

## Fine structure in proton emission from deformed $^{131}\text{Eu}$

E. Maglione<sup>1</sup> and L. S. Ferreira<sup>2</sup>

<sup>1</sup>*Departimento di Fisica "G. Galilei," Via Marzolo 8, I-35131 Padova, Italy  
and Istituto Nazionale di Fisica Nucleare, Padova, Italy*

<sup>2</sup>*Centro de Física das Interações Fundamentais and Departamento de Física, Instituto Superior Técnico, Av. Rovisco Pais,  
P-1049-001 Lisboa, Portugal*

(Received 1 December 1999; published 21 March 2000)

Proton emission from the drip line nucleus  $^{131}\text{Eu}$ , to the first excited  $2^+$  state of the daughter nucleus  $^{130}\text{Sm}$  is analyzed as decay from a deformed nucleus, and the half-lives are evaluated exactly. It is found that the Nilsson state  $K=3/2^+$  for a deformation  $0.34 > \beta > 0.27$  describes both the fine structure and the decay to the ground state of  $^{130}\text{Sm}$ .

PACS number(s): 23.50.+z, 21.10.Tg, 25.70.Ef, 27.60.+j

Important aspects of nuclear structure can be learned from proton decay. It is a powerful tool not only to probe small components of the wave functions of the decaying states, but also to determine the deformation of nuclei and the angular momentum of the ground state. In a recent work [1] we have studied proton emission from odd-even deformed decaying nuclei, and interpreted the decay within our model [2] that leads to an exact evaluation of the half-lives. The angular momentum of the decaying state and the deformation of the parent nucleus is determined from the condition of reproducing the experimental half-lives. There are some cases where more than one state or different values of  $\beta$  could reproduce the data. Therefore, it is necessary to have more experimental information besides that related to ground-state decay, to impose extra consistency bounds on the theoretical interpretation.

Due to energy considerations, proton decay should proceed mainly to the ground state of the daughter nucleus. However, in rotational nuclei the first excited state could be very low in energy and a sizeable branching ratio could be expected, known as fine structure. Recently a new measurement was reported [3] in the highly deformed proton emitter  $^{131}\text{Eu}$ , and a fine-structure splitting in the radioactive decay from the ground state was identified. In a previous experiment [4], only the ground-state line was observed at 950(8) keV. The recent one reports a second proton peak with energy 811(7) keV,  $t_{1/2} = 23_{-6}^{+10}$  ms, and an updated value for the ground-state energy at 932(7) keV,  $t_{1/2} = 17.8(19)$  ms. Since the half-lives of the two states were almost the same, the new line was interpreted as proton decay from the ground state of  $^{131}\text{Eu}$ , to the first excited  $2^+$  state of the daughter nucleus  $^{130}\text{Sm}$ , with a branching ratio of 0.24(5).

From the theoretical point of view, decay to excited states should be reproduced with the same deformation and angular momentum used for the ground-state decay. Therefore, it is our intention in this Brief Report to describe within our model the decay of  $^{131}\text{Eu}$  to the  $2^+$  and take into account the new value of the energy for the emission to the ground state. The basic assumption of our model consists in having the emitted proton moving in a deformed single particle Nilsson level. The Schrödinger equation is thus solved for a deformed Woods-Saxon potential with a deformed spin-orbit term and realistic parameters. Imposing regularity at the ori-

gin and outgoing wave boundary conditions,

$$\lim_{r \rightarrow \infty} u_{lj}(r) = N_{lj}(G_l(kr) + iF_l(kr)) \quad (1)$$

all bound states and resonances can be found. In Eq. (1),  $k = \sqrt{2\mu E/\hbar^2}$ , and  $N_{lj}$  is a normalization constant. The functions  $F$  and  $G$  are the regular and irregular Coulomb functions. In practice, the equality in Eq. (1) holds beyond the range of the nuclear interaction, where the Coulomb potential is spherical.

The Nilsson single-particle energies for  $^{131}\text{Eu}$ , obtained from the solution of the Schrödinger equation, are shown in Fig. 1(a) as a function of the deformation parameter  $\beta$ . The calculation of the half-life requires the knowledge of the energy available for the decay. In fact, the dependence of the half-life on the energy of the outgoing proton is given by the well established relation [5],

$$T_{1/2} \propto \exp a/\sqrt{E}. \quad (2)$$

Therefore, we vary the depth of the nuclear potential to have the real part of the resonance at the measured energy.

