Meson-exchange currents in an extended random phase approximation theory applied to quasielastic electron scattering

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We investigate the role of meson exchange currents within an extended random phase approximation framework for inclusive quasielastic electron scattering on nuclei. The scheme is fully antisymmetric. Interference terms between one-body and two-body external excitation operators are considered and discussed. We calculate the transverse response function for nonrelativistic nuclear matter at several momentum transfers. The results show that the inclusion of these interference terms is relevant only in the dip region, while they are of minor importance for the quasielastic peak.

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I. INTRODUCTION

Since the experimental separation of the electromagnetic longitudinal and transverse response function [1–4], the simultaneous description of both responses in medium and heavy nuclei remains as an open problem. The experimental longitudinal response is overestimated and the transverse one is underestimated.

A large body of theoretical work has been done in order to solve this puzzle, leading to the conclusion that the quasielastic excitation is not as simple as it has been thought to be at first sight. This means that for a proper description of the quasielastic dynamics, one has to take into account subtle nuclear effects such as ground and final state interactions (GSI's and FSI's, respectively) [5–12], meson exchange currents (MEC's) [13–15], the Δ-isobar excitation [16–19], etc. Since the treatment of these effects is quite involved, there has been the tendency to simplify the basic nuclear model. For this reason, many works in this field have been done with models describing the nucleus as an infinite system where, due to translational invariance, the single-particle wave functions are plane waves. This allows one to perform analytically part of the calculation, which simplifies the numerics.

Before going on, let us distinguish several energy regions. The quasielastic peak region is associated with direct ejection of nucleons from the nucleus. For lower energies, the spectrum for the excitation of discrete nuclear bound states and giant resonances dominates. At higher energies there is an other peak due to the direct excitation of the $\Delta(1232)$. The dip region is between the quasielastic peak and the Δ.

Our main concern in this paper is the quasielastic peak. As mentioned above there is at present no work which is able to account for both the longitudinal and transverse responses at any momentum transfer. The problem is twofold. First, one has to choose a theory which can select and add a certain set of diagrams which contribute to the response. Second, the residual interaction should be fixed. In reference to the first point, let us mention some of the approaches used in the literature. In Ref. [6] the model for the correlated basis function is developed. The boson loop expansion is presented in Refs. [12,16]. Finally, the extended random phase approximation (ERPA) is shown in Ref. [7]. This list does not pretend to be complete. The three different approaches account both for GSI's and FSI's and should converge to the same result. Obviously, the way the diagrams are added differs from one model to the other. Also a huge amount of diagrams should be evaluated in each scheme.

The problem of the residual interaction requires more attention than was given to it in the past. In fact, the proper residual interaction for this energy-momentum dominion remains unknown. A residual interaction fixed through any low-energy process does not necessarily hold in this region. Among recent works, Ref. [16] attempts to adjust the interaction through the quasielastic response itself.

In recent works (see Refs. [9,10]), we have developed a projected ERPA theory limited to one-body external excitation operators. We extend now our previous ERPA scheme to two-body operators in order to include MEC's. The aim of the present work is to present the scheme and to analyze the magnitude of the new terms from two-body operators. We will not attempt to adjust the interaction. Because of this and because MEC's essentially contribute to the transverse channel, the longitudinal one will be not considered.

The paper is organized as follows. In Sec. II we develop the formalism in which for completeness the projected ERPA scheme is reviewed. In Sec. III we analyze the results for nuclear matter at different momentum transfers and finally in Sec. IV we present some conclusions.

II. FORMALISM

In Refs. [9] and [10], we have developed a formalism which accounts for correlations of the extended random phase approximation type for one-body (OB) external excitation operators. Now we will extend the scheme to include two-body (TB) operators.

The response function per unit volume for inclusive quasielastic electron scattering is given by

$$R(\boldsymbol{q},\hbar\omega) = -\frac{1}{\pi\Omega}\,\mathrm{Im}\langle 0|\mathcal{O}^{\dagger}G(\hbar\omega)\mathcal{O}|0\rangle, \tag{1}$$

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where Ω is the volume, $\hbar\omega$ represents the excitation energy, and q is the magnitude of the three-momentum transfer by the photon. The nuclear ground state is denoted by $|0\rangle$, $\mathcal{O}$ is the external excitation operator, and the polarization propagator is given by

$$G(\hbar\omega)=\frac{1}{\hbar\omega-H+i\eta}-\frac{1}{\hbar\omega+H+i\eta}, \tag{2}$$

where H is the nuclear Hamiltonian. For the external operator we take a sum of OB and TB operators,

$$\mathcal{O}=\mathcal{O}^{OB}+\mathcal{O}^{TB}. \tag{3}$$

We will be concerned with the transverse channel (T). Explicit forms for $\mathcal{O}_T^{OB}$ and $\mathcal{O}_T^{TB}$ excitation operators are given by

$$(\mathcal{O}^{OB})_T=\frac{1}{2mq}G_E(q,\hbar\omega)\sum_{i=1}^{A}\left\{\frac{1+\tau_3(i)}{2}[\boldsymbol{q}\times(\boldsymbol{p}_i+\boldsymbol{p'}_i)]+i\frac{\mu_s+\mu_v\tau_3(i)}{2}\{\boldsymbol{q}\times[\boldsymbol{\sigma}(i)\times\boldsymbol{q}]\}\right\}, \tag{4}$$

where m is the nucleonic mass, $\boldsymbol{p}_i$ and $\boldsymbol{p'}_i$ denote the initial and final momenta of the struck nucleon, and $G_E(q,\hbar\omega)$ is the usual dipole electromagnetic form factor, which will be defined soon. The values of μ_s and μ_v, which are related to the proton and neutron magnetic moments, are $\mu_s=0.88$ and $\mu_v=4.70$. The *TB* piece is due to the exchange of mesons. We have taken into account only the so-called pion in-flight (Π) and seagull (S) contributions

$$(\mathcal{O}^{TB})_\Pi=-i4\pi\frac{f_\pi^2}{m_\pi^2}\frac{1}{q}F_\pi(q,\hbar\omega)\times\sum_{k,l=1;k\neq l}^{A}F_{\pi N}(k_k^2)F_{\pi N}(k_2^2)2(\boldsymbol{q}\times\boldsymbol{k}_l)\times\frac{\boldsymbol{\sigma}(k)\cdot\boldsymbol{k}_k}{k_k^2+m_\pi^2}\frac{\boldsymbol{\sigma}(l)\cdot\boldsymbol{k}_l}{k_l^2+m_\pi^2}(\boldsymbol{\tau}_k\times\boldsymbol{\tau}_l)_3, \tag{5}$$

$$(\mathcal{O}^{TB})_S=-i4\pi\frac{f_\pi^2}{m_\pi^2}\frac{1}{q}F_S(q,\hbar\omega)\times\sum_{k,l=1;k\neq l}^{A}\left[F_{\pi N}(k_l^2)[\boldsymbol{q}\times\boldsymbol{\sigma}(k)]\frac{\boldsymbol{\sigma}(l)\cdot\boldsymbol{k}_l}{k_l^2+m_\pi^2}-F_{\pi N}(k_k^2)\frac{\boldsymbol{\sigma}(k)\cdot\boldsymbol{k}_k}{k_k^2+m_\pi^2}[\boldsymbol{q}\times\boldsymbol{\sigma}(l)]\right](\boldsymbol{\tau}_k\times\boldsymbol{\tau}_l)_3. \tag{6}$$

The values are for $\boldsymbol{k}_k=\boldsymbol{p}'_k-\boldsymbol{p}_k$, $\boldsymbol{k}_l=\boldsymbol{p}'_l-\boldsymbol{p}_l$, and $\boldsymbol{q}=\boldsymbol{k}_k+\boldsymbol{k}_l$.

