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Utility of nucleon-target profile function in cross section calculations

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We test the utility of an effective nucleon (N)-^{12}C profile function in calculating nucleus-^{12}C optical phase shift function in the Glauber theory. A calculation of the complete Glauber amplitude is performed by using wave functions for ^{4}He and ^{6}He projectiles, leading to the reaction and elastic differential cross sections in good agreement with experiment. By relating the N-^{12}C profile function to the NN profile function, we derive a new, simple formula to calculate reaction cross sections which requires only nuclear densities as an input. By applying this formula to various combinations of ^{4}He, ^{6}He, ^{9}Be, ^{12}C, and ^{27}Al, we can reproduce cross sections measured at 800 MeV/nucleon to much higher accuracy than the optical limit approximation.

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The matter distribution in a nucleus is a basic quantity for nuclear models. Since reaction cross sections at energies of more than a few hundred MeV/nucleon reflect the geometrical size of the nucleus, the discovery of cross section enhancement for nuclei near the neutron drip line [1] has revived much interest in the determination of the matter size. Compared to a charge distribution extracted from electron scatterings, the matter size or distribution is more difficult to determine reliably because the reaction involves strong interactions.

Because of its simplicity the optical limit approximation (OLA) of the Glauber theory [2] has routinely been used as a convenient tool for the extraction of the sizes of unstable nuclei as in the case of stable nuclei [3]. Several authors have shown, however, that a treatment beyond the OLA is necessary for a quantitative analysis of the reaction cross sections [4–6] as well as the elastic scattering cross sections [7,8] for loosely coupled nuclei such as halo nuclei because breakup effects are not properly accounted for in the OLA. Little progress has so far been made to calculate physical observables in the Glauber theory by using microscopic wave functions. Extending our recent analysis [9] for N-target systems, we propose in this Rapid Communication an effective method to calculate the phase shift function and demonstrate its predictable power by comparing it to existing data. We derive a simplified formula for the phase shift function, which necessitates only the nuclear density other than the NN profile function Γ_{NN}. The reaction cross sections calculated by this formula are found to be much closer to experimental values than those by the OLA.

The Glauber theory provides us with an excellent framework to describe the high energy reaction in various fields of physics. The nucleus-nucleus elastic scattering amplitude is specified by the optical phase shift function $\chi(\mathbf{b})$ defined by

$$e^{i\chi(\mathbf{b})}=\langle\psi_0\theta_0|\prod_{i\in P}\prod_{j\in T}\left(1-\Gamma_{NN}(\boldsymbol{\xi}_i-\boldsymbol{\eta}_j+\mathbf{b})\right)|\psi_0\theta_0\rangle, \tag{1}$$

where $\mathbf{b}$ is the impact parameter, and $\psi_0(\theta_0)$ is the projectile (target) wave function with its center-of-mass motion being removed. $\boldsymbol{\xi}_i(\boldsymbol{\eta}_i)$ is the two-dimensional coordinate of ith particle of the projectile (target), relative to its center of mass, which lies on the plane perpendicular to the incident momentum of the projectile. The total reaction cross section is obtained by subtracting the elastic cross section from the total cross section: $\sigma_{\rm R}=\int d\mathbf{b}\,(1-|e^{i\chi(\mathbf{b})}|^2)$. Readers may be referred to [10,11] for the multiple scattering theory formulation of the reaction cross section.

The optical phase shift function (1) is a key quantity to calculate both the elastic scattering amplitude and the reaction cross section. Its calculation is complicated and often approximated to first order in the cumulant expansion [2] as

$$e^{i\chi_{\rm OLA}(\mathbf{b})}=\exp\left\{-\int\int d\mathbf{r}\,d\mathbf{s}\,\rho_P(\mathbf{r})\rho_T(\mathbf{s})\Gamma_{NN}(\boldsymbol{\xi}-\boldsymbol{\eta}+\mathbf{b})\right\}, \tag{2}$$

where $\mathbf{r}$ and $\mathbf{s}$ are the three-dimensional coordinates such that their two components perpendicular to the incident momentum of the projectile coincide with $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$, respectively.

