Nuclear matter equation of state based on effective nucleon-nucleon interactions

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A comparison of the nuclear matter equations of state based on Skyrme, Myers-Swiatecki, and Tondeur interactions is given. It is shown that the difference among these equations of state is not significant in most of the relative neutron excess range which is of interest for both heavy ion collisions and supernova explosion calculations. However, if the equation is fitted to the same standard state, the equation based on the Tondeur interaction is softer than others provided the relative neutron excess is not close to 0.

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The nuclear equation of state $e(\rho,\delta)$, which is the energy per nucleon of nuclear matter as a function of the nucleon density ρ and the relative neutron excess δ, is a fundamental quantity in theories of neutron stars and supernova explosions, as well as in theories of nucleus-nucleus collisions at energies where nuclear compressibility comes into play. The main measured quantities which can provide information about the equation of state (EOS) are the binding energies and other data of finite nuclei. As the finite nuclei are in states near the standard nuclear matter state with normal nucleon density ρ_0 and zero neutron excess, $\delta=0$, our knowledge about the EOS can be confirmed experimentally only in a small region around $\rho\sim\rho_0$ and $\delta\sim 0$.

However, there is currently considerable interest in the very neutron rich nuclei and energetic heavy ion collisions where the nuclear matter state is beyond this region. As any direct information beyond this region is difficult to come by, extrapolation is inescapable and, in this case, a nuclear model is required. The model is fitted to binding energies and other data of finite nuclei at first, then applied to nuclear matter to derive the EOS. In this way, the obtained EOS can be considered as being fitted indirectly to a region around the standard state, but its prediction of states beyond this region should be regarded as an extrapolation. In this case, it is worthwhile to make a comparison between the extrapolations given by different EOS.

The purpose of this Brief Report is to discuss the EOS given by Skyrme (SK) [1], Myers-Swiatecki (MS) [2], and Tondeur (TO) [3] interactions. This is a very specific family of effective *NN* interactions which should not be confused with more realistic models of *NN* interactions which are fitted to describe *NN* scattering data. We will discuss the restriction of equilibrium condition on the properties of standard nuclear matter at first, and then present the prediction for nuclear matter away from the standard state, based on these EOS.

For the SK interaction, the parameters t_0, t_3, x_0, x_3, s_1, s_2, and γ appear in the EOS while W_0 does not, where

$$s_1=[t_1(1+x_1/2)+t_2(1+x_2/2)]/4,$$
$$s_2=[t_2(x_2+1/2)-t_1(x_1+1/2)]/4. \tag{1}$$

For the MS interaction, the parameters α, B, $\overline{\gamma}$ (the γ in Ref. [2]), ξ, and ζ appear in the EOS [4,5], while the Yukawa range of force a does not. In the TO interaction, the parameters a, b, c, and γ are relevant to the EOS, while d and η are not.

The EOS based on these interactions can be written formally as

$$e(\rho,\delta)=T\left[D_2(\delta)\left(\frac{\rho}{\rho_0}\right)^{2/3}-D_3(\delta)\left(\frac{\rho}{\rho_0}\right)^{3/3}+D_5(\delta)\left(\frac{\rho}{\rho_0}\right)^{5/3}+D_\gamma(\delta)\left(\frac{\rho}{\rho_0}\right)^{\gamma/3}\right]. \tag{2}$$

The equilibrium condition $\partial e/\partial\rho|_0=0$, by which the standard state $\rho=\rho_0$ at $\delta=0$ is defined, gives the following relationship:

$$2D_2(0)-3D_3(0)+5D_5(0)+\gamma D_\gamma(0)=0. \tag{3}$$

For nuclear matter close to the standard state (ρ_0,0), the EOS can be written approximately as

$$e(\rho,\delta)=-a_1+\frac{1}{18}(K_0+K_s\delta^2)\left(\frac{\rho-\rho_0}{\rho_0}\right)^2+\left[J+\frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right)\right]\delta^2, \tag{4}$$

which is specified by the volume energy a_1, symmetry energy J, incompressibility K_0, density symmetry L, and symmetry incompressibility K_s. They can be expressed in terms of $D_{i0}=D_i(0)$, $D_{i2}=(1/2)[\partial^2 D_i/\partial\delta^2]_0$, $i=2,3,5,\gamma$, and the following formulas can be derived:

$$K_0=15a_1+[3D_{20}+(\gamma-5)(\gamma-3)D_{\gamma 0}]\,T$$
$$=3\gamma a_1+[(\gamma-2)D_{20}-2(\gamma-5)D_{50}]\,T, \tag{5}$$

where T is a constant with dimension of energy.

