Kaon electromagnetic form factor and QCD

Masami Nakagawa
Department of Physics, Meijo University, Tempaku, Nagoya 468-8502, Japan

Keiji Watanabe
Department of Physics, Meisei University, Hino, Tokyo 191-8506, Japan

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The experimental data on kaon electromagnetic form factor are analyzed both for the timelike and the spacelike momentum by using the superconvergent dispersion relation, which we proposed to synthesize the asymptotic property of QCD and the vector meson dominance model in the low momentum region. As in the case of the pion electromagnetic form factor, the experimental data on the kaon form factor are realized by our formula very well by using the three loop approximation for the effective coupling constant. The boson form factor is sensitive to the approximation on the QCD term.

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I. INTRODUCTION

We have analyzed the electromagnetic form factors of hadrons with recourse to the superconvergent dispersion relation, which was proposed to synthesize the low momentum hadronic phenomena and the prediction of QCD in the high momentum region [1,2]. We investigated the nucleon and the pion electromagnetic form factors and obtained reasonable agreement with the experimental data [2,3]. In this paper we recapitulate the kaon electromagnetic form factor by using the superconvergent dispersion relation to show that we are able to reproduce the experimental data of kaon electromagnetic form factor as in the case of pion form factor. Although the data of kaon form factor are not so accurate as compared with that of the pion form factor, the superconvergence dispersion relation leads to a stringent constraint on the asymptotic property of the form factor.

For the low momentum region, $|t| \lesssim 4\ \mathrm{GeV}^2$, t being the squared momentum transfer, the absorptive part of the dispersion integral is taken so as to realize the vector meson dominance model (VMD); it is approximated by the summation of the Breit-Wigner formulas of vector bosons with possible mixing among them. For the asymptotic region, we express the form factor as a power series expansion in the effective coupling constant of QCD. To calculate the QCD contribution to the absorptive part, it is necessary to extend to the timelike momentum of the effective coupling constant $\alpha_S(Q^2)$, being defined for the spacelike momentum $Q^2 = -t > 0$. We perform analytic continuation to the timelike region, $t>0$, by using the spectral representation of the QCD coupling constant [4–6]. Instead of α_S the QCD contribution is expressed in terms of the coupling constant given by the spectral representation, being written as α_R, for which we derived a simple approximate formula [3]. For the timelike momentum α_R becomes complex and the absorptive part of the form factor is obtained by taking the imaginary part of the power series. The absorptive part of the form factor is constructed so that the QCD contribution dominates for the high momentum and becomes very small for the low momentum $t \lesssim 4\ \mathrm{GeV}^2$. On the other hand, the Breit-Wigner terms are dominant in the low momentum and are very small in the high momentum region so that overlapping of the QCD and the resonance contributions are negligibly small. We are thus able to evade the double counting by applying the dispersion relation, which also works as an interpolation of the low and the high momentum regions for the real part of the form factor.

According to the perturbative QCD the boson form factor, being denoted as F, decreases asymptotically as $F(t) \rightarrow \mathrm{const}/t \ln|t|$ for large $|t|$ [7–9]. The asymptotic form of QCD is derived by utilizing the following property of the dispersion integral: Let $\mathrm{Im}\, F(t)$ approach $\mathrm{Im}\, F(t) \rightarrow c/|t|[\ln(t/Q_0^2)]^\gamma$ for $t \rightarrow \pm\infty$ with c, Q_0, and $\gamma>1$ being constants. Then

$$F(t) = \frac{1}{\pi}\int_{s_0}^{\infty} dt' \frac{\mathrm{Im}\, F(t')}{(t'-t)} \rightarrow \frac{c/\pi}{(\gamma-1)t[\ln(|t|/Q_0^2)]^{\gamma-1}},$$

for $t \rightarrow \pm\infty$ provided that $F(t)$ satisfies the superconvergence condition $\int_{s_0}^{\infty} dt'\ \mathrm{Im}\, F(t') = 0$, where s_0 denotes the threshold [3].

