Isoscalar giant quadrupole resonance state in a relativistic approach with the momentum-dependent self-energies

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We study the excitation energy of the isoscalar giant quadrupole resonance by the scaling method in the relativistic many-body framework with the momentum dependent parts of the Dirac self-energies arising from the one-pion exchange. It is shown that this momentum dependence enhances the Landau mass significantly and thus suppresses the quadrupole resonance energy while the Dirac effective mass is kept to a reasonable value.

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The past decades have seen many successes in the relativistic treatment of the nuclear many-body problem. The relativistic framework has big advantages in several aspects [1]: a useful Dirac phenomenology for the description of nucleon-nucleus scattering [2,3], the natural incorporation of the spin-orbit force [1] and the structure of extreme nuclei [4]. These results have shown that there are large attractive scalar and repulsive vector fields, and that the nucleon effective mass becomes small in the medium.

The small effective mass enlarges the Fermi velocity, and it causes troubles in some observables. For example, the isoscalar giant quadrupole resonance (ISGQR) state is predicted at too high an excitation energy due to the large Fermi velocity [5]. In this subject, it is assumed that the momentum dependence of the Dirac fields is negligible in the low energy region, particularly below the Fermi level. In fact, only very small momentum dependence has appeared in the relativistic Hartree-Fock (RHF) calculation though the Fock contribution is not small [6,7]. Thus the Fock contributions are thought to be incorporated into the relativistic Hartree (RH) approximation by introducing complicated density-dependent interaction and fitting parameters [4,6,8].

On the other hand, the vector-fields must become very small in the high energy region to explain the saturation of the optical potential for proton-nucleus elastic scattering [2,9] and the transverse flow in the heavy-ion collisions [10]. A Dirac-Bruckner-Hartree-Fock (DBHF) calculation has shown that the momentum dependence changes the nuclear equation of state noticeably [11]. Furthermore, Weber *et al.* [9] have suggested that the Fermi velocity does not correspond to the effective mass uniquely when introducing the momentum dependence of the Dirac fields.

We can easily suppose that it is the one-pion exchange force which produces the major momentum dependence because the interaction range is largest. In this paper, thus, we introduce the momentum dependence to the Dirac fields due to the one-pion exchange, and discuss how the Fock parts given by the one-pion exchange affects the excitation energy of ISGQR. Actually we use the scaling method in the way of Ref. [5] which is proved to give consistent results with RPA for the giant multipole states in the nonrelativistic framework [12]. This relation has been confirmed also in the relativistic framework for the monopole vibration mode [13].

Let us consider infinite nuclear matter system with the isospin symmetry. The nucleon propagator in the self-energy Σ is given by

$$S^{-1}(p)=\not{p}-M-\Sigma(p), \tag{1}$$

where $\Sigma(p)$ has a Lorentz scalar part $U_s(p)$ and a Lorentz vector part $U_\mu(p)$ as $\Sigma(p)=-U_s(p)+\gamma^\mu U_\mu(p)$. For the future convenience we define the effective mass and the kinetic momentum as follows:

$$M^*(p)=M-U_s(p),$$
$$\Pi_\mu(p)=p_\mu-U_\mu(p). \tag{2}$$

Using the on-mass-shell condition, $\Pi^2(p)-M^{*2}(p)=0$, the single particle energy with momentum $\boldsymbol{p}$ is defined as

$$\varepsilon(\boldsymbol{p})=p_0|_{\text{on-mass-shell}}=\sqrt{\Pi^2(\boldsymbol{p})+M^{*2}(\boldsymbol{p})}+U_0(\boldsymbol{p}). \tag{3}$$

Next we consider the variation of the total energy in the quadrupole deformation in order to discuss ISGQR with the scaling method. Using the scaling method, first, we vary the density-distribution from the normal nuclear matter distribution $\tilde{\rho}_0(\boldsymbol{r})$ as

$$\tilde{\rho}_0(\boldsymbol{r})\to\tilde{\rho}_0\lambda(\boldsymbol{r})=\tilde{\rho}_0(e^{-\lambda}x,e^{-\lambda}y,e^{2\lambda}z). \tag{4}$$

In the uniform nuclear matter it is equivalent to the variation of the momentum distribution $n(\boldsymbol{p})$ as

$$n_0(\boldsymbol{p})\to n(\boldsymbol{p})=n_\lambda(\boldsymbol{p})=n_0(\boldsymbol{p}_\lambda)=n_0(e^\lambda p_x,e^\lambda p_y,e^{-2\lambda}p_z), \tag{5}$$

where $n_0(\boldsymbol{p})=\theta(p_F-|\boldsymbol{p}|)$ with the Fermi momentum p_F at the saturation density ρ_0.

