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Preformation probabilities for light ternary particles in the cold (neutronless) fission of ^{252}Cf

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The preformation amplitudes for α and ^{10}Be clusters in the cold ternary fission of ^{252}Cf are estimated within a microscopic model starting from single particle spherical Woods-Saxon wave functions and with a large space BCS-type configuration mixing. The resulting position of the maximum of cluster preformation probability is situated in the region between the two heavier fragments near the scission point, and approaches the fission axis as the distance between the fragments increases.

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Recently theoretical predictions were made for the yields of cold (neutronless) α- [1] and ^{10}Be-accompanied fragmentation of ^{252}Cf [2]. The same phenomena were directly observed by triple γ coincidence technique with Gammasphere with 72 detectors [3,4]. The main characteristic of cold nuclear fragmentations is the emission of final nuclei with very low or even zero excitation energy and with high kinetic energies pointing thus to rather compact shapes of the fragments for the scission configuration. The fact that the light charged particles (LCP) emitted in ternary fission (including the cold splittings) are focused mainly into the equatorial plane perpendicular to the fission axis seems to indicate that most of the ternary clusters originate from the region of the neck between the two heavier fragments.

A fully microscopic theory of cold ternary fragmentations has to include preformation probabilities for the emitted LCP. A previous attempt was made only in the point-α-particle approximation and with shell-model nucleonic configurations mixed through pairing forces [5]. We present here a formalism for obtaining the preformation amplitudes for α- and ^{10}Be-ternary particles within a microscopic approach, which starts from realistic single-particle spherical Woods-Saxon wave functions and uses a large space BCS-type configuration mixing [6,7]. The LCP internal wave functions are given by the Wildermuth cluster model and are consistent with the corresponding nuclear mean-square radii. A pairing interaction is acting together among neutrons and separately among protons. The strength of the pairing force is fitted so as to reproduce the experimental pairing gaps Δ_n, Δ_p of the initial and final nuclei.

We consider the cold ternary nuclear fragmentation of a parent nucleus $P \rightarrow A+B+c$ and write the LCP preformation amplitude as

$$F(\mathbf{R}_{Ac},\mathbf{R}_{Bc}) = \int d\xi_A d\xi_B d\xi_c \Psi_P(\xi_P)\Psi^*_{ch}, \qquad (1)$$

where the final channel wave function is: $\Psi_{ch} = [\Psi_A(\xi_A)\Psi_B(\xi_B)\phi_c(\xi_c)Y_L(\hat{R}_{Ac})Y_{L'}(\hat{R}_{Bc})]_{J_P M_P}$. Here, $\Psi_i(i=P,A,B)$ and ϕ_c are the internal wave functions of the initial nucleus of the two heavier final fragments and that of the ternary cluster, respectively, all antisymmetrized and normalized to unity. The ξ's are the internal coordinates of the involved nuclei (the c.m. coordinate being extracted) and $\mathbf{R}_{Ac},\mathbf{R}_{Bc}$ are the relative distances between the LCP and the heavier fragments. For the moment, we study the ground-state fragmentation of even-even nuclei into even-even products so that all J and L angular momenta are zero.

Since we are interested in evaluating the LCP preformation amplitudes around the scission point, we substitute the parent nucleus wave function Ψ_P with the linear combination $[\Psi_{A+c}(\xi_{A+c})\Psi_B(\xi_B)+\Psi_A(\xi_A)\Psi_{B+c}(\xi_{B+c})]_{J_P M_P}$ corresponding to the two possible binary splittings. Thus, the ternary cluster will be preformed either from the fragment $A+c$ or from the fragment $B+c$. Then, after integrating on the internal coordinates ξ_B (ξ_A) in the first (second) term on the right side of Eq. (1), the preformation amplitude becomes

$$\begin{aligned} F(R_{Ac},R_{Bc},\gamma) &= F_{Ac}(R_{Ac},\alpha)+F_{Bc}(R_{Bc},\beta) \\ &= \int d\xi_A d\xi_c \Psi_{A+c}(\xi_{A+c})\Psi^*_A\phi^*_c \\ &\quad + \int d\xi_B d\xi_c \Psi_{B+c}(\xi_{B+c})\Psi^*_B\phi^*_c, \end{aligned} \qquad (2)$$