The decay to the excited state can be discussed in an analogous manner. The total width for the decay is a sum of partial widths for all possible values  $l_p j_p$  of the emitted proton restricted by angular momentum and parity conservation, i.e.,

$$\Gamma^{J_d} = \sum_{j_p = \max(|J_d - K_i|, K_i)}^{J_d + K_i} \Gamma_{l_p j_p}^{J_d}, \quad (3)$$

where  $K_i$  is the angular momentum of the parent state.

It can be proved [6] that

$$\Gamma_{l_p j_p}^{J_d} = \frac{\hbar^2 k}{\mu} \frac{2(2J_d + 1) \langle J_d, 0, j_p, K_i | K_i, K_i \rangle^2}{(2K_i + 1)} |N_{l_p j_p}|^2 u_{K_i}^2, \quad (4)$$

where  $u_{K_i}^2$  is the probability that the single-particle level in the daughter nucleus is empty, evaluated with the pairing residual interaction in the BCS approach.

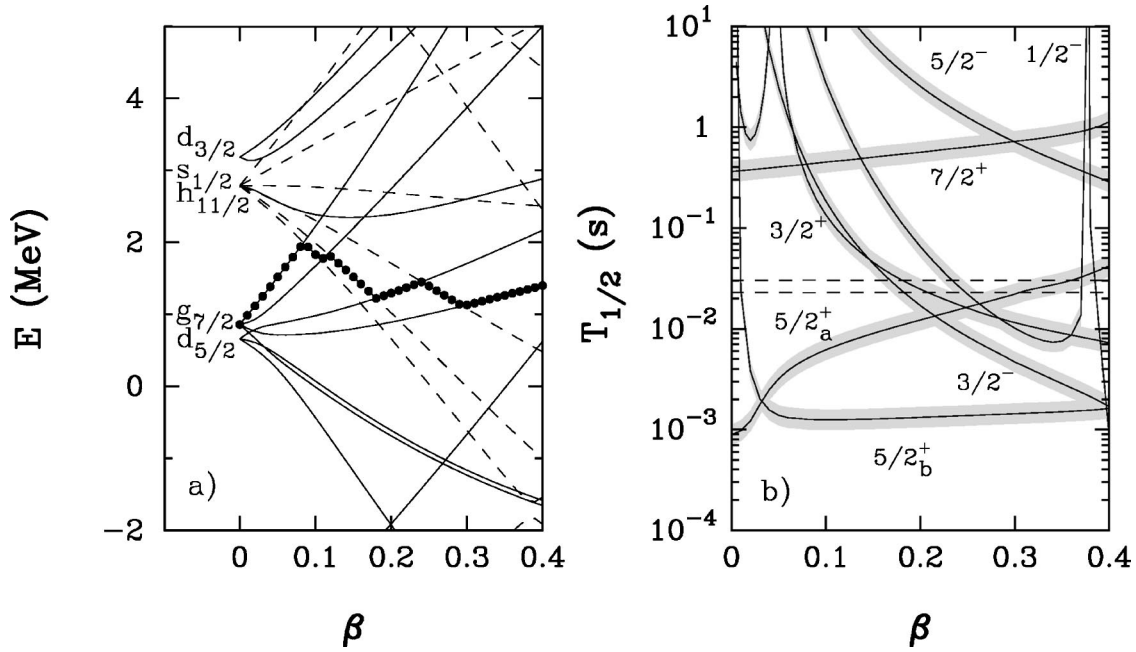


FIG. 1. (a) Proton Nilsson levels corresponding to  $^{131}_{63}\text{Eu}$ . The dotted lines indicate the Fermi level. (b) Half-life of the resonances in  $^{131}_{63}\text{Eu}$  as a function of the deformation  $\beta$  (full lines). The shadowed area represents the uncertainty due to the experimental error bar on the energy. The curve labeled  $K=5/2_a^+$  is coming from the spherical shell  $d_{5/2}$ . The curves labeled  $K=7/2^+$ ,  $K=5/2_b^+$ , and  $K=3/2^+$  are all coming from the spherical shell  $g_{7/2}$ . The curves labeled  $K=5/2^-$ ,  $K=3/2^-$ , and  $K=1/2^-$  are all coming from the  $h_{11/2}$ . The experimental values (dashed lines) are from Ref. [3].

