

Ground- γ band mixing and odd-even staggering in heavy deformed nuclei

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It is proposed that the odd-even staggering (OES) in the γ bands of heavy deformed nuclei can be reasonably characterized by a discrete approximation of the fourth derivative of the odd-even energy difference as a function of angular momentum L . This quantity exhibits a well-developed staggering pattern (zigzagging behavior with alternating signs) in rare-earth nuclei and actinides with long γ bands ($L \geq 10$). It is shown that the OES can be interpreted as the result of the interaction of the γ band with the ground band in the framework of a vector boson model with SU(3) dynamical symmetry. The model energy expression reproduces reasonably the staggering pattern in the considered nuclei up to $L=12-13$. The general behavior of the OES effect in rotational regions is studied in terms of the ground- γ band-mixing interaction, showing that a strong OES effect occurs in regions with strong ground- γ band-mixing interaction. The approach used allows a detailed comparison of the OES in γ bands with the other kinds of staggering effects in nuclei and diatomic molecules.

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I. INTRODUCTION

Various types of deviations of nuclear collective rotations from the well studied (in first approximation) pure rotational motion are known [1,2]. They cause some higher-order effects in the structure of nuclear rotational spectra, such as squeezing, backbending, and staggering. The staggering effects represent bifurcations of rotational bands into sequences of states differing by several units of angular momentum. Such effects are the odd-even staggering observed in the collective γ -bands [1], the $\Delta L=1$, $\Delta L=2$, and $\Delta L=4$ staggering in superdeformed nuclear bands [3–6], and the $\Delta L=2$ staggering in the ground-state bands of normally deformed nuclei [7]. Similar staggering effects have been identified recently in the rotational bands of diatomic molecules [5,8].

In particular, the odd-even staggering (OES) effect represents the relative displacement of the odd angular momentum levels of the γ band with respect to their neighboring levels with even angular momentum. This is a long known effect which is clearly established in even-even nuclei [1]. It, therefore, allows one to test various collective models [9]. On the other hand, the model interpretation of OES in the γ bands could be of use for the understanding of staggering effects in rotational spectra as a whole.

In some studies the OES has been interpreted as a result of the interaction between the even levels of the γ band and corresponding levels of a β band [9–14]. This consideration has been addressed to the SU(3) limit of the interacting boson model (IBM), in which the lowest β - and γ -rotational bands interact in the framework of the same irreducible rep-

resentation (irrep), $(\lambda, \mu=2)$, of the group SU(3) [15].

It is known that this approach encounters several complications [9]:

(i) In some nuclei the β band, which should be responsible for the OES in the γ band, is not observed experimentally [14], or in other cases it is not long enough [16,17].

(ii) The model description of OES is limited in dependence on whether the γ band lies above the β band or not.

(iii) The model analyses suggest increasing staggering in the γ band with increasing separation between the γ and β bands, which is not expected if staggering is due to the interaction between these two bands.

Some of the above items could be dealt with in the SU(3) limit of the IBM by using (for example) the four-body symmetry conserving interactions introduced in [18] as well as by using in an appropriate way the higher-order interactions introduced recently in [19].

On the other hand, it would be natural to extend the OES investigation beyond the IBM classification scheme. In this respect the vector boson model (VBM) with SU(3) dynamical symmetry [20–22] would be appropriate.

As an essential, distinct from IBM, the vector boson model classification scheme unites the γ band together with the ground band into the same split SU(3) multiplet. The similarities and the differences between these two models as well as their mutual complementary in different regions of rotational nuclei have been outlined in Ref. [23]. An important feature of the vector boson model (VBM) scheme is that it provides a relevant way to study the interaction between the ground and the γ band [24].

It is, therefore, reasonable to check in the VBM framework whether the OES in the γ band could be interpreted as the result of the ground- γ band-mixing interaction. Such an approach is strongly motivated by the circumstance that items (i) and (ii) above will be automatically removed. Indeed in most of rotational nuclei the even angular momentum levels of the γ band have their counterparts in the ground band. Also, the γ -band levels are always placed

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above the corresponding levels of the ground band, which allows an equal treatment of the OES effect in all deformed even-even nuclei. Moreover, recent investigations of the SU(3) dynamical symmetry in deformed nuclei show a systematic behavior of the ground- γ band interaction in dependence on the observed SU(3) splitting, i.e., on the mutual disposition of the two bands [23,24]. On this basis one could study the possible dependence of the OES effect on the ground- γ band separation. Such an analysis would be of use for elucidating item (iii).

In the present work we respond to the above consideration. Our purpose is to study the OES in the γ bands of rotational nuclei in terms of the ground- γ band-mixing interaction. We apply the VBM formalism in order to describe this effect as well as to analyze its general behavior in rotational regions. Below it will be seen that in the ground- γ band coupling scheme of the VBM the OES effect appears in a rather natural way. It will be shown that in this approach the staggering effect exhibits a reasonable dependence on the ground- γ band splitting. In addition, the study gives an insight into the recently suggested [23,24] transition between the ground- γ band coupling scheme of the VBM and the β - γ scheme of the IBM.

We remark that traditionally the OES is considered in terms of a plot of the moment-of-inertia parameter versus the angular momentum L [10,12]. On the other hand, the $\Delta L = 2$ staggering effects in nuclei and molecules have been established by introducing a new relevant characteristic of rotational spectra, which is the discrete approximation of the fourth derivative of the energy difference between two levels with $\Delta L = 2$ as a function of L . In the present work we suggest that an analogous characteristic, corresponding to $\Delta L = 1$, will be appropriate for the case of OES in the γ bands. As will be seen below, this is a rather convenient way to provide the analyses of the OES ($\Delta L = 1$) effect in deformed even-even nuclei.

In Sec. II the $\Delta L = 1$ discrete derivation is introduced as an OES effect characteristic. In Sec. III the vector boson model (VBM) is briefly presented and a relevant model expression for the staggering quantity is obtained. The respective experimental and theoretical staggering patterns for rare-earth nuclei and actinides are presented in Sec. IV. The results obtained are discussed in Sec. V and some concluding remarks are given in Sec. VI.