Table I compares the reaction cross sections calculated in the OLA with experiment. The parameters of Γ_{NN} at 800 MeV, taken from [5], are consistent with those of the systematic analysis of [10]. The Coulomb effect on the reaction cross section is ignored. The densities of ^{4}He, ^{9}Be, ^{12}C, and ^{27}Al were taken from Table 2 of Ref. [5]. The density of ^{6}He was taken from a microscopic calculation [12], and fitted by a sum of Gaussians: $\rho(r)=\Sigma_{i=1}^4 C_i\exp[-(r/a_i)^2]$, where $C_1=0.242\,375$, $C_2=2.545\,367\times10^{-2}$, $C_3=1.330\,079\,7\times10^{-3}$, $C_4=5.634\,404\,8\times10^{-5}$ in units of fm^{-3}, and $a_1=1.414\,237$, $a_2=2.314\,934$, $a_3=3.79652$, $a_4=5.773\,427$ in units of fm. The OLA cross sections are larger than the measured cross sections. In consistency with other calculations the difference is rather modest for stable nuclei but becomes fairly large in the halo nucleus of ^{6}He: it is about 10% for ^{12}C and ^{27}Al targets. This overestimation of the cross section by the OLA tends to predict smaller radii for halo nuclei [6].

Considerable efforts have been directed toward going beyond the OLA. No correlated motion of wave functions shows up in the OLA. The use of a wave function is certainly

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TABLE I. A comparison of the theoretical reaction cross sections, in units of mb, with the interaction cross sections measured at 800 MeV/nucleon [1]. The phase shift functions are calculated in three different approximations [Eqs. (2), (8), and (9)].

Target/projectile		^{4}He	^{6}He	^{9}Be	^{12}C
^{9}Be	Eq. (2)	488	716	805	
	Eq. (8)	461	660	765	
	Eq. (9)	453	672	765	
	Exp.	485±4	672±7	755±5	
^{12}C	Eq. (2)	520	782	854	896
	Eq. (8)	490	707	804	856
	Eq. (9)	487	732	813	856
	Exp.	503±5	722±6	806±9	856±9
^{27}Al	Eq. (2)	800	1165	1218	1265
	Eq. (8)	760	1049	1156	1217
	Eq. (9)	760	1096	1170	1219
	Exp.	780±13	1063±8	1174±10	

important. We have recently proposed a method of calculating the optical phase shift function completely [9] and applied it to the analysis of $p+{}^6$He scatterings by using microscopic ^{6}He wave functions. The calculation has reproduced very well the angular distribution measured at 717 MeV with no adjustable parameters, leading to the conclusion that the size of ^{6}He is about 2.51 fm. Though this method can straightforwardly be applied to calculate Eq. (1) for a general case, it would require enormous computer time when one uses microscopic wave functions for both the projectile and the target. It is, therefore, undoubtedly necessary to further develop an effective method where one can avoid heavy computational loads, keeping high accuracy. To this end we consider the nucleon-target (NT) scattering as an elementary vehicle in the Glauber theory, assuming the target as a scatterer, and introduce a profile function Γ_{NT} for the NT scattering. In this formalism the various effects such as the Fermi motion, Pauli correlations, short range dynamic correlations, etc., would be automatically included to some extent in the NT amplitude. Al-Khalili *et al.* [6] started from the NT s matrix in order to calculate the phase shift function in the few-body approach. Other authors [13,14] used $p+{}^4$He profile function to calculate $p+{}^{12}$C, $p+{}^{16}$O, and ^{4}He$+{}^{12}$C elastic differential cross sections by assuming α-cluster model for ^{12}C and ^{16}O. Contrary to these studies, we will use a full projectile wave function.