In our calculation we have employed the following expressions for the form factors:

$$G_E(q,\omega)=\left[1+\frac{q^2-\omega^2}{(839\text{ MeV})^2}\right]^{-2}, \tag{7}$$

$$F_\pi(q,\omega)=\frac{1}{[1+(q^2-\omega^2)/m_\rho]^2}, \tag{8}$$

and

$$F_{\pi N}(k^2)=\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2+k^2}. \tag{9}$$

Following Ref. [9] we introduce three projection operators P, Q, and R, where P $(Q,\ R)$ projects into $npnh$ configurations with $n=0$, 1 (n even, n odd greater or equal to 3). It is easy to verify that $P+Q+R=1$, $P^2=P$, $Q^2=Q$, $R^2=R$, and $PQ=QP=PR=RP=QR=RQ=0$.

Inserting the identity into Eq. (1),

$$R=R_{PP}+R_{PQ}+R_{QP}+R_{QQ}+R_{PR}+R_{RP}+R_{RR}+R_{QR}+R_{RQ}. \tag{10}$$

The expression for R_{PP} is given by

$$R_{PP}(\boldsymbol{q},\hbar\omega)=-\frac{\text{Im}}{\pi\Omega}\langle 0|\mathcal{O}^\dagger PG(\hbar\omega)P\mathcal{O}|0\rangle, \tag{11}$$

and similar expressions can be written for R_{PQ}, etc. Obviously R_{PQ} is equal to R_{QP} and the same holds for R_{PR} (R_{QR}) and R_{RP} (R_{RQ}). Explicit evaluation of each contribution was done in Refs. [9] and [10]. For convenience, let us reproduce the nonvanishing contributions

$$R_{PP}=-\frac{\text{Im}}{\pi\Omega}\times\left\langle 0\left|\mathcal{O}^\dagger P\frac{1}{\hbar\omega-H-\Sigma^{PQP}+\text{Re}\,\Sigma^{PRP}+i\eta}P\mathcal{O}\right|0\right\rangle, \tag{12}$$

$$R_{PQ}=-\frac{\text{Im}}{\pi\Omega}\left\langle 0\left|\mathcal{O}^\dagger P\frac{1}{\hbar\omega-H_0+i\eta}PH_{res}Q\times\frac{1}{\hbar\omega-H_0+i\eta}Q\mathcal{O}\right|0\right\rangle, \tag{13}$$

and

$$R_{QQ}=-\frac{\text{Im}}{\pi\Omega}\left\langle 0\left|\mathcal{O}^\dagger Q\frac{1}{\hbar\omega-H_0+i\eta}Q\mathcal{O}\right|0\right\rangle, \tag{14}$$

where in the last two equations we have separated the total Hamiltonian H into a one-body part H_0 and a residual interaction H_{res}. In the above expressions some third- and higher-order contributions in the residual interaction were neglected. The operator Σ^{PQP} $[\Sigma^{PRP}]$ stands for two-particle–two-hole $(2p2h)$ $[3p3h]$ self-energy insertions,

$$\Sigma^{PQP}=PH_{res}Q\frac{1}{\hbar\omega-H_0+i\eta}QH_{res}P \tag{15}$$

and

$$\Sigma^{PRP}=PH_{res}R\frac{1}{\hbar\omega-H_0+i\eta}RH_{res}P. \quad (16)$$

For simplicity we have shown forward going contributions only. We finally establish the structure of the ground state. By including ground state correlation perturbatively up to first order in the residual interaction, one gets

$$|0\rangle=|HF\rangle-H_0^{-1}QH_{res}P|HF\rangle, \quad (17)$$

with $|HF\rangle$ being the Hartree-Fock (HF) ground state.

As mentioned, contributions to the response function stemming from $\mathcal{O}^{OB}$ were studied in Refs. [9] and [10]. Using only $\mathcal{O}^{TB}$ in Eq. (1), three contributions should be considered. The one to the PP channel was analyzed in Ref. [14] within the shell model formalism and it is known to be negligibly small. In the PQ channel the lowest nonvanishing contribution is of third order in the nuclear interaction and it will not be considered in our scheme. Finally, contributions in the QQ channel, designed as $(R_{QQ})_{TB\text{-}TB}$, were already included and extensively study in Ref. [13]. They are included within the results and we will refer to them as pure MEC contributions.

We focus in the present work on the interference terms between $\mathcal{O}^{OB}$ and $\mathcal{O}^{TB}$. From Eqs. (12)–(14), the relevant contributions are

$$(R_{PP})_{OB\text{-}TB}=-2\frac{\text{Im}}{\pi\Omega}\left\langle 0\left|\mathcal{O}^{OB\dagger}P\frac{1}{\hbar\omega-H_0+i\eta}P\mathcal{O}^{TB}\right|0\right\rangle, \quad (18)$$

$$(R_{PQ})_{OB\text{-}TB}=-\frac{\text{Im}}{\pi\Omega}\left\langle 0\left|\left(\mathcal{O}^{OB\dagger}P\frac{1}{\hbar\omega-H_0+i\eta}\times PH_{res}Q\frac{1}{\hbar\omega-H_0+i\eta}Q\mathcal{O}^{TB}+\mathcal{O}^{TB\dagger}P\frac{1}{\hbar\omega-H_0+i\eta}PH_{res}Q\times\frac{1}{\hbar\omega-H_0+i\eta}Q\mathcal{O}^{OB}\right)\right|0\right\rangle, \quad (19)$$

and

$$(R_{QQ})_{OB\text{-}TB}=-2\frac{\text{Im}}{\pi\Omega}\left\langle 0\left|\mathcal{O}^{OB\dagger}Q\frac{1}{\hbar\omega-H_0+i\eta}Q\mathcal{O}^{TB}\right|0\right\rangle. \quad (20)$$

Graphic representations for these equations are given in Figs. 1–3. The action of $\mathcal{O}^{OB}$ is to create (or destroy) a $1p1h$ configuration or to scatter a particle or a hole. That is, the HF piece of the ground state is connected only to the P space. Once $\mathcal{O}^{TB}$ comes into play, this new set of diagrams together with the above-mentioned pure MEC ones, should be considered. Now a $2p2h$ configuration from the HF term of the ground state can be created (or destroyed). Contributions to R_{PP} (Fig. 1) were studied in Ref. [15] and the same holds for R_{QQ} (Fig. 3) in [5] (but with a different interaction). Some new terms to R_{QP} (Fig. 2) should be considered.

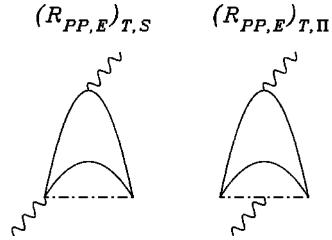

FIG. 1. Goldstone diagrams stemming from Eq. (18). In every diagram the solid line is either a particle or a hole, and the wavy line represents the external probe with energy momentum $(\omega,\boldsymbol{q})$, while a wavy line together with a dashed dotted line is the *TB* external probe. Diagram $(R_{PP,E})_{T,S}$ is the interference between *OB* and seagull and $(R_{PP,E})_{T,\Pi}$ between *OB* and pion in flight. Subscript *E* indicates the Pauli exchange character of the diagram.

In fact, in Fig. 2 we have shown only terms stemming from the first term on the right-hand side (RHS) of Eq. (19). The first nonvanishing contribution from the second term is of third order and will be not considered. Our projected ERPA scheme gives a systematic way to account for all these contributions. In the next section each contribution will be analyzed in detail.

III. RESULTS

In this section the transverse response will be evaluated and discussed. Particular emphasis is put on interference terms $(R_{PP})_{OB\text{-}TB}$, $(R_{PQ})_{OB\text{-}TB}$ and $(R_{QQ})_{OB\text{-}TB}$ from Eqs. (18)–(20), respectively.

Let us first establish the residual interaction. This is based on previous calculations, where a systematic analysis of several low-lying states in ^{48}Ca, as well as high-spin states of ^{208}Pb were carried out [11]. The interaction contains the Landau-Migdal g_0 and g_0' constants together with a long-range component generated by the π- and ρ-meson-exchange potentials,

$$H_{res}(k)=\frac{f_\pi^2}{\mu_\pi^2}\Gamma_\pi^2(k)[g_0\ \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'+\tilde{g}'(k)\boldsymbol{\tau}\cdot\boldsymbol{\tau}'\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}+\tilde{h}'(k)\boldsymbol{\tau}\cdot\boldsymbol{\tau}'\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}}\boldsymbol{\sigma}'\cdot\hat{\boldsymbol{k}}], \quad (21)$$

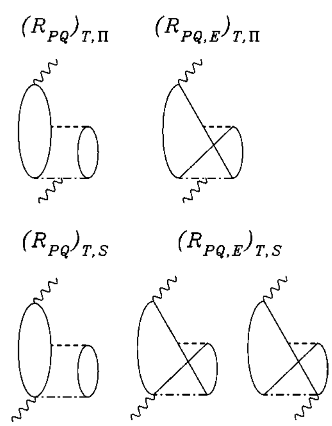

FIG. 2. The same as Fig. 1 but for the R_{PQ} channel from Eq. (19). The dashed line is H_{res}.

