Equation of state of hot polarized nuclear matter using the generalized Skyrme interaction

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We used the generalized Skyrme potential to study the equation of state of polarized nuclear matter in the frame of the Thomas-Fermi model. The critical temperature of the liquid-gas phase transition is found to be $T_c = 16.2$ MeV. This critical temperature was found to decease with the asymmetry, spin, and spin-isospin excess parameters. The isothermal compressibility of polarized nuclear matter was also studied. The volume compressibility K_v was found to decrease with temperature. The symmetry compressibility K_x, the spin symmetry compressibility K_y, and the spin-isospin symmetry compressibility K_z were found to have a little increasing behavior with temperature.

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I. INTRODUCTION

In the last few years, the study of the equation of state (EOS) of nuclear matter has attracted considerable interest in nuclear physics and astrophysics [1–3]. The EOS is closely related to the study of nuclear fission, heavy ion collisions, supernovas, and hot neutron stars. In the temperatures and density domains of the liquid-gas phase transition occurring in nuclear matter, it is desirable to derive the nuclear matter EOS from the nucleon-nucleon interaction. The calculated critical temperature T_c of this phase transition using various kinds of nucleon-nucleon interactions is about 17 ± 3 MeV [4,5]. Most of the calculations of hot nuclear matter have considered the symmetric case $(N=Z)$, and only a few considered the asymmetric case $(N \neq Z)$. For the asymmetric case, T_c was found to decrease with the asymmetry parameter.

It is well known that nuclear fluid is compressible and the magnitude of its compressibility coefficient K plays an important role in the determination of the nuclear EOS. In principle, the value of K can be extracted from the experimental energies of the giant monopole resonance (GMR) in nuclei. In a semiempirical approach the analysis is based upon a leptodermous expansion of compressibility [5]. The used expansion formula does not contain any spin contribution coefficients. The spin coefficients may be related to the energies of the GMR [6].

It is well known that the Skyrme interaction is a simple and useful potential for describing the thermostatic properties of nuclear matter [7]. Recently, Farine *et al.* [8] proposed a generalized Skyrme force that took into account the masses of finite nuclei, breathing mode energies, and masses of neutron-rich nuclei.

In the present work we will extend the generalized Skyrme interaction to the case of polarized nuclear matter in the T^4 approximation. The dependence of the critical temperature T_c on the asymmetry, spin, and spin-isospin excess parameters is studied. The isothermal compressibility of polarized nuclear matter is also studied. In Sec. II, we give the theory of our calculations. Results are analyzed in Sec. III.

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II. THEORY AND CALCULATIONS

Polarized nuclear matter (PNM) is composed of $N_\uparrow(N_\downarrow)$ numbers of spin-up (spin-down) neutrons and $Z_\uparrow(Z_\downarrow)$ numbers of spin-up (spin-down) protons with corresponding densities $\rho_{n\uparrow}$, $\rho_{n\downarrow}$, $\rho_{p\uparrow}$, and $\rho_{p\downarrow}$, respectively. Thus

$$\rho = \sum_{\tau,s} \rho_{\tau,s}, \tag{1}$$

$$\rho = \rho_{n\uparrow} + \rho_{n\downarrow} + \rho_{p\uparrow} + \rho_{p\downarrow}. \tag{2}$$

For PNM, one usually defines the following parameters [6,9]: the neutron excess parameter $X = (\rho_n - \rho_p)/\rho$, the neutron spin-up excess parameter $\alpha_n = (\rho_{n\uparrow} - \rho_{n\downarrow})/\rho$, and the proton spin-up excess parameter $\alpha_p = (\rho_{p\uparrow} - \rho_{p\downarrow})/\rho$.