The optical phase shift function is now calculated by

$$e^{i\tilde{\chi}(\mathbf{b})}=\langle\psi_0|\prod_{i\in P}(1-\Gamma_{NT}(\boldsymbol{\xi}_i+\mathbf{b}))|\psi_0\rangle, \qquad (3)$$

where Γ_{NT} may be parametrized as

$$\Gamma_{NT}(\mathbf{b})=\sum_{j=1}^{k}\left(\frac{1-i\alpha_j}{2\pi}\omega_j\sigma_j\right)e^{-\omega_j\mathbf{b}^2}, \qquad (4)$$

and the parameters σ_j, ω_j, and α_j can be determined by fitting the experimental elastic angular distribution and the total cross section of NT scatterings. The $p+{}^{12}$C elastic differential cross section at $T_p=800$ MeV [15] was fitted by two terms: the parameters $\sigma_j(\mathrm{fm}^2)$, $\omega_j(\mathrm{fm}^{-2})$, α_j are 52.89, 0.25378, $-0.111\,682$ for $j=1$ and -18.78, 0.46576, 0.014 945 5 for $j=2$, respectively. The fit to the angular distribution is very good up to about 22°. The reaction cross section at 800 MeV is predicted to be 249 mb. No experimental data are available at 800 MeV, but there are two data [16,17] at about 870 MeV. Though they seem to disagree with each other, our prediction is very consistent with 255 mb, which is a value extrapolated using the cross section (262 mb) of Ref. [17]. We performed a complete calculation of Eq. (3) for ^{4}He$+{}^{12}$C and ^{6}He$+{}^{12}$C systems by using the same ^{6}He microscopic wave function [18] as in Ref. [9]. As seen in Table II, both of ^{4}He$+{}^{12}$C and ^{6}He$+{}^{12}$C reaction cross sections are in excellent agreement with the experiment. A theoretical prediction for ^{4}He$+{}^{12}$C and ^{6}He$+{}^{12}$C scatterings at 800 MeV/nucleon will be shown later. Figure 1 compares ^{4}He$+{}^{12}$C elastic differential cross section at energy of 342 MeV/nucleon with experiment. Since no $p+{}^{12}$C elastic scattering data at $T_p=342$ MeV are available, we determined the Γ_{NT} as follows. First we calculated the $p+{}^{12}$C elastic scattering cross section in the OLA by using the ^{12}C density and the Γ_{NN} parameters at 325 MeV [10]. Then we corrected this OLA cross section by examining the extent to which the $p+{}^{12}$C data measured at 398 MeV [20] differ from the OLA cross sections calculated at the same energy. Finally this corrected elastic cross section was considered ''experimental'' and used to determine the Γ_{NT} at 325 MeV. Despite this unsatisfactory determination of Γ_{NT} the agreement between theory and experiment is very reasonable, even better than the phenomenological fit of [14].

TABLE II. A comparison of the theoretical reaction cross sections, in units of mb, on ^{12}C target with the interaction cross sections measured at 800 MeV/nucleon [1]. The phase shift function is calculated by Eqs. (3) or (5). The wave function of ^{6}He is obtained by a microscopic $\alpha+n+n$ three-cluster model [18], whereas that of ^{4}He is assumed to be given by the simplest shell model. See also [9].

Projectile	Eq. (3)	Eq. (5)	Exp.
^{4}He	514	494	503±5
^{6}He	736	717	722±6
^{9}Be		814	806±9
^{12}C		869	856±9

It is interesting to see how good the OLA is at this stage. We approximate the right-hand side of Eq. (3) as

$$e^{i\tilde{\chi}_{\mathrm{OLA}}(\mathbf{b})}=\exp\left(-\int d\mathbf{r}\,\rho_P(\mathbf{r})\Gamma_{NT}(\boldsymbol{\xi}+\mathbf{b})\right). \qquad (5)$$

The reaction cross section calculated with this equation, listed in Table II, is only a few percent smaller than the reference value with Eq. (3). The σ_{R} value for ^{6}He$+{}^{12}$C is 717 mb, much closer to the experimental value of 722±6 mb than 782 mb obtained by Eq. (2) of the usual OLA. This encourages us to calculate reaction cross sections for other