To ensure our approximation on α_R we compare it with the experimental data obtained by $e\bar{e}$ collider experiments by taking the QCD scale parameter Λ as an adjustable parameter. We find that the coupling constant agrees with the experimental data very well for the three loop approximation with $\Lambda = 0.2 \sim 0.3$ GeV and the number of active flavor $n_f = 4$ and 5. For the one loop approximation the experimental data are also reproduced very well, but the QCD scale parameter is determined as $\Lambda = 0.09 \sim 0.20$ GeV for $n_f = 4$ and 5 to reproduce the results of collider experiments. The value of Λ for the three loop approximation agrees with that which is obtained from the analysis of deep inelastic processes, $\Lambda_{\overline{\mathrm{MS}}}^{(4)} = 305 \pm 25 \pm 50$ MeV [10], but Λ becomes a little small for the one loop approximation.

In this paper we analyze the experimental data on the kaon electromagnetic form factor by applying the superconvergent dispersion relation. We take on the three loop approximation for α_R and keep the terms up to the order $O(\alpha_R^3)$ in the power series for the QCD part of form factor. The number of active flavor is fixed at $n_f = 3$ and the QCD

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scale parameter Λ is taken as an adjustable parameter. It is shown that, by using three loop approximation, we are able to reproduce the experimental data very well. If one restricts to the one loop approximation for the QCD effective coupling constant and keeps only the $O(\alpha_R)$ term for the QCD part of the form factor, the calculated result does not agree with experiment. Although the existing data of boson form factor are limited to low momentum regions, they provide a stringent constraint on the asymptotic property of the absorptive part of the form factor because of the superconvergence condition to which there is large contribution from the integration over high momentum region. It is, therefore, possible to get information on QCD by investigating the boson electromagnetic form factor.

The organization of this paper is given as follows: In Sec. II we summarize the formulas that are used in this calculation. In Sec. III we discuss on property of the effective coupling constant of QCD. We give the analytically regularized effective coupling constant and its analytic continuation to the timelike momentum. We compare α_R with the experimental results of the collider experiments. Numerical results of our analysis are summarized in Sec. IV. The final section is devoted to general discussions.

II. DISPERSION RELATION FOR THE BOSON FORM FACTOR

According to the perturbative QCD, the kaon electromagnetic form factor $F_K(t)$ approaches asymptotically [7–9,11]

$$F_K(t) \to -32\pi^2 f_K^2/\{\beta_0 t \ln(|t|/\Lambda^2)\} \quad (t\to\infty), \tag{1}$$

where t is the squared momentum of the virtual photon, f_K is the kaon decay constant, Λ is the QCD scale parameter, and β_0 is the one loop approximation of the β function in the renormalization group, $\beta_0 = 11 - 2n_f/3$ with n_f being the number of flavor. Therefore, the unsubtracted dispersion relation holds for the form factor

$$F_K(t) = \frac{1}{\pi}\int_{s_0}^{\infty} dt' \frac{\mathrm{Im}\, F_K(t')}{t'-t}, \tag{2}$$

where s_0 denotes the threshold. As is mentioned in the previous section $F_K(t)$ realizes the QCD prediction provided that $\mathrm{Im}\, F_K(t)$ satisfies the asymptotic condition

$$t\, \mathrm{Im}\, F_K(t) \to -32\pi^3 f_K^2/\{\beta_0 \ln^2(|t|/\Lambda^2)\} \quad (t\to\infty) \tag{3}$$

and the superconvergence condition

$$\frac{1}{\pi}\int_{s_0}^{\infty} dt'\, \mathrm{Im}\, F_K(t') = 0. \tag{4}$$

We approximate the form factor by addition of resonance contribution F_R with possible mixing among resonances F_{mix} and the QCD term F_{QCD}. The form factor is written as

$$F_K(t) = F_R(t) + F_{mix}(t) + F_{\mathrm{QCD}}(t). \tag{5}$$

Here F_R and F_{mix} are

$$F_R(t) = \sum_j \frac{c_j M_j^2}{M_j^2 - t - i\gamma_j}, \tag{6}$$

$$F_{mix}(t) = \sum_{j<k} \frac{c_{jk}\gamma_j\gamma_k}{(M_j^2 - t - i\gamma_j)(M_k^2 - t - i\gamma_k)}, \tag{7}$$

where M_j is the mass of the jth resonance and $\gamma_j = M_j\Gamma_j$ with Γ_j being the width. We neglect the kinematical factor of the reduced widths γ_j and take them independent of t as the dependence on t is negligibly small. The mixing part is the same as that of [12]. The imaginary parts of these amplitudes are given as follows:

$$\mathrm{Im}\, F_R(t) = \sum_j \frac{c_j M_j^2 \gamma_j}{(M_j^2 - t)^2 + \gamma_j^2}, \tag{8}$$

$$\mathrm{Im}\, F_{mix}(t) = \sum_{j<k} c_{jk}\left[\frac{\alpha_{jk}^I (M_j^2 - t) + \alpha_{jk}^R \gamma_j}{(M_j^2 - t)^2 + \gamma_j^2} - \frac{\alpha_{jk}^I (M_k^2 - t) + \alpha_{jk}^R \gamma_k}{(M_k^2 - t)^2 + \gamma_k^2}\right], \tag{9}$$

where the suffices stand for the resonances and α_{jk}^R and α_{jk}^I are

$$\alpha_{jk}^R = -\frac{\gamma_j\gamma_k(M_j^2 - M_k^2)}{(M_j^2 - M_k^2)^2 + (\gamma_j - \gamma_k)^2}, \tag{10}$$

$$\alpha_{jk}^I = -\frac{\gamma_j\gamma_k(\gamma_j - \gamma_k)}{(M_j^2 - M_k^2)^2 + (\gamma_j - \gamma_k)^2}. \tag{11}$$

For the QCD contribution we assume that F_{QCD} is expressed in terms of a function $\hat{F}_{\mathrm{QCD}}(t)$, a power series in the regularized effective coupling constant $\alpha_R(t)$ defined in the next section

$$\hat{F}_{\mathrm{QCD}}(t) = \sum_{j\geq 1} c_j^{\mathrm{QCD}}\{\alpha_R(t)\}^j, \tag{12}$$

multiplied by a function $h(t)$ which is incorporated phenomenologically to assure the threshold behavior and the convergence of the integral $\int_{s_0}^{\infty}\mathrm{Im}\, F_{\mathrm{QCD}}(t')dt'$. $\mathrm{Im}\, F_{\mathrm{QCD}}$ is then given in terms of the imaginary part of Eq. (12),

$$\mathrm{Im}\, F_{\mathrm{QCD}}(t) = \mathrm{Im}[\hat{F}_{\mathrm{QCD}}(t)]h(t). \tag{13}$$

In this calculation we take

$$h(t) = [(t - s_0)/(t + t_1)]^{3/2} t_0/(t + t_0). \tag{14}$$

The function $h(t)$ corresponds to the cutoff and the convergence factor for the QCD part of the nucleon form factors used by Mergell *et al.* [13]. Here we avoid the discontinuity due to the cutoff by taking it as a smooth function of t for $t > s_0$.

The effective coupling constant becomes complex for the timelike region $t>0$, and $\mathrm{Im}\,\hat{F}_{\mathrm{QCD}}$ is obtained by taking the imaginary part of $\alpha_R(t)^n$, $n=1,2,\ldots$. We have

$$\begin{aligned}\mathrm{Im}\,F_{\mathrm{QCD}}(t)=\{&c_1^{\mathrm{QCD}}\,\mathrm{Im}[\alpha_R(t)]\\&+2c_2^{\mathrm{QCD}}\,\mathrm{Re}[\alpha_R(t)]\mathrm{Im}[\alpha_R(t)]\\&+c_3^{\mathrm{QCD}}\{3(\mathrm{Re}[\alpha_R(t)])^2\\&-(\mathrm{Im}[\alpha_R(t)])^2\}\mathrm{Im}[\alpha_R(t)]+\cdots\}h(t).\end{aligned}\tag{15}$$

We analyze the experimental data for the kaon form factor for the timelike and spacelike regions by using the dispersion relation (2), where the absorptive parts of the form factor is expressed in terms of the imaginary parts (8), (9), and (15). The parameters appearing in our formula are determined by analyzing the experimental data under the constraints of the normalization $F(0)=1$ and the superconvergence condition (4). The lowest order of the QCD term is given through (3), (13), (14) as follows:

$$c_1^{\mathrm{QCD}}=-8\pi f_K^2/t_0.\tag{16}$$

Here use is made of the asymptotic form of $\mathrm{Im}\,\alpha_R(t)$

$$\mathrm{Im}\,\alpha_R(t)\to 4\pi^2/\beta_0\ln^2(t/\Lambda^2),$$

as is shown by Eq. (25).