II. THE ODD-EVEN STAGGERING IN A FORM OF $\Delta L = 1$ DISCRETE DERIVATION

In nuclear physics the transition energies between levels differing by one or several units of angular momentum are experimentally well-determined quantities. In particular the transition energy

$$\Delta E(L) = E(L+1) - E(L), \quad (1)$$

with $\Delta L = 1$, carries essential information about the structure of various nuclear collective bands such as the γ bands and

some negative parity bands of heavy deformed nuclei. Its deviation from the rigid rotor behavior can be measured by the quantity

$$\text{Stg}(L) = 6\Delta E(L) - 4\Delta E(L-1) - 4\Delta E(L+1) + \Delta E(L+2) + \Delta E(L-2). \quad (2)$$

In the case of a rigid rotor one can easily see that $\text{Stg}(L)$ is equal to zero. Moreover the terms of the second power in $L(L+1)$ also give zero in Eq. (2). This is due to the fact that $\text{Stg}(L)$ is the discrete approximation of the fourth derivative of the function $\Delta E(L)$, i.e., the fifth derivative of the energy $E(L)$. Therefore, any nonzero values of the quantity $\text{Stg}(L)$ will indicate the presence of order higher than $[L(L+1)]^2$ from the regular rotational motion of the nuclear system.

The above expression is introduced by analogy with the case of the $\Delta L = 2$ staggering in superdeformed nuclei [3,4] and $\Delta L = 2, 4$ staggering in diatomic molecules [5]. The respective quantities have been used properly in various theoretical proposals for the explanation of these effects [25–31], some of them [32–37] using symmetry arguments which could be of applicability to other physical systems as well.

On the above basis it is natural to apply the quantity $\text{Stg}(L)$ to study the OES in the γ bands of heavy deformed nuclei, i.e., to interpret this effect in the form of $\Delta L = 1$ discrete derivation. Such an approach could be very useful in providing a unified analysis of the different kinds of staggering effects as well as in comparing their physical explanations.

Since for the γ bands the experimental energy values are known, Eq. (2) can be written in the form

$$\text{Stg}(L) = 10E(L+1) + 5E(L-1) + E(L+3) - [10E(L) + 5E(L+2) + E(L-2)], \quad (3)$$

i.e., the quantity $\text{Stg}(L)$ is simply determined by six band level energies (with $L-2, L-1, \dots, L+3$). (Note that an analogous expression would not be useful for the staggering effects in superdeformed nuclei and diatomic molecules where only the transition energies are experimentally measured.

As will be seen in Sec. IV, Eq. (3) provides a well-developed staggering pattern [zigzagging behavior of the function $\text{Stg}(L)$] for the experimentally observed γ bands in the rare earth region and in the actinides.

III. ODD-EVEN STAGGERING IN THE VBM

Theoretically, the structure of the γ bands of deformed nuclei is well reproduced in the framework of the vector boson model (VBM) with SU(3) dynamical symmetry [20–23].

The VBM is founded on the assumption that the low-lying collective states of deformed even-even nuclei can be described with the use of two distinct kinds of vector bosons, whose creation operators ξ^+ and η^+ are O(3) vectors and in addition transform according to two independent SU(3) irreps of the type $(\lambda, \mu) = (1, 0)$. The vector bosons provide

relevant constructions of the SU(3) angular momentum and quadrupole operators like the bosons in the Schwinger realization of SU(2) [38]. Also, they can be interpreted as quanta of elementary collective excitations of the nucleus [22].

The VBM Hamiltonian is constructed as a linear combination of three basic O(3) scalars from the enveloping algebra of SU(3):

$$V = g_1 L^2 + g_2 L \cdot Q \cdot L + g_3 A^+ A, \quad (4)$$

where g_1 , g_2 , and g_3 are free model parameters; L and Q are the angular momentum and quadrupole operators, respectively; and $A^+ = \xi^{+2} \eta^{+2} - (\xi^+ \cdot \eta^+)^2$.

The Hamiltonian (4) includes high, third- ($L \cdot Q \cdot L$), and fourth- ($A^+ A$) order effective interactions and reduces the SU(3) symmetry to the rotational group SO(3). It incorporates in a reasonable way the most important collective properties of heavy deformed nuclei determined by their angular momenta and quadrupole moments.

The model basis states

$$\left| \begin{array}{l} (\lambda, \mu) \\ \alpha, L, M \end{array} \right\rangle, \quad (5)$$

corresponding to the SU(3) \supset O(3) group reduction, are constructed by means of the vector-boson operators and are known as the basis of Bargmann-Moshinsky [39,40]. The quantum number α distinguishes the various O(3), O(2) irreps, (L, M), appearing in a given SU(3) irrep (λ, μ) and labels the different bands of the multiplet.

In the VBM the ground and the lowest γ band belong to one and the same SU(3) multiplet, in which λ and μ are even and $\lambda \geq \mu$. These bands are labeled by two neighboring integer values of the quantum number α . The so-defined multiplet is split with respect to α .

It is important to remark that while in the SU(3) limit of the IBM the irreducible representations (irreps) (λ, μ) are restricted by the total number of bosons describing the specific nucleus, in the VBM the possible (λ, μ) irreps of SU(3) are *not restricted* by the underlying theory. On the other hand, recently it has been shown [23] that some favored regions of (λ, μ) multiplets could be outlined through the numerical analysis of the experimental data available for the ground and the γ -collective bands of even-even deformed nuclei. For the favored irreps the VBM scheme gives a good description of the energy levels and of the $B(E2)$ transition ratios within and between the bands. It should be emphasized that in the VBM the other collective bands, in particular the lowest β band, do not belong to the same SU(3) irrep.

We remark that in the rare-earth region and in the actinides the best model descriptions correspond to (favored) SU(3) multiplets with ($\lambda, \mu = 2$).

It is, therefore, reasonable to try to reproduce the fine characteristics of the ground- γ band-mixing interaction in these nuclei for $\mu = 2$. That is why the ($\lambda, \mu = 2$) multiplets can be naturally applied for the description and the interpretation of the OES effect in the framework of the VBM.

In this case the model formalism allows one to obtain simple analytic expressions for the ground- and the γ -band energy levels. For the ($\lambda, 2$) irreps the ground and the γ bands are the only possible ones appearing in the corresponding SU(3) multiplets. For the even angular momentum states the Hamiltonian matrix is always two-dimensional, while for the odd states of the γ band one has single matrix elements.