For the SK interaction, Eq. (3) yields

$$\frac{9}{8}\frac{\rho_0 t_0}{T}+\frac{\gamma}{16}\frac{\rho_0^{\gamma/3}t_3}{T}+3\left(\frac{3\pi^2}{2}\right)^{2/3}\frac{\rho_0^{5/3}}{T}\left(s_1+\frac{1}{2}s_2\right)+\frac{6}{5}=0. \tag{6}$$

TABLE I. Readjusted SK interaction parameters. Input values are $r_0=1.140$ fm, $a_1=15.97$ MeV, $K_0=236.07$ MeV, $J=29.25$ MeV, $L=58.50$ MeV, and $K_s=-67.92$ MeV.

Force	SIII	Ska	SkM	SkM*	RATP
γ	6	4	7/2	7/2	18/5
t_0 (MeV fm^3)	−1405.521	−1792.320	−2372.518	−2372.518	−2179.119
t_3 (MeV fm$^\gamma$)	−14 402.55	12 794.56	12 584.33	12 584.33	11 940.95
x_0	0.069 56	0.137 35	0.197 59	0.197 59	0.180 18
x_3	0.383 68	0.383 68	0.383 68	0.383 68	0.383 68
s_1 (MeV fm^5)	642.825	−42.389	71.813	71.813	55.499
s_2 (MeV fm^5)	−473.186	84.802	−8.196	−8.196	5.090

Therefore, among seven parameters t_0, t_3, x_0, x_3, s_1, s_2, and γ, only six of them are free. Considering this relation, it can be shown that a_1, K_0, J, L, and K_s are independent of each other, in the SK EOS. Besides, a relation connecting t_3 to a_1 and K_0 can be obtained from Eq. (5),

$$K_0=15a_1+\frac{9}{5}T+\frac{(\gamma-5)(\gamma-3)}{16}\rho_0^{\gamma/3}t_3. \tag{7}$$

If $t_3=0$, as $a_1\sim16$ MeV and $T\sim37$ MeV are well known from measurements, this formula gives the estimation $K_0\sim306$ MeV. Hence, in order to have K_0 lower than 306 MeV, the fourth term $(\rho/\rho_0)^{\gamma/3}$ in the SK EOS is needed.

For the MS interaction, $D_\gamma^{MS}(\delta)=0$, the equilibrium condition (3) can be transformed into the following relation among α, B, and $\overline{\gamma}$:

$$5\alpha-10B-4(1-\overline{\gamma})=0. \tag{8}$$

In this case, there are only four independent interaction parameters in the MS EOS, α, B, ξ, and ζ, if $\overline{\gamma}$ is solved from Eq. (8) as a function of α and B. Correspondingly, there are only four independent variables among a_1, K_0, J, L, and K_s in the MS EOS. Actually, the following relationship can be derived:

$$\frac{K_s}{T}=\frac{4B(1+\overline{\gamma})}{4B+\overline{\gamma}}\left[1-\frac{10B+\overline{\gamma}}{2B(1+\overline{\gamma})}\,\frac{3J-L}{T}\right], \tag{9}$$

where $B=5(K_0-6a_1)/18T$ and $\overline{\gamma}=1-5(K_0-15a_1)/9T$. Furthermore, formula (5) for the MS EOS becomes

$$K_0=15a_1+\frac{9}{5}(1-\overline{\gamma})T. \tag{10}$$

For $\overline{\gamma}=0$, the MS interaction is reduced to the Seyler-Blanchard interaction [6] and this formula gives the estimation $K_0\sim306$ MeV, the same as that discussed for the SK interaction. Therefore, $\overline{\gamma}$-dependent terms in the MS EOS are required, in order to obtain K_0 lower than 306 MeV [4].

