Higher order polarizabilities of the proton

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Compton scattering results are used to probe proton structure via measurement of higher order polarizabilities. Values for α^p_{E2}, β^p_{E2}, $\alpha^p_{E\nu}$, and $\beta^p_{E\nu}$ determined via dispersion relations are compared to predictions based upon chiral symmetry and from the constituent quark model. Extensions to spin polarizabilities are also discussed.

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I. INTRODUCTION

Recently the availability of high intensity electron facilities and tagged photon beams has allowed proton structure to be probed by means of Compton scattering [1]. In the case of photons with wavelength much larger than the size of the target, only the overall charge is resolvable. Then, to lowest order the effective Hamiltonian is

$$H^{(0)}_{eff}=\frac{(\vec{p}-e\vec{A})^2}{2M}, \tag{1}$$

and the spin-averaged amplitude for Compton scattering on the proton is given simply by the familiar Thomson form

$$\mathrm{Amp}^{(0)}=-\frac{e^2}{M}\hat{\epsilon}_1\cdot\hat{\epsilon}_2, \tag{2}$$

where e and M represent the proton charge and mass and $\hat{\epsilon}_1,\hat{\epsilon}_2$ and $k_1^\mu=(\omega,\vec{k}_1),k_2^\mu=(\omega,\vec{k}_2)$ specify the polarization vectors and four-momenta of the initial and final photons, respectively. At higher energies (and shorter wavelengths) the structure of the system begins to be observable. The corresponding effective Compton scattering Hamiltonian must be quadratic in the vector potential and be gauge invariant, so it must be written in terms of the electric and magnetic fields. It must also be a rotational scalar and invariant under parity and time reversal transformations. Consequently, the simplest form is [2]

$$H^{(2)}_{eff}=-\frac{1}{2}4\pi\alpha^p_E\vec{E}^2-\frac{1}{2}4\pi\beta^p_M\vec{H}^2, \tag{3}$$

and with the definitions of the electric and magnetic dipole moments,

$$\vec{p}=-\frac{\delta H^{(2)}_{eff}}{\delta\vec{E}}=4\pi\alpha^p_E\vec{E},\quad \vec{\mu}=-\frac{\delta H^{(2)}_{eff}}{\delta\vec{H}}=4\pi\beta^p_M\vec{H}, \tag{4}$$

we recognize α^p_E and β^p_M as the electric and magnetic polarizabilities, respectively, which measure the response of the proton to quasistatic electric and magnetizing fields. The corresponding $\mathcal{O}(\omega^2)$ Compton scattering amplitude becomes

$$\mathrm{Amp}^{(2)}=\hat{\epsilon}_1\cdot\hat{\epsilon}_2\left(\frac{-e^2}{M}+\omega^2 4\pi\alpha^p_E\right)+\hat{\epsilon}_1\times\hat{k}_1\cdot\hat{\epsilon}_2\times\hat{k}_2\omega^2 4\pi\beta^p_M+\mathcal{O}(\omega^4), \tag{5}$$

and the resultant differential scattering cross section is

$$\frac{d\sigma}{d\Omega}=\left(\frac{\alpha}{M}\right)^2\left[\frac{1}{2}(1+\cos^2\theta)-\frac{M\omega^2}{\alpha}\left(\frac{1}{2}(\alpha^p_E+\beta^p_M)(1+\cos\theta)^2+\frac{1}{2}(\alpha^p_E-\beta^p_M)(1-\cos\theta)^2\right)+\cdots\right], \tag{6}$$

where $\alpha=e^2/4\pi$ is the fine structure constant. Thus by measurement of the differential Compton scattering cross section one can extract the electric and magnetic polarizabilities, provided (i) the energy is large enough that such terms are significant with respect to the Thomson contribution, but (ii) not so large that higher order effects begin to dominate. This extraction via the $\gamma p\rightarrow\gamma p$ reaction has been accomplished using measurements in the energy regime 50 MeV$<\omega<100$ MeV, yielding [1]

$$\alpha^p_E=(12.1\pm0.8\pm0.5)\times10^{-4}\ \ \mathrm{fm}^3,$$

$$\beta^p_M=(2.1\mp0.8\mp0.5)\times10^{-4}\ \ \mathrm{fm}^3. \tag{7}$$

Note that in practice one generally uses the results of unitarity and the validity of the forward ($t=0$) scattering dispersion relation, which yields the Baldin sum rule [3]

$$\alpha^p_E+\beta^p_M=14.2\pm0.5\quad (\text{Ref. [4]})$$

$$=13.69\pm0.14\quad (\text{Ref. [5]}) \tag{8}$$

as a constraint, since the uncertainty associated with the integral over the photoabsorption cross section $\sigma_{\mathrm{tot}}(\omega)$ is smaller than that associated with the polarizability measurements.

