PHYSICAL REVIEW C, VOLUME 61, 051303(R)

Three-body analysis of the occurrence of Efimov states in $2n$ halo nuclei such as ^{19}B, ^{22}C, and ^{20}C

I. Mazumdar,[1] V. Arora,[2] and V. S. Bhasin[2]
[1]*Department of Physics, SUNY at Stony Brook, Stony Brook, New York 11794-3800*
[2]*Department of Physics and Astrophysics, University of Delhi, Delhi-1100 07, India*

(Received 1 September 1999; published 13 April 2000)

By employing separable potentials for the binary, i.e., n-n and n-core subsystems, we compute the three-body integral equations to solve for the $2n$-separation energies in the ground and excited states of halo nuclei like ^{19}B, ^{22}C, and ^{20}C. We show through numerical analysis and also from analytical considerations that Borromean-type halo nuclei like ^{19}B and ^{22}C, where n-n and n-core are both unbound, are much less vulnerable to respond to the existence of the Efimov effect. On the contrary, those nuclei, like ^{20}C in which the halo neutron is supposed to be in the intruder low lying bound state with the core, appear to be the promising candidates to search for the occurrence of Efimov states at energies below the n-(nc) breakup threshold which can be within the experimental limits to measure.

PACS number(s): 21.45.+v, 21.10.Dr, 21.60.Gx, 27.20.+n

With the currently available facilities for producing intense radioactive nuclear beams, the properties of halo nuclei near the neutron dripline have revealed several striking features [1]. Certainly, halo clusterization of nuclear matter into a normal core plus a low density veil of loosely bound neutrons is one of the most common features. In the case of two neutron halo nuclei, most are characterized by what is now known as the Borromean property, which implies that while the binary subsystems, such as n-core and n-n, are unbound the three-body system comprising of the n-n core gives rise to a bound state. For such nuclei, therefore, the three-body techniques offer natural premises to study their detailed structures. In this context an intriguing question is whether these two neutron halos, characterized by large spatial extension and very low separation energy of the halo neutrons, actually attain the Efimov limit so as to produce the Efimov states near and below the three-body breakup threshold. Quantitatively, as pointed out by Efimov [2], so long as

$$r_0\sqrt{|E|} \ll 1 \quad \text{and} \quad r_0 \ll a, \tag{1}$$

where r_0 is the range of the two-body interaction, E is the energy of the three-body system (of equal mass particles), and a represents the scattering length of the s state of the two particles, the existence of resonating s-wave potentials results in the appearance of a three-body long range interaction which goes as $1/R^2$, where

$$R^2 = r^2 + \rho^2, \quad \vec{r} = \vec{r}_1 - \vec{r}_2, \quad \text{and} \quad \vec{\rho} = \frac{2}{\sqrt{3}}\left(\vec{r}_3 - \frac{\vec{r}_1 + \vec{r}_2}{2}\right). \tag{2}$$

The existence of such a long range force should respond by producing a large number of states, particularly near the three-body break-up threshold. The conditions for occurrence of Efimov states in two-neutron halos have recently been investigated [3] and discussed [4] employing the Faddeev equations in coordinate space. In a recent publication [5] (henceforth referred to as [B]), we extended the three-body model proposed originally for ^{11}Li [6] (henceforth referred to as [A]), employing separable potentials for the binary subsystems to study the Efimov effect in the halo ^{14}Be nucleus. The present work extends the investigation to include recently discovered $2n$ halo nuclei such as ^{19}B, ^{22}C, and ^{20}C where the data [7] on $2n$ separation energies as well as on the n-core bound/unbound states are now available [8]. The objective here is threefold:

(1) To show that for such nuclei the three-body equations with the realistic two-body potentials provide a natural framework to explain their binding energies, momentum distributions, etc.

(2) To undertake a detailed numerical analysis by computing the integral equation for three-body systems to search for the occurrence of the Efimov states with the realistic two-body potentials as input.

(3) To show from analytical considerations that for the Borromean-type halo nuclei there is a remote possibility for the occurrence of the Efimov states except when the scattering length for the virtual n-core systems is sufficiently large (of the order of a few hundred fermis) and the three-body energy approaches almost zero. On the other hand, for those nuclei in which the binary subsystem is bound, the probability for the occurrence of such states is quite high.