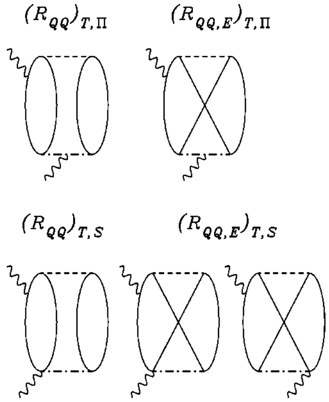

FIG. 3. The same as Fig. 2 but for the R_{QQ} channel from Eq. (20). Note that here the action of the *OB* operator is to scatter a particle or a hole. This is at variance with Figs. 1 and 2, where it creates (or destroy) a particle-hole pair.

with

$$\tilde{g}'(k) = g_0' - \frac{\Gamma_\rho^2(k)}{\Gamma_\pi^2(k)} C_\rho \frac{k^2}{k^2+\mu_\rho^2}, \tag{22}$$

$$\tilde{h}'(k) = -\frac{k^2}{k^2+\mu_\pi^2} + \frac{\Gamma_\rho^2(k)}{\Gamma_\pi^2(k)} C_\rho \frac{k^2}{k^2+\mu_\rho^2}, \tag{23}$$

where $\mu_\pi \hbar c$ ($\mu_\rho \hbar c$) is the pion (rho) rest mass and $C_\rho = 2.18$. For the form factor of the πNN (ρNN) vertex we have taken

$$\Gamma_{\pi,\rho}(k) = \frac{\Lambda_{\pi,\rho}^2 - (\mu_{\pi,\rho}\hbar c)^2}{\Lambda_{\pi,\rho}^2 + (\hbar c k)^2}, \tag{24}$$

with $\Lambda_\pi = 1.3$ GeV and $\Lambda_\rho = 1.75$ GeV. The static limit to the $(\pi+\rho)$-meson exchange interaction has been taken, where k is the momentum transfer.

The Landau-Migdal parameters g_0 and g_0' account for short-range correlations. They were adjusted in order to reproduce the energies and B values of the two 1+ states in ^{208}Pb at 5.85 and 7.30 MeV, respectively. Their values are $g_0 = 0.47$ and $g_0' = 0.76$. Our main concern is to complete the projected ERPA framework with the inclusion of *TB* excitation operators and this interaction is particularly suitable for this. Note, however, that the current conservation establishes a relationship between the nuclear current and the interaction [20]. Within the present model for the residual interaction, to fully satisfy this relationship one should add some additional terms to the *TB* excitation operator due to the exchange of heavier mesons, even though, they are quantitatively much less important [21]. For this reason, we have kept only the pion-exchange contribution to the *TB* excitation operator as defined in Eqs. (5) and (6).

From Eqs. (18)–(20), performing the summation over spin and isospin and making the conversion of sums over momenta to integrals, explicit expressions for $(R_{PP})_{OB\text{-}TB}$, $(R_{PQ})_{OB\text{-}TB}$, and $(R_{QQ})_{OB\text{-}TB}$ are obtained. The ones for $(R_{PP})_{OB\text{-}TB}$ can be found in [15]. Note that those are in fact Pauli-exchange contributions. In the Appendix we present expressions for $(R_{PQ})_{OB\text{-}TB}$ and $(R_{QQ})_{OB\text{-}TB}$, following the ordering and notation of Figs. 2 and 3. While the direct going contributions are proportional to g_0' plus the $\pi+\rho$ meson, the Pauli-exchange ones are proportional only to the g_0 term. The first point comes out as a consequence of spin summation. For exchange terms the isospin summation cancels any isovector term of the interaction. It is worth mentioning here that the absence of exchange terms in $(R_{QQ})_{TB\text{-}TB}$ is due to the isospin summation.

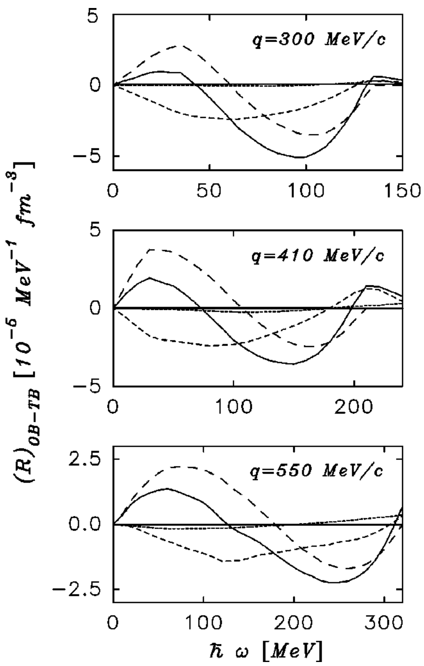

FIG. 4. Interference contribution between *OB* and *TB* external probes at several momentum transfers. The long-dashed line is the $(R_{PP})_{OB\text{-}TB}$ contribution, the short-dashed is the $(R_{PQ})_{OB\text{-}TB}$ one, and the dotted line is the $(R_{QQ})_{OB\text{-}TB}$ result. The solid line is the sum of all contributions.

Let us mention that the $(R_{QQ})_{OB\text{-}TB}$ contributions were already analyzed in Ref. [5], but without the inclusion of exchange terms. We understand that the $(R_{QP})_{OB\text{-}TB}$ term is contained in Ref. [17] (also without exchange terms). But in this last work only one momentum transfer was considered and no discussion of each contribution was shown.

We perform now the numerical evaluation of all contributions for nonrelativistic nuclear matter using a Fermi momentum value of $k_F = 268$ MeV/c. The multiple integrations have been done using a Monte Carlo technique. In Fig. 4, we present the results for the interference terms at momentum transfer $q =$ 300, 410, and 550 MeV/c. The $(R_{PP})_{OB\text{-}TB}$ is the main contribution to the response function, where its action is limited to the quasielastic peak region. At variance with $(R_{PQ})_{OB\text{-}TB}$ and $(R_{QQ})_{OB\text{-}TB}$, it has no dependence on the residual interaction. Within the present approximation we consider the *TB* excitation operator and H_{res} as independent quantities. In this sense, $(R_{PP})_{OB\text{-}TB}$ depends only on *OB* and *TB* excitation operators, while $(R_{PQ})_{OB\text{-}TB}$ and $(R_{QQ})_{OB\text{-}TB}$ are first order in H_{res}. The $(R_{QP})_{OB\text{-}TB}$ term is also important. In this case the intensity is spread over a

TABLE I. Interference terms $(R_{PQ})_{OB\text{-}TB}$ and $(R_{QQ})_{OB\text{-}TB}$ to the response function in nuclear matter for $k_F=268$ MeV/c at momentum transfer $q=410$ MeV/c. The energy is given in MeV, while the response function is in units of 10^{-5} MeV^{-1} fm^{-3}. The second column, Π, is the sum of all direct interference terms between the pion in flight and the transverse operator. The third column represents the corresponding Pauli-exchange contribution, while columns 4 and 5 have the same meaning but for the seagull operator. Column Tot. collects the sum of all contributions.