On the theoretical side, at the crudest level, we observe that the size of α_E^p reveals the feature that the nucleon is strongly bound. Indeed for the hydrogen atom the electric polarizability is of the order of the atomic volume [6]

$$\alpha_E^{\text{H atom}}=\frac{27}{8\pi}\left(\frac{4}{3}\pi a_0^3\right), \tag{9}$$

where $a_0=1/m_e\alpha$ is the Bohr radius. On the other hand, Eq. (7) shows that for the proton

$$\alpha_E^p\sim 4\times 10^{-4}\times\left(\frac{4}{3}\pi\langle r_p^2\rangle^{3/2}\right). \tag{10}$$

More quantitative investigations generally involve one of two techniques. The first involves use of a nonrelativisitic constituent quark picture of the proton and the quantum mechanical sum rule [7]

$$\alpha_E^p=\frac{\alpha}{3M}\left\langle 0\left|\sum_{i=1}^{3} e_i(\vec{r}_i-\vec{R}_{c.m.})^2\right|0\right\rangle + 2\alpha\sum_{n\neq 0}\frac{\left|\left\langle n\left|\sum_{i=1}^{3} e_i(\vec{r}_i-\vec{R}_{c.m.})_z\right|0\right\rangle\right|^2}{E_n-E_0}, \tag{11}$$

where α is the fine structure constant, while e_i and $\vec{r}_i$ denote the charge (measured in units of the proton charge) and position of the ith constituent quark, and $|0\rangle$ represents the ground state. In this case the simple harmonic oscillator model of nucleon structure is found to be somewhat too simplistic, since when the oscillator frequency is fitted to the charge radius via

$$\omega_0=\sqrt{\frac{3}{M\langle r_p^2\rangle}}\approx 180\ \text{MeV} \tag{12}$$

the predicted size of the polarizability

$$\alpha_E^p=\frac{2\alpha M}{9}\langle r_p^2\rangle^2\approx 35\times 10^{-4}\ \text{fm}^3 \tag{13}$$

is a factor of 3 or so too large.

The failure here is associated with the low value of the oscillator frequency given by Eq. (12), and use of a more realistic excitation energy $\omega_0\simeq 300$ MeV yields a value in the right ballpark. However, the real solution to this problem requires going beyond the simple constituent quark picture of the proton to consider meson cloud structure [8]—i.e., a proper treatment of the pionic degrees of freedom—and suggests the efficacy of the second approach—heavy baryon chiral perturbation theory (HBχPT) [9]. Using this technique, one finds at $\mathcal{O}(p^3)$ in the chiral expansion [10]

$$\alpha_E^p=10K_p=12.7\times 10^{-4}\ \text{fm}^3,\quad \beta_M^p=K_p=1.3\times 10^{-4}\ \text{fm}^3, \tag{14}$$

where $K_p=\alpha g_A^2/192\pi F_\pi^2 m_\pi$. Here $g_A\simeq 1.266$ is the axial coupling constant in neutron beta decay and $F_\pi \simeq 92.4$ MeV is the pion decay constant. This $\mathcal{O}(p^3)$ calculation represents only the leading result for α_E^p and β_M^p in HBχPT but gets the qualitative features of the polarizabilities right and even agrees with experiment. The results diverge as $1/m_\pi$ in the chiral limit, giving support to the idea that at these low energies the photon interacts primarily with the long-range pion cloud of the nucleon. Of course, one must include higher order terms in order to properly judge the convergence behavior of the series, and such a calculation at $\mathcal{O}(p^4)$ has been performed by Bernard, Kaiser, Schmidt, and Meißner (BKSM) [11]. At this order counterterms are required, which were estimated by BKSM by treating higher resonances—including $\Delta(1232)$—as very heavy with respect to the nucleon, yielding

$$\alpha_E^p=(10.5\pm 2.0)\times 10^{-4}\ \text{fm}^3,$$

$$\beta_M^p=(3.5\pm 3.6)\times 10^{-4}\ \text{fm}^3, \tag{15}$$

where the uncertainty is associated with the counterterm contribution from the $\Delta(1232)$ and from K,η loop effects.

An alternative tack has been pursued by Hemmert *et al.*, who have developed a chiral expansion—the small scale or ''ϵ'' expansion—wherein the $\Delta(1232)$ is included as an *explicit* degree of freedom and which involves taking $\Delta\equiv M_\Delta-M_N$ as an additional ''small'' parameter [12]. In this approach, one finds new contributions to the $\mathcal{O}(p^3)$ predictions [13]:

$$\delta\alpha_E^p=\frac{L_p}{6}\left(\frac{9\Delta}{\Delta^2-m_\pi^2}+\frac{\Delta^2-10m_\pi^2}{(\Delta^2-m_\pi^2)^{3/2}}\ln R\right),$$