With the variation of the momentum distribution as $n_0(\boldsymbol{p})\to n_0(\boldsymbol{p})+\delta n_{\boldsymbol{p}}$, the single-particle energy $\varepsilon(\boldsymbol{p})$ is obtained in the Hartree-Fock framework by

$$\varepsilon(\boldsymbol{p})=\frac{\delta E_T}{\delta n_{\boldsymbol{p}}}=\sqrt{\Pi(p)^2+M^{*2}(p)}+U_0(p)|_{p_0=\varepsilon(\boldsymbol{p})}. \tag{6}$$

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From this relation we get the following equations:

$$\left.\frac{\partial E_T}{\partial \lambda}\right|_{\lambda=0} = 4\Omega \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left\{ \left.\frac{\partial n(\boldsymbol{p})}{\partial \lambda}\right|_{\lambda=0} \right\} \varepsilon(\boldsymbol{p}) = 0, \tag{7}$$

$$\left.\frac{\partial^2 E_T}{\partial \lambda^2}\right|_{\lambda=0} = 4\Omega \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left\{ \left.\frac{\partial n(\boldsymbol{p})}{\partial \lambda}\right|_{\lambda=0} \right\} \{ D_\lambda \varepsilon(\boldsymbol{p})|_{\lambda=0} \}. \tag{8}$$

The derivative of the single particle energy ε can be written as

$$D_\lambda \varepsilon(\boldsymbol{p}_\lambda, \lambda) = \frac{\partial \boldsymbol{p}_\lambda}{\partial \lambda} \nabla_{\boldsymbol{p}_\lambda} \varepsilon + \frac{\partial \varepsilon}{\partial \lambda}, \tag{9}$$

where the total derivative $\nabla_{\boldsymbol{p}}$ is defined on the on-mass-shell condition: $p_0 = \varepsilon(\boldsymbol{p})$. In this equation the second term of the right-hand side $\partial\varepsilon/\partial\lambda$ corresponds to the derivative with the variation of the self-energies at the fixed momentum by changing the momentum distribution. This term holds the spherical symmetry at the limit of $\lambda \to 0$, and does not contribute to the integral of the right-hand side in Eq. (8).

Substituting Eq. (9) into Eq. (8), hence, the restoring force of ISGQR C_Q becomes

$$\begin{aligned} C_Q &= \left.\frac{\partial^2 E_T/A}{\partial \lambda^2}\right|_{\lambda=0} \\ &= -\frac{4\Omega}{A} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \delta(|\boldsymbol{p}| - p_F) \left(\frac{\boldsymbol{p}^2 - 3p_z^2}{|\boldsymbol{p}|} \right) \left(\frac{\boldsymbol{p}^2 - 3p_z^2}{M_L^*} \right) \\ &= \frac{12}{5} \frac{p_F^2}{M_L^*}, \end{aligned} \tag{10}$$

where

$$M_L^* = \left. \left(2 \frac{d}{d\boldsymbol{p}^2} \varepsilon(\boldsymbol{p}) \right)^{-1} \right|_{|\boldsymbol{p}|=p_F}, \tag{11}$$

which is the so-called ''*Landau mass*'' corresponding to the effective mass in the nonrelativistic framework.

In Ref. [5] the mass parameter of ISGQR is given as $B_Q = 2\varepsilon_F \langle \boldsymbol{r}^2 \rangle$ with the Fermi energy ε_F, and then the frequency of ISGQR is obtained as

$$\omega_Q = \sqrt{\frac{C_Q}{B_Q}} = \sqrt{\frac{6p_F^2}{5M_L^* \varepsilon_F \langle \boldsymbol{r}^2 \rangle}}. \tag{12}$$

This expression is the same as that of Ref. [5] with RH except the Landau mass; $M_L^* = \sqrt{p_F^2 + M^{*2}}$ in RH.

