Limitations of the number self-consistent random phase approximation

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The quasiparticle random phase approximation (QRPA) equations are solved taking into account the Pauli principle at the expectation value level, and allowing changes in the mean field occupation numbers to minimize the energy while having the correct number of particles in the correlated vacuum. The study of Fermi pn excitations in ^{76}Ge using a realistic Hilbert space shows that the pairing energy gaps in the modified mean field are diminished up to one half of the experimental value when strong proton-neutron correlations are present. Additionally, the Ikeda sum rule for Fermi transitions is violated due to the lack of scattering terms in the phonon operators. These results call for a critical revision of the double β decay half-lives estimated using the QRPA extensions when standard QRPA calculations collapse.

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I. INTRODUCTION

The random phase approximation (RPA) and its quasiparticle generalization (QRPA) have been widely used in the last decades to study electromagnetic transitions and β decays in medium and heavy nuclei [1,2]. The proton-neutron quasiparticle random phase approximation (*pn*-QRPA) has been extensively employed in the description of single and double β decays in vibrational nuclei. However the RPA develops a collapse, i.e., it presents imaginary eigenvalues for strengths beyond a critical value of the force [3–6].

A whole family of extensions of the QRPA, called renormalized QRPA (RQRPA) are known that do not develop any collapse by implementing the Pauli principle in a consistent way, beyond the simplest quasiboson approximation [7–13]. However, in its simplest versions there is a violation of the nonenergy weighted Ikeda sum rule [14]. Calculations to determine the amount of the violation and some improvements to the RQRPA, in order to restore the sum rule, have been presented [15]. It has been shown that treating simultaneously BCS and QRPA equations one can fulfil the Ikeda sum rule for the Fermi case when a schematic model is used [16].

In recent articles we studied the expectation values of the particle and quasiparticle numbers, the particle number fluctuations and the number of particle pairs with $J=0,\ T=1$ and $J=1,\ T=0$ in the ground state of ^{76}Ge as a function of the residual proton-neutron interaction, using realistic Hilbert spaces [17,18]. We found an important amount of particle number fluctuations in the RQRPA ground state beyond the QRPA collapse, pointing out a source of uncertainty in the RQRPA results. The analysis of the number of pairs showed that the isoscalar-isovector phase transition found in exact calculations is causing the QRPA collapse and is missed in the RQRPA formalism.

In the present work we go a step further, studying ^{76}Ge with a renormalized RPA where at the same time the mean field is changed by minimizing the energy and fixing the number of particles in the correlated ground state. While particle number fluctuations are smaller than in the previous cases, they still exhibit a clear increase after the point of collapse. More remarkably, the pairing gap is strongly reduced in comparison with its experimental value, and the Ikeda sum rule is violated. Both results cast serious doubts about the double β decay half-lives estimated using the QRPA extensions, in particular for those nuclei where standard QRPA calculations collapse [9,10,12]. Together with the conclusions obtained in Refs. [17,18], we get a clear picture of the limitations associated with the QRPA extensions, which at the end are the same common sense ask for: you cannot allow the residual proton-neutron interaction to dictate the composition of the ground state wave function without missing contact with actual nuclei, even if the formalism allows you to overcome the collapse.

In Sec. II the renormalized gap and number equations are introduced, whose relationship with the RQRPA equations is shown in Sec. III. Some relevant expectation values and the Ikeda sum rule are discussed in Sec. IV, results for ^{76}Ge are presented in Sec. V and the conclusions in Sec. VI.

II. RENORMALIZED GAP AND NUMBER EQUATIONS

In this section the gap and number equations are obtained minimizing the Hamiltonian expectation value $\langle 0|H|0\rangle$ in the correlated vacuum $|0\rangle$. The Hamiltonian is

$$H=H_p+H_n+H_{p,n}. \tag{1}$$

The first two terms refer to the proton and neutron Hamiltonians

$$H_t=\sum_{\mathbf{t}}(e_t-\lambda)a^\dagger_{\mathbf{t}}a_{\mathbf{t}}+\frac{1}{2}\sum_{\mathbf{t}'s}\langle \mathbf{t}_1\mathbf{t}_2|V|\mathbf{t}_3\mathbf{t}_4\rangle a^\dagger_{\mathbf{t}_1}a^\dagger_{\mathbf{t}_2}a_{\mathbf{t}_4}a_{\mathbf{t}_3}, \tag{2}$$

where the single particle energies are denoted by e_t, the chemical potential by λ, and the last term corresponds to the proton-neutron interaction