The resolution of the standard eigenvalue equation gives the following expressions for the even energy levels $E^g(L)$ and $E^\gamma(L_{\text{even}})$ of the ground and the γ band, respectively:

$$E^g(L) = B + (A - BC)L(L+1) - |B| \sqrt{1 + aL(L+1) + bL^2(L+1)^2}, \quad (6)$$

$$E^\gamma(L_{\text{even}}) = B + (A - BC)L(L+1) + |B| \sqrt{1 + aL(L+1) + bL^2(L+1)^2}. \quad (7)$$

The odd γ -band levels, $E^\gamma(L_{\text{odd}})$ are obtained in the form

$$E^\gamma(L_{\text{odd}}) = 2B + AL(L+1), \quad (8)$$

where

$$A = g_1 - (2\lambda + 5)g_2, \quad (9)$$

$$B = 6(2\lambda + 5)g_2 - 2(\lambda + 3)^2 g_3, \quad (10)$$

$$C = \frac{g_3}{B}, \quad (11)$$

and

$$a = -\frac{4}{B^2} \{ (\lambda + 3)[(\lambda + 3)g_3 - 6g_2]g_3 - 3(g_3 - 6g_2)g_2 \}, \quad (12)$$

$$b = \frac{1}{B^2} (g_3 - 6g_2)^2, \quad (13)$$

with g_1 , g_2 , and g_3 being the parameters of the effective interaction (4). We remark that the application of the VBM in rare-earth nuclei and actinides [23] provides $B > 0$ in Eq. (10) so that $|B| = B$.

Further, we rewrite Eqs. (6), (7), and (8) consistently, so as to obtain a unified expression for all the γ -band energy levels:

$$E^g = AL(L+1) - B[\sqrt{1 + aL(L+1) + bL^2(L+1)^2} + CL(L+1) - 1], \quad (14)$$

$$E^\gamma = 2B + AL(L+1) + B[\sqrt{1 + aL(L+1) + bL^2(L+1)^2} - CL(L+1) - 1] \left(\frac{1 + (-1)^L}{2} \right). \quad (15)$$

So, E^γ can be simply written

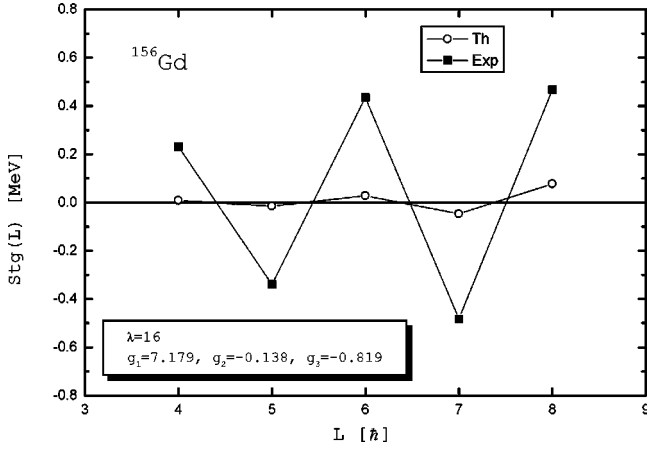


FIG. 1. The experimental and the theoretical values of the quantity $Stg(L)$, Eqs. (3) and (18), respectively, obtained for the γ band of ^{156}Gd are plotted as functions of the angular momentum. The experimental data are taken from [17].

$$E^\gamma = 2B + AL(L+1) + \frac{1}{2}BR(L)[1 + (-1)^L], \quad (16)$$

where

$$R(L) = \sqrt{1 + aL(L+1) + bL^2(L+1)^2} - CL(L+1) - 1. \quad (17)$$

The last factor in Eq. (16) switches over E^γ between the odd and the even states of the γ band. In such a way the VBM gives a natural possibility to reproduce the parity effects in the γ -band structure. Here, it is important to remark that such a result is a direct consequence of the $SU(3)$ dynamical symmetry mechanism.

Now we are able to apply the above model formalism to reproduce theoretically the $\Delta L=1$ discrete derivatives (2) and (3). After introducing Eq. (16) into Eq. (3) of the previous section, we obtain the following model expression for the function $Stg(L)$:

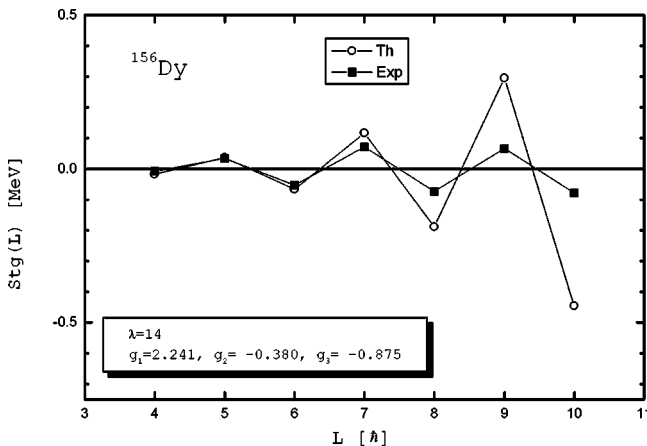


FIG. 2. The same as Fig. 1, but for the γ band of ^{156}Dy .

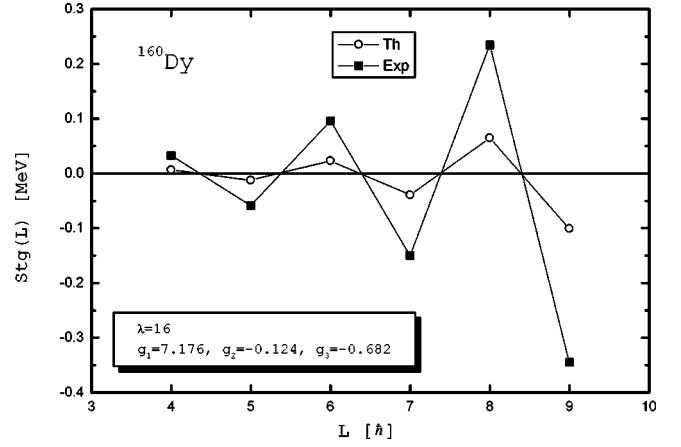


FIG. 3. The same as Fig. 1, but for ^{160}Dy .

$$\begin{aligned} Stg(L) = & \frac{B}{2} [10R(L+1) + 5R(L-1) + R(L+3)] \\ & \times [1 + (-1)^{L+1}] - \frac{B}{2} [10R(L) + 5R(L+2) \\ & + R(L-2)] [1 + (-1)^L]. \end{aligned} \quad (18)$$

One can easily verify that the right-hand side of Eq. (18) has alternative signs as a function of the angular momentum values $L=2,3,4, \dots$, i.e., it gives a regular model staggering pattern. In addition, the amplitude of the staggering increases monotonously with L . The signs and the amplitude are determined by the terms $BR(x)$, ($x=L-2, L-1, \dots, L+3$) which depend on the higher order effective interactions in Eq. (4).