At the saturation density, the Fermi energy agrees with the total energy per nucleon whose value is almost the same as that of nucleon mass: $\varepsilon_F = E_T/A \approx M$, In addition, nuclear radii are scaled to be proportional to $A^{1/3}$ as $\langle \boldsymbol{r}^2 \rangle = 3/5 r_0^2 A^{2/3}$ and then we get the frequency ω_Q as

$$\omega_Q \approx \sqrt{\frac{4\langle T_K^{nr} \rangle}{M_L^* r_0^2}} A^{-1/3}, \tag{13}$$

where $\langle T_K^{nr} \rangle$ is the nonrelativistic averaged kinetic energy as $\langle T_K^{nr} \rangle = \langle \boldsymbol{p}^2/(2M) \rangle$. This expression completely coincides with that of the nonrelativistic model [14]

We substitute the empirical experimental values $\omega_Q \approx 63 A^{-1/3}$ MeV, $\langle T_K^{nr} \rangle \approx 25$ MeV and $r_0 \approx 1.125$ fm into Eq. (13) and obtain the value of the Landau mass [15] as

$$M_L^*/M \approx 0.85. \tag{14}$$

Consequently we do not find any difference in the expression of ISGQR between the relativistic and nonrelativistic frameworks. The main problem is whether we can give the above value of M_L^* while keeping consistency to other observables; the usual analyses indicate that $M^*/M = 0.55 - 0.7$, which gives $M_L^* \approx 0.6 - 0.75$ in RH.

As a next step we explain the details of our calculation in this work. We consider a model of the usual σ-ω modelplus the Fock part of the one-pion exchange.

Along this line we define a Lagrangian density in the system as

$$\begin{aligned} \mathcal{L} = &\bar{\psi}(i\partial\!\!\!/ - M)\psi + \frac{1}{2}\partial_\mu \phi_a \partial^\mu \phi_a - \frac{1}{2} m_\pi^2 \phi_a \phi_a \\ &+ \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \tilde{U}[\sigma] - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\ &+ i \frac{f_\pi}{m_\pi} \bar{\psi} \gamma_5 \gamma^\mu \tau_a \psi \partial_\mu \phi_a + g_\sigma \bar{\psi}\psi\sigma - g_\omega \bar{\psi}\gamma_\mu \psi \omega^\mu, \end{aligned} \tag{15}$$

with $\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, where ψ, ϕ, σ and ω are the nucleon, pion, sigma-meson and omega-meson fields, respectively. In the above expression we use the pseudovector coupling form as an interaction between nucleon and pion. The self-energy potential of the σ field $\tilde{U}[\sigma]$ is taken here [10] as

$$\tilde{U}[\sigma] = \frac{\frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} B_\sigma \sigma^3 + \frac{1}{4} C_\sigma \sigma^4}{1 + \frac{1}{2} A_\sigma \sigma^2}. \tag{16}$$

The symbols m_π, m_σ, and m_ω are the masses of π, σ, and ω mesons, respectively.

Next we calculate the nucleon self-energies. The nucleon self-energies are separated into the local part and the momentum-dependent part as $U_\alpha(p) = U_\alpha^L + U_\alpha^{MD}(p)$, where $\alpha = s, \mu$. The σ- and ω-meson exchange parts produce only very small momentum dependence of nucleon self-energies [6,7] as their masses are large. In fact the RH and RHF approximations do not give any different results in nuclear matter properties after fitting parameters of σ and ω exchanges [6]. On the other hand the one-pion exchange force is a long range one, and makes for a large momentum dependence while it does not contribute to the local part in the spin-saturated system. Thus we make the local part by RH of the σ- and ω-meson exchanges, and the momentum-dependent part by RHF of the pion exchange. Such a separated method can keep the consistency for the energy-momentum tensor [9].

In this model the local part of the self-energies are given as

$$U_s^L = g_\sigma \langle \sigma \rangle, \tag{17}$$

$$U_\mu^L = \delta_{0\mu} \frac{g_\omega{}^2}{m_\omega^2} \rho_H, \tag{18}$$

where $\langle \sigma \rangle$ is the scalar mean-field obtained as

$$\frac{\partial}{\partial \langle \sigma \rangle} \tilde{U}[\langle \sigma \rangle] = g_\sigma \rho_s. \tag{19}$$