III. THE EFFECTIVE COUPLING CONSTANT OF QCD

In order to perform the analytic continuation to the timelike region of the QCD effective coupling constant $\alpha_S(Q^2)$, being defined for the spacelike momentum $Q^2>0$, we define α_R through the spectral representation [4–6]

$$\alpha_R(t)=\frac{1}{\pi}\int_0^\infty dt'\frac{\sigma(t')}{t'-t},\tag{17}$$

where the spectral function σ is given in terms of the discontinuity of α_S along the cut

$$\sigma(t)=\frac{1}{2i}[\alpha_S(e^{-i\pi}t)-\alpha_S(e^{i\pi}t)].\tag{18}$$

For the one loop approximation we have

$$\alpha_S(Q^2)=\frac{4\pi}{\beta_0\ln(Q^2/\Lambda^2)}.\tag{19}$$

The analytic continuation to the timelike region is effected by the replacement $Q^2\to e^{\mp i\pi}t$, namely,

$$\alpha_S(e^{\mp i\pi}t)=\frac{4\pi}{\beta_0}\left[\frac{\ln(t/\Lambda^2)}{\ln^2(t/\Lambda^2)+\pi^2}\pm i\frac{\pi}{\ln^2(t/\Lambda^2)+\pi^2}\right],\tag{20}$$

and σ is obtained to be

$$\sigma(t)=\frac{4\pi}{\beta_0}\frac{\pi}{\ln^2(t/\Lambda^2)+\pi^2}.\tag{21}$$

It must be noticed that substitution of Eqs. (21) to (17) leads to a different coupling constant from the original one for the spacelike momentum as is seen from the consequence that the function defined by the spectral representation has no singularity for $t=-Q^2<0$, while $\alpha_S(Q^2)$ given by Eq. (19) has a pole at $Q^2=\Lambda^2$. Actually, for the one loop approximation α_R becomes

$$\alpha_R(Q^2)=\frac{4\pi}{\beta_0}\left[1/\ln(Q^2/\Lambda^2)-\frac{\Lambda^2}{Q^2-\Lambda^2}\right],\tag{22}$$

which is free from singularity at $Q^2=\Lambda^2$ [5]. Let us discuss the higher order approximation for the effective coupling constant. For the spacelike region the α_S is obtained by solving the equation for the effective coupling constant in the renormalization group through iteration with respect to $\ln(Q^2/\Lambda^2)$. We have

$$\begin{aligned}\alpha_S(Q^2)=\frac{4\pi}{\beta_0}\Bigg[&\ln(Q^2/\Lambda^2)+a_1\ln\{\ln(Q^2/\Lambda^2)\}\\&+a_2\frac{\ln\{\ln(Q^2/\Lambda^2)\}}{\ln(Q^2/\Lambda^2)}+\frac{a_3}{\ln(Q^2/\Lambda^2)}+\cdots\Bigg]^{-1},\end{aligned}\tag{23}$$

where $a_1=2\beta_1/\beta_0^2$, $a_2=4\beta_1^2/\beta_0^4$, $a_3=(4\beta_1^2/\beta_0^4)(1-\beta_0\beta_2/8\beta_1^2)$, and $\beta_i(i=0,1,2)$ are

$$\beta_0=11-\frac{2n_f}{3},\quad \beta_1=51-\frac{19n_f}{3},$$

$$\beta_2=2857-\frac{5033n_f}{9}+\frac{325n_f^2}{27},$$

with n_f being the number of active flavor. We express the effective coupling constant by the Padé-like formula (23) instead of the formula given in Ref. [10] for the following reasons: Eq. (23) is directly derived from the equation for α_S in renormalization group and we can evade the singularity such as $1/\ln^3(Q^2/\Lambda^2)$ at $Q^2=\Lambda^2$. After the analytic continuation to the timelike region via $Q^2\to e^{-i\pi}t$, we have $\alpha_S=\mathrm{Re}[\alpha_S(t)]+i\,\mathrm{Im}[\alpha_S(t)]$ with

