Sensitivity of muon capture to the ratios of nuclear matrix elements

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It is shown that the observables in ordinary muon capture depend very sensitively on the ratios of nuclear matrix elements. This is demonstrated for the case of extracting g_P/g_A from $\gamma\nu$-correlation experiments on ^{28}Si and from the ratio of capture rates from hyperfine states in ^{11}B and ^{23}Na.

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I. INTRODUCTION

The study of $\gamma\nu$-correlation coefficients obtained by two groups [1,2] measuring the capture of polarized muons in ^{28}Si by means of the reaction

$$\vec{\mu}+{}^{28}\mathrm{Si}(0^+,\mathrm{g.s.})\rightarrow{}^{28}\mathrm{Al}(1_3^+,2201)+\nu_\mu$$
$$^{28}\mathrm{Al}(1_3^+,2201)\rightarrow\gamma+{}^{28}\mathrm{Al}(0_1^+,902) \quad (1)$$

suggested that the extracted ratio g_P/g_A is strongly dependent on the nuclear matrix elements used in the analysis. A comparison of the experimental data with the calculations of [3] resulted in $g_P/g_A=0.0\pm3.2$, in strong contradiction to the predictions from the partially conserved axial-vector current (PCAC) hypothesis $g_P/g_A=6.5$ and the experimental result $g_P/g_A=9.2\pm2.0$ obtained from the radiative muon capture (RMC) rate on hydrogen [4]. The calculations of Ref. [3] used the effective muon capture Hamiltonian of [5] in the impulse approximation. The nuclear wave functions corresponded to a many-particle shell model using several variants of full sd-shell model Hamiltonians, including the Wildenthal Hamiltonian [6]. All nuclear models considered in [3] gave very similar results for g_P/g_A in comparison to the experimental data. However, in a recent calculation [7] using renormalized operators for the nuclear effective muon capture Hamiltonian, a change of roughly 10% of the matrix elements of the transition operators resulted in the value of $g_P/g_A=3.2\pm2.0$ for the experiment [2]. It appears therefore worthwhile to ask in which way the ratio g_P/g_A extracted from experimental ordinary muon capture (OMC) data depends on the nuclear model employed; more specifically, which are the relevant nuclear matrix elements leading to the reported quenching of the weak interaction coupling constants in the nuclear environment? Up to now this problem has not been studied systematically. We are aware of only two approaches [8,9] which do not give a definite and consistent answer (see also Ref. [10]). We show here that for the case of allowed nuclear transitions of physical interest a simple algebraic calculation is able to give the desired answer in a transparent, closed form. Two types of muon capture experiments particularly suited for the determination of the pseudoscalar coupling constant g_P will be examined. These are the measurements of the $\gamma\nu$ correlations in OMC [1,2] and the measurement of the ratio of the partial muon capture rates from the hyperfine states of mesic atoms [11,12].

II. CORRELATION EXPERIMENTS

The following derivation is based on a multipole analysis of the muon capture process fully described in [5,13,14]. The partial OMC for nuclear $0^+\rightarrow1^+$ transitions depends only on two independent nuclear amplitudes $M_1(-1)$ and $M_1(2)$ which are functions of the reduced matrix elements $[kwu]$, $[kwup]$ defined in the Appendix and of the various (*weak*) form factors

$$M_1(-1)=\sqrt{\frac{2}{3}}\Bigg\{-\left(G_A-\frac{1}{3}G_P\right)[101]+\frac{\sqrt{2}}{3}G_P[121]-g_A\frac{\hbar[011p]}{M_pc}+\sqrt{\frac{2}{3}}g_V\frac{\hbar[111p]}{M_pc}\Bigg\},$$

$$M_1(2)=\sqrt{\frac{2}{3}}\Bigg\{-\frac{\sqrt{2}}{3}G_P[101]+\left(G_A-\frac{2}{3}G_P\right)[121]+\sqrt{2}g_A\frac{\hbar[011p]}{M_pc}+\frac{1}{\sqrt{3}}g_V\frac{\hbar[111p]}{M_pc}\Bigg\}, \quad (2)$$