Recently Amorim *et al.* [9] have analyzed the universal aspects of Efimov states in the case of light halo nuclei within the framework of the three-body renormalizable model in the context of pairwise short range interactions. We shall see that by using separable potentials, our results confirm to the conclusions arrived at in Ref. [9], especially in the case of ^{20}C.

Our starting point is to write down the solution of the three-body Faddeev equations in terms of the spectator functions $F(p)$ and $G(p)$ representing the dynamics of the core, and of the halo neutron which satisfy the coupled integral equation as given in [A]. For the purpose of studying the sensitive computational details of the Efimov effect, we transformed these equations in terms of $\phi(p)$ and $\chi(p)$ as defined in [B] through Eq. (3), thereby involving only the dimensionless quantities. The two coupled equations were finally reduced into one integral equation for $\chi(p)$, which is written as

$$\Lambda_i \chi(\vec{p}) = \int d\vec{q} K_3(\vec{p},\vec{q},\epsilon_3)\tau_c(\vec{q})\chi(\vec{q}) + 2\int d\vec{q}d\vec{q}\,' K_2(\vec{p},\vec{q},\epsilon_3)\tau_n(\vec{q}) \times K_1(\vec{q},\vec{q}\,',\epsilon_3)\tau_c(\vec{q}\,')\chi(\vec{q}\,'). \tag{3}$$

Here the kernels K_1, K_2, K_3 are essentially the same as given in [A] [of Eq. (15)] except that the variables p,q, etc., are now dimensionless quantities:

$$\frac{p}{\beta_1} \to p, \qquad \frac{q}{\beta_1} \to q,$$

and

$$-\frac{mE}{\beta_1^2} \equiv \epsilon_3.$$

The kernels in the integral equation (3) involve the two-body separable potentials for n-n and n-core binary systems. The potential

$$v_{nn} = -\frac{\lambda_n}{2\mu_{nn}} g(p_{nn})g(p'_{nn}), \quad g(p) = \frac{1}{(p^2+\beta^2)} \tag{4}$$

is known to produce the low energy scattering data for n-n reasonably well. For instance the strength and range parameters $\lambda_n = 18.6\alpha^3$ and $\beta = 5.8\alpha$ (where α is the deuteron binding energy parameter, $\alpha^2/m = 2.225$ MeV) yield the value of the singlet scattering length $a_{nn} = -23.69$ fm and that of the effective range $r_{nn} = 3.2$ fm, the values which agree remarkably well with the phase shift analysis of $1s_0$ n-p scattering. For the n-core system, we assume that the core is spinless and the halo neutron exists in a low-lying intruder s orbit giving a virtual or a bound state with the core. With this restriction we assume a separable potential:

$$v_{nc} = -\frac{\lambda_c}{2\mu_{nc}} f(p_{nc})f(p'_{nc}), \quad f(p) = \frac{1}{(p^2+\beta_1^2)}. \tag{5}$$

Here since we only know the bound state or the excitation energy of the system this gives a certain amount of flexibility in the determination of the parameter β_1.

The integral equation (3) is basically the eigenvalue equation in Λ_i. To compute the equation numerically, we employ the same procedure as has been discussed in [B], i.e., for the given n-n and n-core potentials we seek the solution of the above equation for the three-body energy parameter ϵ_3 when the eigenvalue $\Lambda_i \to 1$, accurate to at least four decimal places. It is worth pointing out that here we are basically encountering a limiting procedure, where various factors in the kernels may blow up as the variable $p \to 0$ and the three-body energy parameter approaches extremely small values. From the computational point of view, this, therefore, requires a rather large size of the matrix with double precision so as to minimize the possible truncation errors.