Equation (18) allows one to study the OES effect in terms of the VBM. Moreover, the obtained result provides a reasonable theoretical tool to interpret the OES effect in the meaning of a $\Delta L=1$ staggering effect. In addition one could estimate analytically the behavior of this effect in dependence on the nuclear collective characteristics.

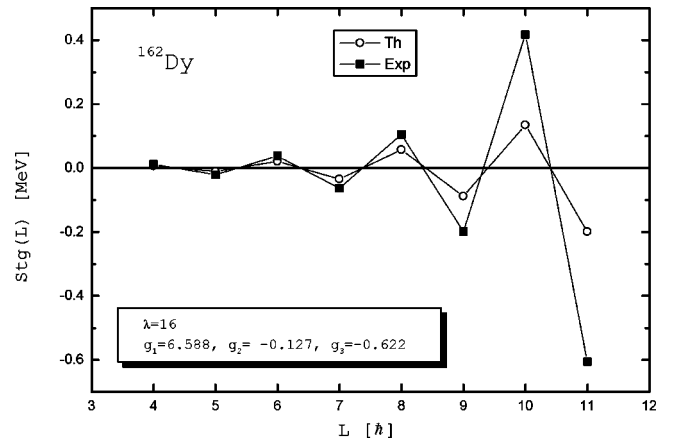
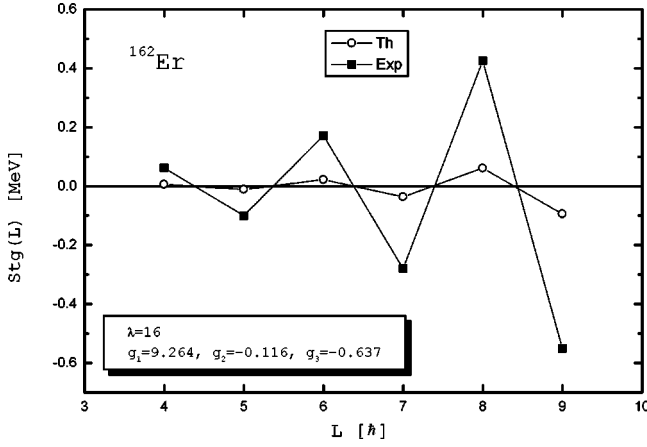


FIG. 4. The same as Fig. 1, but for ^{162}Dy . The experimental data are taken from [16].

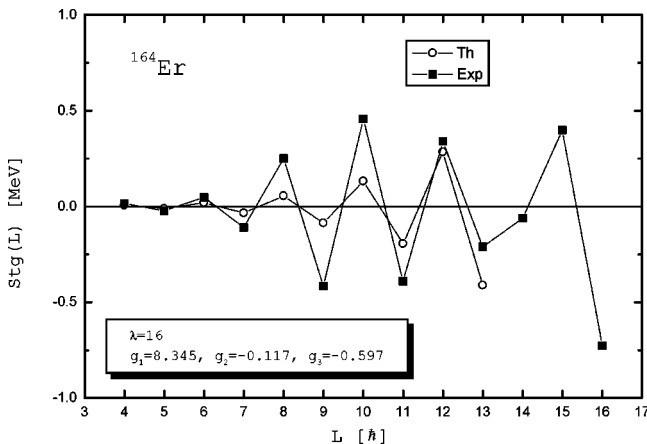
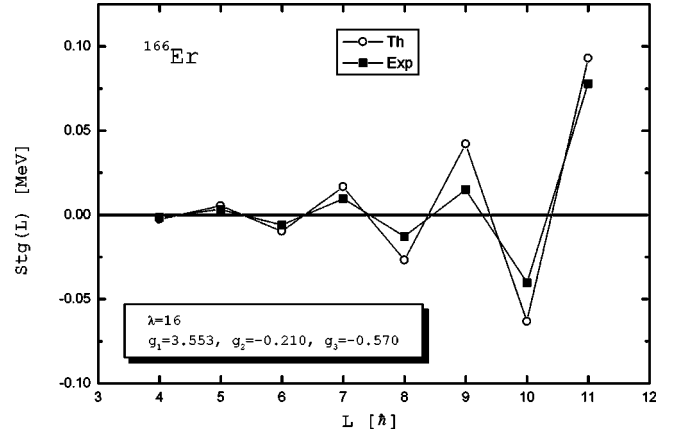
FIG. 5. The same as Fig. 1, but for ^{162}Er .

IV. THE OES EFFECT—EXPERIMENTAL PATTERN AND THEORETICAL DESCRIPTION

We have applied Eq. (3) to the rare-earth nuclei and to the actinides for which the experimentally measured γ bands are long enough ($L \geq 10$), ^{156}Gd [17], $^{156,160}\text{Dy}$ [17], ^{162}Dy [16], $^{162-166}\text{Er}$ [17], ^{170}Yb [41], ^{228}Th [42], ^{232}Th [16]. In all cases we obtain a clearly pronounced staggering pattern, i.e., a zigzagging behavior of the quantity $\text{Stg}(L)$ as a function of angular momentum. This is shown in Figs. 1–10 (lines with squares). Generally, one observes a regular change in the signs of $\text{Stg}(L)$ between the odd and even levels with the amplitude increasing with angular momentum L up to $L = 12$ –13.

In such a way the OES effect in the γ bands appears in the form of a $\Delta L = 1$ staggering, which is consistent with the consideration of other staggering effects in nuclei and diatomic molecules. We remark that for all γ bands under study the experimental uncertainties in the pattern (3) are negligible.

We remark that for the different nuclei the staggering amplitude varies in a rather wide range. For example, for $L = 8$ the quantity $\text{Stg}(8)$ obtains the smallest absolute value 0.013 MeV (for ^{166}Er , Fig. 7) and the largest value 0.467 MeV (for ^{156}Gd , Fig. 1).