where α and β are the angles between the fission axis and the vectors $\mathbf{R}_{Ac}$, $\mathbf{R}_{Bc}$, respectively, while γ is the angle between $\mathbf{R}_{Ac}$ and $\mathbf{R}_{Bc}$. The three Euler angles Ω which are resulting from the transformation of angular coordinates [e.g., $(\hat{R}_{Ac},\ \hat{R}_{Bc}) \rightarrow (\Omega,\ \gamma)$] and which define the orientation of the reaction plane, were omitted from the list of arguments in Eq. (2).

The shell-model configuration mixing is taken into account through the BCS formalism by introducing a residual pairing interaction which couples pairs of neutrons and protons with zero total spin. For example, the internal wave function of the fragment $A+c$ is written as

$$\Psi_{A+c} = \sum_{n,p} X(n)X(p)\prod_{n=1}^{q} |(n_n l_n j_n)^0_2\rangle \prod_{p=1}^{r} |(n_p l_p j_p)^0_2\rangle \cdot \Psi_A, \qquad (3)$$

where $q(r)$ is the number of neutron (proton) pairs in the even-even emitted cluster and the coefficients $X(i) = \langle \Psi_{A+c} | a^{+}_{\gamma_1} a^{+}_{-\gamma_1}, \ldots, a^{+}_{\gamma_t} a^{+}_{-\gamma_t} | \Psi_A \rangle$ are expressed in the framework of the second quantization formalism by using the particle creation operators a^{+}_{γ} for neutrons and protons $(i=n,p)$, respectively, with $\gamma = \{n_i l_i j_i m_i\}$, $-\gamma = \{n_i l_i j_i - m_i\}$ and $t = q(r)$ for neutrons (protons). A similar wave function is used for the fragment $B+c$.

We used realistic single particle wave functions generated in a spherical Woods-Saxon potential and developed in a spherical harmonic oscillator basis [7] with an oscillator parameter α, e.g., for neutrons

$$|(n_n l_n j_n)^0_2\rangle_{WS} = \sum_{\nu_1 \nu_2} c_{\nu_1 n_n} c_{\nu_2 n_n} |(\nu_1 l_n, \nu_2 l_n)^0_2\rangle_{HO}. \quad (4)$$

The parameters chosen for the Woods-Saxon potential were those proposed by Blomqvist and Wahlborn [8].

In order to calculate the preformation probability $|F_{Ac} + F_{Bc}|^2$, we integrate separately over the ξ_A coordinates and ξ_B coordinates, respectively, of the heavier final fragments taken as a core, then transform the individual coordinates of the last c nucleons forming the LCP into their relative (Jacobi) coordinates and further integrate on the ternary cluster ξ_c internal coordinates (for details see Refs. [7,9]). The cluster wave functions ϕ_c are also expressed in relative coordinates. In Ref. [7], we obtained the general expression for the cluster preformation amplitude from a heavier nucleus, as an expansion over the harmonic oscillator radial wave functions $R_{N_{cm}0}$ of the relative motion between the two final nuclei:

$$F_0(R) = \sum_{N_{\mathrm{c.m.}}} B^{(N_{\mathrm{c.m.}})}_{(c)} R_{N_{\mathrm{c.m.}}0}(A_c \alpha R^2), \quad (5)$$

where A_c is the cluster mass, α is the oscillator parameter of the HO basis [Eq. (4)] and $N_{\mathrm{c.m.}}$ is the oscillator radial quantum number.

We wrote the spatial part of the α-particle wave function as $\phi_\alpha(\xi_\alpha) = (\beta/\pi)^{9/4} \exp(-\beta(\rho_1^2 + \rho_2^2 + \rho_3^2)/2)$, where $\rho_1 = (\mathbf{r_1} - \mathbf{r_2})/\sqrt{2}$, $\rho_2 = (\mathbf{r_3} - \mathbf{r_4})/\sqrt{2}$, and $\rho_3 = (\mathbf{r_1} + \mathbf{r_2} - \mathbf{r_3} - \mathbf{r_4})/2$ are the corresponding Jacobi coordinates and the α-particle oscillator parameter is $\beta = 0.57\ \mathrm{fm}^{-2}$. The coefficients of the above expansion, Eq. (5) of the cluster preformation amplitude, resulted as