Table I summarizes the results for the nucleus ^{19}B considered as n-n-^{17}B system with ^{17}B as core, whereas in Table II we present the results for ^{22}C consisting of n-n-^{20}C system with ^{20}C as core. For the n-^{17}B interaction, we note that $\lambda_c = 8.49\alpha^3$ and $\beta_1 = 4.75\alpha$ gives the excitation energy as 514 keV whereas the experimentally determined value is 530 keV [8]. The excitation energy of 514 keV represents a virtual state with scattering length $a_s = -6.515$ fm and the effective range $r_s = 3.234$ fm. With this two-body interaction along with the n-n potential given above, the three-body equation (3) predicts the ground state energy of the n-n-^{17}B system as 500 keV which is rather close to the experimental value of 515 keV [8]. However, with these input parameters of the two-body potentials, the three-body equation does not admit any solution for the excited state. When we change the excitation energy for the n-^{17}B system from 514 keV to about 7.7 keV corresponding to a virtual state with scattering length $a_s = -53.2$ fm, only then does the three-body equation start giving the solution for the excited state. In fact, the Efimov region begins to develop only when the binary interaction of the n-^{17}B system corresponds to a virtual state with a scattering length of the order of a few hundred fermis. From Table II it is clear that almost the same scenario holds in the case of ^{22}C considered as n-n-^{20}C system. For the case of n-^{20}C potential we choose $\lambda_c = 9.82\alpha^3$ and $\beta_1 = 4.9\alpha$ to fit the excitation energy of 320 keV corresponding to the virtual s state of scattering length $a_s = -8.23$ fm and $r_s = 3.02$ fm. Here also with this potential and the same n-n interaction, the three-body equation (3) predicts the ground state energy of the ^{22}C nucleus as 1120 keV, which is in excellent agreement with the experimentally determined value [8]. However, here again the input two-body potential does not allow the three-body equation to admit any solution for the occurrence of the excited states near the breakup threshold. It is only by changing the parameters of the n-^{20}C potential so as

TABLE I. ^{19}B ground and excited states three-body energy for different two-body input paramaters. $\beta_1 = 4.75\alpha$.

n-^{17}B energy (keV)	λ_c/α^3	a_s (fm)	ϵ_0 (keV)	ϵ_1 (keV)	ϵ_2 (keV)
514.8	8.49	−6.515	500		
135.3	9.5	−12.71	728		
48	10.0	−21.16	851		
7.7	10.50	−53.2	978	0.16	
0.67	10.75	−179.6	1042	5.4	0.36

TABLE II. ^{22}C ground and excited states three-body energy for different two-body input parameters. $\beta_1 = 4.9\alpha$.

n-^{20}C energy (keV)	λ_c/α^3	a_s (fm)	ϵ_0 (keV)	ϵ_1 (keV)	ϵ_2 (keV)
319	9.82	−8.23	1120		
127	10.5	−13.02	1287		
48.8	11.0	−21.0	1410		
9.3	11.5	−48.2	1540	0.122	
1.46	11.75	−121.5	1608	4.74	0.198

TABLE III. ^{20}C ground and excited states three-body energy for different two-body input parameters. $\beta_1 = 5.2\alpha$.

n-^{18}C binding energy (keV)	λ_c/α^3	a_s (fm)	ϵ_0 (keV)	ϵ_1 (keV)	ϵ_2 (keV)	N
60	15.51	20.38	3188.03	78.87	65.8	1.01
100	15.89	16.05	3291.54	115.72	100.09	0.94
113.2	16.0	15.15	3317.35	127.41	111.76	0.92
139.60	16.2	13.77	3371.24	150.32	135.29	0.89
168.59	16.4	12.64	3426.03	175.34	163.48	0.86
200	16.6	11.71	3482.95	202.15	194.15	0.84

to reduce the excitation energy to a few keV or corresponding to scattering length to about a hundred fermi that the Efimov region begins to develop and the excited states appear for the ^{22}C system.

Having seen quantitatively the general trend for the appearance of the Efimov states in some specific cases, for Borromean type halo nuclei it would be instructive to investigate from analytical considerations to see how far such a behavior is universal. For this, it is important to realize that the factors which are crucial in governing the dynamics of the binary systems and which play a significant role in describing the kernels of the three-body integral equation (3) are $\tau_n(p)$ and $\tau_c(p)$, which are defined as

$$\tau_n^{-1}(p) = \mu_n^{-1} - \left[\beta_r\left(\beta_r + \sqrt{\frac{p^2}{2a} + \epsilon_3}\right)^2\right]^{-1}, \tag{6}$$

