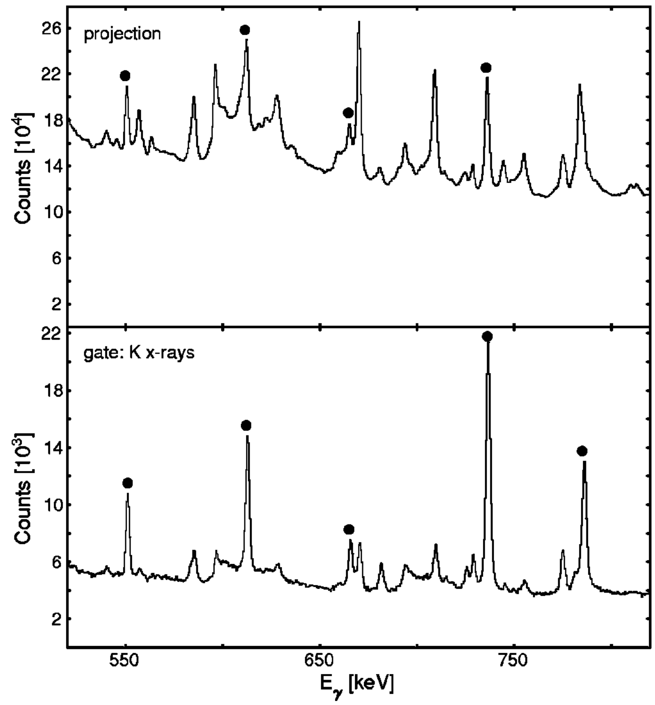

FIG. 1. γ-ray spectra measured in coincidence with γ rays in the energy range up to 2 MeV (upper spectrum) and with the thorium K_α x rays (lower spectrum). The strong lines in ^{232}Th are marked by dots.

^{232}Th. These coincidences do not result predominantly from x rays produced by the internal conversion of coincident γ rays: the strongest line in the lower part of Fig. 1 at 736 keV results from the $2^+_\gamma\rightarrow2^+_g$ transition, which is not in coincidence with K-converted γ rays (see the discussion below). We, therefore, conclude that there is a strong correlation between the Coulomb excitation of levels in ^{232}Th and the production of holes in the K shell of the corresponding thorium nuclei. A summary of the γ rays assigned to ^{232}Th is given in Table I.

The three strongest background lines in Fig. 1, at 670, 709, and 783 keV, result from transitions in ^{38}Ar, (^{30}P/^{41}K), and ^{50}Cr, respectively. We have performed a short test run with a 150 mg/cm^2 thick thorium metal foil of a different origin as the foils used in the main run, to check whether the background lines are produced by the ^{16}O projectiles after leaving the target. In this run the 670 keV line was strongly reduced, whereas the two other background lines remained essentially unchanged. One explanation of this observation could be that the ^{38}Ar is produced in the reaction of ^{16}O with ^{23}Na, which was less abundant in the metal foil used in the test run. We therefore believe that the background is produced predominantly by reactions of the ^{16}O projectiles with target impurities, presumably ^{16}O, ^{23}Na, and ^{40}Ca.

The most interesting result of the $\gamma\gamma$ coincidence measurement is the predominant decay of several of the higher-energy bands to the $K^\pi=0^-$ band, whereas we observe only a few very weak transitions to the β and γ bands. The γ-ray spectra in coincidence with the $1^-\rightarrow2^+$, $3^-\rightarrow4^+$, and $5^-\rightarrow6^+$ transitions from the $K^\pi=0^-$ band to the ground band

TABLE I. Energies and intensities of γ rays in ^{232}Th observed in coincidence with K x rays.

Initial state E (keV)	I^π	Final state [a] E (keV)	I^π	E_γ [b] (keV)	I_γ [b]
333.30	6^+	162.12	4^+	171.18	1000
556.95	8^+	333.30	6^+	223.65	208
730.20	0^+	49.37	2^+	680.83	142
774.34	2^+	0	0^+	774.34	296
	$2^+(3^-)$	49.37	2^+	724.87	88
		162.12	4^+	612.17	711
873.04	4^+	49.37	2^+	823.67	115
785.36	2^+	0	0^+	785.35	761
		49.37	2^+	736.00	1526
890.46	4^+	162.12	4^+	728.34	146
714.42	1^-	49.37	2^+	665.05	273
774.34	$3^-(2^+)$	49.37	2^+	724.87	88
		162.12	4^+	612.17	711
883.74	5^-	333.30	6^+	550.44	328
1042.95	7^-	556.95	8^+	486.00	106
1105.49	3^-	49.37	2^+	1056.12	123

[a]The energies of the 2^+ and 4^+ levels are taken from Ref. [4].
[b]Estimated accuracy ± 0.1 keV for the energies and $\pm 15\%$ for the intensities.

are shown in Fig. 2. Two remarks are necessary in connection with the spectrum in coincidence with the 612 keV $3^- \to 4^+$ transition: (i) the γ rays in coincidence with this transition populate the 3^- level, and not the degenerate 2^+ level: for all γ rays shown in the central part of Fig. 2 a possible coincident 774 keV γ transition to the ground state has an intensity of less than 10% of the 612 keV γ transition to the 2^+ member of the ground band (ii) the Doppler-broadened line at 662 keV and the strong line at 783 keV are background lines from ^{50}Cr ($12^+ \to 11^+$ and $2^+ \to 0^+$ transitions, respectively), resulting from the coincidences with a 610 keV γ ray ($11^+ \to 10^+$ transition). Unfortunately, the 783 keV background line partly masks the 780 keV γ ray (1554 keV $\to$ 774 keV transition) in ^{232}Th. This spectrum thus illustrates the problems caused by the large background encountered in the present work.

The energies and intensities of the γ rays, which can be reliably placed in the level scheme of ^{232}Th, are listed in Table II. Some additional lines assigned to ^{232}Th, but not included in the discussion of the level scheme given below, are collected in Table III. The γ-ray intensities listed in the tables were derived from the $\gamma\gamma$ coincidences ignoring any effects from $\gamma\gamma$ angular correlations. We estimate that such effects might cause uncertainties up to approximately 30% in addition to the statistical errors given in the tables.

For a few lines listed in Tables II and III we consider the assignment to ^{232}Th as tentative, due to problems resulting from the background. As an example we consider the 889.5 keV γ ray, which we observe in the spectrum in coincidence with the 171 keV $6^+ \to 4^+$ transitions, and which is a possible candidate for a transition from the 4^+ member of the second-excited 0^+ band to the 6^+ member of the ground band. This line coincides with a much stronger 889.3 keV background line from ^{46}Ti. However, none of the other lines in the sequence of transitions observed in ^{46}Ti (1121, 1290, and 1598 keV γ rays) is in coincidence with a 171 keV γ ray, and such a transition is also not reported in the literature for ^{46}Ti. Similar considerations lead to the other tentative assignments given in Tables II and III.