There are four Fermi momenta in this general case, namely, $K_n(\lambda_n)$ for neutrons with spin up (spin down) and $K_p(\lambda_p)$ for protons with spin up (spin down). The relations relating these Fermi momenta to the excess parameters are

$$\left.\begin{matrix} K_n^3 \\ \lambda_n^3 \end{matrix}\right\} = K_f^3(1 + X \pm Y \pm Z) \tag{3}$$

and

$$\left.\begin{matrix} K_p^3 \\ \lambda_p^3 \end{matrix}\right\} = K_f^3(1 - X \pm Y \mp Z), \tag{4}$$

where K_f is the Fermi momentum of unpolarized symmetric nuclear matter,

$$Y = \alpha_n + \alpha_p \quad \text{and} \quad Z = \alpha_n - \alpha_p. \tag{5}$$

In our model calculations, we will use the recent form of the generalized Skyrme interaction [8]. This interaction is given in configuration space by

$$V_{ij}=t_0(1+x_0P_\sigma)\delta(r_{ij})+\frac{1}{2}t_1(1+x_1P_\sigma)[K_{ij}^2\delta(r_{ij})+\delta(r_{ij})K_{ij}'^2]+t_2(1+x_2p_\sigma)K_{ij}\delta(r_{ij})K_{ij}'$$
$$+\frac{1}{6}t_{3a}(1+x_{3a}p_\sigma)[\rho_{q_i}(r_i)+\rho_{q_j}(r_j)]^{\alpha_a}\delta(r_{ij})+\frac{1}{6}t_{3b}(1+x_{3b}p_\sigma)[\rho_{q_i}(r_i)+\rho_{q_j}(r_j)]^{\alpha_b}\delta(r_{ij})$$
$$+\frac{1}{2}t_4(1+x_4p_\sigma)(K_{ij}^2[\rho_{q_i}(r_i)+\rho_{q_j}(r_j)]^\beta\delta(r_{ij})+\delta(r_{ij})\{K_{ij}'[\rho_{q_i}(r_i)+\rho_{q_j}(r_j)]^\beta\})+iw_0K_{ij}\delta(r_{ij})K_{ij}'(\sigma_i+\sigma_j). \quad (6)$$

The energy per nucleon of PNM at zero temperature can be written as

$$E_0=E_v+X^2E_x+Y^2E_y+Z^2E_z, \quad (7)$$

where

$$E_v=\frac{3}{8}t_0\rho+\frac{3}{80}\left(\frac{3\pi^2}{2}\right)^{2/3}[3t_1+t_2(5+4x_2)]\rho^{5/3}+\frac{1}{16}(t_{3a}\rho^{\alpha_a+1}+t_{3b}\rho^{\alpha_b+1})+\frac{9}{80}t_4\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{\beta+5/3}, \quad (8)$$

$$E_x=-\frac{t_0}{8}(1+2x_0)\rho+\frac{1}{24}\left(\frac{3\pi^2}{2}\right)^{2/3}[t_2^2(4+5x_2)-3t_1x_1]\rho^{5/3}+\frac{1}{96}t_{3a}[(\alpha_a+1)(\alpha_a+2)(1-x_{3a})+2(2+x_{3a})]\rho^{\alpha_a+1}$$
$$+\frac{1}{96}t_{3b}[(\alpha_b+1)(\alpha_b+2)(1-x_{3b})+2(2+x_{3b})]\rho^{\alpha_b+1}+\frac{1}{8}t_4\left(\frac{3\pi^2}{2}\right)^{2/3}\left(-x_4+\frac{1}{20}\beta(3\beta+13)(1-x_4)\right)\rho^{\beta+5/3}, \quad (9)$$

