$^{146-150}$Nd, Gd, Sm, Dy, and ^{156}Nd within the cranked Hartree-Fock-Bogoliubov scheme

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The energy level schemes and gyromagnetic factors of the excited states of a series of transitional nuclei, $^{146-150}$Nd, Gd, Sm, Dy, and ^{156}Nd have been studied within the framework of cranked Hartree-Fock-Bogoliubov formalism with the inclusion of hexadecapole deformation in the cranking Hamiltonian. The calculated results are discussed with the available experimental findings on these transitional nuclei. It has been observed that the Routhians of these transitional nuclei distinctly indicate deformation structures that can be explained on the basis of the present theoretical model scheme.

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I. INTRODUCTION

The application of both the rotation-particle coupling and quasiparticle-phonon coupling model schemes has been utilized in a few odd-*A* nuclei in the transitional region [1–3]. It has been observed that both the anharmonic version of the core-polarization effect and coriolis coupling of the rotation-particle coupling model have been successful in explaining the spreading of shell-model states in highly excited regions.

As the nuclei $^{146-150}$Nd, Gd, Sm, Dy, and ^{156}Nd possess deformed structures, an attempt has been made to understand energies and *g* factors of the excited states of the nuclei on the basis of the cranked Hartree-Fock-Bogoliubov (CHFB) approach. It can be mentioned that *g* factors provide with the actual testing ground for the validity of the present model on these transitional nuclei. Also the total Routhian surface predicts triaxial nuclear shapes with $\beta=0.25$ and $\gamma=30°$ in this transitional region.

The cranked Hartree-Fock-Bogoliubov approach has been utilized by several authors to understand the excitation spectrum of the deformed nuclei [4,5]. The neutron and proton alignments have been observed to be closely related to the rotational frequency for the change of gyromagnetic factors of the excited states of the deformed nuclei [6–9]. Also single particle energies of the valence neutron and proton states are also responsible for the variation of the *g* factors of the excited states of some rare-earth nuclei [10], as a consequence of which, a significant alignment of proton states is observed. Experimental information [11,12,9,13] also predicts the variation of *g* factors with spin angular momentum along the yrast line in case of rare-earth nuclei. This variation of *g* factors has also been explained on the basis of the CHFB approach [14,15,8,16].

In the present case we have calculated energies and *g* factors of the above transitional nuclei. The input values of the deformation parameters have been calculated for 0^+ spin state of $A=146$ nuclei. These values have been inserted for the diagonalization of the cranking Hamiltonian matrices for the higher spin states of the successive nuclei under study. For the nuclei, e.g., Nd, Dy, Gd, and Sm, the input values of the parameters are for $A=146$. After self-consistency, the energies and *g* factors of the states have been obtained and these values have been reported in Sec III. We have made no attempt to vary the input parameters obtained from the values on 0^+ state.

II. CRANKED HFB APPROACH

The model has already been discussed in recent works on Xe and Ba nuclei [17]. The deviation from the preceding works is the inclusion of hexadecapole term in the cranking Hamiltonian.

A self-consistent HFB calculation including the hexadecapole term [18] has been incorporated into the present calculation. The one body Hamiltonian, which takes care of this deformation, is

$$h_4=\sum D_{4\mu}Q_{4\mu}, \quad \mu=-4,-2,0,2,4, \tag{1}$$

with the following term:

$$D_{4\mu}=\chi_4\sum_{\nu} \alpha_\nu(Q_{4\mu})_\nu=(-1)^\mu D_{4-\mu}. \tag{2}$$

We write for β_4 as

$$\hbar\omega_0\beta_4=D_{40}. \tag{3}$$

TABLE I. Hexadecapole input parameters in Nd, Gd, Sm, and Dy nuclei for the 0^+ state. The mass number $A=146$.

D_{20}	D_{22}	D_{40}	D_{42}	D_{44}
			Nd nucleus	
0.509	0.033	0.032	0.008	0.001
			Gd nucleus	
0.500	0.051	0.079	0.017	0.021
			Sm nucleus	
0.400	0.029	0.020	0.006	0.011
			Dy nucleus	
0.553	0.331	0.200	0.127	0.055

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TABLE II. The energies (E) in MeV and g factors of $^{146-150}$Nd, Gd, Sm, Dy, and ^{156}Nd. The values within the parentheses indicate experimental estimates. In case of ^{156}Nd the experimental and theoretical g factors are $g(I)/g(2)$.