For the TO interaction, $D_5(\delta)=0$, the equilibrium condition (3) now is a relation among a, b, and γ:

$$\frac{3\rho_0 a}{T}+\frac{\gamma\rho_0^{\gamma/3}b}{T}+\frac{6}{5}=0. \tag{11}$$

Therefore, there are only three independent interaction parameters in the TO EOS, e.g., a, c, and γ. Correspondingly, there are only three free variables in a_1, K_0, J, L, and K_s, e.g., a_1, K_0, and J, as it can be shown that

$$L=2J, \quad K_s=-2J. \tag{12}$$

In addition, the following relationship among K_0, a_1, and γ can be written for the TO EOS from Eq. (5):

$$K_0=3\gamma a_1+\frac{3}{5}(\gamma-2)T. \tag{13}$$

From $a_1\sim16$ MeV, $T\sim37$ MeV, and $K_0\sim220$ MeV, the appropriate integer is found to be $\gamma=4$, as given by Ref. [3]. If $\gamma=4$ is chosen, there are only two interaction parameters to be freely adjusted in the data fit, e.g., a and c. Correspondingly, there are only two independent variables in a_1, K_0, J, L, and K_s, e.g., a_1 and J, when K_0 is calculated by Eq. (13). From $a_1\sim16$ MeV, $T\sim37$ MeV, and $\gamma=4$ we can evaluate $K_0\sim236$ MeV [3].

Using the interaction parameters and the nuclear radius constant r_0 given in Refs. [1–3], which will be referred to as the original interaction parameters thereafter, and the physical constants $\hbar c=197.328\,91$ MeV fm, $m=938.905\,95$ MeV/c^2, the calculated values of a_1, K_0, J, L, and K_s are close to each other for the SK, MS, and TO interactions, except SIII where the values of K_0 and K_s are far away from others. The average values of these coefficients over Ska, SkM, SkM*, RATP, MS and TO interactions are $a_1=15.97$ MeV, $K_0=234.4$ MeV, $J=29.25$ MeV, $L=48.63$ MeV, and $K_s=-126.9$ MeV.

The nuclear matter state with zero pressure and minimum energy per nucleon can be solved from the equation $\partial e/\partial\rho=0$. Its solution gives density as function of δ, i.e., $\rho_m=\rho_m(\delta)$. For $\delta=0$, we have $\rho_m(0)=\rho_0$. The incompressibility of nonequilibrium nuclear matter can be defined as $K(\rho,\delta)=9\partial P/\partial\rho$, where $P=\rho^2\partial e/\partial\rho$ is the pressure. Along the line of minimum $\rho_m(\delta)$, this $K(\rho,\delta)$ becomes $K_m(\delta)=9\rho_m^2[\partial^2 e/\partial\rho^2]_m$. At the standard state (ρ_0,0) we have $K_m(0)=K_0$. At the critical point (ρ_c,δ_c), where the maximum and the minimum are coincident, the curvature of

$e(\rho,\delta_c)$ versus ρ changes sign and $K_m(\delta_c)=0$. So $K_m(\delta)$ starts with K_0 and ends at 0 when δ increases along the line of minimum.

Using Eq. (4), the following approximate solutions can be obtained:

$$\rho_m \approx \rho_0[1-(3L/K_0)\delta^2], \quad e_m \approx -a_1 + J\delta^2,$$
$$K_m \approx K_0 + K_s\delta^2, \tag{14}$$

where only the linear term in δ^2 is kept. The systematics of nuclear central densities [7] based on elastic electron scattering data and muonic atom spectroscopy data provide a direct evidence for the first equation of Eqs. (14). In the plot $e(\rho,\delta)$ versus ρ, the standard state is at the minimum point $\rho_m=\rho_0$ with depth a_1 and curvature proportional to K_0. When the minimum is moved with increasing δ from 0, the decrease of ρ_m is controlled by $3L/K_0$, and the increase of depth is controlled by J, while the decrease of curvature is controlled by $-K_s$. In this way, the interaction with different value of these quantities will predict different properties of nuclear matter that are not far away from the standard state. By this understanding, the difference among the EOS based on SK, MS, and TO interactions, except SIII, is not significant at states not far away from the standard one, as their values of a_1, K_0, J, L, and K_s are close to each other.