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$$H_{p,n}=\sum_{\mathbf{p},\mathbf{p}',\mathbf{n},\mathbf{n}'}\langle \mathbf{p},\mathbf{n}|V|\mathbf{p}',\mathbf{n}'\rangle a_{\mathbf{p}}^{\dagger}a_{\mathbf{n}}^{\dagger}a_{\mathbf{n}'}a_{\mathbf{p}'}. \quad (3)$$

The subscripts $\mathbf{t}(t)$ stand for $\mathbf{p}(p)$ (protons) or $\mathbf{n}(n)$ (neutrons), being $\mathbf{t}\equiv t,m_t$, with $t\equiv\{n_t,l_t,j_t\}$ and $m_t\equiv m_j$.

Through the Bogoliubov transformation

$$\alpha_{\mathbf{t}}^{\dagger}=u_t a_{\mathbf{t}}^{\dagger}-v_t a_{\bar{\mathbf{t}}}, \quad (4)$$

with $a_{\bar{\mathbf{t}}}=(-1)^{t+m_t}a_{t,-mt}$, we get [19]

$$H=U+\sum_p 2\Omega_p\epsilon_p\hat{\mathcal{N}}_p+\sum_n 2\Omega_n\epsilon_n\hat{\mathcal{N}}_n$$
$$+H_{22}+H_{40}+H_{04}+H_{13}+H_{31}, \quad (5)$$

being

$$\hat{\mathcal{N}}_{tt'}\equiv\frac{[\alpha_t^{\dagger}\alpha_{\bar{t}'}]^0}{\sqrt{2\Omega_t}},\quad \hat{\mathcal{N}}_t\equiv\hat{\mathcal{N}}_{t=t'},\quad \Omega_t\equiv\frac{(2j_t+1)}{2},$$

and where U and the quasiparticle energies ϵ_t are defined as

$$U=\sum_t\Bigg[2\Omega_t v_t^2(e_t-\lambda)$$
$$+\sum_{t'}\sqrt{\Omega_{t'}\Omega_t}v_t^2v_{t'}^2F(tt,t't',0)-\Omega_t u_t v_t\Delta_t\Bigg], \quad (6)$$

$$\epsilon_t=\Bigg[e_t-\lambda+\sum_{t'}\sqrt{\frac{\Omega_{t'}}{\Omega_t}}v_{t'}^2F(tt,t't',0)\Bigg]$$
$$\times(v_t^2-u_t^2)+2\Delta_t u_t v_t \quad (7)$$

being

$$\Delta_t=-1/2\sum_{t'}\sqrt{\frac{\Omega_{t'}}{\Omega_t}}u_{t'}v_{t'}G(tt,t't',0), \quad (8)$$

the "gap" and F, G the usual particle-hole (PH) and particle-particle (PP) coupled two-particle matrix elements. The terms H_{nm} in Eq. (5) destroy m quasiparticles and creates n quasiparticles, respectively.

To obtain the quasiparticle mean field occupations v_i the ground state energy $\langle 0|H|0\rangle$ is minimized, i.e., it is asked that [1,2]

$$\frac{\partial}{\partial v}\langle 0|H|0\rangle=0, \quad (9)$$

subject to the constrictions

$$\langle 0|\hat{N}|0\rangle=N,\quad \langle 0|\hat{Z}|0\rangle=Z, \quad (10)$$

and the normalization $u_i^2+v_i^2=1$, being $\hat{N}$ and $\hat{Z}$ the neutron and proton particle number operators, respectively.

The standard BCS procedure treats protons and neutrons separately and in Hamiltonian (5) only pairing interactions, refering in general to like particles interacting through the $J=0$ channel, are included. In this case the ground state energy is just U as defined in Eq. (6) and $|0\rangle$ is the BCS ground state. The residual interactions, either between like particles or protons and neutrons, represented by the terms H_{nm} in Eq. (5), are usually included in a second step, most commonly using the QRPA [1,2].

One of the drawbacks of this procedure is that Eq. (10) is only enforced for the BCS vacuum. When residual interactions are present the expectation values of the particle number operators do not coincide with the actual number. The SCRPA [20,11,21] is designed to overcome this difficulty by solving Eqs. (9) and (10) using the RPA vacuum.