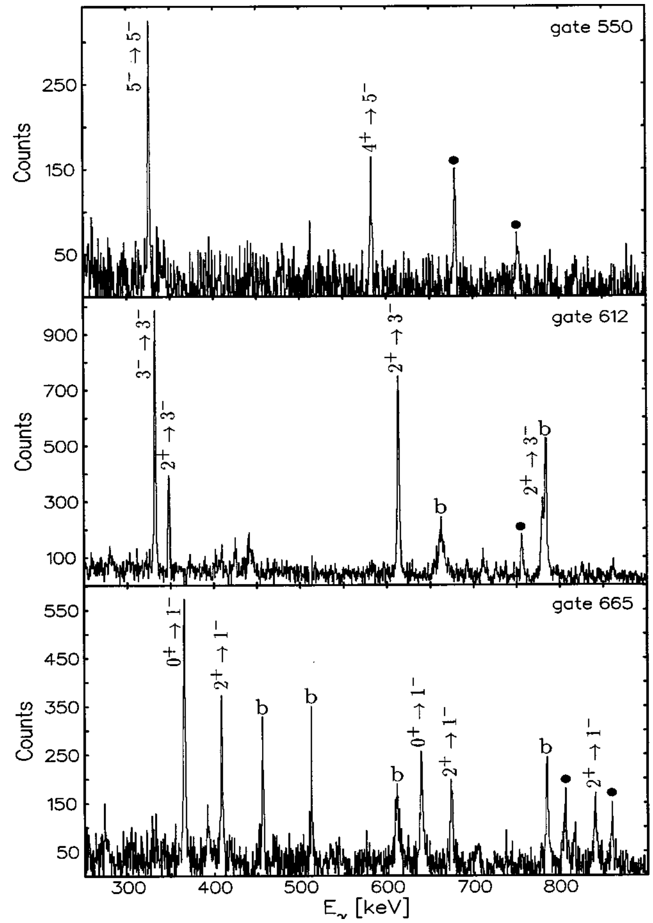

FIG. 2. γ-ray spectra measured in coincidence with 665, 612, and 550 keV γ rays corresponding to the $1^- \to 2^+$, $3^- \to 4^+$, and $5^- \to 6^+$ transitions in ^{232}Th. Background lines are marked by the letter b, lines not included in the level scheme of ^{232}Th (see Table II) by dots.

III. DISCUSSION

In this section we will compare our results with those of the previous Coulomb excitation work with light and medium-heavy projectiles [5–8] and attempt to give answers to some of the questions mentioned in the introduction. In the first two subsections we discuss the low-lying states which are commonly interpreted as vibrational excitations. The additional states observed below the threshold for two-quasiparticle states, which do not fit into the picture of collective vibrations, are discussed in a third subsection.

A. β and γ vibrational bands

The first-excited $K^\pi = 0^+$ and 2^+ bands with bandheads at 730 and 785 keV, respectively, are most strongly popu-

TABLE II. Energies and intensities of γ rays in ^{232}Th observed in the $\gamma\gamma$ coincidence measurement

Band K^{π}	Initial state I^{π}	E (keV)	Final state I^{π}	E (keV)	E_{γ} [a] (keV)	I_{γ} [a]
0_g^+	6^+	333.3	4^+	162.1	171.2	1000
	8^+	556.9	6^+	333.3	223.6	128
	10^+	826.7	8^+	556.9	269.8	9.6
0_1^+	4^+	873.0	4^+	162.1	710.8[b]	13(4)
			6^+	333.3	539.6	22
	6^+	1023.3	4^+	162.1	861.2	30
			6^+	333.3	690.0	9.0
			8^+	556.9	466.7(2)	0.6
2_1^+	2^+	785.4	4^+	162.1	623.1	12.8
	4^+	890.5	4^+	162.1	728.4	207
	6^+	1051.0	4^+	162.1	888.4(5)	5(2)
			6^+	333.3	717.7	20.7
0^-	5^-	883.7	6^+	333.3	550.4	243
	7^-	1042.9	8^+	556.9	486.0	45
			5^-	883.7	159.2	6.9
	9^-	1249.5	10^+	826.7	422.7	3.4
			7^-	1042.9	206.8	2.4
$(2)^-$	3^-	1105.5	4^+	162.1	943.5	34.5
			1^-	714.4	391.3(3)	1.7
			3^-	774.3	331.3	13.4
	5^-	1208.8	4^+	162.1	1046.7	47.3
			6^+	333.3	875.6(2)	0.9(3)
			3^-	774.3	434.3(2)	1.6(5)
			5^-	883.7	325.0	4.5
2_2^+	2^+	1053.7	0^+	730.4	323.2(2)	2.1
			2^+	774.4	279.5(3)	1.7(6)
			2^+	785.3	268.4[b]	<0.7
0_2^+	0^+	1078.6	1^-	714.4	364.2	9.8
	2^+	1121.6	4^+	162.1	959.3(2)	20
			1^-	714.4	407.3	7.4
			3^-	774.3	347.2	6.0
	4^+	1222.8[c]	6^+	333.3	889.5(3)	4.3
0_3^+	0^+	1352.2	1^-	714.4	637.8	7.0
	2^+	1387.0	4^+	162.1	1224.9[b]	9(3)
			0^+	730.4	656.6[b]	<1.5
			1^-	714.4	672.6	7.7
			3^-	774.3	612.7(3)	14(3)
	4^+	1466.4	4^+	162.1	1304.3[b]	<4
			6^+	333.3	1133.5(2)	4.7(9)
			3^-	774.3	691.9(2)	1.6(3)
			5^-	883.7	582.6	4.1
4^+	4^+	1413.8	2^+	785.3	628.5(2)	10.2
			3^+	829.6	584.2(2)	3.0
$(0,1)^+$	2^+	1553.8	1^-	714.4	839.4	5.8
			3^-	774.3	779.6(3)	12(2)

[a]Estimated accuracy ± 0.1 keV for the energies and $\pm 15\%$ for the intensities, unless otherwise noted.
[b]Energies calculated from level energies.
[c]Tentative assignment.

TABLE III. Energies and intensities of γ rays observed in the $\gamma\gamma$ coincidence measurement. These γ rays are assigned to ^{232}Th but not included in the level scheme.

Initial state E (keV)	Final state I^{π}	E (keV)	E_{γ} [a] (keV)	I_{γ} [a]
1362.8[b]	6^+	333.3	1029.5(3)[b]	8.3
1518.8	1^-	714.5	804.3	6.4
1529.4[b]	3^-	774.4	755.0(2)[b]	8.0
1562.0	5^-	883.7	678.3	4.4
1572.7	1^-	714.5	858.3	3.3
1697.4	3^-	774.4	923.0(2)	3.7

[a]Estimated accuracy ± 0.1 keV for the energies and $\pm 15\%$ for the intensities, unless otherwise noted.
[b]Tentative assignment.

lated in Coulomb excitation. We observe the even-spin members of these bands up to the 6^+ states, but do not observe the 3^+ and 5^+ members of the γ band. This is consistent with the observations reported in Ref. [5] for the Coulomb excitation of ^{232}Th with α particles, and is due to the small cross section for excitation of the unnatural-parity states in Coulomb excitation. For example, the cross sections for excitation of the 2^+, 3^+, and 4^+ members of the γ band with 80 MeV ^{16}O projectiles, calculated with the program of Winther and de Boer [12] and the $E2$ transition matrix elements of Ref. [5], are 44, 1.1, and 7.4 mb, respectively. The large difference of the cross sections for the 3^+ and 4^+ states results from the interference of the two main excitation pathways $0_g^+(E2)2_g^+(E2)I_\gamma^+$ and $0_g^+(E2)2_\gamma^+(E2)I_\gamma^+$, which is destructive for the 3^+ level and constructive for the 4^+ level. It is important to note that this is a general property for the Coulomb excitation of the γ band in ^{232}Th, independent of the excitation probabilities and the projectile used for the excitation.