$\hbar\omega$	$(R_{PQ})_{OB\text{-}TB}$ Π	Π,E	S	S,E	Tot.
50.0	−1.56	0.57	−1.13	0.04	−2.08
100.0	−1.97	0.61	−1.05	0.09	−2.32
150.0	−0.77	0.12	−0.51	0.04	−1.12
200.0	0.92	−0.14	0.23	−0.01	1.00
250.0	0.24	−0.08	0.31	−0.02	0.45
$\hbar\omega$	$(R_{QQ})_{OB\text{-}TB}$ Π	Π,E	S	S,E	Tot.
50.0	0.05	−0.03	−0.06	0.00	−0.04
100.0	0.23	−0.17	−0.28	0.01	−0.21
150.0	0.42	−0.21	−0.36	0.01	−0.14
200.0	0.58	−0.19	−0.29	0.02	0.12
240.0	0.51	−0.14	−0.08	0.03	0.32

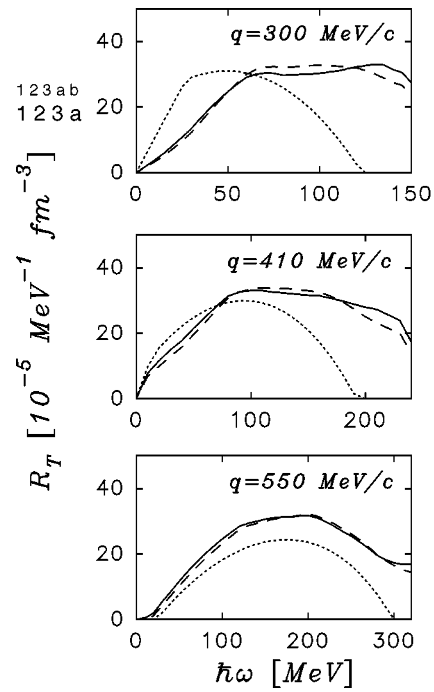

FIG. 5. Total ERPA response with and without MEC contributions for nuclear matter at several momentum transfers. The solid line is the total result while the dashed line contains only *OB* terms. For completeness we show the free response (dotted line).

wider energy region, which obviously includes the dip. If we neglect the energy dependence of the electromagnetic form factors, $(R_{PQ})_{OB\text{-}TB}$ gives no contribution to the non-energy-weight sum rule. This is only suggested in Fig. 4, as we need a wider energy region to show that. The action of $(R_{PQ})_{OB\text{-}TB}$ is to carry the intensity from the low- to the higher-energy region. The values for $(R_{QQ})_{OB\text{-}TB}$ are small and almost negligible.

In Table I we have shown each contribution to $(R_{QP})_{OB\text{-}TB}$ and $(R_{QQ})_{OB\text{-}TB}$ at momentum transfer $q=410$ MeV/c. Our values for $(R_{QQ})_{T\text{-}\Pi}$ and $(R_{QQ})_{T\text{-}S}$ from columns 2 and 4 of Table I can be compared with those of columns 5 and 6 of Table Ib of [5]. Both calculations are in agreement considering that the interaction employed was different. We see that the small value for $(R_{QQ})_{OB\text{-}TB}$ comes from the different signs of $(R_{QQ})_{T\text{-}\Pi}$ and $(R_{QQ})_{T\text{-}S}$. Also from Table I we see that Pauli-exchange contributions are important when compared with theirs corresponding direct ones. Specially $(R_{PQ})_{T\text{-}\Pi}$ has a significant Pauli-exchange contribution at low energies. Pauli-exchange contributions are numerically difficult to evaluate. The behavior of these contributions is essentially the same at the other momentum transfer.

In Fig. 5 we present the results for the full ERPA approximation with and without MEC's. Obviously, MEC's contain the $(R_{QQ})_{TB\text{-}TB}$ contribution (as mentioned, this term does not depend on the residual interaction). We see that the election of this particular residual interaction produces strong ERPA correlation. This election was done in order to study the importance of the interference terms $(R_{PQ})_{OB\text{-}TB}$ and $(R_{QQ})_{OB\text{-}TB}$ under this extreme condition.

We study now how significant the inclusion or not of MEC's is. Let us consider three energy zones. The region for energies lower than 50 MeV is drawn for completeness as low-energy processes are not considered here. At $q=410$ MeV/c the dip region starts at about 200 MeV. In the middle there is the quasielastic region. Considering only the last two regions, from Figs. 4 and 5, we learn that the MEC's are important in the dip region, while its contribution is not significant for the quasielastic peak. As our ERPA correlations are rather big, even when compared with the free response, MEC's do not have a strong influence over the quasielastic peak. At variance, the ERPA correlation coming from *OB* operators is very significant. It is important to stress, as discussed in previous works [9,10], that the ERPA is built up from many contributions among which MEC's are one of them, even though MEC's have a much weaker dependence on the residual interaction: the two main contributions $(R_{QQ})_{TB\text{-}TB}$ and $(R_{PP})_{OB\text{-}TB}$ have no dependence at all, while the remainder are of first order in the residual interaction, being smaller than the previous ones. From the above discussion and this fact, we conclude that MEC's can be neglected in a first step when trying to adjust the quasielastic peak. To put it in other words, if we want to adjust the residual interaction by means of the ERPA scheme in the quasielastic peak, then MEC's could be not considered. This does not hold in the dip region where exchange terms are also significant.

Finally, even the small contribution from MEC's to the quasielastic peak is in agreement with the existing literature for medium and heavy nuclei; for few body systems MEC's give an important effect, increasing the transverse response significantly [21]. The origin of this discrepancy is still not clear. In Ref. [22], for instance, a calculation of the quasielastic response function in ^{4}He based upon a mean-field model used to performed similar calculations in heavier nuclei is presented, the MEC contribution being small. However, an analogous calculation to [21] is not available for medium and heavy nuclei. In [21], FSI's are treated exactly, heavier mesons together with the Δ isobar were considered, and a realistic interaction was employed. In this work we have shown that terms arising from the ERPA are not able to explain the above-mentioned discrepancy.

IV. CONCLUSIONS

We have developed a scheme which extends our previous ERPA approach to two-body excitation operators. The ERPA limited to one-body external operators accounts for initial and final state interactions. A two-body operator represents the MEC's. Diagrammatically, GSI's and MEC's are similar and in principle one expects that these contributions should be equally important. In a unifying manner our formalism contains all the just-mentioned correlations and calls attention to the interference terms $(R_{PQ})_{OB\text{-}TB}$ and $(R_{QQ})_{OB\text{-}TB}$. These contributions are important (as well as the other MEC's), when dealing with the dip region, but the whole effect is not significant for the quasielastic peak.

As mentioned in the Introduction, our final goal is to account for both the longitudinal and transverse response functions. To this end, the residual interaction should not be seen as input, but as a variable that should be adjusted. To do this, it is desirable to reduce the number of diagrams to be evaluated. In this work, we have shown that MEC's can be neglected in a first step in the quasielastic peak region. This was done for a residual interaction that produces strong ERPA correlations. We believe that it is a reasonable assumption that under this condition our values for the MEC terms can be viewed as an upper limit for this contribution.

We have studied the transverse channel for quasielastic electron scattering. In this channel, the Δ isobar is important and should be included. The Δ can be directly excited by the external one-body operator and also contributes to the MEC's. A systematic analysis of the response with the inclusion of the Δ over the parameters entering into the interaction should be done. But this goes beyond the scope of the present work in which we wanted to present the scheme and analyze the importance of MEC's, at least for one interaction.

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APPENDIX

In this appendix, we present explicit expressions for the different graphs to be added to the ERPA scheme of Refs. [9] and [10]. All graphs represent the interference between *OB* and MEC operators.

We have used dimensionless quantities $\boldsymbol{Q}=\boldsymbol{q}/k_F$ and $\nu=\hbar\omega/2\varepsilon_F$, where k_F and ε_F are the Fermi momentum and Fermi energy, respectively.