I^+	E	g
		^{146}Nd
2	0.423(0.454)	0.280(0.250)
4	0.987(1.044)	0.321
6	1.510(1.780)	0.445
8	2.110(2.475)	0.553
10	2.880	0.590
		^{148}Nd
2	0.323(0.302)	0.543(0.560)
4	0.950(0.753)	0.640
6	0.989(1.275)	0.863
8	1.397	0.0412
10	1.443	0.976
		^{150}Nd
2^+	0.145(0.130)	0.3187
4^+	0.561(0.381)	0.168
6^+	0.844(0.720)	0.0156
8^+	1.012	−0.0965
10^+	1.500	−0.0978
		^{156}Nd
2^+	0.067(0.0669)	–
4^+	0.290(0.221)	0.900(0.990)
6^+	0.467(0.460)	1.001(0.920)
8^+	0.880(0.778)	1.100(0.930)
10^+	1.260(1.169)	1.200(0.890)
		^{146}Gd
2^+	1.771(1.971)	−0.035
4^+	2.656(2.611)	0.114
6^+	2.800	0.400
8^+	3.400	0.700
10^+	4.100	0.800
		^{148}Gd
2^+	0.566(0.784)	0.076
4^+	1.233(1.416)	−0.315
6^+	1.890	0.450
8^+	2.100	0.560
10^+	2.870	0.600
		^{150}Gd
2^+	0.588(0.638)	0.040
4^+	0.988(1.288)	−0.067
6^+	1.560(1.936)	0.200
10^+	2.300(2.554)	0.300
		^{146}Sm
2^+	0.647(0.747)	−0.031
4^+	1.231(1.381)	−0.002
6^+	1.800(1.811)	0.100
8^+	2.560(2.737)	0.340
10^+	3.100	0.320
		^{148}Sm
2^+	0.443(0.550)	0.281
4^+	1.001(1.180)	0.233
6^+	1.760(1.900)	0.321
8^+	2.670(2.554)	0.453
10^+	3.121	0.670
		^{150}Sm
2^+	0.338(0.334)	0.188
4^+	0.667(0.773)	0.263
6^+	1.00(1.278)	0.234
8^+	1.440(1.504)	0.310
10^+	1.670	0.550
		^{146}Dy
2^+	0.5880(0.683)	0.800
4^+	1.856(1.608)	0.936
6^+	2.100	1.160
8^+	2.560	1.232
10^+	2.870(2.935)	1.200
		^{148}Dy
2^+	1.500(1.677)	1.150
4^+	2.10(2.426)	1.218
6^+	2.450(2.731)	1.340
8^+	2.667(2.832)	1.560
10^+	2.880	1.640
		^{150}Dy
2^+	0.900(0.803)	0.200
4^+	1.200(1.450)	0.568
6^+	1.500(1.850)	0.630
8^+	2.00(2.400)	1.100
10^+	2.560(3.025)	1.450

So we have now five deformation parameters, which are β_2, γ, β_4, D_{42}, and D_{44}. The pairing and quadrupole force constants are

$$G_p = 27/A \ \text{MeV}, \quad G_n = 22/A \ \text{MeV}, \quad \chi_2 = 70A^{-1.4} \ \text{MeV},$$

$$\hbar\omega_0 = 41.2A^{-1/3} \ \text{MeV}.$$

When hexadecapole force is taken into account, the interaction strengths are altered as

$$\chi_2 = \chi_4 = 70/A^{1.4} \ \text{MeV}, \quad G_p = 28/A \ \text{MeV},$$

$$G_n = 23/A \ \text{MeV}.$$

The g factors have been calculated from $\hat{\mu}_x$, the x component of the magnetic moment operator μ:

$$g_I = \langle \psi_{CHFB} | \hat{\mu}_x | \psi_{CHFB} \rangle / I_x, \tag{4}$$

where

$$\hat{\mu}_x = g_l \sum_i \hat{j}_x(i) + (g_s - g_l) \sum_i \hat{s}_x(i), \tag{5}$$

TABLE III. The theoretical and experimental energies (E) in MeV and g factors of the excited states of Nd nuclei without the inclusion of hexadecapole deformation in the CHFB scheme. The numerical figures within the brackets indicate experimental informations. In the case of ^{156}Nd, g is the ratio of $g(I^{\pi})$ and $g(2^{+})$.