The β and γ bands are closely spaced in ^{232}Th, and thus provide a unique possibility to study the coupling of these bands. In fact this coupling is so strong that it can be observed in the level energies in the form of an odd-even staggering of the levels of the γ band (see, for example, Ref. [13]). These data provide an estimate of the strength of the effective β-γ coupling: with a coupling matrix element of the form $\langle\gamma|H_c|\beta\rangle=\sqrt{2(I-1)I(I+1)(I+2)}h_{\beta\gamma}$ [1], one obtains $|h_{\beta\gamma}|\approx 0.2$ keV. Thus, for example, the 2^+ levels, which are separated by 11 keV, are coupled with a matrix element of ~ 1.4 keV.

A better understanding of the band couplings is usually obtained from the $E2$ branching ratios, which are very sensitive to the mixings due to the different strengths of the intrinsic transition-matrix elements involved. Unfortunately, there is considerable confusion about the experimental γ-ray branchings from the vibrational bands to the ground band, as seen from the summary of the most recent data given in Table IV. However, two statements are possible in connection with the branching ratios from the β band which provides the crucial data for an analysis of the β-γ band mixing:

TABLE IV. Relative γ-ray intensities of the transitions from the one-phonon vibrational bands to the ground band.

Transition $I \rightarrow I_g$	E_γ	I_γ(expt.) Ref. [4]	Ref. [5]	Ref. [7]	This work	I_γ(calc.)[a] Pure K[b]	Band mix.[c]
$K^\pi=0^+_\beta$ band							
$2\rightarrow0$	774.3	100	100	100		100	100
$\rightarrow2$	725.0	$\approx$1.8	20	143 *7*		103	19.9
$\rightarrow4$	612.2	$\approx$43	60	$<$250		79.5	60.9
$4\rightarrow2$	823.7	100	100	100		100	100
$\rightarrow4$	710.9			20 *1*	8.2	43.5	8.1
$\rightarrow6$	539.7	120 *40*	121	13 *1*	13[d]	19.2	13.6
$6\rightarrow4$	861.2	100		100	100	100	100
$\rightarrow6$	690.0			20	30 *6*	26.7	33.8
$\rightarrow8$	466.3	59 *9*		6.5 *4*	2.0 *5*	6.4	4.5
$K^\pi=2^+_\gamma$ band							
$2\rightarrow0$	785.4	56 *5*	58	59 *3*	50 *10*	96.9	66.4
$\rightarrow2$	736.0	100	100	100	100	100	100
$\rightarrow4$	623.2	$\approx$0.8	1.0	$\approx$5		2.2	3.5
$3\rightarrow2$	780.2	100		100		100	100
$\rightarrow4$	667.5	25 *6*		29 *2*		18.3	36.5
$4\rightarrow2$	841.1	18 *4*	18	13.0 *7*		69.7	21.5
$\rightarrow4$	728.3	100	100	100	100	100	100
$\rightarrow6$	557.2	5.0 *16*		9.0 *6*	$<$1	2.3	3.9
$5\rightarrow4$	797.9	100		100		100	100
$\rightarrow6$	627.2	52 *6*		61 *4*		17.1	51.5
$6\rightarrow4$	888.9	100[d]		5.7 *5*	24 *10*	78.5	4.9
$\rightarrow6$	717.7			100	100	100	100
$\rightarrow8$	494.0	$>$53			$<$3	1.6	2.8
$K^\pi=0^-$ band							
$1\rightarrow0$	714.4	16 *2*	17.5	18.6 *21*	18.9 *20*	62.0	18.8
$\rightarrow2$	665.1	100	100	100	100	100	100
$3\rightarrow2$	725.0	$\approx$8	15.5	5.7 *6*	9(3)	125	4.7
$\rightarrow4$	612.2	100	100	100	100	100	100
$5\rightarrow4$	721.6	1.8 *14*		$<$0.3	$<$6	188	0.2
$\rightarrow6$	550.4	100	100	100	100	100	100
$K^\pi=1^-$ band							
$1\rightarrow0$	1078.1		100	100		100	100
$\rightarrow2$	1028.7		39	54 *4*		43.4	39.0
$3\rightarrow2$	1133.4	100	100	100		100	100
$\rightarrow4$	1020.7	30 *4*	25	$>$19		54.8	42.5

[a]Assuming pure $E2$ multipolarity for transitions from the β and γ band with $|\Delta I|<2$.
[b]$B(E\lambda,I\rightarrow I_g)\propto\langle IK\lambda-K|I_g0\rangle^2$.
[c]Generalized intensity relations (see text).
[d]Arbitrarily normalized.

(i) the γ-ray branchings from the 2^+_β level are difficult to determine experimentally, due to the presence of the nearly degenerate 3^- member of the $K^\pi=0^-$ band. However, the data of McGowan and Milner [5] as well as our data, definitely exclude the large γ-ray branching reported for the 725 keV $2^+_\beta\rightarrow2^+_g$ transition in [7] (see, for example, Fig. 2 of Ref. [5] and Fig. 1 of the present work). (ii) the large γ-ray branching for the 466 keV $6^+_\beta\rightarrow8^+_g$ transition listed in the Nuclear Data Sheets [4] is definitely excluded by our data.

In order to obtain a feeling for the information possibly obtainable from the $E2$ branchings we have performed a calculation treating the coupling of the ground, β, and γ bands in first order, and assuming equal intrinsic quadrupole moments for the three bands. In this approximation the $B(E2)$ values from the β and γ bands to the ground band have the form

$$B(E2,I_1\rightarrow I_2)=f(I_1,I_2)\cdot|M_1+g(I_1,I_2)\cdot M_2 + h(I_1,I_2)\cdot M_3|^2, \tag{1}$$

with

$$f(I_1,I_2)=(1+\delta_{K,2})\cdot\langle I_1K2-K|I_20\rangle^2$$

$$g(I_1,I_2)=I_1(I_1+1)-I_2(I_2+1)$$

$$h(I_1,I_2)=[I_1(I_1+1)-I_2(I_2+1)]^2 -2(3-K)[I_1(I_1+1)+I_2(I_2+1)].$$

Equation (1) with $M_3=0$ is the familiar generalized intensity relation, in which the correction term $g(I_1,I_2)\cdot M_2$ takes into account the effects resulting from the coupling of the vibrational bands with the ground band [1]. The mutual coupling of the β and γ bands, with the first-order amplitude of admixture of $\sqrt{2(I-1)I(I+1)(I+2)}\epsilon_{\beta\gamma}$, is included in Eq. (1) by the third term, with the matrix elements $M_3=-\epsilon_{\beta\gamma}\langle 0_g^+|M(E2,-2)|2_\gamma^+\rangle/2\sqrt{6}$ and $\epsilon_{\beta\gamma}\langle 0_g^+|M(E2,0)|0_\beta^+\rangle/2\sqrt{6}$ for the β and γ band, respectively [3].

The γ-ray intensities calculated without band mixing, and assuming pure $E2$ multipolarities for all $(\beta,\gamma)\rightarrow$ ground transitions, are listed in the next to last column of Table IV. The most striking discrepancy with the experimental values is the reduced intensity for the $2_\beta^+\rightarrow 2_g^+$ transition compared to both the $2_\beta^+\rightarrow 0_g^+$ and $2_\beta^+\rightarrow 4_g^+$ transitions, suggested by the experimental data of Ref. [5]. Such a pattern cannot be reproduced with the usual generalized intensity relation but is naturally explained by the inclusion of the M_3 term. The intensities calculated with $M_2/M_1=0.021(-0.025)$ and $M_3/M_1=0.007(-0.0005)$ for the $\beta(\gamma)$ band are listed in the last column of Table IV. Taking into account the large experimental uncertainties the agreement is quite striking, in particular for the β band.