$$\mathrm{Re}[\alpha_S(t)]=\frac{4\pi u}{\beta_0 D(t)},\tag{24}$$

$$\mathrm{Im}[\alpha_S(t)]=\frac{4\pi v}{\beta_0 D(t)},\tag{25}$$

where

$$u=\ln(t/\Lambda^2)+\frac{a_1}{2}\ln(\ln^2(t/\Lambda^2)+\pi^2)+\frac{a_2}{\ln^2(t/\Lambda^2)+\pi^2}\times\left\{\frac{1}{2}\ln(t/\Lambda^2)\ln(\ln^2(t/\Lambda^2)+\pi^2)+\pi\theta\right\}+\frac{a_3\ln(t/\Lambda^2)}{\ln^2(t/\Lambda^2)+\pi^2}, \tag{26}$$

$$v=\pi+a_1\theta-\frac{a_2}{\ln^2(t/\Lambda^2)+\pi^2}\times\left\{\frac{\pi}{2}\ln(\ln^2(t/\Lambda^2)+\pi^2)-\theta\ln(t/\Lambda^2)\right\}-\frac{\pi a_3}{\ln^2(t/\Lambda^2)+\pi^2}, \tag{27}$$

and

$$D=u^2+v^2. \tag{28}$$

Here

$$\theta=\tan^{-1}\left\{\frac{\pi}{\ln(t/\Lambda^2)}\right\}. \tag{29}$$

The spectral function σ is obtained by taking the imaginary part of α_S, that is,

$$\sigma(t)=4\pi v/\beta_0 D, \tag{30}$$

where v and D are defined by Eqs. (27) and (28), respectively.

As a function of t, the effective coupling constant given by Eq. (23) has a pole in the spacelike momentum region at

$$Q^2=Q^{*2}=\Lambda^2 e^{u^*}, \tag{31}$$

with $u^*=0.7659596$ for the number of flavor $n_f=3$. The residue at $Q^2=\Lambda^2 e^{u^*}$ is

$$A^*=4\pi\Lambda^2 e^{u^*}\bigg/\left\{\beta_0\left(1+\frac{a_1}{u^*}-a_2\frac{\ln u^*}{u^{*2}}+\frac{a_2-a_3}{u^{*2}}\right)\right\}. \tag{32}$$

It has branch cuts arising from the logarithmic function in α_S; $t<0$ and $0<t<\Lambda^2$ where the former comes from $\ln(t/\Lambda^2)$ and the latter from the term $\ln(\ln(t/\Lambda^2))$. Therefore, the threshold of the spectral representation (17) is $-\Lambda^2$ if the effective coupling constant is calculated by two or three loop approximation.

It is shown by the direct computation of Eq. (17), by using the spectral function (30) with the threshold kept at $t=0$, that the effective coupling constant (17) is approximately given by the following formula:

$$\alpha_R(Q^2)\approx\alpha_S(Q^2)-A^*/(Q^2-Q^{*2}), \tag{33}$$

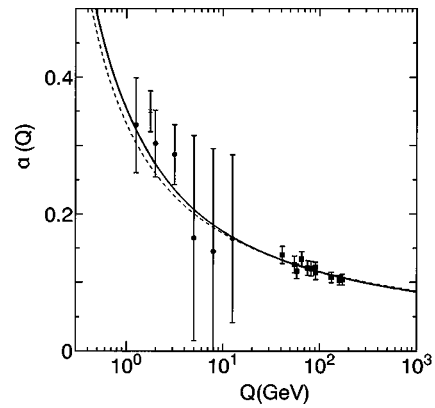

FIG. 1. The effective coupling constant of QCD for $n_f=5$. The solid curve is α_R calculated by the formula (33) and the dashed one denotes α_S the result with ghost (23). $\Lambda=0.194$ GeV for α_R and $\Lambda=0.085$ GeV for α_S. The closed circles are taken from [14], the black square [15,16] and the cross denote the point determined from the τ decay [10].