$$\delta\beta_M^p=\frac{8}{9}\,\frac{b_1^2\alpha}{M^2\Delta}+\frac{L_p}{6}\,\frac{1}{(\Delta^2-m_\pi^2)^{1/2}}\ln R, \tag{16}$$

where $L_p=g_{\pi N\Delta}^2\alpha/9\pi^2F_\pi^2$ with $g_{\pi N\Delta}$ being the $\pi N\Delta$ coupling constant, b_1 the corresponding coupling for radiative $\Delta(1232)$ decay, and

$$R=\frac{\Delta}{m_\pi}+\sqrt{\frac{\Delta^2}{m_\pi^2}-1}. \tag{17}$$

From the experimentally obtained size of the $\Delta\to N\pi$ and $\Delta\to N\gamma$ widths, one determines $g_{\pi N\Delta}=1.05\pm 0.2$, $b_1=-1.93\pm 0.1$. Use of these numbers then results in an increase in the predicted electric polarizability of about 30% and takes us away from experimental agreement at $\mathcal{O}(\epsilon^3)$. However, BKSM have shown that there exists a sizable negative $\mathcal{O}(p^4)$ $N\pi$ loop contribution which tends to cancel this discrepancy.

With respect to the magnetic polarizability, the simple quark model *does* provide a basic understanding. The prediction [7]

$$\beta_M^p = -\frac{\alpha}{2M}\left\langle \sum_i e_i(\vec{r}_i - \vec{R}_{c.m.})^2 \right\rangle - \frac{\alpha}{6}\left\langle \sum_i e_i^2(\vec{r}_i - \vec{R}_{c.m.})^2/m_i \right\rangle + 2\alpha \sum_{n\neq 0} \frac{\left|\left\langle n\left|\sum_i (e_i/2m_i)\sigma_{iz}\right|0\right\rangle\right|^2}{E_n - E_0} \tag{18}$$

involves a substantial diamagnetic recoil contribution

$$^{\text{dia}}\beta_M^p = -10.2\times 10^{-4} \ \ \text{fm}^3, \tag{19}$$

which, when added to the large paramagnetic pole contribution due to the $\Delta(1232)$ [15],

$$^{\Delta}\beta_M^p = 12\times 10^{-4} \ \ \text{fm}^3, \tag{20}$$

yields results in basic agreement with the experimental findings. It is clear then that proper inclusion of the $\Delta(1232)$ degrees of freedom is essential.

The above summary is intended only as a brief review of the subject and is not presumed to represent a substitute for more detailed discussions such as found in Ref. [16]. However, it does reveal how important structure information can be obtained via measurement of the static polarizabilities. The purpose of the present paper is to ask whether it is possible to use Compton scattering in order to provide additional proton structure information via the use of *higher order* polarizabilities. Specifically, in the next section we define and generate theoretical predictions for four new spin-averaged polarizabilities which arise at $\mathcal{O}(\omega^4)$ in the expansion of the Compton scattering amplitude. Then in Sec. III we show how such quantities can be extracted from existing experimental data using fixed-t dispersion relations and confront the values obtained thereby with theoretical expectations. In Sec. IV we extend our discussion to the case of spin polarizabilities, and we conclude with a brief section summarizing our findings.

II. QUADRATIC POLARIZABILITIES

As outlined above, the electric and magnetic polarizabilities arise as $\mathcal{O}(\omega^2)$ corrections to the lowest order (Thomson) scattering amplitude. If one extends the analysis to consider spin-averaged $\mathcal{O}(\omega^4)$ terms, then four new structures are possible which obey the requirements of gauge, P, and T invariance. These can be written in the form [17]

$$H_{eff}^{(4)} = -\frac{1}{2}4\pi\alpha_{E\nu}^p \dot{\vec{E}}^2 - \frac{1}{2}4\pi\beta_{M\nu}^p \dot{\vec{H}}^2 - \frac{1}{12}4\pi\alpha_{E2}^p E_{ij}^2 - \frac{1}{12}4\pi\beta_{M2}^p H_{ij}^2, \tag{21}$$

where

$$E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i), \quad H_{ij} = \frac{1}{2}(\nabla_i H_j + \nabla_j H_i) \tag{22}$$

denote electric and magnetizing field gradients. The physical meaning of the new terms is clear from their definition—Eq. (21). The quantities $\alpha_{E\nu}^p$ and $\beta_{M\nu}^p$ represent dispersive corrections to the lowest order static polarizabilities α_E and β_M and describe the response of the system to time-dependent fields via (in frequency space)

$$\vec{p}(\omega) = 4\pi\alpha_E^p(\omega)\vec{E}(\omega) = 4\pi(\alpha_E^p + \alpha_{E\nu}^p\omega^2 + \cdots)\vec{E}(\omega),$$
$$\vec{\mu}(\omega) = 4\pi\beta_M^p(\omega)\vec{H}(\omega) = 4\pi(\beta_M^p + \beta_{M\nu}^p\omega^2 + \cdots)\vec{H}(\omega). \tag{23}$$