$$B^{(N_{\mathrm{c.m.}})}_{(4)} = f_n f_p \sum_{j_n, j_p} h(j_n) h(j_p) \sum_{\nu_1 \nu_2 \pi_1 \pi_2} c_{\nu_1 n_n} c_{\nu_2 n_n} \times c_{\pi_1 n_p} c_{\pi_2 n_p} A^{(N_{\mathrm{c.m.}})}_{(4)}(\nu_1 l_n, \nu_2 l_n(0) \pi_1 l_p, \times \pi_2 l_p(0) 0, \alpha, \beta), \quad (6)$$

and are formally similar to those obtained earlier by Mang [10] for the overlap integral of the α particle using harmonic oscillator single-particle wave functions. Here the BCS factor f is $f = \Pi_j (U^i_j U^f_j + V^i_j V^f_j)^{\Omega_j}$, with $\Omega = (2j+1)/2$, and i,f superscripts representing the initial and final nuclei, and $h(j) = \Omega_j V^i_j U^f_j / (U^i_j U^f_j + V^i_j V^f_j)$. The coefficients $A^{(N_{\mathrm{c.m.}})}_{(4)}(n_1 l_1, n_2 l_2(0) n_3 l_3, n_4 l_4(0) 0, \alpha, \beta)$ appear when one calculates the overlap integral for the case of the α particle, using harmonic oscillator single-particle wave functions, and for a given configuration of the last four particles [9,10]. When the oscillator parameter α of the HO basis employed for calculating the Woods-Saxon single-particle wave functions of the decaying nucleus and the cluster size parameter β are equal, the $A_{(4)}$ coefficients take the simple expression

$$A^{(N_{\mathrm{c.m.}})}_{(4)} = (\pi/4)^{3/4} \hat{l}_n \hat{l}_p 4^{-N_{\mathrm{c.m.}}} \left(N_{\mathrm{c.m.}}! \left(N_{\mathrm{c.m.}} + \frac{1}{2} \right)! / \prod_{i=1}^{4} n_i! \left(n_i + l_i + \frac{1}{2} \right)! \right)^{1/2}, \quad (7)$$

with the orbital momenta $l_1 = l_2 = l_n$, $l_3 = l_4 = l_p$ coupled to zero, $\hat{l} = \sqrt{2l+1}$, and with the usual condition $N_{\mathrm{c.m.}} = n_1 + n_2 + l_n + n_3 + n_4 + l_p$.

In order to calculate the preformation factor of the ^{10}Be cluster, we obtained its internal function from the usual ansatz prescribed by the cluster-type nuclear structure theories [11] as $\phi_{^{10}\mathrm{Be}} = \Pi_{j=1,2} \phi_{\alpha_j} \chi_{00}(\rho_\nu) \chi_{10}(\rho_{01}) \chi_{20}(\rho_r)$, where both ϕ_α's are as shown above. The function χ_{00} describes the relative motion between the last two neutrons, χ_{10} describes the relative motion with two oscillator quanta between the first α and the di-neutron (as for the ^{6}He case), and the function χ_{20} describes the relative motion with four oscillator quanta between the center of mass of ^{6}He and the

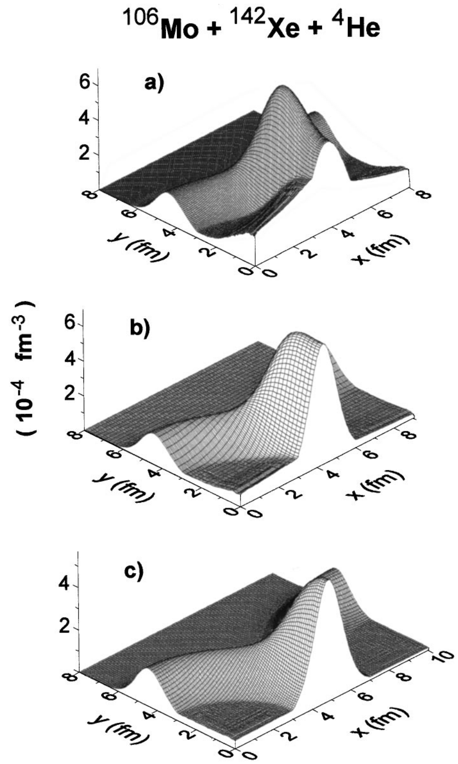

FIG. 1. The preformation probability $|F_{Ac} + F_{Bc}|^2$ of the α cluster as a function of the distance between the two heavier fragments $R_{AB} = 8$ fm (a), 9 fm (b), 10 fm (c).