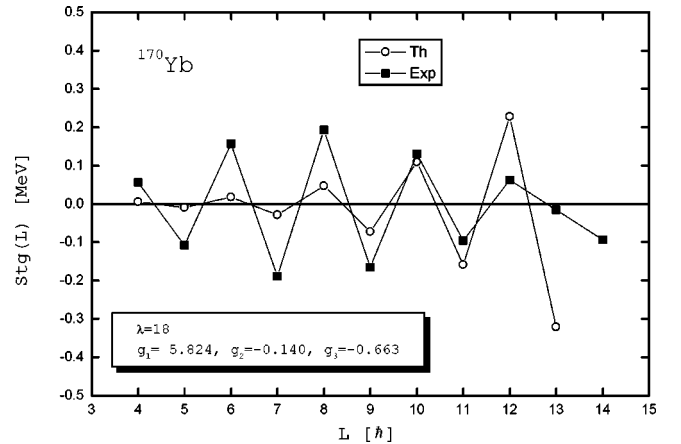
FIG. 6. The same as Fig. 1, but for ^{164}Er .FIG. 7. The same as Fig. 1, but for ^{166}Er .

Interesting staggering patterns are observed in the nuclei ^{164}Er (Fig. 6) and ^{170}Yb (Fig. 8) for which the γ bands are longest, $L_{\text{max}} = 19$ for ^{164}Er [17], $L_{\text{max}} = 17$ for ^{170}Yb [41]. In these cases the staggering amplitude initially increases as a function of angular momentum up to $L = 8$ –10 and then begins to decrease. Further, at $L = 14$, an irregularity in the alternative signs of the quantity $\text{Stg}(L)$ occurs.

We have applied the model expression, Eq. (18), to describe theoretically the staggering pattern of all the nuclei under consideration. For this purpose we use the sets of model parameters g_1 , g_2 , g_3 , and λ , obtained after fitting Eqs. (14) and (15) to the experimental ground- and γ -band levels up to $L = 10$ –12. The values of these parameters are listed in Table I together with the corresponding rms deviations measured by

$$\sigma_E = \sqrt{\frac{1}{n_E} \sum_{L,v} [E_\nu^{\text{Th}}(L) - E_\nu^{\text{Exp}}(L)]^2}, \quad (19)$$

where $n_E = n_g + n_\gamma$ is the total number of the levels used in the fit and $\nu = g, \gamma$ label the ground- and the γ -band levels, respectively.

FIG. 8. The same as Fig. 1, but for ^{170}Yb . The experimental data are taken from [41].

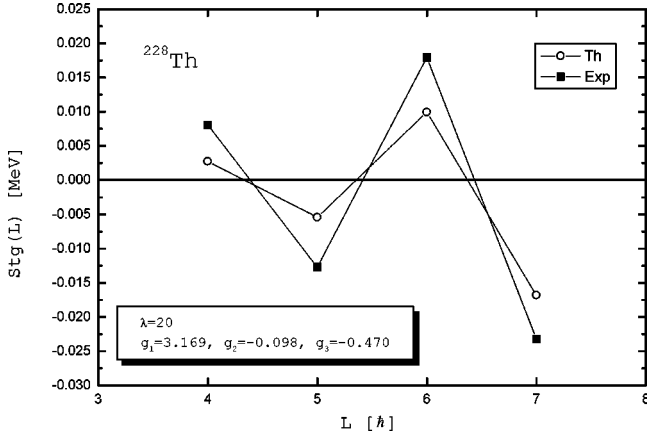


FIG. 9. The same as Fig. 1, but for ^{228}Th . The experimental data are taken from [42].

For all the nuclei the theoretical pattern reproduces the alternating signs as well as the general increase in the staggering amplitude as a function of L up to $L=12-13$.

Good description of the staggering pattern is obtained for the nuclei $^{156,160,162}\text{Dy}$ (Figs. 2–4), ^{166}Er (Fig. 7), ^{228}Th (Fig. 9), ^{232}Th (Fig. 10). Also for the nuclei ^{164}Er (Fig. 6) and ^{170}Yb (Fig. 8) the staggering pattern is well reproduced up to $L=13$ (Figs. 6 and 8, respectively), i.e., up to the appearance of the sign irregularity.

For two of the nuclei, ^{156}Gd and ^{162}Er , the difference between the theoretical and the experimental staggering magnitude is noticeably larger than for the other nuclei under study (Figs. 1 and 5). In the next section it will be shown that such a disagreement could be referred to the rather strongly perturbed rotational structure of the respective ground and γ bands. As is seen from Table I, this circumstance reflects on the quality of the model energy descriptions obtained for both nuclei. The respective rms factors are relatively larger ($\sigma_E=40$ keV for ^{156}Gd and $\sigma_E=32.7$ keV for ^{162}Er) than the ones in the other nuclei.

On the other hand we remark that for the obtained sets of parameters (Table I) our fitting procedures guarantee correct reproduction of all the $B(E2)$ transition rates available for

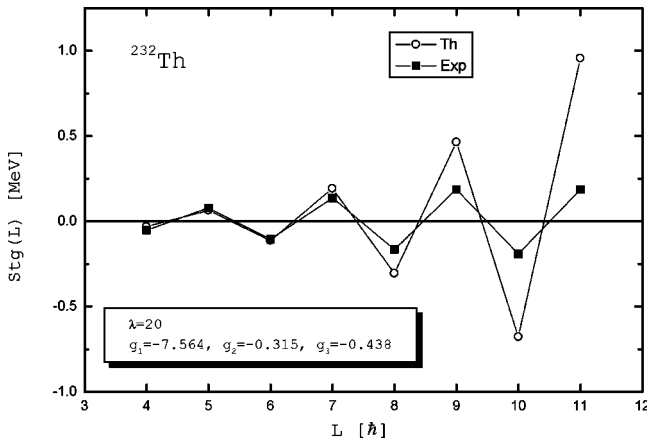


FIG. 10. The same as Fig. 1, but for ^{232}Th . The experimental data are taken from [16].