The parameters α_{E2}^p and β_{M2}^p, on the other hand, represent quadrupole polarizabilities and measure the electric and magnetic quadrupole moments induced in a system in the presence of an applied field gradient via

$$Q_{ij} = \frac{\delta H_{eff}^{(4)}}{\delta E_{ij}} = \frac{1}{6}4\pi\alpha_{E2}^p E_{ij}, \quad M_{ij} = \frac{\delta H_{eff}^{(4)}}{\delta H_{ij}} = \frac{1}{6}4\pi\beta_{M2}^p H_{ij}, \tag{24}$$

where

$$Q_{ij} = \left\langle \psi \left| \sum_k e_k[3(\vec{r}_k - \vec{R}_{\text{c.m.}})_i(\vec{r}_k - \vec{R}_{\text{c.m.}})_j - \delta_{ij}(\vec{r}_k - \vec{R}_{\text{c.m.}})^2] \right| \psi \right\rangle \tag{25}$$

indicates the induced proton electric quadrupole moment and M_{ij} is its magnetic analog. Here $|\psi\rangle$ represents the proton state in the presence of the applied field gradient.

As in the case of the ordinary polarizabilities one can attempt to predict the size of these quantities in two somewhat orthogonal ways. For example, using the sum rules

$$\alpha_{E\nu}^p = 2\alpha\sum_{n\neq 0}\frac{|z_{n0}|^2}{(E_n - E_0)^3}, \quad \alpha_{E2}^p = \frac{\alpha}{2}\sum_{n\neq 0}\frac{|(Q_{zz})_{n0}|^2}{E_n - E_0}, \tag{26}$$

and the simple oscillator picture, one finds the predictions

$$\alpha_{E\nu}^p = \frac{2\alpha M^3}{81}\langle r_p^2\rangle^4, \quad \alpha_{E2}^p = \frac{\alpha M}{9}\langle r_p^2\rangle^3. \tag{27}$$

Similarly one can generate predictions for the corresponding magnetic quantities. However, there is no reason to suspect that this picture should yield any better results here than in the case of the ordinary polarizabilities.

On the other hand, one can also predict the quadratic polarizabilities within heavy baryon chiral perturbation theory using either the $\mathcal{O}(p^3)$ or $\mathcal{O}(\epsilon^3)$ expansion. In the former case one finds [14]

$$\alpha_{E\nu}^p = \frac{9}{10}\frac{K_p}{m_\pi^2}, \quad \beta_{M\nu}^p = \frac{7}{5}\frac{K_p}{m_\pi^2},$$

$$\alpha^p_{E2}=\frac{42}{5}\frac{K_p}{m^2_\pi},\quad \beta^p_{M2}=-\frac{18}{5}\frac{K_p}{m^2_\pi}, \tag{28}$$

while in the latter the values given in Eq. (28) are augmented by the terms

$$\delta\alpha^p_{E\nu}=-\frac{1}{180}\frac{L_p}{m^2_\pi}\left(\frac{\Delta}{(\Delta^2-m^2_\pi)^3}(29\Delta^4-143\Delta^2m^2_\pi-231m^4_\pi)-\frac{3m^2_\pi(\Delta^4-107\Delta^2m^2_\pi-9m^4_\pi)}{(\Delta^2-m^2_\pi)^{7/2}}\ln R\right),$$

$$\delta\beta^p_{M\nu}=\frac{8}{9}\frac{b_1^2\alpha}{M^2\Delta^3}+\frac{1}{360}\frac{L_p}{m^2_\pi}\left(\frac{54\Delta^3-144\Delta m^2_\pi}{(\Delta^2-m^2_\pi)^2}+\frac{3(2\Delta^2m^2_\pi+28m^4_\pi)}{(\Delta^2-m^2_\pi)^{5/2}}\ln R\right),$$

$$\delta\alpha^p_{E2}=\frac{1}{30}\frac{L_p}{m^2_\pi}\left(\frac{22\Delta^3-82\Delta m^2_\pi}{(\Delta^2-m^2_\pi)^2}+\frac{3(6\Delta^2m^2_\pi+14m^4_\pi)}{(\Delta^2-m^2_\pi)^{5/2}}\ln R\right),$$

$$\delta\beta^p_{M2}=-\frac{3}{5}\frac{L_p}{m^2_\pi}\left(\frac{\Delta}{\Delta^2-m^2_\pi}-\frac{m^2_\pi}{(\Delta^2-m^2_\pi)^{3/2}}\ln R\right). \tag{29}$$

It is interesting to note that the sum rules, Eq. (26), require both electric polarizabilities to be positive definite, which is obeyed by the chiral calculation.