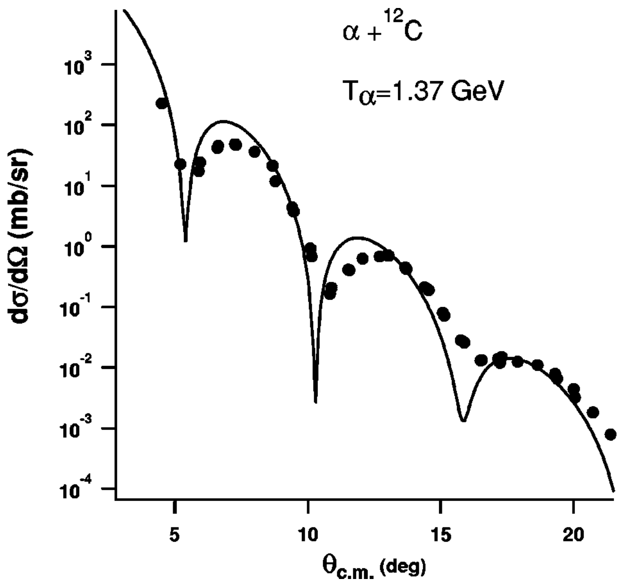

FIG. 1. Elastic differential cross sections for ^{4}He+^{12}C at energy of $T_\alpha=1.37$ GeV. The N-^{12}C profile function was determined as explained in text; $\sigma_j(\mathrm{fm}^2)$, $\omega_j(\mathrm{fm}^{-2})$, α_j are 32.0, 0.25, 0.10 for $j=1$, and -3.7, 1.25, 0.28 for $j=2$, respectively. The data are taken from [19].

nuclei by using densities even when wave functions are not available. The examples of ^{9}Be and ^{12}C projectiles are included in Table II. The cross sections calculated by Eq. (5) improve much better than those calculated by Eq. (2) (see Table I).

In the case where NT scattering data at a given energy are rich enough to determine the Γ_{NT}, our method can thus describe very well the reactions of various projectiles on that target at the same incident energy per nucleon. This serves to examine projectile wave functions with the reservation that the high energy reactions of the type considered here probe primarily the nuclear surface region. In such a case where no appropriate data are available, however, we cannot determine the Γ_{NT}, and one may think that the method would not work. To overcome this difficulty, until the data become available and to make the method more widely applicable, it is useful to relate the Γ_{NT} to the elementary function Γ_{NN} as described below.

Since the Γ_{NT} is such that its Fourier transform gives the NT elastic scattering amplitude, we express it in terms of Γ_{NN} by

$$\Gamma_{NT}(\mathbf{b})=1-\langle\theta_0|\prod_{i\in T}(1-\Gamma_{NN}(\mathbf{b}-\boldsymbol{\eta}_i))|\theta_0\rangle. \tag{6}$$

The use of the cumulant expansion leads to a very simple calculation of the optical phase shift function. By approximating the right-hand side of Eq. (6) as

$$\Gamma_{NT}(\mathbf{b})\approx 1-\exp\left(-\int d\mathbf{s}\,\rho_T(\mathbf{s})\Gamma_{NN}(\mathbf{b}-\boldsymbol{\eta})\right), \tag{7}$$