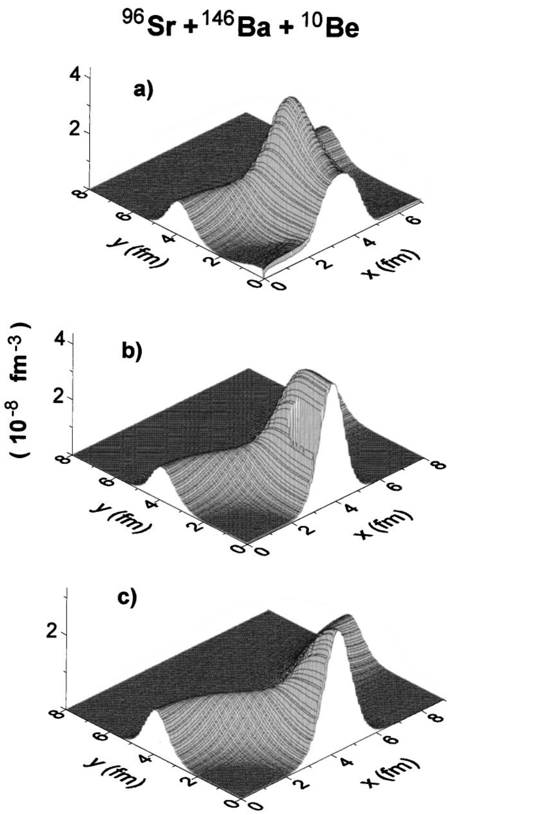

FIG. 2. The preformation probability $|F_{Ac}+F_{Bc}|^2$ of the ^{10}Be cluster as a function of the distance between the two heavier fragments $R_{AB}=7$ fm (a), 8 fm (b), 9 fm (c).

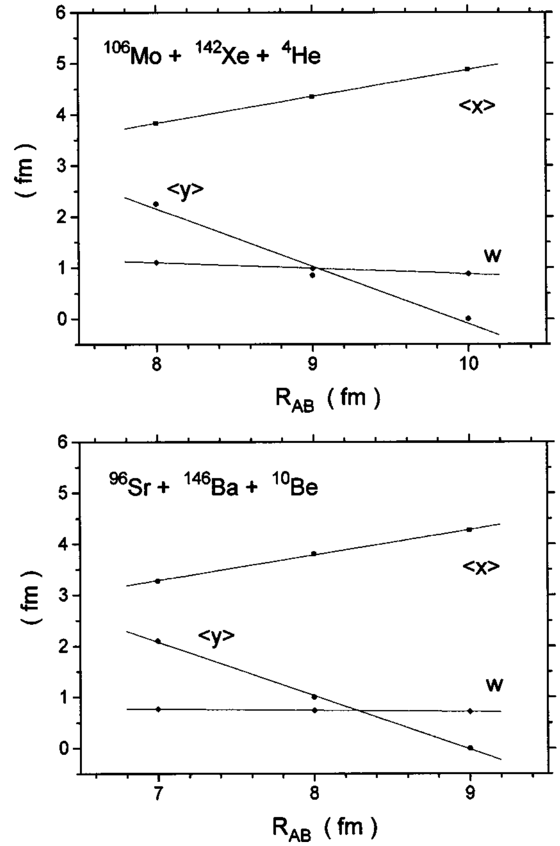

FIG. 3. The coordinates $\langle x\rangle$, $\langle y\rangle$, and the Gaussian width w of the maximum of preformation probability for the α and ^{10}Be cluster, respectively, as a function of the distance R_{AB}.