As to the experimental evaluation of such proton structure probes, it is, of course, in principle possible to extract them directly from Compton cross section measurements, since they modify the scattering amplitude via

$$\delta\mathrm{Amp}^{(4)}=4\pi\omega^4\left\{\hat{\epsilon}_2\cdot\hat{\epsilon}_1\left[\alpha^p_{E\nu}-\frac{1}{12}\beta^p_{M2}+\hat{k}_2\cdot\hat{k}_1\times\left(\beta^p_{M\nu}+\frac{1}{12}\alpha^p_{E2}\right)+(\hat{k}_2\cdot\hat{k}_1)^2\frac{1}{6}\beta^p_{M2}\right]+\hat{\epsilon}_2\cdot\hat{k}_1\hat{\epsilon}_1\cdot\hat{k}_2\left(-\beta^p_{M\nu}+\frac{1}{12}\alpha^p_{E2}-\hat{k}_2\cdot\hat{k}_1\frac{1}{6}\beta^p_{M2}\right)\right\}. \tag{30}$$

However, isolating such pieces from other terms which affect the cross section at energies above $\sim$100 MeV is virtually impossible since additional higher order effects soon become equally important [16]. Thus an alternative procedure is required and is made possible by the validity of dispersion relations, as described in the next section.

III. DISPERSIVE EVALUATION

Assuming the validity of fixed-t dispersion relations it is possible to determine the quadratic polarizabilities in a relatively model-independent fashion. Such a dispersive approach to the calculation of Compton scattering amplitudes has recently been carried out by Drechsel *et al.* [18]. In this method one decomposes the center-of-mass frame Compton amplitude in terms of invariant amplitudes $A_i(\nu,t)$ via

$$\mathrm{Amp}_{\mathrm{c.m.}}=-\hat{\epsilon}_1\cdot\hat{\epsilon}_2\omega^2[A_1(1-\cos\theta)+(A_3+A_6)(1-\cos\theta)]+\hat{\epsilon}_2\cdot\hat{k}_1\hat{\epsilon}_1\cdot\hat{k}_2\omega^2(A_1+A_3+A_6)+i\vec{\sigma}\cdot\hat{\epsilon}_2\times\hat{\epsilon}_1\frac{\omega^3}{M}[-(A_2+A_5)\times(1-\cos\theta)+A_4(1+\cos\theta)]+i\vec{\sigma}\cdot\hat{k}_2\times\hat{k}_1\hat{\epsilon}_2\cdot\hat{\epsilon}_1\frac{\omega^3}{M}(A_5-A_6)+(\vec{\sigma}\cdot\hat{\epsilon}_2\times\hat{k}_1\hat{\epsilon}_1\cdot\hat{k}_2-\vec{\sigma}\cdot\hat{\epsilon}_1\times\hat{k}_2\hat{\epsilon}_2\cdot\hat{k}_1)\frac{\omega^3}{2M}\times(A_6-2A_5-A_4-A_2)+(\vec{\sigma}\cdot\hat{\epsilon}_2\times\hat{k}_2\hat{\epsilon}_1\cdot\hat{k}_1-\vec{\sigma}\cdot\hat{\epsilon}_1\times\hat{k}_1\hat{\epsilon}_2\cdot\hat{k}_2)\frac{\omega^2}{2M}(A_2-A_4-A_6)+\mathcal{O}\left(\frac{\omega^4}{M^2},\frac{\omega^5}{M^3}\right), \tag{31}$$

where $\nu=(s-u)/4M$ and $t=-2k_1\cdot k_2$, with the assumption that the A_i can be represented in terms of once-subtracted dispersion relations at fixed t:

$$A_i(\nu,t)=A_i^{\mathrm{Born}}(\nu,t)+[A_i(0,t)-A_i^{\mathrm{Born}}(0,t)]+\frac{2\nu^2}{\pi}P\int_{\nu_{thr}}^{\infty}d\nu'\frac{\mathrm{Im}\,{}_tA_i(\nu',t)}{\nu'(\nu'^2-\nu^2)}. \tag{32}$$

Here $\mathrm{Im}\,{}_tA_i(\nu',t)$ is evaluated using empirical photoproduction data [we use the pion-photoproduction multipoles from the Haustein-Drechsel-Tiator (HDT) analysis [19]], while the subtraction constant $A_i(0,t)-A_i^{\mathrm{Born}}(0,t)$ is represented via use of t-channel dispersion relations

$$A_i(0,t)-A_i^{\mathrm{Born}}(0,t)=a_i+a_i^{t\,pole}+\frac{t}{\pi}\times\left(\int_{4m^2_\pi}^{\infty}-\int_{-\infty}^{-4Mm_\pi-2m^2_\pi}dt'\times\frac{\mathrm{Im}\,{}_tA_i(0,t')}{t'(t'-t)}\right), \tag{33}$$

with Im ${}_tA_i$ evaluated using the contribution from the $\pi\pi$ intermediate states [$\Delta(1232)$ intermediate states] along the positive [negative] t-channel cut. In principle then there remain six unknown subtraction constants a_i which can be determined empirically. However, in view of the limitations posed by the the data, Drechsel *et al.* note that four of these quantities can be reasonably assumed to obey unsubtracted forward dispersion relations and can be evaluated via

$$a_i=\frac{2}{\pi}\int_{\nu_0}^{\infty}d\nu'\frac{\text{Im }{}_sA_i(\nu',t=0)}{\nu'},\quad i=3,4,5,6. \tag{34}$$

The group treats a_1,a_2 as free parameters which can be determined via a fit to experiment and in this way is able to obtain a very good description of the low energy Compton scattering data.