$$E_y=(2x_0-1)\rho+\frac{1}{24}\left(\frac{3\pi^2}{2}\right)^{2/3}[t_2(4+5x_2)+3t_1x_1]\rho^{5/3}+\frac{1}{96}t_{3a}[(\alpha_a+1)(\alpha_a+2)(1-x_{3a})-2(2-x_{3a})]\rho^{\alpha_a+1}$$
$$+\frac{1}{96}t_{3b}[(\alpha_b+1)(\alpha_b+2)(1-x_{3b})-2(2-x_{3b})]\rho^{\alpha_b+1}+\frac{1}{8}t_4\left(\frac{3\pi^2}{2}\right)^{2/3}\left(x_4+\frac{1}{20}\beta(3\beta+13)(1+x_4)\right)\rho^{\beta+5/3}, \quad (10)$$

and

$$E_z=-\frac{t_0}{8}\rho+\frac{1}{12}\left(\frac{3\pi^2}{2}\right)^{2/3}t_2(2+x_2)\rho^{5/3}+\frac{1}{96}t_{3a}[(\alpha_a+1)(\alpha_a+2)-4]\rho^{\alpha_a+1}$$
$$+\frac{1}{96}t_{3b}[(\alpha_b+1)(\alpha_b+2)-4]\rho^{\alpha_b+1}+\frac{1}{160}t_4\left(\frac{3\pi^2}{2}\right)^{2/3}\beta(3\beta+13)\rho^{\beta+5/3}. \quad (11)$$

In Eq. (7) terms higher than quadratic in X, Y, and Z are neglected.

The pressure of a nuclear system is given by

$$P=\rho^2\frac{\partial E}{\partial\rho}, \quad (12)$$

and the (isothermal) compressibility is

$$K=9\rho^2\left(\frac{\partial^2E}{\partial\rho^2}\right)_T. \quad (13)$$

Using Eq. (7), we get the pressure and compressibility at zero temperature as

$$P_0=P_v+X^2P_x+Y^2P_y+Z^2P_z \quad (14)$$

and

$$K_0=K_v+X^2K_x+Y^2K_y+Z^2K_z. \quad (15)$$

Using the simple analytical equations (8)–(11) for E_v, E_x, E_v, and E_z, we were able to get simple analytical equations for $P_v(K_v)$, $P_x(K_x)$, $P_y(K_y)$, and $P_z(K_z)$.

It is well known from classical thermodynamics that the thermal properties of the system are completely determined if the free energy $F(\rho,T)$ per nucleon is determined:

$$F(\rho,T)=E(\rho,T)-TS(\rho,T), \quad (16)$$

where $E(\rho,T)$ is the energy per nucleon at temperature T and S is the entropy of the system. The energy per nucleon is given by [4]

$$E(\rho,T)=E_0(\rho,T=0)+\frac{1}{\rho_0}\int_0^T dT'\,C_v(T'), \quad (17)$$

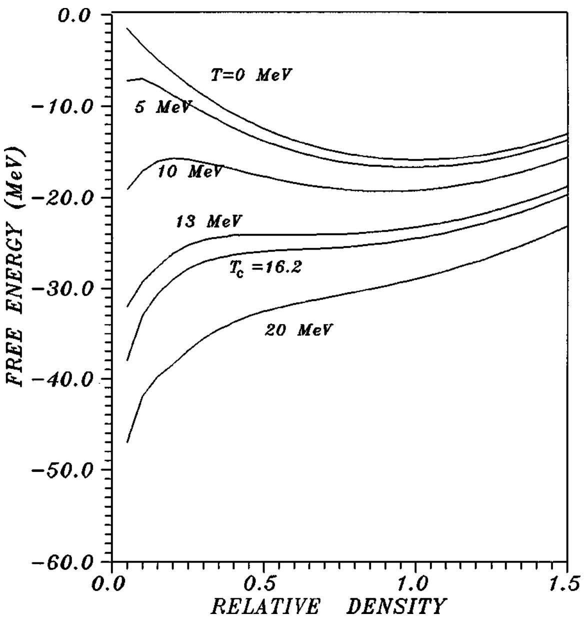

FIG. 1. The free energy as a function of the relative density ρ/ρ_0 for unpolarized symmetric nuclear matter.