both for the timelike and spacelike momentum region for $|t|\gtrsim 1\ \mathrm{GeV}^2$. The term $A^*/(Q^2-Q^{*2})$ works as elimination of the ghost pole from the effective coupling constant in the spacelike momentum region. We note that Eq. (33) is shown to be exact if the threshold of integral in Eq. (17) is taken as $t=-\Lambda^2$. By the direct computation of α_R by Eq. (17) with σ given by Eq. (30) we have the following result: For $|t|\gtrsim 3\ \mathrm{GeV}^2$ the difference of the approximate one (33) and α_R is less than 0.4%. The approximation becomes better as $|t|$ becomes larger. It must be remarked that we have multiplied by the function $h(t)$ to define $\mathrm{Im}\,F^{\mathrm{QCD}}$, therefore, the contribution from low t region is considerably suppressed in the dispersion integral. Our approximate effective coupling constant (33) can be used for the analytically regularized one via the spectral representation. For the timelike region we perform the analytic continuation of the effective coupling constant (33) by the replacement $Q^2\to e^{-i\pi}t$.

Let us compare the regularized effective coupling constant α_R with the experimental data for the one loop (22) and the three loop approximation (33) by taking Λ and n_f as parameters. For $n_f=4$ we have $\Lambda=0.141\pm0.019$ GeV with $\chi^2=3.9$ for the one loop approximation and $\Lambda=0.325\pm0.038$ GeV with $\chi^2=6.1$ for the three loop approximation. In the case of $n_f=5$ we obtain the following results: $\Lambda=0.087\pm0.013$ GeV with $\chi^2=4.4$ for the one loop approximation and $\Lambda=0.194\pm0.027$ GeV with $\chi^2=4.0$ for the three loop approximation. Here we use the data given in [14–16] with the number of points 17. The datum obtained by the τ decay is omitted in the chi square analysis. The value of Λ agrees with the world average for the three loop approximation but for the one loop approximation Λ becomes a little small. We compare in Fig. 1 the calculated running coupling constants α_S (19) and α_R (33), respectively, for the spacelike momentum for the three loop approximation for $n_f=5$ together with the experimental data. The QCD parameter Λ is 0.087 GeV for α_S and 0.194 GeV for α_R. The regularized effective coupling constant α_R agrees with the experimental data very well. To calculate the QCD contribution to the

TABLE I. The residues c_i, the mixing parameter $c_{\omega'-\omega''}$, and the QCD parameters c_i^{QCD} $(i=1,2,3)$, which are determined by our analysis for the cases with and without mixing. Three loop approximation is used for the QCD effective coupling constant. The χ^2 values are given for the spacelike momentum χ^2_{space}, the timelike momentum χ^2_{time}, and the total value $\chi^2_{tot}=\chi^2_{space}+\chi^2_{time}$. The parameters appearing in the threshold function $h(t)$ (14) are fixed at $t_0=t_1=16\,\text{GeV}^2$ [3]. The numbers of experimental data points are 15 and 51 for the spacelike and the timelike momentum regions, respectively. The QCD scale parameter is taken as $\Lambda=0.6$ GeV.

	No mixing	$\omega(1420)-\omega(1600)$ mixing
c_ρ	0.062	0.113
c_ω	0.555	0.527
c_ϕ	0.3457	0.3450
$c_{\omega'}$	−2.670	−0.055
$c_{\omega''}$	0.147	−0.107
$c_{\rho'}$	0.218	0.249
$c_{\phi'}$	−0.09319	−0.06371
$c_{\rho''}$	0.029	0.0530
$c_{\omega'-\omega''}$	0	−3.685
c_1^{QCD} [a]	−0.04011	−0.04011
c_2^{QCD}	−1.057	−1.211
c_3^{QCD}	−0.603	0.469
χ^2_{time}	65.8	62.5
χ^2_{space}	12.6	12.7
χ^2_{tot}	78.4	75.2

[a] c_1^{QCD} is calculated by Eq. (16).

form factor we shall use α_R with three loop approximation (33) which is extrapolated to the timelike momentum; $\text{Im}[\alpha_R]$ is given by Eq. (25) and $\text{Re}[\alpha_R]$ by Eq. (24) with the addition of the term $A^*/(t+Q^{*2})$ to $\alpha_S(Q^2)$.