In this paper we wish to go a step further and attempt to identify *higher order* terms in the expansion of the Compton amplitudes A_i which can be reasonably evaluated by means of the *subtracted* dispersion relations. Defining $a_{i,t},a_{i,\nu}$ as the appropriate derivatives at $t,\nu^2=0$, i.e.,

$$a_{i,\nu}=\frac{2}{\pi}\int_{\nu_{thr}}^{\infty}d\nu'\frac{\text{Im }{}_sA_i(\nu',t=0)}{\nu'^3}, \tag{35}$$

$$a_{i,t}=\frac{1}{\pi}\left(\int_{4m_\pi^2}^{\infty}-\int_{-\infty}^{-4Mm_\pi-2m_\pi^2}dt'\frac{\text{Im }{}_tA_i(0,t')}{t'^2}\right), \tag{36}$$

we have [17]

$$\begin{aligned}
4\pi\alpha_{E2}^p&=-12(a_{3,t}+a_{6,t}+a_{1,t})+\frac{3}{M^2}a_3,\\
4\pi\beta_{M2}^p&=-12(a_{3,t}+a_{6,t}-a_{1,t})+\frac{3}{M^2}a_3,\\
4\pi\alpha_{E\nu}^p&=-a_{3,\nu}-a_{6,\nu}-a_{1,\nu}+a_{3,t}+a_{6,t}+3a_{1,t}\\
&\quad-\frac{1}{4M^2}(a_3+4a_5),\\
4\pi\beta_{M\nu}^p&=-a_{3,\nu}-a_{6,\nu}+a_{1,\nu}+a_{3,t}+a_{6,t}-3a_{1,t}\\
&\quad-\frac{1}{4M^2}(a_3-4a_5).
\end{aligned} \tag{37}$$

(It should be noted that the small nonderivative recoil terms arise from the transformation from the Breit frame, wherein the effective Hamiltonian description is defined, to the center of mass frame, in which we work [17].) From Eq. (37), one finds the values (all in units of 10^{-4} fm^5)

$$\begin{aligned}
\text{DR:}\ \alpha_{E\nu}^p&=-3.84-0.19+0.06,\\
\beta_{M\nu}^p&=+9.29+0.15-0.07,\\
\alpha_{E2}^p&=+29.31-0.10-0.17,\\
\beta_{M2}^p&=-24.33+0.10-0.34.
\end{aligned} \tag{38}$$

The second number on the right-hand side (RHS) side of Eq. (38) indicates the change which results when the SAID99 [20] multipole solutions are substituted for the HDT ones in the s-channel πN intermediate states. The third number indicates the modification in the evaluation of the integral along the negative-t cut in Eq. (35) when, besides the $\Delta(1232)$ intermediate states, nonresonant πN intermediate states are also included. These two additional numbers, when taken together, provide a reasonable estimate for the uncertainty in our dispersive evaluations. It should also be noted that the values in Eq. (38) are in reasonably good agreement with those obtained in Ref. [17] which assumes the validity of an unsubtracted dispersion relation and appends high energy behavior in the cross channel,

$$\begin{aligned}
\text{DR [17]:}\quad &\alpha_{E\nu}^p=-3.8,\quad \beta_{M\nu}^p=9.1,\\
&\alpha_{E2}^p=27.5,\quad \beta_{M2}^p=-22.4.
\end{aligned} \tag{39}$$

The (relatively small) differences between the two evaluations should perhaps be considered as an indication of the uncertainty in the extraction.

Now we can confront these values with the corresponding theoretical calculations. In the case of the chiral predictions at $\mathcal{O}(p^3)$ we have from Eq. (28)

$$\begin{aligned}
\mathcal{O}(p^3):\quad &\alpha_{E\nu}^p=2.4,\quad \beta_{M\nu}^p=3.7,\\
&\alpha_{E2}^p=22.1,\quad \beta_{M2}^p=-9.5.
\end{aligned} \tag{40}$$

We see then that the size of α_{E2}^p is about right, while for both β_{M2}^p and $\beta_{M\nu}^p$ the sign and order of magnitude is correct but additional contributions are called for. The most serious problem lies in the experimental determination of $\alpha_{E\nu}$ which is negative in contradistinction to the chiral prediction and to sum rule arguments which assert its positivity. Of course, the experimental (and theoretical) numbers are small, so perhaps the disagreement lies within the uncertainty of our evaluation. Equivalently it could be that a nonrelativistic constituent quark model approach to subtle details of proton structure is inappropriate. These issues should be addressed in future work.