Full self-consistency requires to consider the proton-neutron interaction contribution in the minimization, but the system of equations which describes this problem is nonlinear and rather complicated, and has only been implemented in schematic models [11,22,23]. In order to perform calculations in realistic Hilbert spaces we will include in this first step only the like particles part of the Hamiltonian, while the ground state $|0\rangle$ will be sensitive to proton-neutron interaction through the RPA equations, following a philosophy close to Ref. [24]. From here on we will refer to this approximation as SRQRPA. We are absolutely aware that our treatment is not fully self-consistent. We are just meeting the requirements of Eq. (10) for the RPA vacuum, and including the modifications in the mean field due to pn correlations at the lowest level. However, as is shown below, even these mild modifications have very important effects in the observables of the system.

Following [21], but including only the modifications of the gap equations due to use of the RPA vacuum instead of the BCS vacuum, i.e., not taking into account the proton-neutron residual interaction explicitly, we arrive to the modified gap equation

$$2(\overline{e_t}-\lambda)v_t u_t-(u_t^2-v_t^2)\tilde{\Delta}_t=0, \quad (11)$$

where

$$\overline{e_t}=e_t+\sum_{t'}\sqrt{\frac{\Omega_{t'}}{\Omega_t}}v_{t'}^2F(tt,t't',0) \quad (12)$$

is the single particle energy corrected by the self-energy and

$$\tilde{\Delta}_t=-1/2\sum_{t'}\sqrt{\frac{\Omega_{t'}}{\Omega_t}}u_{t'}v_{t'}G(tt,t't',0)(1-2\langle 0|\hat{\mathcal{N}}_{t'}|0\rangle), \quad (13)$$

the "renormalized" gap. Notice that the pairing interaction appears in the gap equation renormalized by the presence of proton-neutron residual interactions, which introduce a finite number of quasiparticles in the RPA vacuum [17]. For the sake of simplicity we have dropped some higher order terms in Eq. (11) which come from the interaction between like particles connecting the many quasiparticle components of the correlated ground state. The definition of the renormalized gap (13) is unique at this level of approximation [21]. This is an important definition to be kept in mind, because the factor $(1-2\langle 0|\hat{\mathcal{N}}_{t'}|0\rangle)$ will play a definite role in suppressing the gap when proton-neutron residual interactions are large. What remains is to couple the renormalized gap problem with the RQRPA equations.

III. RQRPA

The nuclear excited states are constructed as [9,15]

$$|\lambda JM\rangle \equiv \Omega^{\dagger}(\lambda JM)|0\rangle, \tag{14}$$

$$\Omega^{\dagger}(\lambda JM)=\sum_{pn}\left[X_{pn}(\lambda J)A^{\dagger}_{pn}(JM)-Y_{pn}(\lambda J)A_{pn}(\overline{JM})\right], \tag{15}$$

where

$$\hat{A}^{\dagger}_{pn}(JM)\equiv[\alpha^{\dagger}_{p}\alpha^{\dagger}_{n}]^{JM}D^{-1/2}_{pn}, \quad D_{pn}\equiv(1-\langle 0|\hat{\mathcal{N}}_p+\hat{\mathcal{N}}_n|0\rangle), \tag{16}$$

are the renormalized two-quasiparticle proton-neutron creation operators, which satisfy

$$\langle 0|[A_{pn}(JM),A^{\dagger}_{p'n'}(J'M')]|0\rangle=\delta_{pp'}\delta_{nn'}\delta_{JJ'}\delta_{MM'}. \tag{17}$$

Here $|0\rangle$ is the RPA correlated ground state, defined by the condition

$$\Omega(\lambda JM)|0\rangle=0. \tag{18}$$

Each amplitude, $X_{pn}(\lambda J)$ and $Y_{pn}(\lambda J)$, is associated with the excitation energy $\omega_{\lambda J}$ of the λth state with angular momentum J. They are the eigenvectors and eigenvalues, respectively, of the RPA equations [1,2]

$$\begin{pmatrix} A(J) & B(J)\\ B^{*}(J) & A^{*}(J)\end{pmatrix}\begin{pmatrix} X(\lambda J)\\ Y(\lambda J)\end{pmatrix}=\omega_{\lambda J}\begin{pmatrix} X(\lambda J)\\ -Y(\lambda J)\end{pmatrix}, \tag{19}$$

where

$$A_{pn,p'n'}(J)=(\epsilon_p+\epsilon_n)\delta_{pp'}\delta_{nn'}+D^{1/2}_{pn}U^{\mathcal{F}}_{pn,p'n'}(J)D^{1/2}_{p'n'},$$