We conclude that there is considerable evidence for the influence of the β-γ mixing on the $(\beta,\gamma)\rightarrow$ ground transition rates, although more reliable and accurate experimental data are required for a complete analysis beyond the first-order approximation. The existing data also suggest that appreciable corrections might be required to derive intrinsic values from the measured $B(E2,0_g^+\rightarrow 2_{\beta,\gamma}^+)$ values. With $M_1^\beta=\langle 0_g^+|M(E2,0)|0_\beta^+\rangle+24\,M_3^\beta$, $M_1^\gamma=\langle 0_g^+|M(E2,-2)|2_\gamma^+\rangle-4\,M_2^\gamma$ [3], and the M_i/M_1 given above one would obtain $B(E2)_{mix}/B(E2)_0=1.83$ and 0.87 for the β and γ band, respectively.

The strong mixing of the β and γ bands in ^{232}Th due to their close spacing was realized more than 30 years ago by Stephens *et al.* [14]. These authors studied the conversion-electron spectrum after Coulomb excitation of ^{232}Th with 80 MeV ^{16}O ions, and observed strong $E0$ deexcitation of the β band to the ground band. In addition they found an $E0$ component for the $2_\gamma^+\rightarrow 2_g^+$ transition and noted that this provides evidence for a mixing of the β and γ bands of "20–25% in the amplitude." A precise measurement of the

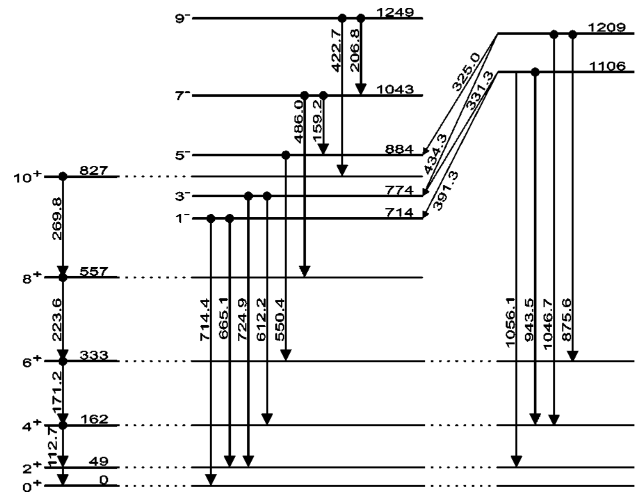

FIG. 3. Partial level scheme of ^{232}Th showing the negative-parity levels observed in the present work. The transitions observed in the $\gamma\gamma$ coincidence measurement are marked by dots. The 1056.1 keV $3^-\rightarrow 2^+$ γ ray is observed in coincidence with K_α x rays.

$E0$ branchings would indeed provide direct information on the mixing amplitudes and would, therefore, be very interesting.

In addition to the one-phonon quadrupole vibrations we also observe a level at 1414 keV, which had earlier been interpreted as the bandhead of the $K^\pi=4^+$ two-phonon $\gamma\gamma$ vibration [9]. This interpretation was questioned by Kröll and Gerl *et al.* [7,8] who suggest that this level might be the bandhead of the $K^\pi=3^-$ octupole vibration. We have calculated the cross sections for the Coulomb excitation of this level by 80 MeV ^{16}O ions using the Coulex program [12] and assuming assignments of $4_{\gamma\gamma}^+$ with $B(E2,2_\gamma^+\rightarrow 4_{\gamma\gamma}^+)=0.22\,e^2b^2$ [9,15] or 3^- with $B(E3,0^+\rightarrow 3^-)=0.2\,e^2b^3$, yielding $\sigma(1414\,\text{keV},I^\pi)/\sigma(891\,\text{keV},4_\gamma^+)=2.9\times10^{-2}$ and 57×10^{-2} for the 4^+ and 3^- assignments, respectively. For the experimental ratio of yields, calculated from the deexciting γ rays, we obtain 5.7×10^{-2}. Thus our result does not contradict the two-phonon interpretation, although a 3^- assignment with a small $B(E3)$ would be equally probable.

B. Octupole vibrational bands

The negative-parity levels observed in the present work are shown in Fig. 3. Additional levels with $I^\pi=1^-$ and 3^- are proposed at 1078 and 1183 keV [5–7], which we do not observe because of the small $B(E3,0^+\rightarrow 3^-)$ values and the predominant decay of both levels to the 0^+ and 2^+ members of the ground band. These two levels, as well as the 3^- and 5^- levels at 1106 and 1209 keV (see Fig. 3), seem well established which lead to the suggestion that they are members of the $K^\pi=1^-$ and 2^- octupole bands, respectively. Naturally, this raises the question of the location of the 2^- members of these two bands. Gerl *et al.* [6] and more recently Kröll [7] suggest levels at 1054 and 1121 keV as 2^- members of the 2^- and 1^- octupole band, respectively, but in accordance with McGowan and Milner we assign $I^\pi=2^+$ to these levels (see discussion in the following section). As already pointed out by McGowan and Milner the unnatural-parity 2^- levels are populated only very weakly in Coulomb excitation. We have calculated the cross sections for the 1054 and 1121 keV levels in Coulomb excitation with 80 MeV ^{16}O ions assuming the 2^- assignment and the $E3$ matrix elements of Ref. [5] with the result $\sigma(1054\,\mathrm{keV},2^-2)=0.3\,\mathrm{mb}$ and $\sigma(1121\,\mathrm{keV},2^-1)=0.02\,\mathrm{mb}$. Again the small cross sections are a general property for Coulomb excitation resulting from the destructive interference of the two pathways for excitation $0^+(E3)3^-(E2)2^-$ and $0^+(E2)2^+(E3)2^-$.

Most of the γ rays shown in Fig. 3 as depopulating the negative-parity levels were already proposed previously. The γ-ray intensities reported by different authors for the $E1$ transitions depopulating the members of the $K^\pi=0^-$ band are summarized in Table IV. For the intensity ratios $I_\gamma[E1,I^-\rightarrow(I+1)^+]/I_\gamma[E2,I^-\rightarrow(I-2)^-]$ from the $K^\pi=0^-$ band the following results are reported in {Ref. [16]/ Ref. [7]/present work}: {5.2(10)/7.0(5)/6.5(13)} and {1.54(23)/1.34(5)/1.42(28)} for $I=7$ and 9, respectively. The 331.3 keV γ ray (3^-; 1106 keV$\rightarrow 3^-$; 774 keV transition) was observed by McGowan and Milner [5] who assume that it populates the 3^- level (and not the degenerate 2^+ level) and derive $>90\%$ $M1$ multipolarity from the measured angular distribution of this γ ray. The $3^-\rightarrow 3^-$ assignment is confirmed by the present work and that of Ref. [7]. The 325.0 keV γ ray (5^-1209 keV$\rightarrow 5^-$; 884 keV transition) is also reported in [7], although with an energy of 324.2 keV.