and

$$\tau_c^{-1}(p) = \mu_c^{-1} - 2a\left[1 + \sqrt{2a\left(\frac{p^2}{4c} + \epsilon_3\right)}\right]^{-2}, \tag{7}$$

where $\mu_n = \pi^2\lambda_n/\beta_1^3$, $\mu_c = \pi^2\lambda_c/2a\beta_1^3$, $a = m_c/m + m_c$, $c = m_c/m_c + 2m$, and $\beta_r = \beta/\beta_1$. Substituting the above factors, Eq. (6) is written more explicitly as

$$\tau_n^{-1} = \frac{\beta_1^3\lambda_n^{-1}}{\pi^2} - \frac{1}{\beta_r\left(\beta_r + \sqrt{\frac{p^2}{2a} + \epsilon_3}\right)^2}. \tag{8}$$

For an unbound system (as, e.g., a virtual state)

$$\frac{\beta^3\lambda^{-1}}{\pi^2} > 1.$$

Since the second term in Eq. (8) is a monotonically decreasing factor as p increases, this term is always going to be less than the first and the difference between the two would increase as p increases. To express λ_c in terms of n-c scattering length, we have the relation

$$\frac{1}{a_{nc}} = \frac{\beta_1}{2} - \frac{\beta_1^4}{2\pi^2\lambda_c} \quad \text{or} \quad \lambda_c^{-1} = \frac{\pi^2}{\beta_1^3}\left(1 - \frac{2}{a_{nc}\beta_1}\right).$$

Thus,

$$\tau_c^{-1} = 2a\left[1 - \frac{2}{a_{nc}\beta_1} - \frac{1}{\left(1 + \sqrt{2a\left(\frac{p^2}{4c} + \epsilon_3\right)}\right)^2}\right].$$

Clearly, so long as a_{nc} is negative (representing a virtual state), and the third term is always smaller than the first term there is hardly any possibility of making $\tau_c^{-1} \to 0$ or $\tau_c \to \infty$, except when a_{nc} approaches a large value and ϵ_3 goes to the zero limit. This is precisely what has been demonstrated through the numerical analysis. On the other hand, if the binary subsystem is bound corresponding to a positive scattering length, there is a clear possibility of reducing τ_c^{-1} or allowing τ_c to be large enough.

A typical example in this case is ^{20}C considered as a bound system of n-n-^{18}C where n-^{18}C is known to be bound with binding energy 160 ± 100 keV [8]. In view of the experimental uncertainties in the determination of the binding energy, we have carried out the calculations for a range of n-^{18}C binding energies varying from 60 to 200 keV choosing the parameter $\beta_1 = 5.2\alpha$. The results are presented in Table III. We find that as the binding energy of the n-^{18}C increases from 60 to 200 keV, the three-body integral equation predicts the ground state energy of ^{20}C increasing from 3.1 to 3.5 MeV, in close agreement with the experimental data [8]. In addition, we get excited states as presented in Table III. The first excited states (ϵ_1) shown in column 5 of Table III have binding energies below the n-(nc) scattering threshold (ϵ_1-ϵ_{n-core}) which can vary from 2 to 19 keV depending on the n-^{18}C input binding energy. In order to analyze these states further, we present the number of bound states (N) for the respective two-body interactions as per Efimov's relations for these quantities [1]. The value of $N \sim 1$ in column 7 is indicative of the occurrence of not more than one Efimov state. We estimate the size [1] of the first excited state for the different two-body interactions to vary between 50 and 55 fm. The rather large size (≈ 50 fm) is also in conformity with expectations for the sizes of Efimov states.

From the three-body equations we also find an evidence for the occurrence of higher excited states above the n-(nc) scattering threshold but below the three-body breakup threshold. Here it is interesting to note that the second ex-