where u is the total angular momentum transferred. In [5,1] it has been shown that the $\gamma\nu$-correlation coefficients are functions of the ratio

$$x=\frac{M_1(2)}{M_1(-1)}. \quad (3)$$

We solve, for the ratio g_P/g_A,

$$M_1(2)-xM_1(-1)=0 \quad (4)$$

or

TABLE I. Nuclear matrix elements (m.e.) and their ratios for reaction (1) calculated for several nuclear models. The following numerical values [15] have been used: $\hbar c = 197.328$ MeV fm, $M_p c^2 = 938.279$ MeV, $m_\mu c^2 = 105.659$ MeV, $g_V = 1.0$, $g_A = -1.264$, and $g_M = 3.706$.

Nuclear model		[101]	[121]	[111p] in fm^{-1}	[011p] in fm^{-1}	g_P/g_A Eq. (7)	g_P/g_A Eq. (9)
[6]	m.e.	0.041	−0.005	0.012	−0.017		
	m.e./[101]	1.0	−0.120	0.297	−0.407	−3.0	−1.2
[6] [a]	m.e.	−0.041	0.005	−0.011	0.017		
	m.e./[101]	1.0	−0.128	0.272	−0.416	−3.4	−1.4
[17]	m.e.	−0.040	0.003	−0.000	0.018		
	m.e./[101]	1.0	−0.080	0.002	−0.439	−0.8	+0.2
[16]	m.e.	−0.053	0.007	−0.008	0.022		
	m.e./[101]	1.0	−0.139	0.144	−0.406	−3.5	−1.3
[18]	m.e.	−0.057	0.007	−0.007	0.025		
	m.e./[101]	1.0	−0.122	0.119	−0.432	−3.0	−1.1

[a] With mass dependence of single-particle energies as described in [16].

$$(x+\sqrt{2})([101]+\sqrt{2}[121])G_P = 3(x[101]+[121])G_A + 3(x+\sqrt{2})\frac{\hbar[011p]}{M_p c}g_A + \sqrt{3}(1-\sqrt{2}x)\frac{\hbar[111p]}{M_p c}g_V. \quad (5)$$

The definitions of G_A and G_P are given at the end of this paper. For the physically interesting nuclear $0^+\to1^+$ transitions with large partial OMC rate, [101] is the dominating matrix element. It differs from the Gamow-Teller matrix element studied in β decay only through its radial part. Isolating a term that does not depend on the nuclear matrix elements we can rewrite Eq. (5) in the following form:

$$\frac{g_P}{g_A} = 1+\frac{g_V}{g_A}+\frac{g_M}{g_A}+\frac{3x}{x+\sqrt{2}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right) + \left\{3\frac{1-\sqrt{2}x}{x+\sqrt{2}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)\frac{[121]}{[101]} + 6\frac{\hbar c}{E_\nu}\frac{[011p]}{[101]}+2\sqrt{3}\,\frac{1-\sqrt{2}x}{x+\sqrt{2}}\,\frac{\hbar c}{E_\nu}\,\frac{g_V}{g_A}\,\frac{[111p]}{[101]}\right\} \times\left(1+\sqrt{2}\,\frac{[121]}{[101]}\right)^{-1}. \quad (6)$$

This equation shows that the extracted ratio g_P/g_A has a weak dependence on g_A. It is, however, very sensitive to the ratios of the nuclear matrix elements [121]/[101], [111p]/[101], and [011p]/[101] (even in the case of the very fast partial OMC transitions where [101] dominates) since they are multiplied by large coefficients. As an example we consider the reaction (1) for which the experimental values are $x=0.25$ [1] and $E_\nu = 97.57$ MeV. Equation (6) becomes, in this case,

$$\frac{g_P}{g_A} = 7.6+\left\{26.8\frac{[121]}{[101]}+12.1\frac{[011p]}{[101]}-2.2\frac{[111p]}{[101]}\right\} \times\left(1+\sqrt{2}\,\frac{[121]}{[101]}\right)^{-1}. \quad (7)$$