The octupole bands in ^{232}Th are uniquely suited for an analysis in terms of Coriolis couplings. The spacing between the $K^\pi=1^-$ and 2^- bands is so small that an almost complete mixing of these bands is expected [5,17]. Unfortunately, their 2^- members are not yet experimentally established and, therefore, a complete calculation is not possible. However, a coupling calculation with the known 1^-, 3^-, and 5^- levels could give limits for the parameters involved and some constraints on the location of the 2^- levels. We have, therefore, performed a calculation assuming unperturbed energies $E(K,I)=E_0(K)+A\cdot I(I+1)$, with a common A parameter for all bands, and interaction matrix elements

$$\langle K+1,I|H_c|KI\rangle=\sqrt{(1+\delta_{K,0})}\cdot\sqrt{(I-K)(I+K+1)}\cdot h_{K,K+1}.$$

In a first step we have ignored the $K^\pi=3^-$ band, leaving six parameters to be determined in the calculation (A, h_{01}, h_{12}, and the bandhead energies of the $K=0$, 1, and 2 bands). We have varied the parameter h_{01} and determined the remaining parameters by reproducing the energies of the experimentally known two 1^- and three 3^- levels. The results for the parameters A and h_{12}, and the 2^- and 5^- levels, for which experimental energies have been suggested, are shown in Fig. 4.

As is apparent from Fig. 4 the location of the two experimentally established 5^- levels requires $|h_{01}|\approx 13\,\mathrm{keV}$ and $|h_{12}|\approx 12\,\mathrm{keV}$. These values are very close to those found in a similar analysis of the octupole bands in ^{228}Th [18]. To check the influence of the $K^\pi=3^-$ band on the results of the calculation we have performed calculations assuming reasonable values for h_{23} and determining the remaining parameters by reproducing the experimentally known 1^-, 3^-, and 5^- levels. With $|h_{23}|=15\,\mathrm{keV}$ this procedure yields $A=7.21\,\mathrm{keV}$, $|h_{01}|=15.0\,\mathrm{keV}$, $|h_{12}|=11.3\,\mathrm{keV}$ and the energies listed in Table V.

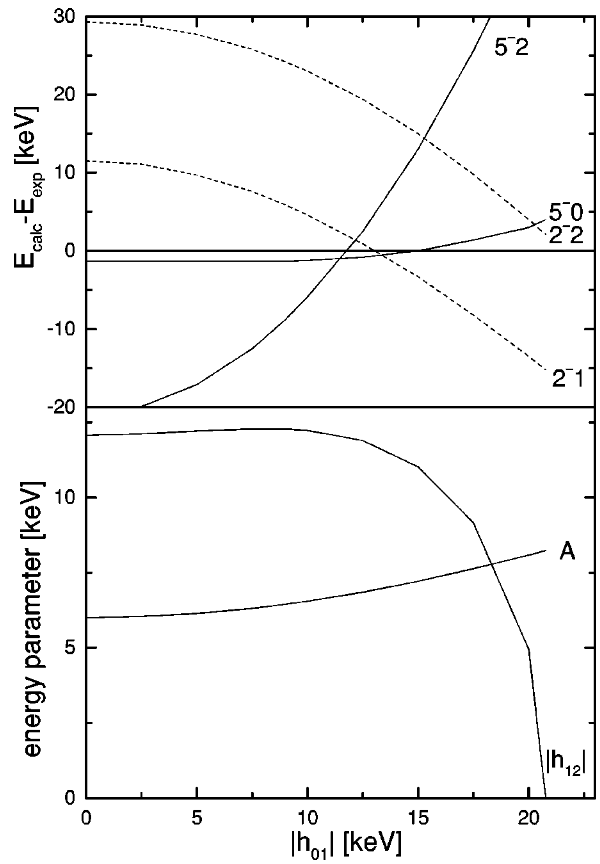

FIG. 4. Results of a calculation of the Coriolis coupling of the octupole bands in ^{232}Th. Lower part: inertial parameter A and Coriolis matrix element h_{12}. Upper part: deviation of the calculated energies from experimental ones proposed at 1054 keV (2^-2), 1122 keV (2^-1), 884 keV (5^-0), and 1209 keV (5^-2).

The following conclusions can be drawn from these calculations: (i) The $K^\pi=1^-$ admixtures to the members of the $K^\pi=0^-$ band are essentially independent of the location of the $K^\pi=3^-$ band and are close to the first-order results. In

TABLE V. Energies of bandheads and rotational levels of the octupole bands, calculated with $A=7.21\,\mathrm{keV}$, $|h_{01}|=|h_{23}|=15\,\mathrm{keV}$, and $|h_{12}|=11.3\,\mathrm{keV}$. The energies marked by a star are the experimental energies which were used to determine the parameters A, $h_{K,K+1}$, and $E_0(K)$.

Quantity (keV)	$K^\pi=0^-$	$K^\pi=1^-$	$K^\pi=2^-$	$K^\pi=3^-$
$E_0(K)$	702.5	1061.2	1047.3	1220.5
E_{calc}; $I^\pi=1^-$	714.4*	1078.1*		
2^-		1121.2	1073.8	
3^-	774.4*	1182.8*	1105.5*	1314.8
4^-		1237.8	1141.4	1382.4
5^-	883.7*	1337.6	1208.8*	1466.6

this approximation the amplitudes of admixture have the form $c_0(I)=\mp\sqrt{2I(I+1)}\epsilon_{01}$ with $\epsilon_{01}=h_{01}/[E_0(K=1)-E_0(K=0)]$. The calculation discussed above yields $|\epsilon_{01}|=0.042$ and $c_{\rm calc}(I)/c_0(I)=0.99(0.99)$, $0.95(0.76)$, and $0.89(0.72)$ for the $K^\pi=0^-(1^-)$ band and $I=1$, 3, and 5, respectively. (ii) With reasonable parameters it is not possible to obtain an energy of less than 1070 keV for the lowest 2^- level. We consider this as additional evidence that the level observed in Coulomb excitation at 1054 keV cannot be the first-excited 2^- level. (iii) The calculated energy of the second-excited 2^- level listed in Table V agrees exactly with that proposed in Ref. [7] for this level. It might thus be that the 1122 keV level observed in the Coulomb excitation is a $(2^+/2^-)$ doublet, although its dominant part must belong to the 2^+ level. (iv) The 5^- member of the $K^\pi=1^-$ band is proposed in Refs. [5] and [7] at 1293.0 and 1293.9 keV, respectively, 45 keV below the calculated energy. We note, however, that the proposed levels are based in both cases on a single observed transition (959.7 keV; $5^-\to6^+$ in Ref. [5] and 410.0 keV; $5^-\to5^-$ in Ref. [7]), which, moreover, at least in the case of the 1293.0 keV level, is placed twice in the level scheme (959.9 keV; $2^+\to4^+$ transition from the 1122 keV; 2^+ level). These assignments must, therefore, be considered as doubtful. (v) An assignment of the 3^- and 5^- levels at 1106 and 1209 keV as members of a $K^\pi=3^-$ band [19] leads to unreasonable values for the coupling parameters and can almost certainly be excluded.

A level at 1073 keV is reported in the literature [4,5], for which only a single depopulating transition to the 2^+ member of the ground band is observed and which was assigned as 2^+ level [4,5]. A 2^- assignment was presumably not considered in the Coulomb excitation study by McGowan and Milner because of the strong excitation of this level derived from the intensity of the depopulating γ ray (see Sec. 3.3 of Ref. [5]). However, in the case of such an assignment the 1073 keV level might be populated by weak but highly converted $M1$ or $E2$ transitions from the strongly excited higher-lying 3^- levels. For example, the total intensity of a 1106 keV→1073 keV rotational $E2$ transition, calculated with the wave functions and transition-matrix elements discussed below, would account for 40% of the population of the 1073 keV level observed in the Coulomb excitation. We, therefore, consider this level as a candidate for the missing bandhead of the $K^\pi=2^-$ band.