TABLE I. The parameters of the fits of the energy levels of the ground and the γ bands [Eqs. (14) and (15)] of the nuclei investigated are listed for the $(\lambda, 2)$ multiplets which provide the best model descriptions. The Hamiltonian parameters g_1 , g_2 , and g_3 [Eq. (4)] and the rms quantities σ_E [Eq. (19)] are given in keV. The numbers of ground- and the γ -band levels used in the fit, n_g and n_γ , respectively, are also given.

Nucl	λ	g_1	g_2	g_3	σ_E	n_g	n_γ
^{156}Gd	16	7.179	-0.138	-0.819	40.0	5	9
^{156}Dy	14	2.241	-0.380	-0.875	47.8	5	9
^{160}Dy	16	7.176	-0.124	-0.682	24.5	5	11
^{162}Dy	16	8.980	-0.059	-0.606	21.3	5	11
^{162}Er	16	9.264	-0.116	-0.637	32.7	5	11
^{164}Er	16	8.345	-0.117	-0.597	21.4	5	11
^{166}Er	16	3.553	-0.210	-0.570	23.4	5	11
^{170}Yb	18	5.824	-0.140	-0.663	26.6	5	11
^{228}Th	20	3.169	-0.098	-0.470	13.6	5	9
^{232}Th	20	-7.564	-0.315	-0.438	22.2	9	11

the ground and the γ bands. This is extensively demonstrated in our previous paper [23] where the advantages of the vector boson model description (compared to other collective models) are pointed out. That is why we do not concentrate on such considerations in the present study. In Table II, as an illustration, we show the theoretical ground- and γ -band energy levels and the attendant $B(E2)$ transition ratios of the nucleus ^{166}Er which correspond to the parameter set given in Table I. The following $B(E2)$ transition ratios are included in our numerical procedures [23]:

$$R_1(L) = \frac{B(E2; L_\gamma \rightarrow L_g)}{B[E2; L_\gamma \rightarrow (L-2)_g]}, \quad L \text{ even}, \quad (20)$$

$$R_2(L) = \frac{B[E2; L_\gamma \rightarrow (L+2)_g]}{B(E2; L_\gamma \rightarrow L_g)}, \quad L \text{ even}, \quad (21)$$

$$R_3(L) = \frac{B[E2; L_\gamma \rightarrow (L+1)_g]}{B[E2; L_\gamma \rightarrow (L-1)_g]}, \quad L \text{ odd}, \quad (22)$$

$$R_4(L) = \frac{B[E2; L_g \rightarrow (L-2)_g]}{B[E2; (L-2)_g \rightarrow (L-4)_g]}, \quad L \text{ even}, \quad (23)$$

where the indices g and γ label the ground- and the γ -band levels, respectively.

As is seen from Table II, very good agreement between the theoretical and experimental data is obtained. This example demonstrates that for the same set of model parameters the staggering effect is reproduced in consistency with the other electromagnetic and energy characteristics of the ground and the γ band.

The results presented show that a relevant description of the OES effect ($\Delta L=1$ staggering) in the γ bands of rare-earth nuclei and actinides is possible in the framework of the VBM with $\text{SU}(3)$ dynamical symmetry.

TABLE II. Theoretical and experimental energy levels and transition ratios [Eqs. (20)–(23)] for the nucleus ^{166}Er , corresponding to the multiplet (16,2) and the set of parameters given in Table I. The experimental data (used in the fits) for the energy levels are taken from [17], while the data for the $E2$ transitions are from [43–45]. The numbers in brackets refer to the uncertainties in the last digits of the experimental ratios.

L	E_g^{Th}	E_g^{Exp}	E_γ^{Th}	E_γ^{Exp}	R_1^{Th}	R_1^{Exp}	R_2^{Th}	R_2^{Exp}	R_3^{Th}	R_3^{Exp}	R_4^{Th}	R_4^{Exp}
2	74.8	80.6	797.8	785.9	1.75	1.86(10)	0.08	0.097(8)				
3			865.7	859.4					0.64	0.72(6)		
4	249.2	265.0	956.4	956.2	4.8	5.72(47)	0.18	0.26(7)			1.39	1.45(16)
5			1069.5	1075.3					1.20	1.43(15)		
6	523.1	545.4	1205.8	1215.9	8.29	12.25(75)	0.28	0.28			1.05	1.12(22)
7			1363.9	1376.0					1.89			
8	896.2	911.2	1546.3	1555.7	13.45	20.9(45)	0.42				0.97	1.05(28)
9			1748.9	1751.1					2.87			
10	1368.0	1349.6	1978.3	1964.0	22.7		0.61				0.92	1.02(27)

V. DISCUSSION

We are now able to analyze several important characteristics of the fine rotational structure of the γ bands together with the respective nuclear collective properties hidden behind them. Although the number of considered nuclei (ten nuclei) does not allow one to provide any detailed systematics, our study leads to a consistent theoretical interpretation of all the available experimental information concerning the $\Delta L=1$ (OES) staggering effect in γ bands. (Note that two newest sets of data are used for ^{170}Yb [41] and ^{228}Th [42].)

The analysis of the experimental $\Delta L=1$ staggering amplitude obtained in rare-earth nuclei shows that it is generally larger for the nuclei placed in the beginning of this rotational region compared to the midshell nuclei. For example, the staggering amplitude in ^{156}Gd is more than one order of magnitude larger than the ones in ^{166}Er (see Figs. 1 and 7). Also, a gradual decrease of the amplitude towards the midshell region is observed for the three Er isotopes, ^{162}Er , ^{164}Er , and ^{166}Er [with $\text{Stg}(8)=0.425$ MeV, $\text{Stg}(8)=0.251$ MeV, and $\text{Stg}(8)=-0.013$ MeV, respectively; see Figs. 5, 6, and 7].

The above observation is consistent with the general behavior of the nuclear rotational properties in the limits of the valence shells. It is well known that towards the midshell region these properties are better revealed so that any kind of deviations from the regular rotational band structures should be smaller. In this respect the weaker $\Delta L=1$ staggering effect observed in the rare-earth midshell isotopes is quite natural.

On the other hand, such a behavior of the staggering effect can be reasonably interpreted in terms of the ground- γ band interaction. It has been shown in the vector boson model framework that this interaction systematically decreases towards the middle of rotational regions [23,24]. Thus the weaker mutual perturbation of these two bands in the midshell region is consistent with the good rotational behavior of the γ band.