The exact solution $\rho_m(\delta)$ depends on the interaction. The analytic solution is possible for SIII, Ska, MS, and TO interactions, while the numerical solution is appropriate for SkM, SkM*, and RATP interactions. A natural question is, what is the difference among these EOS, if the standard state is the same with same location ρ_0, depth a_1, curvature $\sim K_0$, and so on? In order to make this comparison, the interaction parameters should be readjusted according to chosen ρ_0, a_1, K_0, and so on. For the value of ρ_0, we choose $r_0=1.140$ fm determined by the data fit to nuclear charge radii [8]. In addition, we can choose the value of a_1, K_0, J, L, and K_s in an appropriate way. In this case, Eq. (9) should be fulfilled for the MS interaction, while Eqs. (12) and (13) should be fulfilled for the TO interaction. The chosen values used to calculate the interaction parameters are as follows: $a_1=15.97$ MeV, $K_0=236.07$ MeV, $J=29.25$ MeV, and $L=58.50$ MeV; in addition, $K_s=-67.92$ MeV for the SK and MS interactions while -58.50 MeV for the TO interaction. Among these values, a_1 and J are the average values mentioned above; K_0 and L are calculated by Eqs. (12) and (13). K_s is calculated by Eq. (12) in TO's case while by Eq. (9) in MS's case. In SK's case, K_s can be chosen from either MS's or TO's value; there is no significant difference in the calculated result.

The calculated SK interaction parameters are given in Table I. MS interaction parameters are calculated as $\alpha=2.017\,74$, $B=1.039\,79$, $\overline{\gamma}=1.077\,28$, $\xi=-0.170\,12$, $\zeta=0.475\,55$. TO interaction parameters with $\gamma=4$ are calculated as $a=-672.13$ MeV fm^3, $b=799.71$ MeV fm^4, and $c=99.116$ MeV fm^2. These parameters will be referred to as readjusted interaction parameters hereafter.

The plot e versus ρ/ρ_0 for $\delta=0$, calculated by these readjusted interaction parameters, shows that there is almost no difference among these EOS for $0.4<\rho/\rho_0<1.6$. Using these parameters, we can calculate $\rho_m(\delta)$, $e_m(\delta)$, and $K_m(\delta)$ along the equilibrium line. Figure 1 displays (a) ρ_m versus δ, (b) e_m versus δ, and (c) K_m versus δ calculated by readjusted parameters for various SK interactions (solid lines), the MS interaction (dot-dashed line), and the TO interaction (dashed line). The solid lines denoting SK interactions, from top to bottom, correspond to (a) Ska, RATP, SkM, SkM*, and SIII interactions, respectively; SkM and SkM* are the same whereas RATP, SkM, and MS almost overlap; (b) SIII, SkM, SkM*, RATP, and Ska interactions, respectively, where SkM and SkM* are the same; SkM, RATP, and Ska are almost