Keeping only the first order in the small ratios [121]/[101], [011p]/[101], [111p]/[101], i.e., using

$$\left(1+\sqrt{2}\,\frac{[121]}{[101]}\right)^{-1} \approx 1-\sqrt{2}\,\frac{[121]}{[101]}$$

in Eq. (6) we obtain

$$\frac{g_P}{g_A} \approx 1+\frac{g_V}{g_A}+\frac{g_M}{g_A}+\frac{3x}{x+\sqrt{2}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right) + 6\frac{\hbar c}{E_\nu}\,\frac{[011p]}{[101]}+3\frac{1-\sqrt{2}x}{x+\sqrt{2}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)\frac{[121]}{[101]} + 2\sqrt{3}\,\frac{1-\sqrt{2}x}{x+\sqrt{2}}\,\frac{\hbar c}{E_\nu}\,\frac{g_V}{g_A}\,\frac{[111p]}{[101]}. \quad (8)$$

Inserting the numerical values for the above-mentioned case leads to

$$\frac{g_P}{g_A} \approx 7.6+26.8\frac{[121]}{[101]}+12.1\frac{[011p]}{[101]}-2.2\frac{[111p]}{[101]}. \quad (9)$$

This clearly shows that g_P/g_A depends sensitively on the ratios of very small nuclear matrix elements (usually considered to be unimportant) to the dominating one. Each term in Eqs. (7) and (9) contributes to g_P/g_A at the same order of magnitude as the leading constant term. The constant term

TABLE II. Nuclear matrix elements (m.e.) and their ratios for reaction (14) calculated for several nuclear models.

Nuclear model		[101]	[121]	[111p] (fm^{-1})	[011p] (fm^{-1})	[022]	[122]	[112p] (fm^{-1})	g_P/g_A Eq. (15)	g_P/g_A Eq. (22)
(POT) [21]	m.e.	−0.069	0.001	−0.003	0.024	−0.007	−0.010	−0.002		
	m.e./[101]	1.0	−0.014	0.038	−0.347	0.097	0.144	0.025	4.5	4.3
(I) [21]	m.e.	0.076	−0.001	0.004	−0.024	0.007	0.010	0.002		
	m.e./[101]	1.0	−0.010	0.050	−0.318	0.093	0.137	0.024	4.8	4.7
(II) [21]	m.e.	−0.072	0.001	−0.004	0.024	−0.007	−0.010	−0.002		
	m.e./[101]	1.0	−0.010	0.048	−0.337	0.096	0.139	0.025	4.6	4.5
PKUO [22]	m.e.	0.077	0.000	0.007	−0.025	0.007	0.010	0.002		
	m.e./[101]	1.0	0.002	0.088	−0.318	0.092	0.133	0.024	5.1	4.9
(P516T) [22]	m.e.	0.080	−0.001	0.003	−0.025	0.007	0.012	0.002		
	m.e./[101]	1.0	−0.013	0.037	−0.319	0.084	0.144	0.022	4.7	4.6
(P1016T) [22]	m.e.	0.086	−0.002	0.001	−0.025	0.007	0.012	0.002		
	m.e./[101]	1.0	−0.018	0.010	−0.294	0.080	0.136	0.021	4.8	4.7