$$B_{pn,p'n'}(J)=D^{1/2}_{pn}U^{\mathcal{B}}_{pn,p'n'}(J)D^{1/2}_{p'n'}, \tag{20}$$

with

$$\begin{aligned}U^{\mathcal{F}}_{pn,p'n'}(J)=&\,G(pn,p'n',JM)(u_pu_nu_{p'}u_{n'}+v_pv_nv_{p'}v_{n'})\\&+F(pn,p'n',JM)\\&\times(u_pv_nu_{p'}v_{n'}+v_pu_nv_{p'}u_{n'}),\\U^{\mathcal{B}}_{pn,p'n'}(J)=&-G(pn,p'n',JM)(u_pu_nv_{p'}v_{n'}\\&+v_pv_nu_{p'}u_{n'})+F(pn,p'n',JM)\\&\times(v_pu_nu_{p'}v_{n'}+u_pv_nv_{p'}u_{n'}).\end{aligned} \tag{21}$$

The RPA equations (19) depend explicitly on the mean field occupations v_i through the **A** and **B** matrices defined in Eqs. (20) and (21). At the same time, the quasiparticle occupations v_i depend on the RPA amplitudes X,Y, on which depend also $|0\rangle$, through the renormalized gap equation (11) and number equations (10).

IV. EXPECTATION VALUES IN THE CORRELATED RPA VACUUM

Using the quasiboson approximation

$$\begin{aligned}[A_{pn}(JM),A^{\dagger}_{p'n'}(J'M')]&\approx\langle 0|[A_{pn}(JM),A^{\dagger}_{p'n'}(J'M')]|0\rangle\\&=\delta_{pp'}\delta_{nn'}\delta_{JJ'}\delta_{MM'},\end{aligned} \tag{22}$$

the RPA ground state defined by Eq. (18) can be written as

$$|0\rangle=N_0e^{\hat{S}}|\mathrm{BCS}\rangle, \tag{23}$$

with the BCS vacuum defined by the property

$$\alpha_t|\mathrm{BCS}\rangle=0 \tag{24}$$

and

$$\hat{S}=\frac{1}{2}\sum_{pnp'n'J}\sqrt{(2J+1)}C(J)_{pnp'n'}[A^{\dagger}_{pn}(J)A^{\dagger}_{p'n'}(J)]^{0},$$

$$C(J)_{pnp'n'}=\sum_{\lambda}Y(J)^{*}_{pn,\lambda}X(J)^{*\,-1}_{\lambda,p'n'}. \tag{25}$$

The quasiparticle occupations $\langle 0|\hat{\mathcal{N}}_t|0\rangle$ evaluated using the QRPA vacuum (23) have the explicit form [18]

$$\langle 0|\hat{\mathcal{N}}_p|0\rangle=\sum_{\lambda Jn'}\frac{(2J+1)}{2\Omega_p}D_{pn'}|Y(J)_{pn',\lambda}|^2,$$

$$\langle 0|\hat{\mathcal{N}}_n|0\rangle=\sum_{\lambda Jp'}\frac{(2J+1)}{2\Omega_n}D_{p'n}|Y(J)_{p'n,\lambda}|^2. \tag{26}$$

The mean particle numbers of protons and neutrons are [25]

$$\langle Z\rangle\equiv\langle 0|\hat{\mathcal{Z}}|0\rangle=2\sum_p\Omega_pv_p^2+2\sum_p\Omega_p(u_p^2-v_p^2)\langle 0|\hat{\mathcal{N}}_p|0\rangle,$$

$$\langle N\rangle\equiv\langle 0|\hat{\mathcal{N}}|0\rangle=2\sum_n\Omega_nv_n^2+2\sum_n\Omega_n(u_n^2-v_n^2)\langle 0|\hat{\mathcal{N}}_n|0\rangle. \tag{27}$$

When the BCS vacuum is used, the second term in each of these expressions vanishes and one is left with the usual number equation. When the residual interaction is present these terms have a relevant contribution to the particle number [17]. They are included in the present work in a self-consistent way by modifying the mean field occupations v_p,v_n to obtain the correct number of particles in the correlated RPA vacuum.