Expressions for the graphs shown in Fig. 1 can be found in Ref. [15]. Let us consider the graphs from R_{PQ} channel of Fig. 2. In what follows, a superscript p (h) means that the upper nuclear interaction (Fig. 2) scatters a particle (hole) at the left of each graph:

$$(R_{PQ})^{p}_{T,\Pi}=-F_{\pi}(Q,\nu)G_E(Q,\nu)\frac{3Am}{2\pi^5k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3k_2F^3_{\pi N}(k_1)F_{\pi N}(k_2)\,\theta(1-h)\,\theta(|\boldsymbol{h}+\boldsymbol{Q}-\boldsymbol{k}_2|-1)$$
$$\times\theta(|\boldsymbol{h}+\boldsymbol{Q}|-1)[\tilde{g}'(k_2)+\tilde{h}'(k_2)](-2(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)\{[\boldsymbol{h}\cdot\boldsymbol{k}_2-(\boldsymbol{Q}\cdot\boldsymbol{h})(\boldsymbol{Q}\cdot\boldsymbol{k}_2)]/Q^2\}$$
$$+\mu_v[k_2^2Q^2-(\boldsymbol{Q}\cdot\boldsymbol{k}_2)^2])\frac{1}{k_1^2+\mu_\pi^2}\,\frac{1}{k_2^2+\mu_\pi^2}\left\{I(\alpha,k_2)\frac{1}{\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}}+R(\alpha,k_2)\,\delta(\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q})\right\}, \tag{A1}$$

where

$$\alpha=\nu-Q^2/2-k_2^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}+\boldsymbol{Q}\cdot\boldsymbol{k}_2+\boldsymbol{h}\cdot\boldsymbol{k}_2, \tag{A2}$$

$$I(\alpha,k)=\int dk'\,\theta(|\boldsymbol{k}'+\boldsymbol{k}/2|-1)\,\theta(1-|\boldsymbol{k}'-\boldsymbol{k}/2|)\,\delta(\alpha-\boldsymbol{k}'\cdot\boldsymbol{k}), \tag{A3}$$

and

$$R(\alpha,k)=\int dk'\,\theta(|\boldsymbol{k}'+\boldsymbol{k}/2|-1)\,\theta(1-|\boldsymbol{k}'-\boldsymbol{k}/2|)\frac{1}{\alpha-\boldsymbol{k}'\cdot\boldsymbol{k}} \tag{A4}$$

$$(R_{PQ})^{h}_{T,\Pi}=F_{\pi}(Q,\nu)G_{E}(Q,\nu)\frac{3Am}{2\pi^5 k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3k_2 F^3_{\pi N}(k_1)F_{\pi N}(k_2)\,\theta(1-h)\,\theta(|\boldsymbol{h}+\boldsymbol{Q}-\boldsymbol{k}_2|-1)$$
$$\times\theta(1-|\boldsymbol{h}-\boldsymbol{k}_2|)[\tilde{g}'(k_2)+\tilde{h}'(k_2)]\{2(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)[(\boldsymbol{h}-\boldsymbol{k}_2)\cdot\boldsymbol{k}_2-\boldsymbol{Q}\cdot(\boldsymbol{h}-\boldsymbol{k}_2)(\boldsymbol{Q}\cdot\boldsymbol{k}_2)/Q^2]$$
$$+\mu_v[k_2^2Q^2-(\boldsymbol{Q}\cdot\boldsymbol{k}_2)^2]\}\frac{1}{k_1^2+\mu_\pi^2}\,\frac{1}{k_2^2+\mu_\pi^2}\left\{I(\alpha,k_2)\frac{1}{\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}+\boldsymbol{Q}\cdot\boldsymbol{k}_2}+R(\alpha,k_2)\,\delta(\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}+\boldsymbol{Q}\cdot\boldsymbol{k}_2)\right\},\tag{A5}$$

where

$$\alpha=\nu-Q^2/2-k_2^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}+\boldsymbol{Q}\cdot\boldsymbol{k}_2+\boldsymbol{h}\cdot\boldsymbol{k}_2;\tag{A6}$$

$$(R_{PQ})^{p}_{T,S}=F_{S}(Q,\nu)G_{E}(Q,\nu)\frac{3Am}{4\pi^5 k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3k_2\,\theta(1-h)\,\theta(|\boldsymbol{h}+\boldsymbol{Q}-\boldsymbol{k}_2|-1)\,\theta(|\boldsymbol{h}+\boldsymbol{Q}|-1)F^2_{\pi N}(k_2)$$
$$\times\left\{2F_{\pi N}(k_2)\frac{1}{k_2^2+\mu_\pi^2}[\tilde{g}'(k_2)+\tilde{h}'(k_2)][hh+\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_2)]-F_{\pi N}(k_1)\frac{1}{k_1^2+\mu_\pi^2}2\tilde{g}'(k_2)[hh-\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_1)]+\tilde{h}'(k_2)\right.$$
$$\left.\times\{2hh(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)/\boldsymbol{k}_2^2-\mu_v[\boldsymbol{k}_2^2\boldsymbol{Q}^2-(\boldsymbol{Q}\cdot\boldsymbol{k}_2)^2]/\boldsymbol{k}_2^2]\right\}\left\{I(\alpha,k_2)\frac{1}{\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}}+R(\alpha,k_2)\,\delta(\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q})\right\},\tag{A7}$$

where

$$\alpha=\nu-\boldsymbol{Q}^2/2-\boldsymbol{k}_2^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}+\boldsymbol{h}\cdot\boldsymbol{k}_2+\boldsymbol{Q}\cdot\boldsymbol{k}_2\tag{A8}$$

and

$$hh=\boldsymbol{h}\cdot\boldsymbol{k}_2-(\boldsymbol{Q}\cdot\boldsymbol{h})(\boldsymbol{Q}\cdot\boldsymbol{k}_2)/Q^2;\tag{A9}$$

$$(R_{PQ})^{h}_{T,S}=F_{S}(Q,\nu)G_{E}(Q,\nu)\frac{3Am}{4\pi^5 k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3k_2\,\theta(1-h)\,\theta(|\boldsymbol{h}+\boldsymbol{Q}-\boldsymbol{k}_2|-1)\,\theta(1-|\boldsymbol{h}-\boldsymbol{k}_2|)F^2_{\pi N}(k_2)$$
$$\times\left\{-2F_{\pi N}(k_2)\frac{1}{k_2^2+\mu_\pi^2}[\tilde{g}'(k_2)+\tilde{h}'(k_2)][hh-\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_2)]-F_{\pi N}(k_1)\frac{1}{k_1^2+\mu_\pi^2}2\tilde{g}'(k_2)[hh+\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_1)]\right.$$
$$\left.+\tilde{h}'(k_2)\{2hh(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)/\boldsymbol{k}_2^2+\mu_v[\boldsymbol{k}_2^2\boldsymbol{Q}^2-(\boldsymbol{Q}\cdot\boldsymbol{k}_2)^2]/\boldsymbol{k}_2^2]\right\}\left\{I(\alpha,k_2)\frac{1}{\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}+\boldsymbol{Q}\cdot\boldsymbol{k}_2}+R(\alpha,k_2)\right.$$
$$\left.\times\,\delta(\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}+\boldsymbol{Q}\cdot\boldsymbol{k}_2)\right\},\tag{A10}$$

where

$$\alpha=\nu-\boldsymbol{Q}^2/2-\boldsymbol{k}_2^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}+\boldsymbol{h}\cdot\boldsymbol{k}_2+\boldsymbol{Q}\cdot\boldsymbol{k}_2\tag{A11}$$

and

$$hh=(\boldsymbol{h}-\boldsymbol{k}_2)\cdot\boldsymbol{k}_2-\boldsymbol{Q}\cdot(\boldsymbol{h}-\boldsymbol{k}_2)(\boldsymbol{Q}\cdot\boldsymbol{k}_2)/Q^2;\tag{A12}$$