In the above equations the scalar density ρ_s and the vector Hartree density ρ_H are given by

$$\rho_s = 4 \int \frac{d^3 \boldsymbol{p}}{(2\pi)^3} n(\boldsymbol{p}) \frac{M_\alpha^*(p)}{\tilde{\Pi}_0(p)}, \tag{20}$$

$$\rho_H = 4 \int \frac{d^3 \boldsymbol{p}}{(2\pi)^3} n(\boldsymbol{p}) \frac{\Pi_0(p)}{\tilde{\Pi}_0(p)}, \tag{21}$$

where $n(\boldsymbol{p})$ is the momentum distribution, and $\tilde{\Pi}_\mu(p)$ is defined by

$$\tilde{\Pi}_\mu(p) = \frac{1}{2} \frac{\partial}{\partial p^\mu} [\Pi^2(p) - M^{*2}(p)]. \tag{22}$$

As a next step we define the momentum-dependent parts of the self-energies as the Fock parts with the one-pion exchange. When using the pseudovector (PV) coupling the Fock parts do not become zero at the infinite limit of the momentum $|\boldsymbol{p}|$. One usually erases these contributions by introducing the cutoff parameter. In this work, instead of that, we subtract these contributions from the momentum-dependent parts (these contributions can be renormalized into the Hartree parts): $U_\alpha \to U_\alpha - U_\alpha(p \to \infty)$. Thus we obtain the momentum-dependent parts of the self-energies as

$$U_s^{MD}(p) = \frac{3 f_\pi^2}{2} \int \frac{d^3 \boldsymbol{k}}{(2\pi)^3} n(\boldsymbol{k}) \frac{M^*(k)}{\tilde{\Pi}_0(k)} \Delta_\pi(p-k), \tag{23}$$

$$U_\mu^{MD}(p) = -\frac{3 f_\pi^2}{2} \int \frac{d^3 \boldsymbol{k}}{(2\pi)^3} n(\boldsymbol{k}) \frac{\Pi_\mu(k)}{\tilde{\Pi}_0(k)} \Delta_\pi(p-k), \tag{24}$$

where the $\Delta_\pi(q)$ is the pion propagator defined as

$$\Delta_\pi(q) = \frac{1}{q^2 - \mathrm{m}_\pi^2}. \tag{25}$$

In the above vector self-energies we omit the tensor-coupling part involving $[\Pi(k) \cdot (p-k)](p-k)_\mu$. This term is very small if the self-energy is independent of momentum [6], and their momentum dependence is actually very small as shown later.

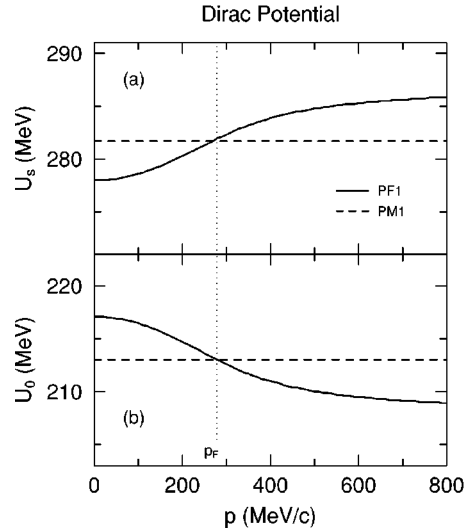

FIG. 1. Momentum dependence of the scalar (a) and vector (b) self-energies. The solid and dashed lines indicate the results in PF1 and PM1, respectively. The dotted line denotes the position of the Fermi momentum at $\rho_B = \rho_0$.

Using the above formulation we get the total energy density of the spin-isospin saturated nuclear matter with the momentum distribution $n(\boldsymbol{p})$ as

$$E_T/\Omega = 4 \int \frac{d^3 \boldsymbol{p}}{(2\pi)^3} n(\boldsymbol{p}) \varepsilon(\boldsymbol{p}) + \tilde{U}[\sigma] + 2 \int \frac{d^3 \boldsymbol{p}}{(2\pi)^3} n(\boldsymbol{p}) \frac{M^*(p) U_s^{MD}(p) - \Pi_\mu(p) U^\mu(p)}{\tilde{\Pi}_0(p)}, \tag{26}$$

with the system volume Ω.

Let us show the calculated results in our model using the momentum-dependent Dirac self-energies with the one-pion exchange. In this calculation we fit the parameters (PF1) for the σ and ω exchanges to reproduce the saturation properties that the binding energy $BE = 16$ MeV, the incompressibility $K = 200$ MeV and the effective mass $M^*/M = 0.7$ at the saturation density $\rho_0 = 0.17\ \mathrm{fm}^{-3}$.