The Fermi transition operators are written in terms of the pair creation and annihilation operators as [3,4]

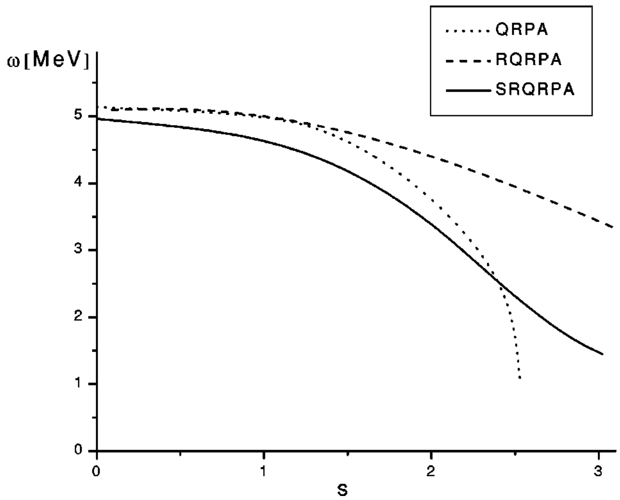

FIG. 1. Lowest $J^{\pi}=0^{+}$ excitation energies in ^{76}As, calculated from the ^{76}Ge ground state, for the QRPA, RQRPA, and SRQRPA approximations, as function of the residual interaction parameter *s*.

$$\tau^{-}=\sum_{pn}\ [v_p u_n[\alpha_p^{\dagger}\alpha_n^{\dagger}]^{00}+u_p v_n[\alpha_p\alpha_n]^{00}],\qquad \tau^{+}=\{\tau^{-}\}^{\dagger}, \tag{28}$$

which are *not* the exact expressions because they are missing the scattering terms which create a proton (neutron) quasiparticle and annihilates a neutron (proton) quasiparticle.

The total strengths $S_{\pm}$ associated with these transition operators are

$$S_{\pm}=\sum_{\lambda}\ |\langle\lambda J=0|\tau^{\pm}|0\rangle|^2. \tag{29}$$

The Ikeda sum rule states that, when the exact operators $\tau^{\pm}$ are used, and when the set of states $|\lambda\rangle$ is a complete one, including all the states in the odd-odd nuclei which can be connected with the ground state $|0\rangle$ of the even-even nuclei through the transition operators, then

$$S_{-}-S_{+}=N-Z. \tag{30}$$

However, in the present case the Fermi transition operators are truncated, and the strengths difference has a more complicated form [25]

$$S_{-}-S_{+}=\langle 0|\hat{N}-\hat{Z}|0\rangle-\sum_{pn}\ (u_n^2-v_p^2)\langle 0|\hat{\mathcal{N}}_n-\hat{\mathcal{N}}_p|0\rangle. \tag{31}$$

The first term is equivalent to $N-Z$ due to the constrictions (10). The second term gives rise to the violation of the Ikeda sum rule (30). When the BCS vacuum is employed, this term has no contribution. For this reason the standard *pn*-QRPA fulfils the Ikeda sum rule. There are some special cases in which the proton and neutron quasiparticle occupations in the ground state are equal, and this term also vanishes, as it was found in the SO(5) model [16] and in the single mode model of the RQRPA [15]. Only when the sum in Eq. (26) is restricted to $J=0$, forcing $j_p=j_n$ and $\Omega_p=\Omega_n$, it is true that $\langle 0|\hat{\mathcal{N}}_n-\hat{\mathcal{N}}_p|0\rangle=0$ for each level. In the present case, when a realistic Hilbert space is employed and the sum over different J's is included in Eq. (26), those occupations are not equal and the Ikeda sum rule is violated. It is worth to mention that, while in general the quasiparticle occupations for protons and neutrons are different, the expectation value of the total number of quasiparticles $\langle N_{qp}\rangle\equiv\Sigma_{\mathbf{t}}\langle 0|\hat{\mathcal{N}}_t|0\rangle$ is the same, as can be easily seen making the sum in Eq. (26).