7.6 in Eqs. (7) and (9), lying between the PCAC value of 6.5 and the value 9.2 obtained from RMC, is reduced by −8.8. The main part of this shift is related to the ratio [011p]/[101]. Inclusion of higher order terms shifts the extracted value of g_P/g_A roughly from −1 to −3. On the other hand, the partial OMC rate for fast allowed transitions is mainly determined by $g_A\cdot$[101] and has very little sensitivity to the other matrix elements. One can therefore conclude that reproducing the partial OMC rate does not guarantee a correct extraction of g_P/g_A from the experimental value of x. A discussion of the dependence of the total capture rates in light nuclei on the various matrix elements can be found in [10,14]. Table I shows the nuclear matrix elements calculated within a many-particle shell model using various Hamiltonians. The extracted ratios g_P/g_A according Eqs. (6) and (8), given in the last column, show a slight model dependence as do the ratios of the nuclear matrix elements themselves. The values of g_P/g_A within these models vary only between −3 and −3.5. The model of Ref. [17], on the other hand, demonstrates nicely the strong dependence of g_P/g_A on the ratios of the nuclear matrix elements. The values of the matrix elements and their ratios are quite different from those of the other authors. Equations (6) or (8) also explains the findings of Ref. [7]. If, as quoted in [7], a renormalization of the nuclear matrix elements reduces the magnitudes of the matrix elements (roughly by 10% for [101] and 30% for the other matrix elements), then the overall reduction of the ratios amounts to almost 20%, thus shifting the extracted value of g_P/g_A up.

In two recent papers [19] terms of order $v^2_{\rm nucl}/c^2$ of the effective OMC Hamiltonian have been investigated. As a result the effective form factors G_A and G_P entering the nuclear amplitudes $M_1(-1)$ and $M_1(2)$ are changed into

$$G_A\to G_A-\frac{1}{2}\left[g_A+(g_V+2g_M)\left(\frac{m_\mu c^2}{E_\nu}-1\right)\right]\left(\frac{E_\nu}{2M_pc^2}\right)^2,$$

$$G_P\to G_P-\frac{1}{2}(g_P-g_A-g_V-2g_M)\left(\frac{m_\mu c^2}{E_\nu}-1\right)\left(\frac{E_\nu}{2M_pc^2}\right)^2,$$

and additional velocity-dependent matrix elements of order $v^2_{\rm nucl}/c^2$ appear. For Eqs. (6) and (8) the resulting contributions of these second order terms have a common factor $E_\nu/2M_pc^2\leq 0.05$ and can therefore be neglected in the present context.

III. OMC FROM HYPERFINE STATES

If the target nucleus has nonzero spin I_i, the muon in the K orbit of the muonic atom may exist in two hyperfine states with spin $\mathcal{F}^+=I_i+1/2$ and $\mathcal{F}^-=I_i-1/2$. The ratio Λ^+/Λ^- of the partial OMC rates from these hyperfine states is known to be sensitive to g_P/g_A [5,14]. The general theoretical framework for the description of the partial $\Lambda^\pm$ rates has been described in [5,14] and in [20]. In [20] some improvements in the interference terms between different-order forbidden transitions have been performed as compared with Eqs. (2.151)–(2.152) of [5].

We start from the simplest case. The only nonvanishing nuclear matrix elements for a $1^+\to 0^+$ transition are those of the rank-1 tensor operators. In this case according to [5,14] we have

$$y=\frac{\Lambda^+}{\Lambda^-}=\frac{\frac{3}{2}M_1^2(2)}{3M_1^2(-1)} \tag{10}$$

or

$$M_1^2(2)-2yM_1^2(-1)=0,$$

$$[M_1(2)-\sqrt{2y}M_1(-1)][M_1(2)+\sqrt{2y}M_1(-1)]=0. \tag{11}$$

Replacing x by $\sqrt{2y}$ ($\sqrt{2y}$ positive) we can read off immediately from Eq. (8) the solutions to Eq. (11) (we show here only the linearized version):

$$\frac{g_P}{g_A}\approx 1+\frac{g_V}{g_A}+\frac{g_M}{g_A}\;\frac{+3\sqrt{y}}{1+\sqrt{y}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)+6\frac{\hbar c}{E_\nu}\frac{[011p]}{[101]}+\frac{3}{\sqrt{2}}\frac{1-2\sqrt{y}}{1+\sqrt{y}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)\times\frac{[121]}{[101]}+\sqrt{6}\frac{1-2\sqrt{y}}{1+\sqrt{y}}\frac{\hbar c}{E_\nu}\frac{g_V}{g_A}\frac{[111p]}{[101]} \tag{12}$$

and

$$\frac{g_P}{g_A}\approx 1+\frac{g_V}{g_A}+\frac{g_M}{g_A}-\frac{3\sqrt{y}}{1-\sqrt{y}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)+6\frac{\hbar c}{E_\nu}\frac{[011p]}{[101]}+\frac{3}{\sqrt{2}}\frac{1+2\sqrt{y}}{1-\sqrt{y}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)\times\frac{[121]}{[101]}+\sqrt{6}\frac{1+2\sqrt{y}}{1-\sqrt{y}}\frac{\hbar c}{E_\nu}\frac{g_V}{g_A}\frac{[111p]}{[101]}. \tag{13}$$