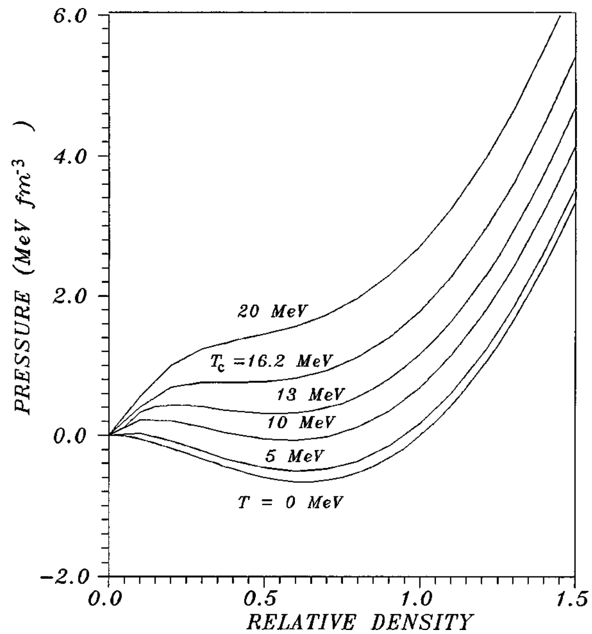

FIG. 2. The isothermal pressure as a function of the relative density ρ/ρ_0 for unpolarized symmetric nuclear matter.

where $E_0(\rho,T=0)$ is the energy per nucleon at zero temperature and is given by Eq. (7) and C_v is the specific heat per unit volume and is given by

$$C_v=\rho T\left(\frac{\partial S}{\partial T}\right)_\rho . \tag{18}$$

Using the Fermi-liquid approximation of Landau, the entropy of the system is calculated in terms of the Fermi integrals, which are temperatures dependent. Expanding the Fermi integrals in temperatures up to T^4 and following Ref. [10], we get the entropy of PNM as

$$S(\rho,T)=\frac{T}{3}\left(\frac{3\pi^2}{2}\right)^{1/2}\left(\frac{2m^*T}{\eta^2}\right)\rho^{-2/3}\left\{1-\frac{1}{9}(X^2+Y^2+Z^2)\right\} -\frac{7}{1080}\left(\frac{2m^*T}{\eta^2}\right)^3\rho^{-2}(1+X^2+Y^2+Z^2), \tag{19}$$

where m^* is the nucleon effective mass and is given by [8]

$$\frac{\hbar^2}{2m^*_{\tau,s}}=\frac{\hbar^2}{2m_{\tau,s}}+\frac{1}{8}\{t_1(2+x_1)+t_2(2+x_2)\}\rho +\frac{1}{8}\{t_2(1+2x_2)-t_1(1+2x_1)\}\rho_{\tau,s} +\frac{1}{8}t_4\left\{(2+x_4)\rho^\beta(\rho-\rho_{\tau,s})+\frac{1}{2}(1-x_4)\times(2\rho_{\tau,s})^{\beta+1}\right\}. \tag{20}$$

It is clear from Eqs. (17)–(19) that the key quantity of the thermal properties of the system is the entropy. The only dependence on the effective interaction is through the dependence of the entropy on the effective mass. This conclusion was also pointed out in Ref. [10].

Using Eqs. (17)–(19), we can write

$$F=E_0+E_T, \tag{21}$$

where E_0 is the zero-temperature part of the energy [and is given by Eq. (7)] and E_T is the temperature-dependent part of the energy and is given by

$$E_T(\rho,T)=-\frac{T^2}{6}\left(\frac{3\pi^2}{2}\right)^{1/3}\rho^{-2/3}\left(\frac{2m^*}{\hbar^2}\right)\times\left\{1-\frac{1}{9}(X^2+Y^2+Z^2)\right\}+\frac{7T^4}{4320}\left(\frac{2m^*}{\hbar^2}\right)^3\rho^{-2}\times(1+X^2+Y^2+Z^2). \tag{22}$$

The pressure and compressibility at finite temperature can be written as

$$P(\rho,T)=P_0(\rho,T=0)+P_T(\rho,T) \tag{23}$$

and

$$K(\rho,T)=K_0(\rho,T=0)+K_T(\rho,T), \tag{24}$$

where P_T and K_T are the temperature-dependent parts of the pressure and compressibility. They are directly derived from Eq. (22).