In Fig. 1 we draw the momentum dependence of the scalar self-energy $U_s(p)$ and the time component of the vector self-energy $U_0(p)$. It can be seen that the variation of the momentum-dependent self-energies is only 2.5% at most below Fermi level, which looks very small.

In Fig. 2 we show the density dependence of the Dirac self-energies U_s and U_0 on the Fermi surface (a) and the Landau mass (b) with the parameter sets of PF1 and PM1. Though two results of U_s and U_0 almost agree together, we can see rather large difference in the Landau mass: the value at $\rho_B = \rho_0$ is $M_L^*/M = 0.85$ in PF1 which is consistent with the value expected by the analysis of ISGQR as shown previously. On the contrary, the momentum-independent calculation (PM1) gives $M_L^*/M = 0.74$ which overestimates the excitation energy of ISGQR. Hence it is shown that the very small momentum dependence in the nucleon self-energies enhances the Fermi velocity about 15%, and gives a significant difference in the Landau mass. Furthermore we can also see an interesting behavior of M_L^* in PF1, namely, its value agrees with the bare mass at $\rho_B \approx 0.5\rho_0$ and becomes larger with the decrease of the density. Effects of small Dirac effective mass are largely canceled at low density by the momentum dependence created by the one-pion exchange. This fact implies that the nonlocality of the self-energies affects nuclear surface properties such as the isovector magnetic moment, whose value is still larger than the Schmidt value [16].

Here we should give a further comment. Benz *et al.* have shown in Ref. [17] that the Landau mass is reduced by the one-pion exchange, which is opposite to ours. In this calculation Benz *et al.* have used the pseudoscalar (PS) coupling, and the sign of U_μ^{MD} was taken to be opposite to ours. The full HF calculation with the PS coupling makes too large a contribution to the Dirac self-energies [3] while Bentz *et al.* calculated the Fock term the perturbative way. Thus a calculation with the PV coupling must be more reliable than that with the PS coupling.

We still have some ambiguities in this work. For example the bulk density of finite nuclei is smaller than the saturation density, so that we may discuss the value of the Landau mass at lower density. In Ref. [18], furthermore, it has been reported that the time-dependent mean-field calculations have explained the excited energies of ISGQR for ^{16}O and ^{40}Ca. These results are inconsistent with the macroscopic theory. The treated nuclei may be too small or their calculations involve other correlation beyond the macroscopic theory. Thus we should not make a quantitative conclusion on the ISGQR state before investigating it in finite nuclei. Though we still have ambiguities, nevertheless, we can conclude that the momentum dependence largely affects the Fermi velocity, particularly in the low density region.

Here we should note that although the introduction of the momentum dependence changes the Landau mass, the depth of self-energies and hence the density dependence of the total energy are affected very little. Thus the approximation to neglect the nonlocality of the Dirac field should be correct in discussions of many aspects of the nuclear structure in the Dirac approach. However, as for some physical quantities such as Fermi velocity, we will have to take account of the nonlocality effects in the Dirac approach. This effect cannot be involved even if the density-dependent parameters are introduced into the RH approximation [4,8].

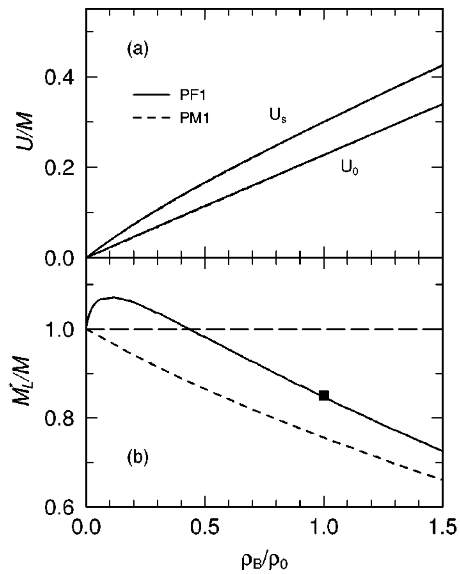

FIG. 2. Density dependence of the Dirac self-energies U_s and U_0 on the Fermi surface (a) and the Landau mass (b). The solid and dashed lines indicate the results for PF1 and PM1, respectively, and the full square in (b) denotes the value expected empirically from ISGQR.

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