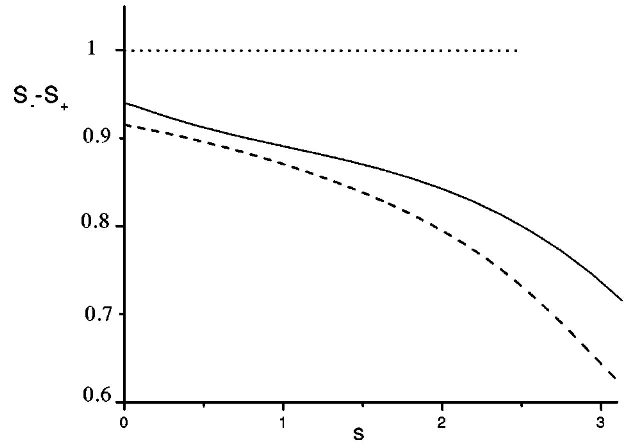

FIG. 2. Fermi sum rule $S_{-}-S_{+}$, normalized to the Ikeda value $N-Z$, as function of *s* with the same line conventions used in Fig. 1.

V. RESULTS

In the present work we studied Fermi β excitations in ^{76}Ge. We adopted a δ-type residual interaction used previously [17], and our Hilbert space has six single particle energy levels, including all the single-particle orbitals from oscillator shells $3\hbar\omega$ plus $1g_{9/2}$ and $1g_{7/2}$ from the $4\hbar\omega$ oscillator shell. They were obtained using a Coulomb-corrected Woods-Saxon potential. Their numerical values for ^{76}Ge are tabulated in Table 1 of Ref. [26]. We include $J^{\pi}=0^{\pm},\ldots,3^{\pm}$ in the sums in Eq. (26).

In order to describe the dependence of the various observables on the proton-neutron residual interaction we use the parameter

$$s=\frac{v_{\mathbf{pn}}^{pp}}{v^{\text{pair}}},$$

which is the ratio between the coupling constant in the proton-neutron $J=0$ particle-particle channel and the pairing force constant $v^{\text{pair}}=(v_{\mathbf{pp}}^{\text{pair}}+v_{\mathbf{nn}}^{\text{pair}})/2$. We compare results obtained within the usual QRPA with those coming from the RQRPA (no mean field modification), and the SRQRPA.

In Fig. 1 we show the lowest $J^{\pi}=0^{+}$ excitation energies ω (in MeV) in ^{76}As, calculated from the ^{76}Ge ground state, for the QRPA (dotted lines), RQRPA (dashed lines), and SRQRPA (full lines) approximations, as a function of the residual interaction parameter *s*. It can be clearly seen that the excitation energy goes to zero, and collapses around $s=2.5$ for the QRPA, while in the RQRPA and SRQRPA

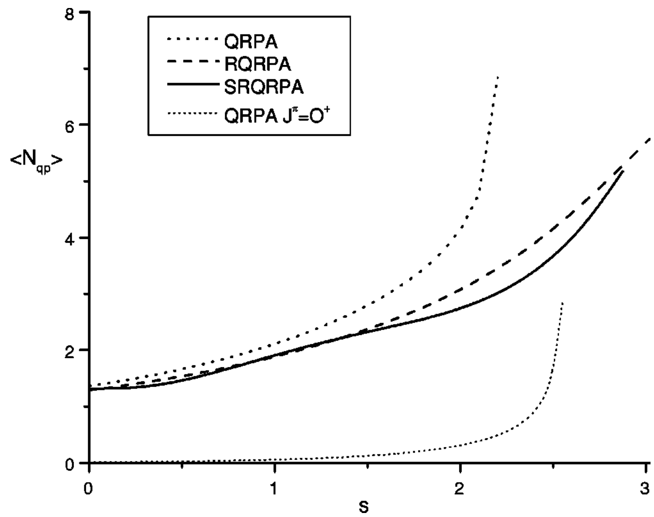

FIG. 3. Total quasiparticle number (equal for protons and neutrons) evolution as function of s. The result for the QRPA case when the sums in Eq. (26) are restricted to $J^{\pi}=0^{+}$ is also shown.

formalisms the collapse is avoided, although the excitation energies predicted after the collapse in these two approaches differ by a factor of 2.

In Fig. 2 the sum rule $S_{-}-S_{+}$, normalized to the Ikeda value $N-Z$, is presented as a function of s with the same convention of Fig. 1. While in the QRPA the Ikeda sum rule is always fulfilled, in the other cases it is violated. Notice that, having the correct number of protons and neutrons in