The second solution corresponds to negative $M_1(2)/M_1(-1)$ and leads to a very large negative ratio g_P/g_A. Therefore only the case with positive $M_1(2)/M_1(-1)$ should be considered.

In the following we treat the case of two experiments in which the ratio of OMC rates from hyperfine states have been measured. In [12] the reaction

$$\mu^-+{}^{11}\mathrm{B}\left(\frac{3^-}{2},\mathrm{g.s.}\right)\rightarrow\nu_\mu+{}^{11}\mathrm{Be}\left(\frac{1^-}{2},320\right) \tag{14}$$

has been studied and a value of $\Lambda^+/\Lambda^-=0.028\pm0.021(\mathrm{stat.})\pm0.003(\mathrm{syst.})$ was obtained. In [11] the hyperfine dependence of partial OMC rates in ${}^{23}\mathrm{Na}(\frac{3}{2}^+,\mathrm{g.s.})$ has been investigated for several allowed partial nuclear transitions. The most interesting partial transitions in OMC on ^{23}Na are those having the same angular momentum and parity selection rules as the transitions in ^{11}B. Therefore we concentrate in the following on the transitions $(3/2)^\pm\rightarrow(1/2)^\pm$. In these transitions the only nonvanishing nuclear matrix elements are those of tensor operators of rank 1 and 2. From [5,14] we find

$$y=\frac{\Lambda^+}{\Lambda^-}=\frac{\frac{1}{10}[\sqrt{15}M_1(2)-M_2(2)]^2+\frac{8}{3}M_2^2(-3)}{\frac{8}{3}M_1^2(-1)+\frac{1}{6}[M_1(2)+\sqrt{15}M_2(2)]^2}. \tag{15}$$

We proceed as before and keep only ratios of small matrix elements to first order. Therefore we omit in Eq. (15) the squares of rank-2 amplitudes $M_2(2)$ and $M_2(-3)$. $M_2(2)$ is given by [5,14]

$$M_2(2)=\sqrt{\frac{2}{5}}\left\{\sqrt{2}G_V[022]-G_A[122]-\sqrt{\frac{5}{3}}g_V\frac{\hbar[112p]}{M_pc}\right\}. \tag{16}$$

If we introduce the notation $t=M_1(2)/M_1(-1)$ and $s=M_2(2)/M_1(-1)$, we obtain, from Eq. (15),

$$9t^2-6\sqrt{\frac{3}{5}}st-y(16+t^2+2\sqrt{15}st)=0,$$

which has the positive solution

$$t=\sqrt{15}\left(\frac{\frac{3}{5}+y}{9-y}\right)s+\sqrt{16\frac{y}{9-y}+15\left(\frac{\frac{3}{5}+y}{9-y}\right)^2s^2}. \tag{17}$$

After dropping the term proportional to s^2 under the square root in Eq. (17) we obtain the equation

$$M_1(2)=\sqrt{\frac{16y}{9-y}}M_1(-1)+\sqrt{\frac{3}{5}}\frac{3+5y}{9-y}M_2(2). \tag{18}$$