and substituting it into Eq. (5), we obtain

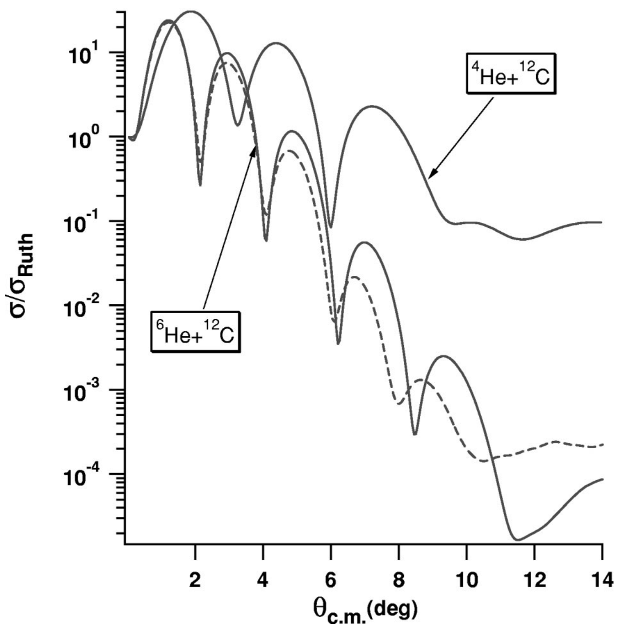

FIG. 2. Elastic differential cross sections in Rutherford ratio for ^{6}He+^{12}C and ^{4}He+^{12}C at energy of 800 MeV/nucleon. The solid curves are obtained by calculating the complete Glauber amplitudes, while the dashed one is the approximation with Eq. (9).

$$e^{i\chi_{\mathrm{eff}}(\mathbf{b})}=\exp\left[-\int d\mathbf{r}\,\rho_P(\mathbf{r})\times\left\{1-\exp\left(-\int d\mathbf{s}\,\rho_T(\mathbf{s})\Gamma_{NN}(\boldsymbol{\xi}-\boldsymbol{\eta}+\mathbf{b})\right)\right\}\right]. \tag{8}$$

This formula is very appealing because it requires only the densities of the projectile and the target. If the integral of $\rho_T\Gamma_{NN}$ is small enough compared to unity, then Eq. (8) reduces to the usual OLA formula of Eq. (2), otherwise the effect of multiple scatterings of the projectile nucleon with the target is included to some extent. Since the role of the projectile and the target is interchangeable in the calculation of the elastic scattering amplitude as well as the reaction cross section, it may be possible to symmetrize Eq. (8) as follows:

$$\begin{aligned}e^{i\chi_{\mathrm{eff}}(\mathbf{b})}=\exp\left[-\frac{1}{2}\int d\mathbf{r}\,\rho_P(\mathbf{r})\right.&\\ \times\left\{1-\exp\left(-\int d\mathbf{s}\,\rho_T(\mathbf{s})\Gamma_{NN}(\boldsymbol{\xi}-\boldsymbol{\eta}+\mathbf{b})\right)\right\}\Bigg]&\\ \times\exp\left[-\frac{1}{2}\int d\mathbf{s}\,\rho_T(\mathbf{s})\right.&\\ \times\left\{1-\exp\left(-\int d\mathbf{r}\,\rho_P(\mathbf{r})\Gamma_{NN}(\boldsymbol{\eta}-\boldsymbol{\xi}+\mathbf{b})\right)\right\}\Bigg].&\end{aligned} \tag{9}$$

The reaction cross sections calculated with Eq. (8) or its symmetrized form (9) are listed in Table I. We see that Eq. (8) or Eq. (9) produces only a little difference. This formula gives the reaction cross sections which are much closer to experiment than the OLA; the deviation from experiment is, at most, only a few percent, so that the formula can be more reliably used than the OLA. It is gratifying that the density, which reproduces the charge radius, reproduces the reaction cross section consistently with experiment. Figure 2 displays ^{6}He$+{}^{12}$C and ^{4}He$+{}^{12}$C elastic scatterings at 800 MeV/nucleon. The cross sections at small angles are significantly enhanced compared to the Rutherford cross sections. The difference in diffraction patterns between the two cases is due to the different kinematics and the different structure of ^{6}He and ^{4}He as well. The dashed curve represents the cross sections calculated by the optical phase shift function of Eq. (9), and follows very well the solid curve of the full calculation at small angles.

In summary, we have demonstrated the utility of an effective nucleon-target profile function in the calculation of the projectile-target optical phase shift function. Combining this profile function with the formulation [9] of calculating the complete Glauber amplitude, a good agreement between theory and experiment has been obtained for the reaction cross section as well as the elastic differential cross section. This will provide us with the possibility of examining projectile wave functions, in particular, near the surface region against high-energy scattering data. A simple formula for the reaction cross section has been derived by relating the nucleon-target profile function to the nucleon-nucleon profile function. Many examples have confirmed that the formula gives more accurate cross sections than the conventional optical limit approximation. Because of its simplicity we hope that the present approach will be applied to high energy reactions of complex systems encountered in other fields of physics as well.

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