We can now move on to consider whether corrections from $\Delta(1232)$ degrees of freedom can help to address any discrepancies found above. We find

$$\begin{aligned}
\mathcal{O}(\epsilon^3):\quad &\alpha_{E\nu}^p=1.7,\quad \beta_{M\nu}^p=7.5,\\
&\alpha_{E2}^p=26.2,\quad \beta_{M2}^p=-12.3.
\end{aligned} \tag{41}$$

Except for the sign problem with $\alpha_{E\nu}^p$ indicated above, which remains in the ϵ expansion, the changes are generally helpful, although the magnetic quadrupole polarizability is still somewhat underpredicted.

IV. HIGHER ORDER SPIN POLARIZABILITIES

One can also analyze higher order contributions to spin polarizabilities. In this case the leading order—$\mathcal{O}(\omega^3)$—effective Lagrangian reads

$$H_{eff}^{(3)}=-\frac{1}{2}4\pi(\gamma_{E1}^{p}\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}}+\gamma_{M1}^{p}\vec{\sigma}\cdot\vec{H}\times\dot{\vec{H}}$$
$$-2\gamma_{E2}^{p}E_{ij}\sigma_i H_j+2\gamma_{M2}^{p}H_{ij}\sigma_i E_j) \quad (42)$$

and the chiral predictions for the spin polarizabilities at $\mathcal{O}(p^3)$ are found to be [21]

$$\gamma_{E1}^{p}=-\frac{10K_p}{\pi m_\pi},\quad \gamma_{M1}^{p}=-\frac{2K_p}{\pi m_\pi},\quad \gamma_{E2}^{p}=\frac{2K_p}{\pi m_\pi},$$
$$\gamma_{M2}^{p}=\frac{2K_p}{\pi m_\pi}. \quad (43)$$

Numerical values are given below in units of 10^{-4} fm^4:

$$\mathcal{O}(p^3):\quad \gamma_{E1}^{p}=-5.8,\quad \gamma_{M1}^{p}=-1.2,$$
$$\gamma_{E2}^{p}=1.2,\quad \gamma_{M2}^{p}=1.2. \quad (44)$$

It is interesting to note that γ_{E1} is nearly an order of magnitude larger than the other spin polarizabilities. It should be noted that before comparison with experiment is made these terms must be augmented by contributions from the ''anomaly'' (i.e., pion pole graph):

$${}_a\gamma_{E1}^{p}=-{}_a\gamma_{M1}^{p}=-{}_a\gamma_{E2}^{p}={}_a\gamma_{M2}^{p}=\frac{24K_p}{\pi m_\pi g_A}. \quad (45)$$

Thus far, experiments utilizing a polarized target and beam, which are necessary in order to directly measure the spin polarizabilities, have not been performed. However, one can compare with dispersion relation predictions, as done above. Since each involves spin-flip amplitudes, unsubtracted integrals are expected to converge and one finds values

$$\text{DR [18]:}\quad \gamma_{E1}^{p}=-4.31+0.21,\quad \gamma_{M1}^{p}=+2.93+0.09,$$
$$\gamma_{E2}^{p}=+2.20-0.01,\quad \gamma_{M2}^{p}=-0.02-0.01, \quad (46)$$

where the second number in the different spin polarizabilities is again due to the uncertainty when using the SAID99 multipoles instead of the HDT ones. The values of Eq. (46) are in reasonable agreement with the numbers

$$\text{DR [17]:}\quad \gamma_{E1}^{p}=-3.4,\quad \gamma_{M1}^{p}=2.7,$$
$$\gamma_{E2}^{p}=1.9,\quad \gamma_{M2}^{p}=0.3, \quad (47)$$

extracted in Ref. [17]. Again the sign discrepancy in the small term γ_{M2}^{p} is perhaps an indication of the overall precision which one can expect via the dispersive procedure. In comparing with the chiral numbers—Eq. (44)—we observe that the predictions for both electric multipoles are quite satisfactory. However, there is a clear problem in the comparison for γ_{M1}^{p}, suggesting the necessity of including the contributions from the $\Delta(1232)$, which are found to be

$$\delta\gamma_{E1}^{p}=\frac{L_p}{12}\left(\frac{\Delta^2+5m_\pi^2}{\Delta^2-m_\pi^2}\right)\left(\frac{1}{\Delta^2-m_\pi^2}-\frac{\Delta}{(\Delta^2-m_\pi^2)^{3/2}}\ln R\right),$$

$$\delta\gamma_{M1}^{p}=\frac{4}{9}\frac{b_1^2\alpha}{M^2\Delta^2}-\frac{L_p}{12}\left(\frac{1}{\Delta^2-m_\pi^2}-\frac{\Delta}{(\Delta^2-m_\pi^2)^{3/2}}\ln R\right),$$

$$\delta\gamma_{E2}^{p}=\delta\gamma_{M2}^{p}=\frac{L_p}{12}\left(\frac{1}{\Delta^2-m_\pi^2}-\frac{\Delta}{(\Delta^2-m_\pi^2)^{3/2}}\ln R\right). \quad (48)$$