IV. NUMERICAL RESULTS ON THE KAON FORM FACTOR

We determine the parameters appearing in our formulas by comparing with the experimental data. We are able to reproduce the experimental results both for the spacelike and the timelike regions. We summarize in Table I the parameters determined by the analysis. We take the threshold of the dispersion relation as $s_0=4m_\pi^2$, m_π being the pion mass and fix the kaon decay constant at $f_K=0.1598$ GeV [10]. The parameters appearing in $h(t)$ (14) are taken as $t_0=t_1=16\,\text{GeV}^2$ [3]. We remark that the result is not sensitive to the values of s_0, t_0, and t_1. The QCD scale parameter Λ is treated as an adjustable parameter, and the number of active flavor is fixed at $n_f=3$. For the vector bosons we restrict ourselves to the established ones, $\rho(770)$, $\omega(780)$, $\phi(1060)$, $\omega'(1420)$, $\rho(1450)$, $\omega''(1600)$, $\phi'(1680)$, $\rho(1700)$ with the masses and widths kept at the experimental values given in Ref. [10].

In Figs. 2(a) and 2(b) we compare our result with the data on kaon form factor for the timelike momentum [17] and in Fig. 3 $|t||F_K(t)|^2$ is compared with experiment for the spacelike momentum [18]. We are able to realize the experimental data for the three loop approximation both for the timelike and spacelike momentum. The value of the QCD scale parameter is determined to be in the range $\Lambda=0.3\sim2.0$ GeV. The best fit is attained for $\Lambda\sim0.6$ GeV. The dashed curve in Fig. 2(a) stands for the case without mixing among particles. The theoretical curve seems to be a little smaller than the experimental data around the energy $2E\approx1.5$ GeV, where $2E=\sqrt{t}$. To investigate if we are able to improve the result we considered the particle mixing of ω'-ω''. We did not consider the mixing of the other resonances as the data for the kaon electromagnetic form factor are not sufficiently accurate to determine more than two mixing parameters. We find that the result is improved a little by the particle mixing ω'-ω''. We enter the results for VMD with the $\rho(770)$, $\omega(780)$, and $\phi(1060)$ [17] in Figs. 2 and 3 by the dotted curve. Taking the one loop approximation for the effective coupling constant (22) with $O(\alpha_R)$ for $\text{Im}\,F^{\text{QCD}}$, we do not have a good result. In this case the QCD scale parameter is

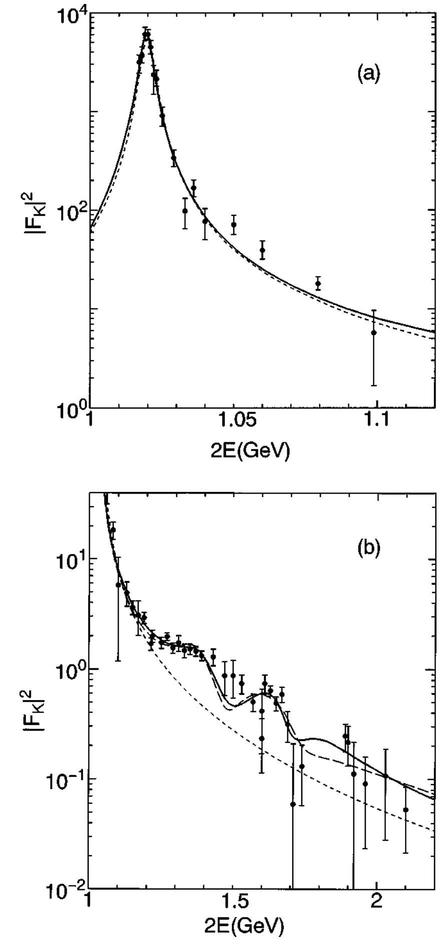

FIG. 2. $|F_K(t)|^2$ for the timelike momentum, where $2E=\sqrt{t}$ with $t>0$. The solid curve is the result with mixing between ω' and ω'' and the dotted one denotes the result for the simple VMD with $\rho(770)$, $\omega(782)$, and $\phi(1020)$. The dashed curve in (b) denotes the case without mixing among vector bosons. The experimental data is taken from [17]. (a) $2E\leqslant1.2$ GeV; (b) $2E\geqslant1.0$ GeV.