$$(R_{PQ,E})^{p}_{T,\Pi}=-F_{\pi}(Q,\nu)G_{E}(Q,\nu)\frac{3Am}{8\pi^{5}k_{F}^{2}\Omega}\left(\frac{f_{\pi}^{2}}{4\pi\hbar c}\right)^{2}\frac{1}{\mu_{\pi}^{4}}\int d^{3}h\int d^{3}h'\int d^{3}k_{2}F_{\pi N}(k_{1})F_{\pi N}(k_{2})\,\theta(1-h)\,\theta(1-h')$$
$$\times\theta(|\boldsymbol{h}+\boldsymbol{k}_{2}|-1)\,\theta(|\boldsymbol{h}'+\boldsymbol{Q}-\boldsymbol{k}_{2}|-1)\,\theta(1-|\boldsymbol{h}'-\boldsymbol{k}_{2}|)F_{\pi N}^{2}(|\boldsymbol{h}-\boldsymbol{h}'+\boldsymbol{k}_{2}|)(g_{0}\{2(\boldsymbol{k}_{1}\cdot\boldsymbol{k}_{2})[\boldsymbol{k}_{2}\cdot(\boldsymbol{h}'-\boldsymbol{k}_{2})-\boldsymbol{Q}\cdot(\boldsymbol{h}'-\boldsymbol{k}_{2})$$
$$\times(\boldsymbol{Q}\cdot\boldsymbol{k}_{2})/Q^{2}]+\mu_{v}[\boldsymbol{k}_{2}^{2}\boldsymbol{Q}^{2}-(\boldsymbol{k}_{2}\cdot\boldsymbol{Q})^{2}]\})\frac{1}{k_{1}^{2}+\mu_{\pi}^{2}}\,\frac{1}{k_{2}^{2}+\mu_{\pi}^{2}}\left\{\delta[\nu-(\boldsymbol{Q}^{2}/2+\boldsymbol{k}_{2}^{2}+\boldsymbol{h}'\cdot\boldsymbol{Q}-\boldsymbol{Q}\cdot\boldsymbol{k}_{2}-\boldsymbol{h}'\cdot\boldsymbol{k}_{2}+\boldsymbol{h}\cdot k_{2})]\right.$$
$$\left.\times\frac{1}{\nu-\boldsymbol{h}'\cdot\boldsymbol{k}_{2}+\boldsymbol{h}\cdot\boldsymbol{k}_{2}}-\delta[\nu-(\boldsymbol{Q}^{2}/2+\boldsymbol{h}'\cdot\boldsymbol{Q}-\boldsymbol{Q}\cdot\boldsymbol{k}_{2})]\frac{1}{\boldsymbol{k}_{2}^{2}-\boldsymbol{h}'\cdot\boldsymbol{k}_{2}+\boldsymbol{h}\cdot\boldsymbol{k}_{2}}\right\};\tag{A13}$$

$$(R_{PQ,E})^{h}_{T,\Pi}=F_{\pi}(Q,\nu)G_{E}(Q,\nu)\frac{3Am}{8\pi^{5}k_{F}^{2}\Omega}\left(\frac{f_{\pi}^{2}}{4\pi\hbar c}\right)^{2}\frac{1}{\mu_{\pi}^{4}}\int d^{3}h\int d^{3}h'\int d^{3}k_{2}F_{\pi N}(k_{1})F_{\pi N}(k_{2})\,\theta(1-h)\,\theta(1-h')$$
$$\times\theta(|\boldsymbol{h}'+\boldsymbol{k}_{2}|-1)\,\theta(|\boldsymbol{h}'+\boldsymbol{Q}-\boldsymbol{k}_{2}|-1)\,\theta(|\boldsymbol{h}+\boldsymbol{Q}|-1)F_{\pi N}^{2}(|\boldsymbol{h}-\boldsymbol{h}'+\boldsymbol{Q}-\boldsymbol{k}_{2}|)(g_{0}\{2(\boldsymbol{k}_{1}\cdot\boldsymbol{k}_{2})$$
$$\times[\boldsymbol{k}_{2}\cdot\boldsymbol{h}-(\boldsymbol{Q}\cdot\boldsymbol{h})(\boldsymbol{Q}\cdot\boldsymbol{k}_{2})/Q^{2}]+\mu_{v}[\boldsymbol{k}_{2}^{2}\boldsymbol{Q}^{2}-(\boldsymbol{k}_{2}\cdot\boldsymbol{Q})^{2}]\})\frac{1}{k_{1}^{2}+\mu_{\pi}^{2}}\,\frac{1}{k_{2}^{2}+\mu_{\pi}^{2}}\left\{\delta[\nu-(\boldsymbol{Q}^{2}/2+\boldsymbol{k}_{2}^{2}+\boldsymbol{h}\cdot\boldsymbol{Q}-\boldsymbol{h}\cdot\boldsymbol{k}_{2}-\boldsymbol{Q}\cdot\boldsymbol{k}_{2}\right.$$
$$\left.+\boldsymbol{h}'\cdot k_{2})]\frac{1}{\boldsymbol{k}_{2}^{2}-\boldsymbol{h}\cdot\boldsymbol{k}_{2}-\boldsymbol{Q}\cdot\boldsymbol{k}_{2}+\boldsymbol{h}'\cdot\boldsymbol{k}_{2}}-\delta[\nu-(\boldsymbol{Q}^{2}/2+\boldsymbol{h}.\boldsymbol{Q})]\frac{1}{\boldsymbol{k}_{2}^{2}-\boldsymbol{h}\cdot\boldsymbol{k}_{2}-\boldsymbol{Q}\cdot\boldsymbol{k}_{2}+\boldsymbol{h}'\cdot\boldsymbol{k}_{2}}\right\};\tag{A14}$$

$$(R_{PQ,E})^{p}_{T,S}=F_{S}(Q,\nu)G_{E}(Q,\nu)\frac{3Am}{8\pi^{5}k_{F}^{2}\Omega}\left(\frac{f_{\pi}^{2}}{4\pi\hbar c}\right)^{2}\frac{1}{\mu_{\pi}^{4}}\int d^{3}h\int d^{3}h'\int d^{3}k_{2}\,\theta(1-h)\,\theta(1-h')\,\theta(|\boldsymbol{h}+\boldsymbol{k}_{2}|-1)$$
$$\times\theta(|\boldsymbol{h}'+\boldsymbol{Q}-\boldsymbol{k}_{2}|-1)\,\theta(1-|\boldsymbol{h}'-\boldsymbol{k}_{2}|)F_{\pi N}^{2}(|\boldsymbol{h}-\boldsymbol{h}'+\boldsymbol{k}_{2}|)g_{0}\left\{F_{\pi N}(k_{2})\frac{1}{k_{2}^{2}+\mu_{\pi}^{2}}[hh+\mu_{v}(\boldsymbol{Q}\cdot\boldsymbol{k}_{2})]\right.$$
$$\left.+F_{\pi N}(k_{1})\frac{1}{k_{1}^{2}+\mu_{\pi}^{2}}[hh-\mu_{v}(\boldsymbol{Q}\cdot\boldsymbol{k}_{1})]\right\}\left\{\delta[\nu-(\boldsymbol{Q}^{2}/2+\boldsymbol{k}_{2}^{2}+\boldsymbol{h}'\cdot\boldsymbol{Q}-\boldsymbol{Q}\cdot\boldsymbol{k}_{2}-\boldsymbol{h}'\cdot\boldsymbol{k}_{2}+\boldsymbol{h}\cdot k_{2})]\frac{1}{\boldsymbol{k}_{2}^{2}-\boldsymbol{h}'\cdot\boldsymbol{k}_{2}+\boldsymbol{h}\cdot\boldsymbol{k}_{2}}\right.$$
$$\left.-\delta[\nu-(\boldsymbol{Q}^{2}/2+\boldsymbol{h}'\cdot\boldsymbol{Q}-\boldsymbol{Q}\cdot\boldsymbol{k}_{2})]\frac{1}{\boldsymbol{k}_{2}^{2}-\boldsymbol{h}'\cdot\boldsymbol{k}_{2}+\boldsymbol{h}\cdot\boldsymbol{k}_{2}}\right\},\tag{A15}$$

where

$$hh=(\boldsymbol{h}'-\boldsymbol{k}_{2})\cdot\boldsymbol{k}_{2}-\boldsymbol{Q}\cdot(\boldsymbol{h}'-\boldsymbol{k}_{2})(\boldsymbol{Q}\cdot\boldsymbol{k}_{2})/Q^{2};\tag{A16}$$