Also, it has been established that the ground- γ band-mixing interaction in heavy deformed nuclei is correlated with the energy separation between the two bands. In the

SU(3) dynamical symmetry framework this separation corresponds to the splitting of the SU(3) multiplet and is measured by the ratio

$$\Delta E_L = \frac{E_L^\gamma - E_L^g}{E_2^g}, \quad (24)$$

which characterizes the magnitude of the energy differences between the even angular momentum states of the ground and the γ band.

For example, the experimental ΔE_2 ratios vary within the limits $5 \leq \Delta E_2 \leq 20$ for the nuclei of rare-earth region, and $13 \leq \Delta E_2 \leq 25$ for the actinides.

It has been found that the ΔE_L ratio generally increases towards the middle of a given rotational region in consistency with the decrease in the ground- γ band-mixing interaction and corresponds to an increase in the SU(3) quantum number λ .

On the above basis, it has been established that for nuclei with a weak SU(3) splitting ($\Delta E_2 \leq 12$ for rare-earth nuclei and $\Delta E_2 \leq 15$ for actinides) the ground and the γ bands are strongly coupled in the framework of the SU(3) dynamical symmetry, with λ obtaining favored values in the region $\lambda = 14-20$.

Now we remark that the nuclei of the present study indeed belong to this kind of SU(3) dynamical symmetry nuclei. In such a way the relatively strong ground- γ band interaction in these nuclei can be considered as the reason causing the observed OES ($\Delta L=1$ staggering).

For the nuclei with strong SU(3) splitting ($\Delta E_2 > 12$ for rare-earth nuclei, and $\Delta E_2 > 15$ for actinides) the ground and the γ bands are weakly coupled in the framework of the SU(3) dynamical symmetry, with $\lambda > 60$. Typical examples for this kind of nuclei are ^{172}Yb (with $\Delta E_2 = 17.6$) and ^{238}U (with $\Delta E_2 = 20.3$). It has been suggested that for these nuclei the ground and the γ band could be separated in different SU(3) irreps. Then the OES can be possibly interpreted as the interaction of the γ band with the β band as it is done in the framework of the interacting boson model [15,9]. We

remark that in these cases the γ bands are not long enough in order to study the OES in the form of $\Delta L = 1$ staggering.

The above consideration is strongly consistent with the possibility for a transition [23,24] from the ground- γ band coupling scheme of the present VBM, which is more appropriate near the ends of the rotational regions, to the IBM classification scheme [15] with β - γ band coupling, which is more relevant in the midshell nuclei. It clearly illustrates that both model schemes are mutually complementary in the different regions of rotational nuclei. Moreover, we remark that for the nuclei considered in the present study the proposed ground- γ band mixing interpretation of the OES effect is *unique*.

At this point it is reasonable to discuss the general behavior of the OES effect in heavy deformed nuclei in terms of the SU(3) dynamical symmetry characteristics.

Here we refer to the so-called SU(3) contraction limits, in which the algebra of SU(3) goes to the algebra of the semi-direct product $T_5 \wedge SO(3)$, i.e., $SU(3) \rightarrow T_5 \wedge SO(3)$ (T_5 is the group of five-dimensional translations generated by the components of the SU(3)-quadrupole operators) [46–51]. (Generally, the contraction limit corresponds to a singular linear transformation of the basis of a given Lie algebra. The transformed structure constants approach well-defined limits and a new Lie algebra, called contracted algebra, results [46]. The original and the contracted algebras are not isomorphic.)

In the VBM collective scheme the SU(3) contraction corresponds to the following two limiting cases:

- (i) $\lambda \rightarrow \infty$, with μ finite;
- (ii) $\lambda \rightarrow \infty$, $\mu \rightarrow \infty$, with $\mu \leq \lambda$.

It has been shown that in these limits the ground- γ band mixing gradually disappears so that the corresponding SU(3) multiplets are disintegrated into distinct noninteracting bands [24]. It follows that all fine spectroscopic effects based on the band-mixing interactions, such as the OES effect, should be reduced towards the SU(3) contraction limits.

Indeed, one can easily verify that in the limit (i) the OES effect should not be observed. In fact, for this case we have deduced analytically that the terms of the type $BR(L)$, which determine the quantity $Stg(L)$ in Eq. (18), go to zero as

$$\lim_{\lambda \rightarrow \infty} BR(L) = \lim_{\lambda \rightarrow \infty} \frac{3(g_2 g_3 - 3g_2^2)}{g_3 \lambda^2} L(L+1) = 0. \quad (25)$$

[This analytic limit is obtained by using the explicit expressions (10) and (17) with Eq. (12), (13), and (11).] It follows that the staggering amplitude goes to zero when λ increases to infinity. This is illustrated in Fig. 11, where the model staggering pattern is plotted for $\lambda = 20$, $\lambda = 40$, and $\lambda = 60$ and fixed (overall) values of the model parameters $g_2 = -0.2$ and $g_3 = -0.25$.

We remark that the first limiting case, $\lambda \rightarrow \infty$ with $\mu = 2$, is physically reasonable for midshell nuclei, which are characterized by large values of the quantum number λ (with $\mu = 2$) and strong ground- γ band splitting. However, we emphasize that for the nuclei (under study) where odd-even staggering is observed the SU(3) scheme of the vector boson model works far from the SU(3) contraction limits.

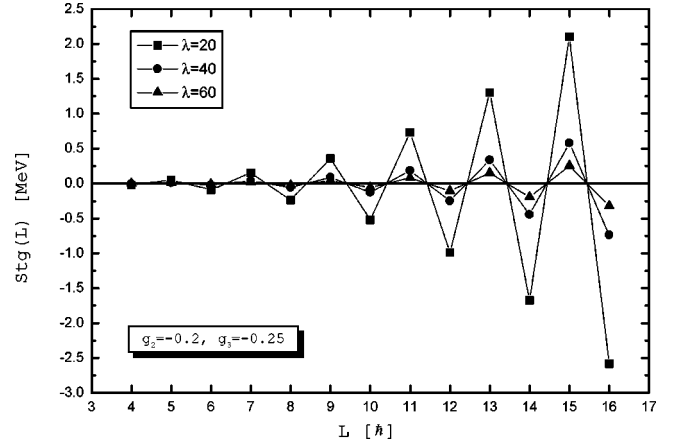


FIG. 11. The theoretical values of the quantity $Stg(L)$, Eq. (18), obtained for three different values of the SU(3) quantum number λ ($\lambda = 20$, $\lambda = 40$, and $\lambda = 60$) and fixed (overall) values of the model parameters ($g_2 = -0.2$ and $g_3 = -0.25$) are plotted as functions of the angular momentum.