Using the definition of the amplitudes (2) and (16) we get

$$\left(1+\sqrt{2}\frac{[121]}{[101]}\right)G_P=3\frac{\sqrt{8y}}{\sqrt{9-y}+\sqrt{8y}}G_A+3\frac{\sqrt{9-y}}{\sqrt{2(9-y)}+\sqrt{16y}}\frac{[121]}{[101]}G_A+3\frac{\hbar[011p]}{M_pc[101]}g_A+\sqrt{3}\frac{\sqrt{9-y}-\sqrt{32y}}{\sqrt{2(y-9)}+\sqrt{16y}}\frac{\hbar[111p]}{M_pc[101]}g_V-\frac{9}{5}\frac{3+5y}{9-y}\frac{\sqrt{9-y}}{\sqrt{9-y}+\sqrt{8y}}\frac{[022]}{[101]}G_V+\frac{9}{5}\frac{3+5y}{9-y}\frac{\sqrt{9-y}}{\sqrt{2(9-y)}+\sqrt{16y}}\frac{[122]}{[101]}G_A+3\sqrt{\frac{3}{5}}\frac{3+5y}{9-y}\frac{\sqrt{9-y}}{\sqrt{2(9-y)}+\sqrt{16y}}\frac{\hbar[112p]}{M_pc[101]}g_V.$$

With the abbreviation $a=\sqrt{16y/(9-y)}$ and $b=\sqrt{\frac{3}{5}}(3+5y)/(9-y)$ we obtain

$$\frac{g_P}{g_A}\approx 1+\frac{g_V}{g_A}+\frac{g_M}{g_A}+\frac{3a}{a+\sqrt{2}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)+\Bigg\{6\frac{\hbar c}{E_\nu}\frac{[011p]}{[101]}+3\frac{1-a\sqrt{2}}{\sqrt{2}+a}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)\frac{[121]}{[101]}$$
$$+2\sqrt{3}\,\frac{1-a\sqrt{2}}{\sqrt{2}+a}\frac{\hbar c}{E_\nu}\frac{g_V}{g_A}\frac{[111p]}{[101]}-\sqrt{\frac{54}{5}}\frac{b}{a+\sqrt{2}}\left(\frac{2M_pc^2}{E_\nu}+1\right)\frac{g_V}{g_A}\frac{[022]}{[101]}+\sqrt{\frac{27}{5}}\frac{b}{a+\sqrt{2}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)\frac{[122]}{[101]}$$
$$+\frac{6b}{a+\sqrt{2}}\frac{\hbar c}{E_\nu}\frac{g_V}{g_A}\frac{[112p]}{[101]}\Bigg\}\left(1+\sqrt{2}\,\frac{[121]}{[101]}\right)^{-1}. \tag{19}$$

Linearization as in the previous sections leads to

$$\frac{g_P}{g_A}\approx 1+\frac{g_V}{g_A}+\frac{g_M}{g_A}+\frac{3a}{a+\sqrt{2}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)+6\frac{\hbar c}{E_\nu}\frac{[011p]}{[101]}+3\frac{1-a\sqrt{2}}{\sqrt{2}+a}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)\frac{[121]}{[101]}$$
$$+2\sqrt{3}\,\frac{1-a\sqrt{2}}{\sqrt{2}+a}\frac{\hbar c}{E_\nu}\frac{g_V}{g_A}\frac{[111p]}{[101]}-\sqrt{\frac{54}{5}}\frac{b}{a+\sqrt{2}}\left(\frac{2M_pc^2}{E_\nu}+1\right)\frac{g_V}{g_A}\frac{[022]}{[101]}+\sqrt{\frac{27}{5}}\frac{b}{a+\sqrt{2}}\left(\frac{2M_pc^2}{E_\nu}-\frac{g_V}{g_A}-\frac{g_M}{g_A}\right)\frac{[122]}{[101]}$$
$$+\frac{6b}{a+\sqrt{2}}\frac{\hbar c}{E_\nu}\frac{g_V}{g_A}\frac{[112p]}{[101]}. \tag{20}$$