$$(R_{PQ,E})^{h}_{T,S}=-F_{S}(Q,\nu)G_{E}(Q,\nu)\frac{3Am}{8\pi^{5}k_{F}^{2}\Omega}\left(\frac{f_{\pi}^{2}}{4\pi\hbar c}\right)^{2}\frac{1}{\mu_{\pi}^{4}}\int d^{3}h\int d^{3}h'\int d^{3}k_{2}\,\theta(1-h)\,\theta(1-h')\,\theta(|\boldsymbol{h}'+\boldsymbol{k}_{2}|-1)$$
$$\times\theta(|\boldsymbol{h}+\boldsymbol{Q}-\boldsymbol{k}_{2}|-1)\,\theta(|\boldsymbol{h}+\boldsymbol{Q}|-1)F_{\pi N}^{2}(|\boldsymbol{h}-\boldsymbol{h}'+\boldsymbol{Q}-\boldsymbol{k}_{2}|)g_{0}\left\{F_{\pi N}(k_{2})\frac{1}{k_{2}^{2}+\mu_{\pi}^{2}}[hh+\mu_{v}(\boldsymbol{Q}\cdot\boldsymbol{k}_{2})]\right.$$
$$\left.+F_{\pi N}(k_{1})\frac{1}{k_{1}^{2}+\mu_{\pi}^{2}}[hh-\mu_{v}(\boldsymbol{Q}\cdot\boldsymbol{k}_{1})]\right\}\left\{\delta[\nu-(\boldsymbol{Q}^{2}/2+\boldsymbol{k}_{2}^{2}+\boldsymbol{h}\cdot\boldsymbol{Q}-\boldsymbol{h}\cdot\boldsymbol{k}_{2}-\boldsymbol{Q}\cdot\boldsymbol{k}_{2}-\boldsymbol{h}'\cdot k_{2})]\right.$$
$$\left.\times\frac{1}{\boldsymbol{k}_{2}^{2}-\boldsymbol{h}\cdot\boldsymbol{k}_{2}-\boldsymbol{Q}\cdot\boldsymbol{k}_{2}+\boldsymbol{h}'\cdot\boldsymbol{k}_{2}}-\delta[\nu-(\boldsymbol{Q}^{2}/2+\boldsymbol{h}\cdot\boldsymbol{Q})]\frac{1}{\boldsymbol{k}_{2}^{2}-\boldsymbol{h}\cdot\boldsymbol{k}_{2}-\boldsymbol{Q}\cdot\boldsymbol{k}_{2}+\boldsymbol{h}'\cdot\boldsymbol{k}_{2}}\right\},\tag{A17}$$

where

$$hh=\boldsymbol{h}\cdot\boldsymbol{k}_{2}-(\boldsymbol{Q}\cdot\boldsymbol{h})(\boldsymbol{Q}\cdot\boldsymbol{k}_{2})/Q^{2}.\tag{A18}$$

We present now graphs from R_{QQ} channel of Fig. 3. In what follows, a superscript p (h) means that the OB external operator (Fig. 3) scatters a particle (a hole):

$$(R_{QQ})^{p}_{T,\Pi}=-F_{\pi}(Q,\nu)G_{E}(Q,\nu)\frac{3Am}{2\pi^5 k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3k_1 F^3_{\pi N}(k_1)F_{\pi N}(k_2)\,\theta(1-h)\,\theta(|\boldsymbol{h}+\boldsymbol{Q}-\boldsymbol{k}_1|-1)$$
$$\times\theta(|\boldsymbol{h}-\boldsymbol{k}_1|-1)I(\alpha',k_1)\Big\{[\tilde{g}'(k_1)+\tilde{h}'(k_1)](2(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)\{[(\boldsymbol{h}-\boldsymbol{k}_1)\cdot\boldsymbol{k}_1-\boldsymbol{Q}\cdot(\boldsymbol{h}-\boldsymbol{k}_1)(\boldsymbol{Q}\cdot\boldsymbol{k}_1)]/Q^2\}$$
$$+\mu_v[k_1^2Q^2-(\boldsymbol{Q}\cdot\boldsymbol{k}_1)^2])\frac{1}{\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}+\boldsymbol{Q}\cdot\boldsymbol{k}_1}\,\frac{1}{k_1^2+\mu_\pi^2}\,\frac{1}{k_2^2+\mu_\pi^2}, \tag{A19}$$

where

$$\alpha'=\nu-Q^2/2-k_1^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}+\boldsymbol{Q}\cdot\boldsymbol{k}_1+\boldsymbol{h}\cdot\boldsymbol{k}_1; \tag{A20}$$

$$(R_{QQ})^{h}_{T,\Pi}=F_{\pi}(Q,\nu)G_{E}(Q,\nu)\frac{3Am}{2\pi^5 k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3k_1 F^3_{\pi N}(k_1)F_{\pi N}(k_2)\,\theta(1-h)\,\theta(|\boldsymbol{h}+\boldsymbol{Q}-\boldsymbol{k}_1|-1)$$
$$\times\theta(1-|\boldsymbol{Q}+\boldsymbol{h}|)I(\alpha'',k_1)\Big\{[\tilde{g}'(k_1)+\tilde{h}'(k_1)](-2(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)\{[\boldsymbol{h}\cdot\boldsymbol{k}_1-(\boldsymbol{Q}\cdot\boldsymbol{h})(\boldsymbol{Q}\cdot\boldsymbol{k}_1)]/Q^2\}$$
$$+\mu_v[k_1^2Q^2-(\boldsymbol{Q}\cdot\boldsymbol{k}_1)^2])\frac{1}{\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}}\,\frac{1}{k_1^2+\mu_\pi^2}\,\frac{1}{k_2^2+\mu_\pi^2}, \tag{A21}$$

where

$$\alpha''=\nu-Q^2/2-k_1^2/2+\boldsymbol{h}\cdot\boldsymbol{Q}-\boldsymbol{Q}\cdot\boldsymbol{k}_1-\boldsymbol{h}\cdot\boldsymbol{k}_1; \tag{A22}$$

$$(R_{QQ})^{p}_{T,S}=-F_{S}(Q,\nu)G_{E}(Q,\nu)\frac{3Am}{4\pi^5 k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3k_1\,\theta(1-h)\,\theta(|\boldsymbol{h}-\boldsymbol{Q}+\boldsymbol{k}_1|-1)\,\theta(|\boldsymbol{h}+\boldsymbol{k}_1|-1)F^2_{\pi N}(k_2)$$
$$\times\Big\{2F_{\pi N}(k_2)\frac{1}{k_2^2+\mu_\pi^2}[\tilde{g}'(k_2)+\tilde{h}'(k_2)][hh-\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_2)]-2F_{\pi N}(k_1)\frac{1}{k_1^2+\mu_\pi^2}2\tilde{g}'(k_2)[hh+\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_1)]$$
$$+\tilde{h}'(k_2)\{2hh(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)/\boldsymbol{k}_2^2+\mu_v[\boldsymbol{k}_2^2\boldsymbol{Q}^2-(\boldsymbol{Q}\cdot\boldsymbol{k}_2)^2]/\boldsymbol{k}_2^2\}\Big\}\frac{1}{\nu+Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}-\boldsymbol{Q}\cdot\boldsymbol{k}_1}I(\nu-k_1^2/2-\boldsymbol{h}\cdot\boldsymbol{k}_1,k_2), \tag{A23}$$

where

$$hh=(\boldsymbol{h}-\boldsymbol{k}_2)\cdot\boldsymbol{k}_2-\boldsymbol{Q}\cdot(\boldsymbol{h}-\boldsymbol{k}_2)(\boldsymbol{Q}\cdot\boldsymbol{k}_2)/Q^2; \tag{A24}$$

$$(R_{QQ})^{h}_{T,S}=-F_{S}(Q,\nu)G_{E}(Q,\nu)\frac{3Am}{4\pi^5 k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3k_1\,\theta(1-h)\,\theta(|\boldsymbol{h}+\boldsymbol{k}_1|-1)\,\theta(1-|\boldsymbol{h}+\boldsymbol{Q}|)F^2_{\pi N}(k_2)$$
$$\times\Big\{2F_{\pi N}(k_2)\frac{1}{k_2^2+\mu_\pi^2}[\tilde{g}'(k_2)+\tilde{h}'(k_2)][hh'+\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_2)]-F_{\pi N}(k_1)\frac{1}{k_1^2+\mu_\pi^2}2\tilde{g}'(k_2)[hh'-\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_1)]$$
$$+\tilde{h}'(k_2)\{2hh'(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)/\boldsymbol{k}_2^2-\mu_v[\boldsymbol{k}_2^2\boldsymbol{Q}^2-(\boldsymbol{Q}\cdot\boldsymbol{k}_2)^2]/\boldsymbol{k}_2^2\}\Big\}\frac{1}{\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}}I(\nu-k_1^2/2-\boldsymbol{h}\cdot\boldsymbol{k}_1,k_2), \tag{A25}$$