I^{π}	E	g
		^{146}Nd
2^+	0.411(0.454)	0.180(0.250)
		^{148}Nd
2^+	0.320(0.302)	0.468(0.568)
		^{156}Nd
2^+	0.055(0.067)	-
4^+	0.250(0.221)	0.880(0.990)
6^+	0.400(0.460)	0.700(0.920)
8^+	0.812(0.778)	1.500(0.930)
10^+	1.200(1.169)	1.414(0.890)

and the free-particle g factors are

$$g_l=1, \quad g_s=5.586 \ \text{ for protons,}$$

$$g_l=0, \quad g_s=-3.826 \ \text{ for neutrons.}$$

The adopted g values which are the spin g factors can be written as

$$g_s=0.6g_s \ \text{(free)}.$$

So we have

$$g_I=[\langle \hat{I}_x^p\rangle+2.351(\langle \hat{s}_x^p\rangle-0.98\langle \hat{s}_x^n\rangle)]/\sqrt{I(I+1)}. \qquad (6)$$

III. RESULTS

In the present calculation we have taken oscillator shells $N=4$ and 5 for protons and $N=5$ and 6 for neutrons as basis states. ^{110}Zr$_{40,70}$ has been taken as inert core [17]. The coupling constants, e.g., the input values for λ_p, λ_n, Δ_p, and Δ_n, have been taken from Ref. [19]. We have shown the relevant hexadecapole parameters that have been taken as input parameters for the nuclei under studies in Table I. These have been chosen from self-consistency conditions for each nucleus having a 0^+ state. We have attempted no optimization of these hexadecapole parameters. The inclusion of the hexadecapole deformation term into the cranking Hamiltonian is justified because of the fact of the close proximity of the theoretical g factors with the available experimental ones. The physics is optimistic as the nuclei lie in the fringe of the well-deformed rare-earth region. The alignment of neutron-proton angular momentum on the symmetry axis is closely connected with deformation. This particular behavior has also been observed on Xe-Ba nuclei.

The energies and g factors of 2^+-10^+ states of $^{146-150}$Nd, Gd , Sm, and ^{156}Nd are depicted in Table II. The calculated results have been compared with the experimental energies of the excited states [13,20–25] and the available g factors in this mass region. Further, in order to exhibit the validity of the CHB approach with the inclusion hexadecapole term in the cranking Hamiltonian, the nuclear g factors of $^{146-150,156}$Nd have been compared with the experimental g values of the excited states. The g factors almost coincide with the experimental values in this transitional region.

The calculated g factors of the present nuclei show the particular alignment of the conserved angular momentum along the symmetry axis. Consequently the g factors of these transitional nuclei can be explained on the basis of the present approach.

To justify the relevance of the hexadecapole terms, we applied the CHFB approach without the inclusion of the same to calculate energies and g factors of the 2^+ excited states of Nd isotopes and have compared the calculated results with experiment. Comparison of the two tables (Tables II and III) clearly indicates the justification of the inclusion deformation term in the cranking Hamiltonian. The physics behind the inclusion of the hexadecapole term within the cranking Hamiltonian is justified because of the alignment of the neutron-proton spin along the symmetry axis for obtaining the proximity of the calculated g factor with the available experimental results.

Let us now clarify the present results with the reported theoretical results concerning the deformed structures of the nuclei lying in the fringe of the deformed region. Within the framework of the static mean field calculations, Woods-Saxon [26], Hartree-Fock + BCS with Skyrme effective parameters [27], a series of nuclei with A ranging from 132 to 136 has been investigated. These deformed nuclei have the quadrupole deformation parameter 0.28–0.40. In the present case for the highest spin state in A-150 nuclei the values lie below 0.1. The strongly prolate shaped nuclei whose Z values range within 53–69 also have the same deformation parameter within 0.30–0.35 and it has been reported from relativistic Hartree-Bogoliubov calculations with Gognoy interaction [28]. The cranked relativistic Hartree-Bogoliubov calculation with pairing interaction of this two body Gogny type [28] has also been applied to superdeformed bands of the nuclei $A=190$ claiming a larger value for the said deformation parameter. The neutron pairing gap parameter from the experimental findings on deformed nuclei ($A=130-150$) has been found to be within 0.6–0.9 [29] and it has also been corroborated from theoretical findings on cranked Hartree-Fock-Bogoliubov calculations [30]. The values of the gap parameter as well as the deformation parameter are also justified from the present calculation. The shell correction procedure has been adopted for the study of ^{150}Gd [31]. Within the framework of the Strutinsky approach [32], the pairing correlation for any general triaxial-shaped nuclei [31] has been applied to the Hartree-Fock-Bogoliubov method. In this calculation a comparatively large value for the neutron gap parameter for very high spin states has been reported. In the present case, for all the spin states we have obtained neutron-proton gap parameters that lie within 0.55–0.88 MeV and the corresponding quadrupole deformation parameters range within 0.3–0.5. This proves the justification of our calculated results. These quoted parameter values are obtained from the self-consistent iterative method leading to