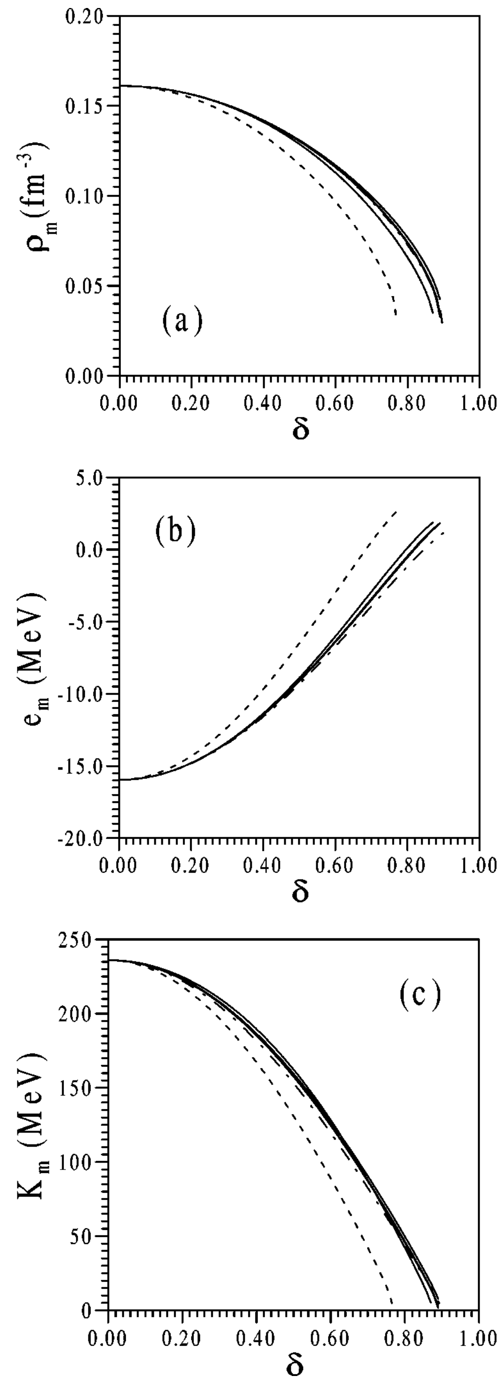

FIG. 1. (a) ρ_m versus δ, (b) e_m versus δ, and (c) K_m versus δ.

TABLE II. The critical point values (ρ_c,δ_c) predicted by SK, MS, and TO interactions. For each item, the first (second) line is given by original (readjusted) interaction parameters.

	SIII	Ska	SkM	SkM*	RATP	MS	TO
r_0 (fm)	1.180	1.154	1.142	1.142	1.143	1.140	1.145
	1.140	1.140	1.140	1.140	1.140	1.140	1.140
δ_c	0.8385	0.8647	0.8390	0.8421	0.8303	0.8213	0.8732
	0.8772	0.8980	0.8908	0.8908	0.8920	0.8988	0.7697
ρ_c (fm^{-3})	0.071 73	0.024 16	0.023 45	0.024 20	0.038 92	0.030 39	0.030 81
	0.027 32	0.029 69	0.028 25	0.028 25	0.028 51	0.026 43	0.031 31
e_c (MeV)	3.9019	1.5852	1.2572	1.2814	1.9898	1.1031	2.6142
	1.8894	1.8505	1.7025	1.7025	1.7311	1.1280	2.6304

coincident; (c) Ska, RATP, SkM, SkM*, and SIII interactions, respectively; SkM and SkM* are the same. The critical point values (ρ_c,δ_c) are listed in Table II.

Most of the discussion about the nuclear EOS, up to now, concentrates on states around the standard one, i.e., about the quantities a_1, J, L, K_0, and K_s, especially K_0 in supernova explosion and neutron star calculations and K_s in heavy ion collisions. However, even these quantities or equivalently the interaction parameters were well determined by the measured data of nuclei, the extrapolation to states far away from the standard one is still an open problem. The numerical result given above presents an example which shows that the asymmetry dependence of the critical point depends on the model used to perform the extrapolation. When the EOS is fitted to same standard state, SK's and MS's δ_c are close to each other; especially δ_c does not depend sensitively on the choice of γ in the SK interaction. On the other hand, TO's δ_c is well below others. This is because the value of TO's δ_c depends sensitively on the interaction parameters. In this context, in order to make a choice among these interactions for the extrapolation, experiments which can provide direct or even indirect information about nuclear matter with large asymmetry δ and low density ρ are required.

In summary, a comparison of nuclear matter EOS based on SK, MS, and TO interactions is given in this Brief Report. It is shown that the difference among these EOS is not significant in most of the relative neutron excess range which is of interest for both heavy ion collisions and supernova explosion calculations. However, if the equation is fitted to the same standard state, the equation based on the TO interaction is softer than others provided the relative neutron excess is not close to 0.

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