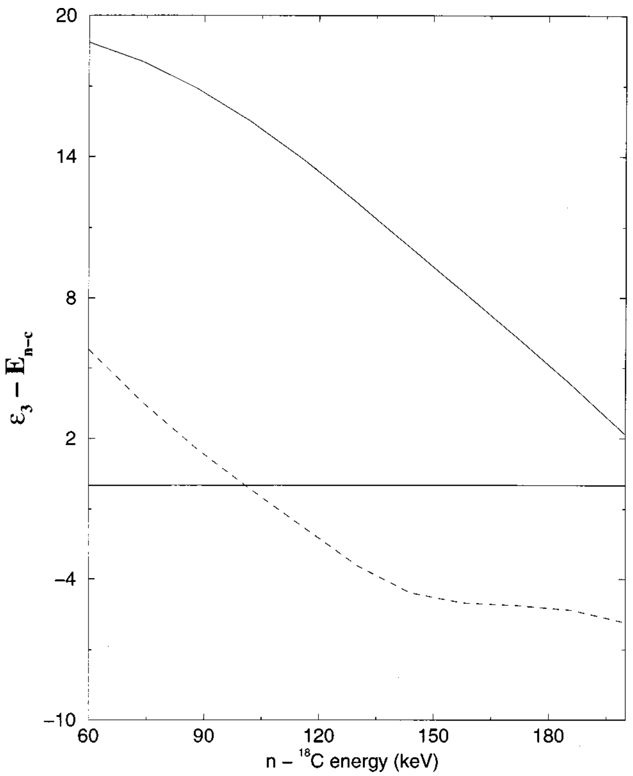

FIG. 1. The plot of three-body excited states, ϵ_1 (solid line) and ϵ_2 (broken line), relative to the elastic threshold vs the n-^{18}C binding energy.

cited state is below the two-body (n-^{18}C) bound state only for 60 keV. At 100 keV the excited state is just at the breakup threshold. For higher bound state energy the second excited state is above the two-body breakup. These states (ϵ_2) are also presented in Table III though this would still need a detailed investigation incorporating the breakup effect explicitly in the structure of the integral equations. Figure 1 presents the plot of ^{20}C Efimov states (ϵ_1 and ϵ_2) relative to the elastic threshold as a function of the n-^{18}C binding energy. As discussed earlier the region of 60 keV $< E_{n-c}$ <200 keV supports the existence of one Efimov state (ϵ_1, solid line). However, for $E_{n-c} > 100$ keV the second excited state (ϵ_2, broken line) gets destroyed. The conclusions arrived here are in overall qualitative agreement with the analysis of Amorim *et al.* [9] who also conclude the presence of not more than one Efimov state with an estimated binding energy less than 14 keV below the n-(nc) scattering threshold. The size of the first excited state obtained by us is also in qualitative agreement with the value of Amorim *et al.* [9], who estimate the size to be at least 35 fm.

In conclusion, the main thrust of this investigation is to establish both analytically as well as quantitatively through the numerical analysis the physical situations for looking into the possibility of the occurrence of Efimov states in 2n-halo nuclei considered as a three-body system. We have shown that Borromean type halo nuclei, where n-n and n-core are both unbound, are much less vulnerable to respond to the existence of the Efimov effect. On the other hand, those nuclei, in which the halo neutron is in the intruder low lying bound state, appear to be the most promising candidates to search for the occurrence of Efimov states at energies which may well be within experimental limits to measure.

V.A. and V.S.B. gratefully acknowledge financial support from the University Grants Commission for the research project.

[1] P.G. Hansen, A.S. Jensen, and B. Jonson, Annu. Rev. Nucl. Part. Sci. **45**, 591 (1995); V. Efimov, Nucl. Phys. **A210**, 157 (1973).
[2] V.N. Efimov, Phys. Rev. C **47**, 1876 (1993), and references therein.
[3] D.V. Fedorov and A.S. Jensen, Phys. Rev. Lett. **71**, 4103 (1993).
[4] D.V. Fedorov, A.S. Jensen, and K. Riisager, Phys. Rev. Lett. **73**, 2817 (1994); Phys. Rev. C **49**, 201 (1994).
[5] I. Mazumdar and V.S. Bhasin, Phys. Rev. C **56**, R5 (1997).
[6] S. Dasgupta, I. Mazumdar, and V.S. Bhasin, Phys. Rev. C **50**, R550 (1994).
[7] The values we quote as experimental data are taken from the systematics of Audi and Wapstra [8] prepared by the least square fitting of all available experimental data.
[8] G. Audi and A.H. Wapstra, Nucl. Phys. **A565**, 66 (1993).
[9] A.E.A. Amorim, T. Frederico, and L. Tomio, Phys. Rev. C **56**, R2378 (1997).