second α. Only $0s$ quantum numbers for the relative motion between pairs of nucleons are considered in the emitted cluster. The cluster wave function is normalized to unity with regard to the internal relative coordinates ξ_c. Finally we obtain the coefficients

$$B_{(10)}^{(N_{\rm c.m.})}=\sum_{N_{R_1},N_{R_\nu}} B_{(4)}^{(N_{R_1})}B_{(4)}^{(N_{R_2})}B_{(2)}^{(N_{R_\nu})}\cdot\langle 1,N_{R_{01}}|N_{R_1},N_{R_\nu}\rangle \times\langle 2,N_{\rm c.m.}|N_{R_{01}},N_{R_2}\rangle, \quad (8)$$

with the usual energy conserving HO relations $N_{R_{01}}=N_{R_1}+N_{R_\nu}-1$ and $N_{R_2}=N_{\rm c.m.}-N_{R_{01}}+2$. Here, $\langle n,N|n_1,n_2\rangle$ is short-hand notation for the Moshinsky coefficients generalized for pairs of particles with unequal masses and with all angular momenta equal to zero. The first bracket corresponds to $D=m_1/m_2=2$ and the second bracket corresponds to $D=1.5$. We employed an oscillator parameter of 0.39 fm^{-2} for the ^{10}Be cluster.

We normalized the position-dependent cluster preformation amplitudes F_{Ac},F_{Bc} with the help of the cluster spectroscopic factor which we defined as $S=\int dRR^2|F(R)|^2$. We used the cluster spectroscopic factors deduced by Blendowske *et al.* [12], namely $S=0.63\times10^{-2}$ for the α particle and 0.25×10^{-6} for the ^{10}Be particle.

It is well known [6,7,10,13] that the maximum for the cluster preformation amplitudes $F(R)$ in heavy nuclei is reached for radial distances R about 1 to 1.5 fm beneath the nuclear surface, both for spherical and deformed nuclei. In Fig. 1 we show the preformation probability $|F_{Ac}+F_{Bc}|^2$, (2), for the cold ternary splitting ^{106}Mo+^{142}Xe+α as a function of the coordinates (x,y) of the α cluster for the interfragment distance $R_{AB}=8$, 9, and 10 fm, respectively. The center of the fragment ^{106}Mo is placed in the origin while the center of the fragment ^{142}Xe is situated at the other extremity of the x axis. In Fig. 2, the spatial distribution of the preformation probability for the cold ternary splitting ^{96}Sr+^{146}Ba+^{10}Be is drawn for the interfragment distance $R_{AB}=7$, 8, and 9 fm, respectively. We can observe that the highest cluster preformation probabilities are reached in the region between the two heavier fragments. Non-negligible preformation probabilities are also obtained for the clusters in the interior region of the two fragments and towards their outer poles. Nevertheless, the potential barrier between the cluster and the heavier fragments is much higher for these cases resulting in very low cluster emission rates.

We studied the dynamics of the distribution for the ternary cluster preformation probability along the fission trajectory. The preformation probability distributions around the maxima were fitted with Gaussians. As the two heavier fragments separate, the position of the top of the LCP preformation probability distribution moves closer to the fission axis $(y=0)$. In Fig. 3(a), the coordinates $\langle x\rangle$ and $\langle y\rangle$ of this point are shown as a function of the interfragment distance R_{AB}, together with the Gaussian width w of the α-particle preformation probability distribution; in Fig. 3(b) the same coordinates are shown for the ^{10}Be cluster preformation probability distribution. One can see that while the distance $\langle y \rangle$ of the preformation probability peak relative to the fission axis decreases to zero, the peak width w is nearly constant (1.0 fm for the α cluster and 0.8 fm for the ^{10}Be cluster).

In conclusion, within a consistent treatment of the cold ternary fragmentations, the microscopic cluster preformation probabilities could indicate the LCP initial conditions, namely the distribution of (x,y) initial positions and the Gaussian distribution widths w, thus replacing any arbitrary hypothesis regarding these distributions. Furthermore, with these initial conditions, it will be possible to perform a multidimensional potential barrier penetrability calculation and, finally, to obtain the LCP trajectories and the corresponding angular and kinetic energy distributions, to compare to the experimental data.

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