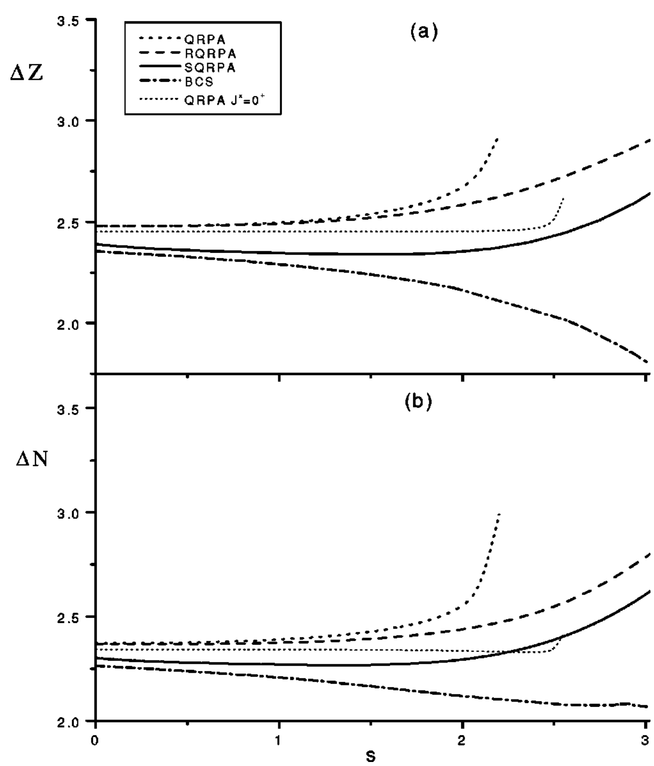

FIG. 4. Proton (a) and neutron (b) number fluctuations obtained using the BCS, QRPA with only $J^{\pi}=0^{+}$ and all the J's in Eq. (26), RQRPA and SRQRPA descriptions.

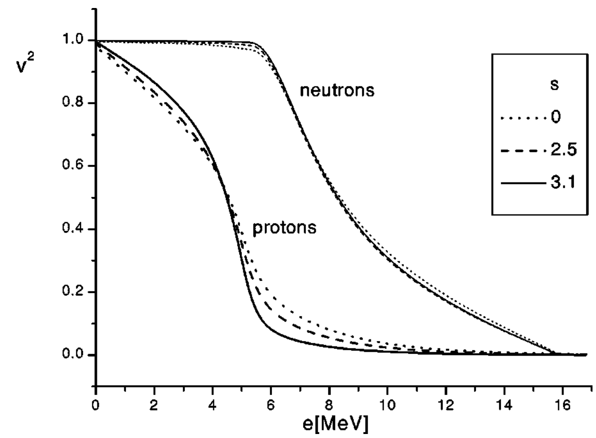

FIG. 5. Single particle occupation numbers v_p^2 (thick lines), and v_n^2 (thin lines) as a smoothed function of the single particle energies e, for the values $s=0.0$, 2.5, and 3.1 of the residual interaction parameter.