In this way the present model approach completely resolves the problem (iii) stated in Sec. I. Indeed, we obtain that the increase in the separation between the ground and the γ band [i.e., the SU(3) splitting] is correlated with the respective decrease in the magnitude of the OES effect. In addition, the lack of staggering in the SU(3) contraction limit [where the SU(3) multiplets are disintegrated] endorses our conclusion that for the nuclei under study this effect should be due to the SU(3) coupling of the γ band together with the ground band. In this respect the extent to which the observed phenomenon could be interpreted as a result of forced ground- γ band-mixing becomes clear.

On the other hand, the area of applicability of the present VBM scheme is also clear. Besides the contraction limit, our model interaction comes to another restriction which appears naturally towards the transition region. This is indicated by the circumstance that the model suffers in the reproduction of the large staggering amplitudes in the nuclei ^{156}Gd and ^{162}Er [$Stg(8) \sim 0.4\text{--}0.5$ MeV, see Figs. 1 and 5], which are placed far from the middle of the rotational region. (More precisely, ^{156}Gd is close to the beginning of the region, while ^{162}Er is in the beginning of the respective group of rotational isotopes.) Actually, our analyses suggest that in these cases the ground- γ mixing is far stronger than a small perturbation to the respective rotational band structures. This is supported by the observation of large experimental ground- γ interband transition probabilities [24]. In such a way, the above difficulty of our model description could be referred to the standard problem of the strong perturbation interactions.

Also, it is important to remark that the staggering pattern could be influenced by the presence of β - γ interaction. Although for the nuclei considered the latter should be essentially weaker than the ground- γ interaction, its involvement in a more general study would be of interest. Since the respective quantitative analysis oversteps the capacity of the present vector boson scheme (which does not include the β -rotational band), here we only give an idea about two future possible approaches to the problem:

(1) A symplectic $Sp(6, R)$ extension of the present VBM with involvement of appropriate mixing between the different $SU(3)$ multiplets;

(2) IBM analysis with consistent application of higher-order interactions in both the $SU(3)$ limit (with the standard β - γ coupling) [18] and the recently proposed $O(6)$ scheme [19] with a cubic quadrupole interaction $(\hat{Q} \times \hat{Q} \times \hat{Q})^{(0)}$. The latter seems to be very promising for a relevant IBM treatment of the ground- γ band interaction in deformed nuclei.

So, in these ways one could try to improve the theoretical description of the observed staggering patterns. Of course, the price to be paid is the more complicated model formalism and the larger number of parameters. Also, one should not expect essential improvement in the patterns with large experimental amplitudes [$Stg(8) \sim 0.4$ – 0.5 MeV] where the problems are due (as has already been mentioned) to the generally stronger perturbed rotational structure of the γ band.

Let us now consider the two long staggering patterns in ^{164}Er (Fig. 6) and ^{170}Yb (Fig. 8) for which sign irregularities are observed at $L=14$. It is known that at this angular momentum the structure of the γ band of ^{164}Er is changed. This is interpreted as the result of a crossing with another band known as a super band [52,53]. The same phenomenon is considered to be responsible for the backbending effect observed in this γ band [54]. On the above basis our analysis suggests that a similar situation is realized in the ^{170}Yb γ band [41]. In fact the backbending effect is beyond the scope of the presently used VBM with $SU(3)$ dynamical symmetry, which explains the reason why our theoretical description is restricted up to $L \leq 12$ – 13 . It is however remarkable that the experimentally determined quantity $Stg(L)$ gives an excellent indication for the presence of bandcrossing effects.

It is important to emphasize the meaning of the introduction of the $\Delta L=1$ characteristics of the γ bands. Indeed the moment-of-inertia versus the angular momentum analyses reasonably indicate the presence of the OES effect [10,12]. On the other hand, the use of the fourth derivative of the odd-even energy differences gives a rather accurate quantitative measure to estimate the magnitude of this effect for a given angular momentum or given region of angular momenta. In such a way the role of the band-mixing interactions could be correctly taken into account. Moreover, the well determined staggering amplitudes together with the clearly established alternating signs pattern allow one to provide various quantitative analyses (as the present one) of the fine structure of nuclear collective bands as a whole.

VI. CONCLUSION

We have studied the OES effect in the γ bands of even-even deformed nuclei in terms of the discrete approximation

of the fourth derivative of the $\Delta L=1$ (odd-even) energy difference. The staggering pattern obtained in several rare-earth nuclei and actinides is clearly pronounced (with the respective experimental uncertainties being negligible) and can be referred to as $\Delta L=1$ staggering. Its form is essentially similar to the one seen for the other kinds of staggering observed in nuclei and diatomic molecules. The most common feature of all staggering patterns is the initial increase of the amplitude as a function of angular momentum followed by its alternations with L as well as by possible occurrence of sign irregularities. Thus in all cases the staggering pattern reflects the fine structure of rotational bands and gives a rather natural indication for some singular changes such as the bandcrossing effects.

We have shown that the OES can be interpreted reasonably as the result of the interaction of the γ band with the ground band in the framework of the vector boson model with $SU(3)$ dynamical symmetry. The model energy expression reproduces adequately the staggering pattern in the considered nuclei below the backbending region $L=12$ – 13 . On the above basis we were able to study the general behavior of the OES effect in rotational regions in terms of the ground- γ band-mixing interaction. As a result we have established that the increase in the separation between the ground and the γ band towards the midshell region is correlated with the respective decrease in the magnitude of the OES effect. Thus we explained the presence of the well-developed OES patterns in nuclei with relatively weak $SU(3)$ splitting (strong ground- γ band coupling) as well as the decrease in the staggering amplitude (even the absence of the effect) for the strongly split $SU(3)$ multiplets in midshell nuclei.

The approach presented gives a rather general prescription for the analysis of various fine characteristics of rotational motion in quantum-mechanical systems. In this respect it allows a detailed comparison of the different kinds of staggering effects in nuclei and diatomic molecules. A future unified interpretation and/or treatment of these fine effects could be possible on this basis.

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