The Coriolis coupling of the octupole bands is known to also strongly modify the matrix elements for the transitions connecting these bands with the ground band. Its dramatic influence on the $B(E3,0^+\to3^-)$ values is discussed in Refs. [5,17] (we note here, with reference to the discussion in Sec. 4.1 of Ref. [5], that the calculation presented in Table V yields $c_{\rm admixed}/c_{\rm main}=\pm0.60$ for the amplitudes of the $K=1$ and 2 components of the 1106 and 1183 keV 3^- levels). For the $E1$ transitions only the $K^\pi=0^-\to0^+$ and $K^\pi=1^-\to0^+$ transition-matrix elements enter if one neglects K admixtures to the ground band. Since the mutual mixing of the $K^\pi=0^-$ and 1^- bands is close to the first-order approximation, the $B(E1)$ values from these bands to the ground band are most transparently expressed in terms of the reduced mixing amplitude ϵ_{01} discussed above and the ratios of intrinsic transition-matrix elements $R_0=\langle0^+|M(E1,-1)|1^-\rangle/\langle0^+|M(E1,0)|0^-\rangle$ and $R_1=1/R_0$ as

$$\frac{B(E1,I^-K\to(I+1)^+K)}{B(E1,I^-K\to(I-1)^+K)}=\frac{I+1-K}{I+K}\left|\frac{1-(I+K)\cdot z_K}{1+(I+1-K)\cdot z_K}\right|^2 \tag{2}$$

with $z_K=(-)^K\sqrt{2}\cdot\epsilon_{01}\cdot R_K$. The γ-ray intensities of the $E1$ transitions depopulating the 714 keV 1^- level yield $z_0=-0.176(10)$. The intensities calculated with this value are listed in the last column of Table IV. Using a value of $|\epsilon_{01}|=0.04(1)$ one obtains $|R_0|=3.1(8)$. This result differs from that derived in Ref. [5] by approximately a factor of 2, which is mainly due to the fact that the amplitude of admixture used in Ref. [5] is 1.8 times as large as ours.

The γ-ray intensities of the $E1$ transitions from the 1078 and 1183 keV members of the $K^\pi=1^-$ band, calculated with $z_1=-2\epsilon_{01}^2/z_0=0.018$, are listed in the last column of Table IV. They are in fair agreement with the experiment, thus supporting the small value of z_1.

The absolute value of the $K^\pi=0^-\to0^+$ $E1$ transition-matrix element can be derived from the experimental $E2/E1$ γ-ray intensity ratios. Taking into account a slight decrease of the z_0 value with increasing spin, as suggested by the coupling calculation, one obtains from the I_γ ratios quoted above for the transitions from the 7^- and 9^- levels a consistent ratio of $\langle0^+|M(E1,0)|0^-\rangle/\langle0^-|M(E2,0)|0^-\rangle=(2.5\pm0.5)\times10^{-5}\ \mathrm{fm}^{-1}$. Assuming an equal intrinsic electric quadrupole moment for the $K^\pi=0^-$ band as for the ground band ($Q_0=962\ \mathrm{fm}^2$) one obtains

$$\begin{aligned}|\langle0^+|M(E1,0)|0^-\rangle|&=(7.6\pm1.5)\times10^{-3}\mathrm{e\ fm}\\&=(4.9\pm1.0)\times10^{-3}\mathrm{W.u.},\\|\langle0^+|M(E1,0)|1^-\rangle|&=(2.3\pm0.7)\times10^{-2}\mathrm{e\ fm}\\&=(1.5\pm0.5)\times10^{-2}\mathrm{W.u.}\end{aligned}$$

The small value for the $0^-\to0^+$ matrix element reflects the octupole vibrational character of the negative-parity levels in ^{232}Th, in contrast to the lighter Th isotopes which are expected to be octupole deformed (see, for example, Fig. 17 of Ref. [16]). The $1^-\to0^+$ matrix element corresponds to $B(E1,0^+0\to1^-1)=2\times10^{-4}$W.u. which is fairly close to the value of 5×10^{-5}W.u. calculated within the quasiparticle-phonon nuclear model for the corresponding band in ^{228}Th [3].

Finally, we should comment on the signs of the parameters involved in the coupling calculation. The signs of the $h_{K,K+1}$ enter in the wave functions, but not in the energies. Consequently, any statement on the signs must rely on theoretical arguments. In the spherical limit the Coriolis matrix elements have the form $h_{K,K+1}=-A\sqrt{(3-K)(3+K+1)}$ and the theoretical calculations give values close to this limit [17]. With $A=7.21$ the spherical limits are $h_{K,K+1}=-25.0$, -22.8, and -17.7 keV for $K=0$, 1, and 2, repectively. The results of our coupling calculation are close in

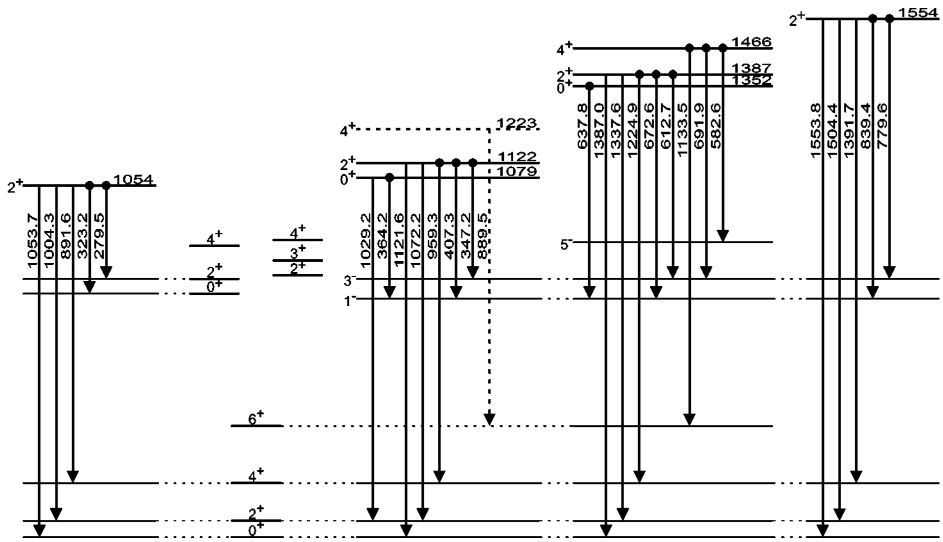

FIG. 5. Partial level scheme of ^{232}Th showing the second- and higher-excited $K^\pi=0^+$ and 2^+ bands observed in the present work (for completeness the first-excited bands are also indicated). Transitions observed in the $\gamma\gamma$ coincidences are marked by dots. The additional transitions shown are reported in Ref. [5]; the listed γ-ray energies are the differences of the level energies.

magnitude to these values suggesting that the $h_{K,K+1}$ are negative. In this case ϵ_{01} is negative and the two $E1$ matrix elements have the same sign. This is also supported by the $B(E3)$ values as discussed by McGowan and Milner [5].