III. RESULTS AND DISCUSSION

With the model described above, we now survey the thermostatic properties of PNM. We wish to emphasize that the motivation of the present work is not to reconsider symmet-

TABLE I. The values of the asymmetry spin-symmetry and spin-isospin symmetry energies and compressibility.

	SK200	SK220	SK240	SKKM
K_f (fm^{-3})	1.32	1.315	1.31	1.335
F_v	−15.86	−15.81	−15.79	−15.86
F_x	30	30	30	30
F_y	6.8	17.53	34.9	25.8
F_z	33.8	34.18	34.8	31.2
K_v	200	220	240	220
K_x	−353	−466	−523	−242
K_y	−889	−87	−296	−128
K_z	−411	−154	−90	−97

ric unpolarized nuclear matter, but to study the effect of spin on asymmetric nuclear matter. We carried out our calculations for the parameter sets of interactions SK200, SK220, SK240, and SKKM (see Ref. [8] for the parameters of these sets). Our results of the calculations are nearly the same for these sets. We shall show our results for SK220 only and point out any differences between the other sets.

In Fig. 1 we show the free energy F as a function of the relative density ρ/ρ_0 at different temperatures for unpolarized symmetric nuclear matter. It should be noted that at $T=0$ MeV our result was the usual saturation curve for the energy with a minimum at $\rho/\rho_0=1$. As the temperature increases, the free energy decreases and an inflection point on the curve occurs. This behavior reflects a liquid-gas phase transition that ceases at a critical temperature. Above this critical temperature the free energy isotherms are monotonically increased and only one phase of nuclear matter can exist. The pressure isotherms confirm this liquid-gas phase

TABLE II. The critical temperature at different values of X, Y, and Z.

X, Y, or Z	$T_c(X)$	$T_c(Y)$	$T_c(Z)$
0.1	15.9	15.7	15.3
0.2	15.2	15.5	15
0.3	14.5	15	14.2
0.4	13.3		
0.5	11.6		

transition as shown in Fig. 2. The critical temperature is found from the pressure isotherms, the temperature at which the maximum and minimum in the pressure curve coalesce into a turning point. The critical temperature T_c for SK200, SK220, and SK240 is found to be $T_c=16.2$ MeV. This result could be explained by the fact that the parameter sets of these interactions fit the same properties of finite nuclei, except for the compressibility K_v of symmetric unpolarized NM. In Table I we list the values of E_x, E_y, and E_z using SK200, SK220, and SK240. The values of the spin symmetry energy E_y and the spin-isospin symmetry energy E_z are uncertain to some extent. An empirical estimate of E_x, E_y, and E_z may be obtained with the help of the empirical values of the Landau parameters [11] lead for $K_f=1.36$ fm^{-1} to $E_x=27$ MeV, $E_y=34.5$ MeV, and $E_z=39$ MeV. Dabrowski [6] calculated E_x, E_y, and E_z using the K-matrix theory with the Brueckner-Gammel-Thelar and soft core potentials, and obtained $E_x=26.5$ MeV, $E_y=32.5$ MeV, and $E_z=38$ MeV. The generalized hydrodynamics model of Uberall [12], which gives $(E_z/E_x)^{1/2}\simeq 1.1$, fixes E_z for a given value of E_x. Maheswari *et al.* [13], using the Seyler-Blanachard potential, performed calculations with different