Taking the values of Ref. [12] for y and E_ν obtained from OMC on ^{11}B ($y=0.028$, $E_\nu=92.83$ MeV) we get

$$\frac{g_P}{g_A}\approx 7.1+\left\{30.0\frac{[121]}{[101]}+12.8\frac{[011p]}{[101]}-2.4\frac{[111p]}{[101]}+9.1\frac{[022]}{[101]}+9.2\frac{[122p]}{[101]}-1.7\frac{[112p]}{[101]}\right\}\left(1+\sqrt{2}\,\frac{[121]}{[101]}\right)^{-1} \tag{21}$$

and

$$\frac{g_P}{g_A}\approx 7.1+30.0\frac{[121]}{[101]}+12.8\frac{[011p]}{[101]}-2.4\frac{[111p]}{[101]}+9.1\frac{[022]}{[101]}+9.2\frac{[122p]}{[101]}-1.7\frac{[112p]}{[101]} \tag{22}$$

in linear approximation. Table II gives the extracted values of g_P/g_A for various nuclear Hamiltonians. As can be seen the matrix elements as well as the relevant ratios calulated with different effective interactions are rather similar. This is reflected in the almost model-independent values for g_P/g_A (4.3–4.9). Equations (20) and (19) lead to almost identical results. The exact solution of Eq. (15) leads to values of g_P/g_A only roughly (2–4 %) above those of Eq. (22).

IV. CONCLUSIONS

We showed in this paper that the generally held opinion that the velocity-dependent terms in OMC observables are of little importance is not true for the ratios of observables considered. These terms influence strongly the result of g_P/g_A if extracted from ratios of OMC observables. According to Eqs. (7) and (21), $[011p]$ has the biggest impact on the ratio g_P/g_A obtained from the experiments [1,2]. The nuclear matrix elements used in this work and in Refs. [3,7] have been evaluated within a many-particle shell model using various effective interactions. The many-body wave functions $|J_{i,f}\rangle$ and the one-body transition densities have been calculated with the computer program OXBASH [23] with single-particle harmonic oscillator wave functions. This choice may overestimate the radial integrals of the gradient operator in the velocity-dependent terms. Further theoretical efforts are desirable to further improve the nuclear structure part for the muon capture process.

APPENDIX

The nuclear matrix elements used in the calculations of nuclear OMC are defined by [5,13,14]

$$[0uu]=\langle J_f\|\sqrt{1/(4\pi)}\sum_{k=1}^{A}\phi_\mu(r_k)\tau_k^- j_u(E_\nu r_k)\times Y_u(\hat{r}_k)\|J_i\rangle/\sqrt{2J_f+1},$$

$$[1wu]=\langle J_f\|\sqrt{3/(4\pi)}\sum_{k=1}^{A}\phi_\mu(r_k)\tau_k^- j_w(E_\nu r_k)\times[\sigma,Y_w(\hat{r}_k)]_u\|J_i\rangle/\sqrt{2J_f+1},$$

$$[1wup]=\langle J_f \| \sqrt{3/(4\pi)} \sum_{k=1}^{A} \phi_\mu(r_k)\tau_k^- j_w(E_\nu r_k) \times [Y_w(\hat{r}_k),\nabla_k]_u \| J_i\rangle / \sqrt{2J_f+1},$$

$$[0uup]=\langle J_f \| \sqrt{1/(4\pi)} \sum_{k=1}^{A} \phi_\mu(r_k)\tau_k^- j_u(E_\nu r_k) \times Y_u(\hat{r}_k)(\vec{\nabla}_k,\vec{\sigma}_k) \| J_i\rangle / \sqrt{2J_f+1}. \tag{A1}$$

Here τ^- is defined through $\tau^-|p\rangle=|n\rangle$. The effective form factors appearing in the amplitudes $M_u(\kappa)$ are given by the following combinations of the weak hadronic form factors:

$$G_V=g_V(q^2)\left(1+\frac{E_\nu}{2M_pc^2}\right),$$

$$G_A=g_A(q^2)-[g_V(q^2)+g_M(q^2)]\frac{E_\nu}{2M_pc^2},$$

$$G_P=[g_P(q^2)-g_A(q^2)-g_V(q^2)-g_M(q^2)]\frac{E_\nu}{2M_pc^2}. \tag{A2}$$

As is usually done for light nuclei we have approximated the muon wave function $\phi_\mu(r)$ by the square root of the muon density averaged over the nuclear volume.

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