g values and energies mentioned in the following tables. Finally, for inclusion of the hexadecapole term, we have seen that transitional nuclei possess nonzero or small negative hexadecapole deformations and γ and β_4 degrees of freedom cannot be separated from each other [33]. Also shape deformations at a correct mass number can only be explained if we take care of hexadecapole deformations [34]. Also transitional nuclei in this mass region ($A=145$) do possess triaxial nuclear shape as has been mentioned at the outset. So both from the coincidence of available experimental g factors with the present theoretical ones as well as the indication that shape deformation does exist in the transitional region, the physics is justified for the contribution of the hexadecapole term in the cranked Hamiltonian matrix.

[1] R. Majumdar, Acta Phys. Slov. **43**, 323 (1993).
[2] R. Majumdar, J. Phys. G **15**, 47 (1989).
[3] R. Majumdar, J. Phys. G **13**, 1429 (1987).
[4] P. Ring and P. Schuck, *The Nuclear Many Body Problem* (Springer, Berlin, 1980); A. Faessler *et al.*, Prog. Part. Nucl. Phys. **5**, 79 (1981).
[5] M. J. A. de Vogt *et al.*, Rev. Mod. Phys. **55**, 949 (1983); K. Tanabe and K. Sugawara-Tanabe, Phys. Lett. B **259**, 12 (1991).
[6] S. Frauendorf, Phys. Lett. **100B**, 219 (1981).
[7] Y. S. Chen and S. Fraundorf, Nucl. Phys. **A393**, 135 (1983).
[8] K. Sugawara-Tanabe and K. Tanabe, Phys. Lett. B **207**, 243 (1988).
[9] H. R. Andrews *et al.*, Phys. Rev. Lett. **45**, 1835 (1980).
[10] P. Kleinheinz *et al.*, Z. Phys. A **290**, 279 (1979).
[11] C. E. Doran *et al.*, Z. Phys. A **325**, 285 (1986).
[12] H. R. Andrews *et al.*, Nucl. Phys. **A383**, 509 (1982).
[13] P. Raghavan, At. Data Nucl. Data Tables **42**, 189 (1989).
[14] A. N. Mantri *et al.*, Phys. Rev. Lett. **47**, 308 (1981).
[15] A. Ansari *et al.*, Nucl. Phys. **A415**, 215 (1984).
[16] A. Ansari, Phys. Rev. C **41**, 782 (1990).
[17] Ramen Majumdar, Phys. Rev. C **55**, 745 (1997), and references therein.
[18] A. Ansari, Phys. Rev. C **38**, 323 (1988), and references therein.
[19] K. Kumar and M. Baranger, Nucl. Phys. **A110**, 529 (1968).
[20] L. K. Peker *et al.*, Nucl. Data Sheets **82**, 243 (1997).
[21] L. K. Peker, Nucl. Data Sheets **42**, 124 (1984).
[22] E. derMateosian *et al.*, Nucl. Data Sheets **75**, 835 (1995).
[23] C. W. Reich, Nucl. Data Sheets **81**, 759 (1997).
[24] H. R. Andrews *et al.*, Phys. Rev. Lett. **45**, 1835 (1980).
[25] A. E. Stuchbery *et al.*, Z. Phys. A **338**, 135 (1991).
[26] R. Wyss *et al.*, Phys. Lett. B **215**, 211 (1988).
[27] R. Redon *et al.*, Phys. Rev. C **38**, 550 (1988).
[28] G. A. Lalazissis *et al.*, Nucl. Phys. **A650**, 133 (1998).
[29] S. Perries *et al.*, Phys. Rev. C **60**, 064313 (1999).
[30] K. Tanabe *et al.*, Phys. Lett. B **259**, 12 (1991).
[31] A. Ansari *et al.*, Nucl. Phys. **A334**, 73 (1980).
[32] I. M. Brak *et al.*, Rev. Mod. Phys. **44**, 320 (1972).
[33] F. T. Baker, Nucl. Phys. **A371**, 68 (1981); **A258**, 43 (1976); **A331**, 39 (1979).
[34] U Gott *et al.*, Nucl. Phys. **A192**, 1 (1972).