C. Additional low-lying positive-parity excitations

A partial level scheme showing the positive-parity bands observed in the present work above the β and γ bands is shown in Fig. 5. McGowan and Milner [5] propose four additional 2^+ levels below 1.4 MeV which could not be observed in the present work due to their small $B(E2)$ strength and their predominant decay to the 0^+ and 2^+ members of the ground band. One more 2^+ level at 1477 keV is reported in Ref. [5] to be excited approximately half as strongly as the 1387 keV 2^+ level, and to decay to the 2^+ member of the β band by a 30% branch with a 703 keV γ ray. We do not observe this γ ray in coincidence with the 774 keV transition depopulating the 2^+_β level, with an intensity limit at least a factor of 4 below the intensity reported in Ref. [5]. In the following we will discuss the levels-bands shown in Fig. 5 in detail.

(a) 1054 keV level. We observe two γ rays populating the 0^+ and 2^+ members of the β-band, which result from a level at 1053.7 keV. We assume that this level is identical with those previously observed at 1054.0 keV and assigned as 2^+ and 2^- levels in Refs. [5] and [7,19], respectively. The transition to the 0^+_β level excludes the 2^- assignment. We note here that Gerl *et al.* [8] report the discovery of a new state at 1055.3 keV decaying to the 0^+_β level, which corresponds to a γ-ray energy of 324.7 keV in definite disagreement with the energy of the corresponding γ-ray observed in the present work (323.2 keV).

The structure of the 1054 keV 2^+ level is not entirely clear, although the low intensity of the γ transition to the 4^+ member of the ground band [5] might indicate a $K^\pi=2^+$ assignment. If correct, it would be the bandhead of a second-excited $K^\pi=2^+$ band located $\Delta E=268$ keV above the first-excited one. Similar bands are known in the neighboring nuclei ^{228}Th, ^{230}Th, and ^{234}U with $\Delta E=185$, 228, and 199 keV, respectively. In the case of ^{234}U, where this band was observed first, it was interpreted as a highly anharmonic two-phonon $\beta\gamma$ vibration [20]. But this interpretation has been questioned considering the low excitation energy of this band [2].

One further important point should be mentioned: if the $K^\pi=2^+$ assignment is correct, the 1054 keV level could de-excite by a strong, as yet unobserved, $E0$ transition to the first-excited $I^\pi K=2^+2$ level, as established in the neighboring nuclei. This would then increase the $B(E2)$ value derived in Ref. [5] from the intensities of the depopulating transitions, possibly by up to a factor of 10 (see, for example, Ref. [21]).

(b) Second-excited $K^\pi=0^+$ band. The levels at 1078.6 and 1121.6 keV are interpreted as the lowest members of a second-excited 0^+ band [4,19]. The most interesting feature of this band is the comparatively strong $E1$ branching to the $K^\pi=0^-$ band. We observe a 364.2 keV γ-ray populating the 714.4 keV 1^- level (see Fig. 2) which fixes the energy of the depopulated level at 1078.6 keV. This energy is in accordance with that given previously for the 0^+ level [4] and excludes its placement as depopulating the 1078.2 keV 1^- level proposed in Ref. [7]. The 347.2 keV γ ray ($2^+\to3^-$ transition) was previously observed by McGowan and Milner [5] and Kröll [7], although in the latter work with $E_\gamma=348.4$ keV. The 407.3 keV γ-ray ($2^+\to1^-$ transition) is reported in Ref. [7] but not in Ref. [5].

The assignment of $I^\pi=2^+$ to the 1122 keV level, in contrast to a 2^- assignment, also proposed in the literature (see discussion in Sec. III B), is supported by the observation of the γ transition to the 4^+ member of the ground band both in the present work and that of Ref. [5]. The γ-ray intensities reported there for the $E1$ and $E2$ transitions from the 1122 keV level to the ground and $K^\pi=0^-$ bands support the $K=0$ assignment: the $E2$ branching to the 0^+ and 4^+ levels can be explained for $K=0$ with the generalized intensity relation with a reasonable ratio of the M_1 and M_2 matrix elements, whereas this ratio would be less reasonable for a $K=2$ assignment. The $E1$ transitions would be K forbidden for $K=2$, whereas the calculated intensity ratio $I_\gamma(347\,\mathrm{keV})/I_\gamma(407\,\mathrm{keV})=0.93$ for $K=0$ is in agreement with the observed ratio.

The inclusion of the 407 keV transition in the Coulomb excitation analysis of McGowan and Milner increases the $B(E2,0^+\rightarrow 2^+)$ for the 1122 keV level by $\approx 15\%$. However, it has to be emphasized again that the $B(E2,0^+\rightarrow 2^+)$ reported in Ref. [5] has been derived ignoring a possible 1122 keV$\rightarrow$49 keV$(2^+\rightarrow 2^+)E0$ transition which can be significant [22].

(c) Third-excited $K^\pi=0^+$ band. This band with its bandhead at 1352 keV was first proposed by Kröll and Gerl *et al.* [7,8]. The energy of its 2^+ member obtained in the present work agrees with that proposed for a 2^+ level observed in earlier Coulomb excitation (1387.2 keV) [5], suggesting that this latter 2^+ level is the 2^+ member of this 0^+ band. The $K^\pi=0^+$ assignment for the 2^+ level observed in Ref. [5] is supported by the intensities reported there for the γ rays depopulating the 2^+ level to the 0^+, 2^+, and 4^+ members of the ground band. In Refs. [7,8] and the present work only the $E1$ transitions to the $K^\pi=0^-$ band were observed. The intensity ratios $I_\gamma[I^+\rightarrow(I+1)^-]/I_\gamma[I^+\rightarrow(I-1)^-]$ from (Ref. [7])/(present work)/(calculation with $K=0$) are 1.5(5)/1.8(5)/1.13 and 1.3(2)/2.6(6)/0.75 for $I=2$ and 4, respectively. The experimental results are in reasonable agreement. The discrepancies with the calculations can be explained by the $K^\pi=1^-$ admixtures to the 0^- band, but a quantitative analysis is not justified due to the large uncertainties of the experimental results.

The arrangement of the levels at 1352, 1387, and 1466 keV as 0^+, 2^+, and 4^+ members of a $K^\pi=0^+$ band was proposed by Kröll, Gerl *et al.* [7,8], who assign in addition levels at 1586 and 1740 keV as 6^+ and 8^+ members of this band. It is interpreted by these authors as a two-phonon octupole vibration, based on (1) its bandhead energy, which is close to twice the energy of the $K^\pi=0^-$ octupole vibration $[2\cdot E_0(K^\pi=0^-)=1405\,\mathrm{keV}]$ and (2) an intrinsic $E3$ matrix element connecting the one-phonon and two-phonon vibrations of 1.2(4) e b$^{3/2}$ close to $\sqrt{2}\cdot\langle 0_g^+|M(E3,0)|0^-\rangle=1.0(1)\,e\,\mathrm{b}^{3/2}$ [5] as predicted in the harmonic limit. However, this $E3$ matrix element was obtained assuming a pure double-$E3$ excitation of the two-phonon vibration via the one-phonon vibration, neglecting the direct $E2$ excitation. This assumption seems questionable in view of the comparatively strong $E2$ excitation of the 1387 keV 2^+ level observed in Coulomb excitation with ^{4}He ions [5], where double $E3$ excitation is expected to be negligible.