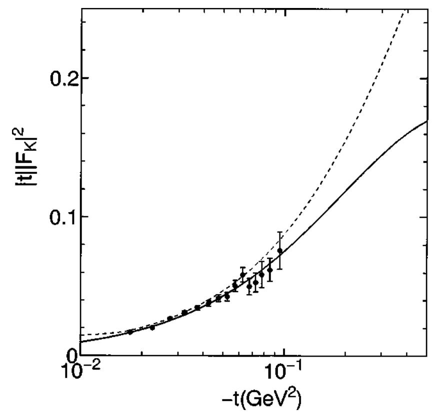

FIG. 3. $|t||F_K(t)|^2$ for the spacelike momentum. The solid curve is the result with mixing between ω' and ω'' and the dotted one denotes the result for the simple VMD with $\rho(770)$, $\omega(782)$, and $\phi(1020)$. The experimental data are taken from [18].

determined as $\Lambda = 10 \sim 20\,\mathrm{GeV}$ with the value of chi square as large as $\chi^2 \approx 300$. Even if the particle mixing of ω'-ω'' is considered, the situation is not improved appreciably.

V. DISCUSSIONS

By using the unsubtracted dispersion relation with the superconvergence constraint we analyzed the experimental data of the kaon electromagnetic form factor and obtained agreement with the experiments both for the spacelike and timelike momenta. The absorptive part is given as the summation of the Breit-Wigner formulas for the resonances and the QCD term. The latter is expressed by the power series expansion in the effective coupling constant of QCD. For the absorptive part of the form factor the QCD contribution dominates in the high t region and is negligibly small for $t \lesssim 10\,\mathrm{GeV}^2$, while the resonance contribution is dominant in the low momentum region $t \lesssim 10\,\mathrm{GeV}^2$ and becomes very small above 10 GeV^2 so that there is no danger of double counting. The dispersion relation with the superconvergence condition works as an interpolation of the high and low momentum regions for the real part of the form factor.

To synthesize the VMD and QCD we have alternatives of the addition and the multiplication models. In the former $\mathrm{Im}\,F$ is given as the summation of the Breit-Wigner formulas as we have done in this paper and in the latter $\mathrm{Im}\,F$ is expressed as a product of the Breit-Wigner and the QCD terms [19]. Imposing the superconvergence condition on the form factor, we obtain qualitatively similar results for both models. In this paper we take on the addition model because the physical meaning is clearer and the better result is obtained.

For the simple VMD with $\rho(770)$, $\omega(782)$, and $\phi(1020)$, the result is not good especially for the timelike region. If we take account of higher resonances, we are able to improve the result. However, the QCD constraint is not satisfied, as the form factor decreases as a polynomial, $F_K(t) \sim t^{-n}$ with $n > 0$ being an integer.

Klingl *et al.* investigated the boson electromagnetic form factor based on the chiral SU(3) [20]. They examined the pion and kaon form factors both for the timelike and spacelike momentum and they were able to reproduce the experimental data. For the timelike region, there are small deviations from the data for the momentum around 1.1 GeV and 1.4 GeV for the kaon electromagnetic form factor. This implies that vector bosons with larger mass than the ϕ meson are necessary for the electromagnetic form factor as we have assumed in this paper.

In our analysis the QCD scale parameter is determined as $\Lambda \sim 0.6\,\mathrm{GeV}$ which is a little larger than that obtained by the deep inelastic processes, $\Lambda \sim 0.3\,\mathrm{GeV}$. We have analyzed the kaon form factor so that the number of active flavor is $n_f = 3$, while for the deep inelastic processes $n_f = 4 \sim 5$. The result seems to support the implication that Λ becomes smaller as the number of flavor increases [21].

The experiments on the boson electromagnetic form factor are advantageous in investigating subhadronic interactions at high energy in the following respects: First, we are able to study processes with small number of active flavor. Secondly, it is possible to estimate effects of resonances directly, because they contribute to the form factor as real processes in contrast to the nucleon form factor, in which contribution of vector bosons is indirect as most of resonances are below the $N\bar{N}$ threshold. Therefore, by the experimental data in the timelike region the low momentum part of $\mathrm{Im}\,F$ is nearly determined. As we have used the unsubtracted dispersion relation with the superconvergence condition, for which there is large contribution from high momentum part of $\mathrm{Im}\,F$, we are able to study on the hadron interaction at very high energy. Precise measurement of the boson form factor for the large momentum transfer, especially for the spacelike momentum, is expected to give important data in exploring interactions of subhadronic system.

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