where

$$hh'=\boldsymbol{h}\cdot\boldsymbol{k}_2-(\boldsymbol{Q}\cdot\boldsymbol{h})(\boldsymbol{Q}\cdot\boldsymbol{k}_2)/Q^2; \tag{A26}$$

$$(R_{QQ,E})^{p}_{T,\Pi}=F_{\pi}(Q,\nu)G_E(Q,\nu)\frac{3Am}{8\pi^5k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3h'\int d^3k_2F_{\pi N}(k_1)F_{\pi N}(k_2)\,\theta(1-h)\,\theta(1-h')$$
$$\times\theta(|\boldsymbol{h}+\boldsymbol{k}_2|-1)\,\theta(|\boldsymbol{h}'+\boldsymbol{Q}-\boldsymbol{k}_2|-1)\,\theta(|\boldsymbol{h}'-\boldsymbol{k}_2|-1)\,\delta[\nu-(\boldsymbol{Q}^2/2+\boldsymbol{k}_2^2+\boldsymbol{h}'\cdot\boldsymbol{Q}-\boldsymbol{Q}\cdot\boldsymbol{k}_2-\boldsymbol{h}'\cdot\boldsymbol{k}_2+\boldsymbol{h}\cdot k_2)]$$
$$\times(g_0\{2(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)[\boldsymbol{k}_2\cdot(\boldsymbol{h}'-\boldsymbol{k}_2)-\boldsymbol{Q}\cdot(\boldsymbol{h}'-\boldsymbol{k}_2)(\boldsymbol{Q}\cdot\boldsymbol{k}_2)/Q^2]+\mu_v[\boldsymbol{k}_2^2\boldsymbol{Q}^2-(\boldsymbol{k}_2\cdot\boldsymbol{Q})^2]\})$$
$$\times\frac{1}{k_1^2+\mu_\pi^2}\,\frac{1}{k_2^2+\mu_\pi^2}\,\frac{1}{\nu-Q^2/2-\boldsymbol{h}'\cdot\boldsymbol{Q}+\boldsymbol{Q}\cdot\boldsymbol{k}_2}F^2_{\pi N}(|\boldsymbol{h}-\boldsymbol{h}'+\boldsymbol{k}_2|);\tag{A27}$$

$$(R_{QQ,E})^{h}_{T,\Pi}=-F_{\pi}(Q,\nu)G_E(Q,\nu)\frac{3Am}{8\pi^5k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3h'\int d^3k_2F_{\pi N}(k_1)F_{\pi N}(k_2)\,\theta(1-h)\,\theta(1-h')$$
$$\times\theta(1-|\boldsymbol{h}+\boldsymbol{Q}|)\,\theta(|\boldsymbol{h}'+\boldsymbol{Q}-\boldsymbol{k}_2|-1)\,\theta(|\boldsymbol{h}'+\boldsymbol{k}_2|-1)\,\delta[\nu-(\boldsymbol{Q}^2/2+\boldsymbol{k}_2^2+\boldsymbol{h}'\cdot\boldsymbol{Q}-\boldsymbol{Q}\cdot\boldsymbol{k}_2-\boldsymbol{h}'\cdot\boldsymbol{k}_2+\boldsymbol{h}\cdot k_2)]$$
$$\times(g_0\{2(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)[\boldsymbol{k}_2\cdot\boldsymbol{h}'-(\boldsymbol{Q}\cdot\boldsymbol{h})(\boldsymbol{Q}\cdot\boldsymbol{k}_2)/Q^2]+\mu_v[\boldsymbol{k}_2^2\boldsymbol{Q}^2-(\boldsymbol{k}_2\cdot\boldsymbol{Q})^2]\})\frac{1}{k_1^2+\mu_\pi^2}\,\frac{1}{k_2^2+\mu_\pi^2}\,\frac{1}{\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}}F^2_{\pi N}$$
$$\times(|\boldsymbol{h}-\boldsymbol{h}'+\boldsymbol{k}_2|);\tag{A28}$$

$$(R_{QQ,E})^{p}_{T,S}=-F_S(Q,\nu)G_E(Q,\nu)\frac{3Am}{8\pi^5k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3h'\int d^3k_2\,\theta(1-h)\,\theta(1-h')\,\theta(|\boldsymbol{h}+\boldsymbol{k}_2|-1)$$
$$\times\theta(|\boldsymbol{h}'+\boldsymbol{Q}-\boldsymbol{k}_2|-1)\,\theta(|\boldsymbol{h}'-\boldsymbol{k}_2|-1)\,\delta[\nu-(\boldsymbol{Q}^2/2+\boldsymbol{k}_2^2+\boldsymbol{h}'\cdot\boldsymbol{Q}-\boldsymbol{Q}\cdot\boldsymbol{k}_2-\boldsymbol{h}'\cdot\boldsymbol{k}_2+\boldsymbol{h}\cdot\boldsymbol{k}_2)]g_0$$
$$\times\left\{F_{\pi N}(k_2)\frac{1}{k_2^2+\mu_\pi^2}[hh+\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_2)]+F_{\pi N}(k_1)\frac{1}{k_1^2+\mu_\pi^2}[hh-\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_1)]\right\}\frac{1}{\nu-Q^2/2-\boldsymbol{h}'\cdot\boldsymbol{Q}+\boldsymbol{Q}\cdot\boldsymbol{k}_2}$$
$$\times F^2_{\pi N}(|\boldsymbol{h}-\boldsymbol{h}'+\boldsymbol{k}_2|),\tag{A29}$$

where

$$hh=(\boldsymbol{h}'-\boldsymbol{k}_2)\cdot\boldsymbol{k}_2-\boldsymbol{Q}\cdot(\boldsymbol{h}'-\boldsymbol{k}_2)(\boldsymbol{Q}\cdot\boldsymbol{k}_2)/Q^2;\tag{A30}$$

$$(R_{QQ,E})^{h}_{T,\Pi}=-F_S(Q,\nu)G_E(Q,\nu)\frac{3Am}{8\pi^5k_F^2\Omega}\left(\frac{f_\pi^2}{4\pi\hbar c}\right)^2\frac{1}{\mu_\pi^4}\int d^3h\int d^3h'\int d^3k_2\,\theta(1-h)\,\theta(1-h')\,\theta(1-|\boldsymbol{Q}+\boldsymbol{h}|)$$
$$\times\theta(|\boldsymbol{h}'+\boldsymbol{Q}-\boldsymbol{k}_2|-1)\,\theta(|\boldsymbol{h}+\boldsymbol{k}_2|-1)\,\delta[\nu-(\boldsymbol{Q}^2/2+\boldsymbol{k}_2^2+\boldsymbol{h}'\cdot\boldsymbol{Q}-\boldsymbol{Q}\cdot\boldsymbol{k}_2-\boldsymbol{h}'\cdot\boldsymbol{k}_2+\boldsymbol{h}\cdot k_2)]g_0$$
$$\times\left\{F_{\pi N}(k_2)\frac{1}{k_2^2+\mu_\pi^2}[hh+\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_2)]+F_{\pi N}(k_1)\frac{1}{k_1^2+\mu_\pi^2}[hh-\mu_v(\boldsymbol{Q}\cdot\boldsymbol{k}_1)]\right\}\frac{1}{\nu-Q^2/2-\boldsymbol{h}\cdot\boldsymbol{Q}}F^2_{\pi N}(|\boldsymbol{h}-\boldsymbol{h}'+\boldsymbol{k}_2|),\tag{A31}$$

where

$$hh=\boldsymbol{h}\cdot\boldsymbol{k}_2-(\boldsymbol{Q}\cdot\boldsymbol{h})(\boldsymbol{Q}\cdot\boldsymbol{k}_2)/Q^2.\tag{A32}$$

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