In the case of transitions to the ground state,  $J_d=0$ , and Eqs. (3), (4) reduce to

$$\Gamma_{l_p, K_i}^0 = \frac{\hbar^2 k}{\mu} \frac{2}{(2K_i+1)} |N_{l_p, K_i}|^2 u_{K_i}^2. \quad (5)$$

In a previous calculation [1], the half-lives for the decay to the ground state were determined using the old value of 950(8) keV for the energy, and presented in Fig. 2(c) of Ref. [1]. We used a spectroscopic factor  $u^2 \approx 0.5$ , which is adequate for levels around the Fermi surface. Dividing the theoretical value obtained for the half-life by this quantity, the only state that could reproduce the experimental half-life with a large deformation, and at the same time was close to the Fermi surface, was the  $K=5/2^+$  coming from the spherical  $d_{5/2}$  state. This result was compatible with the prediction of Ref. [4] that quoted the states  $K=5/2^+$  and  $K=3/2^+$  as possible candidates.

Since a new value for the energy of this transition was reported, we just have to look at the states that fulfill the new data. The half-life for proton emission for all levels that lie, or are close to the Fermi surface up to a deformation  $\beta=0.4$ , are shown in Fig. 1(b). This figure is essentially Fig 2(c) of Ref. [1], but scaled by a factor of  $\approx 1.85$ . This is expected from Eq. (2) since the experimental energy is now 18 keV lower.

Using the spectroscopic factor  $u^2 \approx 0.5$  there are three states  $K=5/2_a^+$ ,  $K=3/2^-$ , and  $K=3/2^+$  that could describe the half-life for the decay. The first state describes the half-

life with  $\beta$  in the range of 0.15–0.28. Similarly, for the states  $K=3/2^-$  and  $K=3/2^+$ , the range for  $\beta$  is 0.2–0.24 and 0.24–0.34, respectively.

From the various terms in Eq. (3) the dominant ones correspond to situations where  $N_{l_p, j_p}$  are large or the centrifugal barrier low, i.e.,  $l_p$  small. The last condition is usually the most important one. If  $K_i = l_p + 1/2$ , there is only one large term in the sum, the one with the same  $l_p, j_p$  as the decay to the  $0^+$ . In this case, the ratio of the decay width to the  $2^+$  with respect to the one to the  $0^+$ , depends practically on the energy of the  $2^+$  and on  $l_p, j_p$  only, while very little dependence on deformation is observed, since  $N_{l_p, j_p}$  cancels out in the ratio. A strong dependence on deformation can be observed only if  $N_{l_i, K_i}$  is much smaller than the other components, or if  $K_i = l_p - 1/2$ , because two channels with the same centrifugal barrier can contribute.

The branching ratio for the decay to the  $2^+$  is shown in Fig. 2. The spectroscopic factor  $u^2$ , present in the numerator and denominator of this ratio, is the same and cancels out in the final result. As it can be seen from the figure, the branching ratio for the  $K=5/2_a^+$  state is practically constant. The centrifugal barrier of the  $j_p = g_{7/2}$  and  $g_{9/2}$  components is very high, and  $j_p = d_{5/2}$  dominates the decay. This is not true for the  $K=3/2^+$  state, where the most important component, almost a factor of 20–40 larger, is the  $d_{5/2}$ , that has the same centrifugal barrier of the  $d_{3/2}$ , but a larger  $N$ .