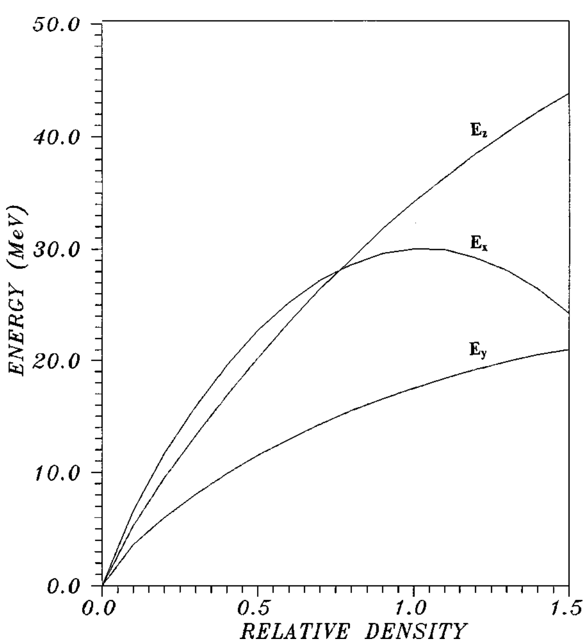

FIG. 3. The symmetry energy E_x, the spin symmetry energy E_y, and the spin-isospin symmetry energy E_z as a function of the relative density ρ/ρ_0.

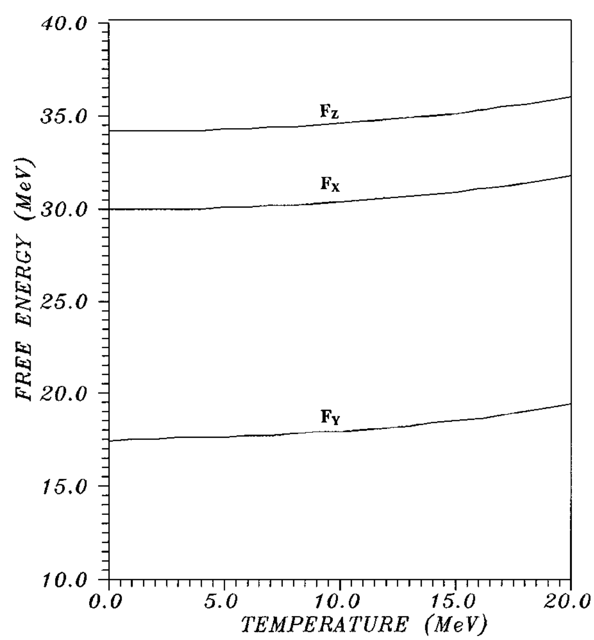

FIG. 4. The symmetry free energy F_x, the spin symmetry free energy F_y, and the spin-isospin free energy F_z as a function of the temperature T.

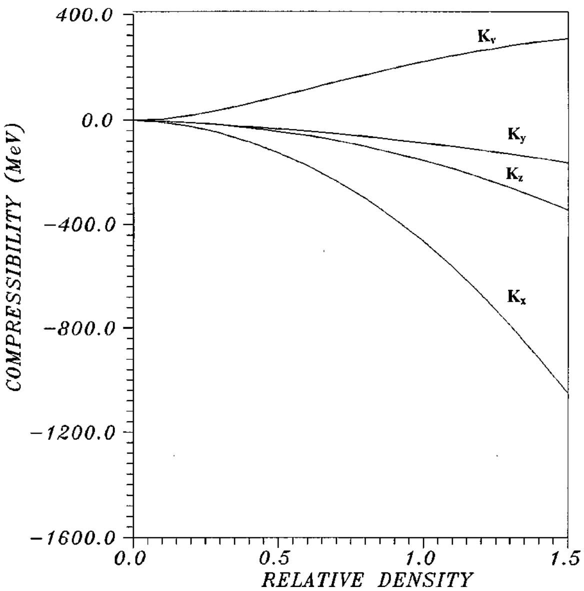

FIG. 5. The symmetry incompressibility K_x, the spin symmetry incompressibility K_y, and the spin-isospin incompressibility K_z as a function of the relative density ρ/ρ_0.