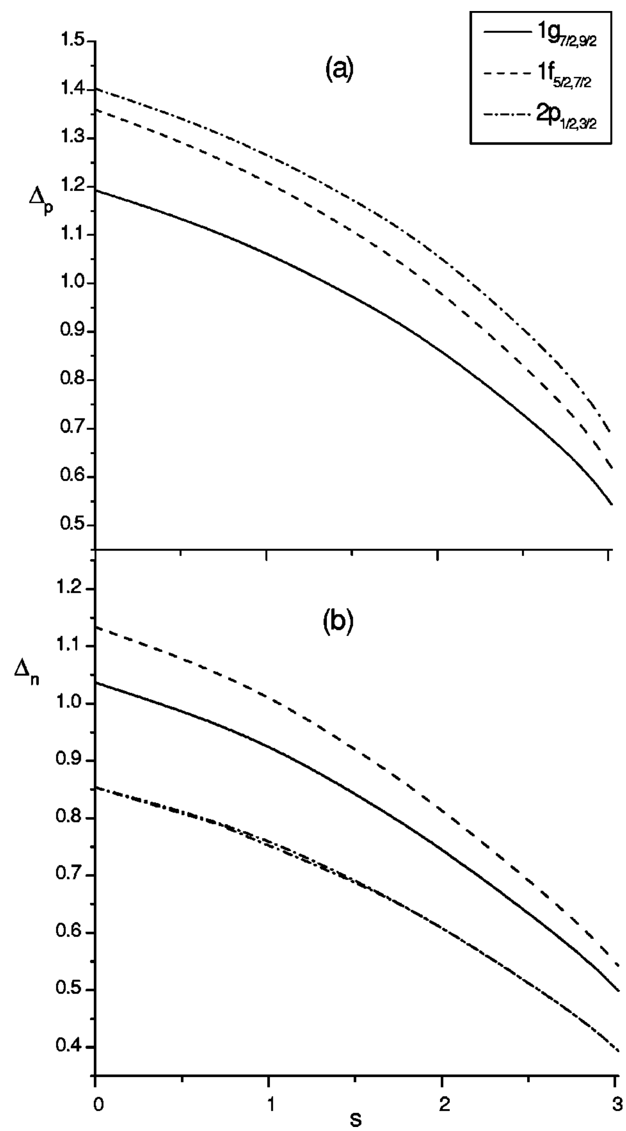

FIG. 6. Evolution of the pairing energy gap as a function of s for protons (a) and for neutrons (b).

the correlated ground state, the SRQRPA departs *less* from $N-Z$ than the RQRPA, but the departure is anyway noticeable. It is important to point out that in the present calculations, even when $s=0$ the other proton-neutron residual interaction channels for $J\neq 0$ are present and have a finite value. For this reason there is a violation of the Ikeda sum rule at $s=0$.

To describe in more detail the above mentioned point we present in Fig. 3 the total number of quasiparticles as a function of s. When only $J=0$ states are included in the sum in Eq. (26) the number of quasiparticles goes to zero when s is zero. In the other three cases the residual interaction in the other channels is present and increase this number.

The fluctuations in the particle number $\Delta N \equiv \sqrt{\langle 0|(\hat{N}-N)^2|0\rangle}$ represent a warning about the region of applicability of the different QRPA models [17,18]. They are shown in Fig. 4(a) (for protons) and Fig. 4(b) (for neutrons) as a function of s. It is interesting to note that the pure BCS fluctuations $\Delta N=2\sqrt{\Sigma_t \Omega_t^2 u_t^2 v_t^2}$ diminish when the pn residual interaction increases, due to the sharpening in the particle distribution when the mean field is varied to keep the number of particles in their right value. In the SRQRPA these fluctuations diminish slightly at first, dominated by this mean field effect, to increase in a noticeable way after the QRPA collapse. The QRPA results for the total quasiparticle number and the particle number fluctuations explode for $s \approx 2.5$ when only the $J^\pi=0^+$ contribution is included in Eq. (26). When all the angular momentum contributions are taken into account the $J^\pi=1^+$ component collapses around $s=2.3$, and these expectation values diverge.

The single particle occupation numbers v_p^2, v_n^2 are presented in Fig. 5 as a function of the single particle energies. While only six levels are in use, the curves represent their smoothed values. It can be seen that both protons and neutrons have their distributions sharpened, i.e., the larger s the more they resemble a step distribution, although this effect is more evident for protons.

The behavior of the pairing energy gap as a function of s is shown in Fig. 6(a) (for protons) and Fig. 6(b) (for neutrons). As advanced in the previous section, the pairing gaps, while different for the different single particle levels, all behave the same way. They show a strong reduction when s increases, down to one half of the original value. This is not a curiosity: the pairing energy gap is observable, and any attempt to properly describe the β decay using the QRPA extensions must keep the descriptive power of the simplest formalism, the QRPA, where the pairing constants are adjusted to reproduce the observed gap.

VI. CONCLUSIONS

A study of the β decay of ^{76}Ge using the SRQRPA was presented. The mean field gap and number equations were introduced and solved together with the RQRPA equations, but the self-consistency was only included to guarantee the good number of particles in the correlated ground state.

The Ikeda sum rule was studied in some detail. It is known to be fulfilled in standard pn-QRPA calculations [3,4], violated in RQRPA ones [14], and recovered when self-consistent RQRPA calculations are performed in simple models [15,16]. In the present work it was shown that the Ikeda sum rule is violated when a realistic Hilbert space is used in spite of using the number self-consistent QRPA approach.

The most remarkable phenomenon found in the calculation was the strong reduction of the pairing gap. In the presence of the proton-neutron residual interaction the mean field changes needed to have the correct number of particles generate sharper distributions for the occupations of single particle states, visible when plotting v^2 as a function of the single particle energies for different values of s. At the same time, it strongly reduces the energy gap, which can be as small as a half of the observed value.

Having in mind the various merits of the renormalized and self-consistent extensions of the quasiparticle random phase approximation, we conclude that it must be used with extreme caution in the region where the standard QRPA collapse. Not only the particle number fluctuations have a clear increase, but observable quantities depart from their measured values.

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