It is also clear from Fig. 2 that the  $K=5/2_a^+$  and  $K=3/2^-$  states can be disregarded, since the theory predicts a branching ratio one order of magnitude smaller than the experimental value. The only state capable of reproducing the

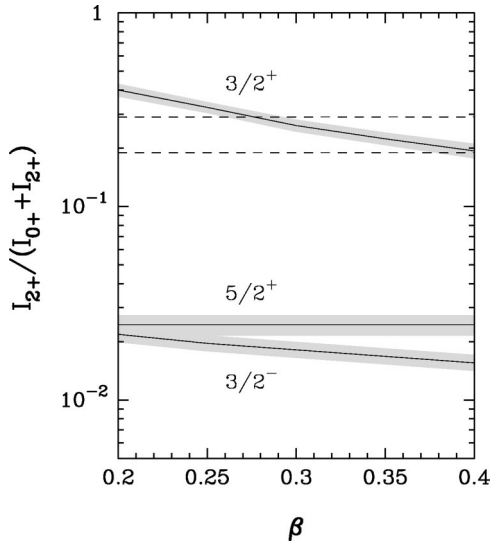


FIG. 2. Branching ratio for the decay to the  $2^+$  state of  $^{130}\text{Sm}$  as a function of deformation.

data is the  $K=3/2^+$ , with deformation larger than 0.27. Therefore, the interpretation of the fine-structure data allows an unambiguous determination of the angular momentum of the ground state, and deformation of  $^{131}_{63}\text{Eu}$ . Our result and assignment are in perfect agreement with the findings of Ref. [3].

In order to derive Eq. (4), we have assumed that daughter

and parent nuclei have the same deformation. Therefore, one expects that  $^{130}\text{Sm}$  has a deformation also larger than 0.27. There is a well-known empirical relation [7], between the energy of the  $2^+$  state and the deformation,  $E_{2^+} \approx 1225/A^{7/3}\beta^2$  MeV, followed by all even-even nuclei. This relation suggests for  $^{130}\text{Sm}$ ,  $\beta \approx 0.34$  since the  $2^+$  excitation lies at 121(3) MeV, in agreement with our hypothesis, and providing another consistency test of our calculation.

Another nucleus where the fine structure could be observed is  $^{141}\text{Ho}$ . In this case there is no doubt on the  $K=7/2^-$  assignment of the state, as explained in Ref. [1]. Even small variations of the experimental energy would not alter this conclusion. Since the energy of the  $2^+$  in  $^{140}\text{Dy}$  is not known, we have calculated the branching ratio as a function of energy. In this case there is no strong dependence on deformation, since the  $h_{11/2}$  width is a factor 50 smaller due to the centrifugal barrier.

Using a deformation of  $\beta=0.29$  [8,9] the  $2^+$  of  $^{140}\text{Dy}$  should be at 140 keV giving a branching ratio around 5%, a value probably not reachable with present experimental apparatus.

In conclusion, we have shown in this work that our model can calculate exactly the decay from single-particle Nilsson levels and is able to describe quite accurately and consistently the data on proton emission from the ground state of  $^{131}\text{Eu}$  and the fine structure for the decay to the first excited  $2^+$  state of the daughter nucleus. The deformation and angular momentum  $J$  of the decaying nuclei are in agreement with the predictions of Refs. [8,9].

- [1] E. Maglione, L.S. Ferreira, and R.J. Liotta, Phys. Rev. C **59**, R589 (1999).  
 [2] E. Maglione, L.S. Ferreira, and R.J. Liotta, Phys. Rev. Lett. **81**, 538 (1998).  
 [3] A.A. Sonzogni, C.N. Davids, P.J. Woods, D. Seweryniak, M.P. Carpenter, J.J. Ressler, J. Schwartz, J. Uusitalo, and W.B. Walters, Phys. Rev. Lett. **83**, 1116 (1999).  
 [4] C.N. Davids, P.J. Woods, A.A. Sonzogni, J.C. Batchelder, C.R. Bingham, T. Davinson, D.J. Henderson, R.J. Irvine, G.L. Poli, J. Uusitalo, and W.B. Walters, Phys. Rev. Lett. **80**, 1849

(1998).

- [5] V.I. Kukulín, V.M. Krasnopolsky, and J. Horacek, *Theory of Resonances* (Klover, Boston, 1989).  
 [6] E. Maglione and L.S. Ferreira, in Proceedings of the PROCON99 Symposium, Oak Ridge, 1999 (in press).  
 [7] L. Grodzins, Phys. Lett. **2**, 88 (1962).  
 [8] P. Möller, J.R. Nix, W.D. Myers, and W.J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).  
 [9] P. Möller, R.J. Nix, and K.L. Kratz, At. Data Nucl. Data Tables **66**, 131 (1997).