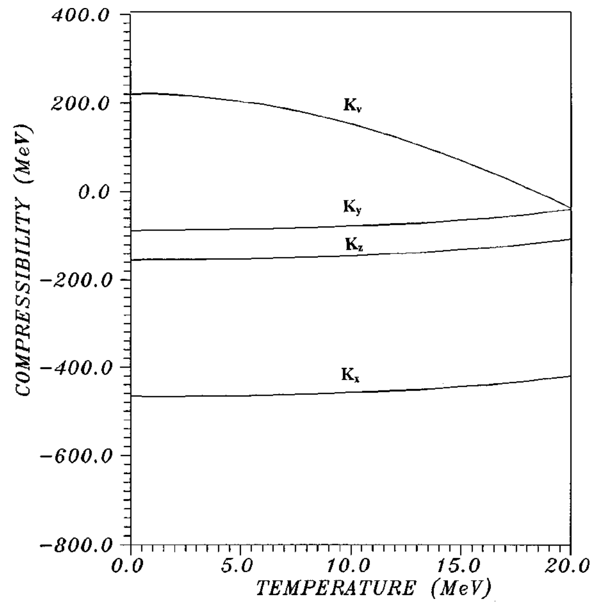

FIG. 6. The same quantities as in Fig. 5, but as a function of the temperature T.

values of E_y and found that the observed maximum neutron star masses and surface magnetic field are best explained with $E_x = 33.4$ MeV, $E_y = 15$ MeV, and $E_z = 36.5$ MeV. These values are in reasonable agreement with our values. This may be due to fitting the generalized Skyrme with the masses of neutron-rich nuclei. The energies E_x, E_y, and E_z as a function of the relative density ρ/ρ_0 are shown in Fig. 3. An increasing behavior of E_y and E_z (with density) is noticed. The symmetry energy E_x has a maximum value at $\rho \cong \rho_0$.

The critical temperature decreases when increasing the asymmetry parameters X, Y, and Z as shown in Table II. There is a small increasing behavior for F_x, F_y, and F_z with increasing temperatures as shown in Fig. 4.

The symmetry, spin symmetry, and the spin-isospin symmetry compressibility (K_x, K_y, and K_z) are shown in Fig. 5 as a function of the relative density ρ/ρ_0. One way to determine the compressibility of nuclear matter from the giant monopole resonance data is to use a leptodermous expansion of compressibility [5,14]. The leptodermous expansion used does not contain K_y and K_z. Their values may affect the GMR data [6]. The values of K_x, K_y, and K_z obtained in our calculations are presented in Table I.

The effect of temperature on the compressibility terms is shown in Fig. 6. The temperature has a little increasing effect on K_x, K_y, and K_z. The volume compressibility K_v was found to decrease with increasing temperatures. Many authors have obtained a similar behavior for K_v: in Refs. [15,16] using the SKM* force, in Refs. [17,18] using the Gogny D1 interaction, in Ref. [19] using the Brink-Poeker interaction, in Refs. [20,21] using the Seyler-Blanchard interaction, and in Ref. [22] using the Paris interaction.

In summary, we have studied the EOS of PNM, focusing attention on the critical temperature of the liquid-gas phase transition and compressibility. The critical temperature was found to decrease with X, Y, and Z. We have calculated the spin symmetry energy E_y and the spin-isospin symmetry energy E_z and compared our results with the available data. We have also calculated the spin symmetry compressibility and the spin-isospin symmetry compressibility. The effect of temperature was also studied. The temperature has a minor increasing effect on E_x, E_y, E_z, K_x, K_y, and K_z. The volume compressibility K_v was found to decrease with increasing temperature.

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