To check on the nature of the excitation of the 1387 keV 2^+ level we have performed Coulex calculations [12] of the cross section for single $E2$ excitation (denoted σ_{E2}) or double $E3$ excitation ($\sigma_{E3,E3}$). For the transition-matrix elements the following values—derived from the results of Ref. [5] taking into account the additional depopulation of the 2^+ level by the 613 keV γ ray—were used:

$$\langle 0^+|M(E2,0)|0_g^+\rangle=0.115\,\mathrm{e\,b},$$

$$\langle 0^-|M(E3,0)|0_g^+\rangle=0.735\,\mathrm{e\ b}^{3/2},$$

and

$$\langle 0^+|M(E3,0)|0^-\rangle=\sqrt{2}\cdot\langle 0^-|M(E3,0)|0_g^+\rangle.$$

These calculations yield $\sigma_{E3,E3}(1387\,\mathrm{keV},2^+)/\sigma_{E2}(1387\,\mathrm{keV},2^+)=5.2\times10^{-4}$, 2.5×10^{-2}, and 7.2×10^{-2} for 18 MeV ^{4}He, 80 MeV ^{16}O, and 265 MeV ^{58}Ni projectiles, respectively, and $\sigma_{E2}(1387\,\mathrm{keV},2^+)/\sigma(891\,\mathrm{keV},4_\gamma^+)=0.13$ for 80 MeV ^{16}O projectiles. The latter result agrees with the corresponding ratio of yields of 0.15, derived from the intensities of the depopulating γ rays listed in Table II. We, therefore, conclude that the 1352 keV 0^+ band is populated in Coulomb excitation by direct $E2$ transitions with only small contributions from double $E3$ excitations via the $K^\pi=0^-$ band.

Another somewhat disturbing property of the 0_3^+ band is its moment of inertia. The rotational energy splitting $E(2^+)-E(0^+)$ has values of 49.4, 44.1, 43.0, and 34.8 keV for the ground, 0_1^+, 0_2^+, and 0_3^+ band, respectively. The increase of the moments of inertia of the excited bands compared to the ground band is a general property of these bands in the actinide region (see, for example, Refs. [22,23], and references given there), but the value for the 0_3^+ band seems somewhat too high. Kröll suggests that the reason for this small energy splitting might be the interaction of the 0^+ band with a higher-lying 1^+ band [7]. An interesting possibility for such a band would be one with the 1554 keV level discussed below as its 2^+ member.

(d) 1554 keV level. This level was observed previously in the ^{232}Th$(n,n'\gamma)$ reaction and in Coulomb excitation with α particles [4,5] with $I^\pi=2^+$ assigned on the basis of its γ decay to the ground band. McGowan and Milner report a deexcitation of this level, in addition to that shown in Fig. 5, by γ transitions to the 0_β^+ and 2_β^+ levels with $B(E2)$ values of $\sim$45 s.p.u. and to the 2_γ^+ level with 16 s.p.u. [5]. From an analysis of the expected $\gamma\gamma$ coincidences we obtain intensity limits for these transitions, in the normalization of Table II, of $I_\gamma<0.5$, thus excluding the γ-ray intensities reported in Ref. [5] (we would like to mention that the three γ rays in question were assigned doubly in Ref. [5]). The $B(E\lambda)$ values quoted in Ref. [5] for the 1554 keV level have to be reduced by approximately a factor of 2. Nevertheless, they are still the largest ones for all levels discussed in this section.

From the γ-ray intensities of the $E2$ transitions depopulating the 1554 keV level to the ground band reported in Coulomb excitation and in the ^{232}Th$(n,n'\gamma)$ reaction one obtains ratios of $B(E2,2^+\rightarrow 4^+)/B(E2,2^+\rightarrow 0^+)\approx 2.2$ and 1.3(5), respectively [4,5]. The corresponding theoretical values are 2.57, 1.14, and 0.07 for $K=0$, 1, and 2, respectively, and thus a $K=2$ assignment seems unlikely. For the $E1$ transitions to the $K^\pi=0^-$ band our data agree with those of Ref. [5] yielding $I_\gamma(780\,\mathrm{keV};2^+\rightarrow 3^-)/I_\gamma(839\,\mathrm{keV};2^+\rightarrow 1^-)=2.0$, compared to theoretical ratios of 1.2 and 0.53 for $K^\pi=0^+$ and 1^+, respectively. However, as discussed above, McGowan and Milner interpret the 780 keV γ ray as a doublet populating both the degenerate 2^+ and 3^- levels at 774 keV. If we assume that this γ ray results from the 2^+

$\rightarrow 3^-$ transition alone the above quoted intensity ratio, derived from the singles γ-ray intensities of Ref. [5], increases by a factor of ~ 3, a not yet understood discrepancy (the 780 keV γ-ray observed in the singles γ-ray sectrum might contain a small contribution from a $3^+_\gamma \rightarrow 2^+_g$ transition, but its intensity is probably too small to explain the discrepancy). We conclude that the experimental data favor a $K=0$ assignment, although $K=1$, which would be particularly interesting, can not be excluded.

IV. CONCLUSION

The vibrational excitations of ^{232}Th were populated in Coulomb excitation induced by ^{16}O ions. γ-γ coincidences were measured with the detector array YRAST Ball yielding information on a number of controversial assignments of levels and transitions in ^{232}Th.

The quadrupole and octupole vibrations in ^{232}Th are uniquely suitable for a study of their couplings due to the small energy spacings between the interacting bands. From a detailed discussion of these couplings we conclude that the available experimental information is still too incomplete to allow a reliable analysis. In particular, additional information on the γ decay of the β band as well as a location of the 2^- members of the $K^\pi = 1^-$ and 2^- bands would be highly desirable. Such information could possibly be obtained from new investigations of the β decay of ^{232}Ac or the ^{232}Th$(n,n'\gamma)$ reaction.

Above the 2^+_β and 2^+_γ levels at 774 and 785 keV, respectively, nine additional 2^+ states were previously proposed, between 1054 and 1554 keV, from a study of single γ rays in Coulomb excitation induced by ^{4}He ions [5]. We observe four of these 2^+ levels, find conflicting results for one of the remaining ones (1477 keV level) and propose that one more might have $I^\pi = 2^-$ (1073 keV level). We tentatively propose $I^\pi K = 2^+2$ for the 1054 keV level in accordance with established excitations in the neighboring Th and U nuclei. Our data are in agreement with previously proposed assignments of the 2^+ levels at 1122 and 1387 keV as members of $K^\pi = 0^+$ bands, with bandheads at 1079 and 1352 keV, respectively. For the latter 0^+ band a two-octupole-phonon vibrational structure has been proposed [7,8], which is not supported by the present work. Finally, our data show that the previously proposed strong $E2$ transitions from the 2^+ state at 1554 keV to members of the β and γ bands do not exist. These transitions had been of particular concern since they lead to $B(E2)$ values an order of magnitude larger than vibrational $B(E2)$ values, which would be difficult to explain with our present understanding of collectivity.

Our data confirm the existence of at least three $K^\pi = 0^+$ bands below the threshold for two-quasiparticle excitations in ^{232}Th. The results from Coulomb excitation with ^{4}He ions [5] indicate that several more 0^+ bands might exist in ^{232}Th at these energies, which are connected to the ground state with small $B(E2)$ strength. A similar variety of 0^+ excitations has only been reported for ^{228}Th [3]. This supports the statement by Sood *et al.* [2] that many missing 0^+ states remain to be located in the actinide nuclei.

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