There does exist then a significant contribution to γ_{M1}^{p} from the $\Delta(1232)$ pole diagram as well as small contributions to the other spin polarizabilities from $\Delta\pi$ loop effects. When these are appended to the $N\pi$ loop predictions given in Eq. (44) we find the results

$$\mathcal{O}(\epsilon^3):\quad \gamma_{E1}^{p}=-6.1,\quad \gamma_{M1}^{p}=1.0,$$
$$\gamma_{E2}^{p}=1.1,\quad \gamma_{M2}^{p}=1.1, \quad (49)$$

which are in quite reasonable agreement with the numbers obtained dispersively above.

However, it is also possible to study *higher order* polarizability contributions to the spin-dependent Compton scattering amplitude, which contribute at $\mathcal{O}(\omega^5)$. There are eight such new terms, which can be expressed in terms of the effective Hamiltonian

$$H_{eff}^{(5)}=-\frac{1}{2}4\pi[\gamma_{E1\nu}^{p}\vec{\sigma}\cdot\dot{\vec{E}}\times\ddot{\vec{E}}+\gamma_{M1\nu}^{p}\vec{\sigma}\cdot\dot{\vec{H}}\times\ddot{\vec{H}}$$
$$-2\gamma_{E2\nu}^{p}\sigma_i\dot{E}_{ij}\dot{H}_j+2\gamma_{M2\nu}^{p}\sigma_i\dot{H}_{ij}\dot{E}_j+4\gamma_{ET}^{p}\epsilon_{ijk}\sigma_i E_{jl}\dot{E}_{kl}$$
$$+4\gamma_{MT}^{p}\epsilon_{ijk}\sigma_i H_{jl}\dot{H}_{kl}-6\gamma_{E3}^{p}\sigma_i E_{ijk}H_{jk}$$
$$+6\gamma_{M3}^{p}\sigma_i H_{ijk}E_{jk}], \quad (50)$$

where

$$(E,H)_{ijk}=\frac{1}{3}[\nabla_i\nabla_j(E,H)_k+\nabla_i\nabla_k(E,H)_j+\nabla_j\nabla_k(E,H)_i]$$
$$-\frac{1}{15}[\delta_{ij}\nabla^2(E,H)_k+\delta_{jk}\nabla^2(E,H)_i$$
$$+\delta_{ik}\nabla^2(E,H)_j] \quad (51)$$

are the (spherical) tensor gradients of the electric and magnetizing fields. (For completeness, we note that these higher order spin polarizabilities can be expressed, neglecting recoil terms, in terms of the usual multipole expansion via

$$
\begin{aligned}
4\pi\omega^5\gamma_{ET} &= 3(f_{EE}^{2+}-f_{EE}^{2-}),\\
4\pi\omega^5\gamma_{MT} &= 3(f_{MM}^{2+}-f_{MM}^{2-}),\\
4\pi\omega^5\gamma_{E3} &= 15 f_{ME}^{2+},\\
4\pi\omega^5\gamma_{M3} &= 15 f_{EM}^{2+},\\
4\pi\omega^3(\gamma_{E2}+\omega^2\gamma_{E2\nu}) &= 6 f_{ME}^{1+},\\
4\pi\omega^3(\gamma_{M2}+\omega^2\gamma_{M2\nu}) &= 6 f_{EM}^{1+},\\
4\pi\omega^3(\gamma_{E1}+\omega^2\gamma_{E1\nu}) &= f_{EE}^{1+}-f_{EE}^{1-},\\
4\pi\omega^3(\gamma_{M1}+\omega^2\gamma_{M1\nu}) &= f_{MM}^{1+}-f_{MM}^{1-}.)
\end{aligned}
\tag{52}
$$

We see that, as in the spin-averaged case, four of the new terms are simply dispersive corrections to the $\mathcal{O}(\omega^3)$ spin polarizabilities defined in Eq. (42). However, there exist also new structures which probe the octupole excitation of the system. The modification of the Compton scattering amplitude by such terms is found to be

$$
\begin{aligned}
\delta\,\mathrm{Amp}^{(5)} = -i4\pi\omega^5\Bigg\{ & \vec{\sigma}\cdot\hat{\epsilon}_2\times\hat{\epsilon}_1\Bigg[\gamma^p_{E1\nu}+\gamma^p_{M2\nu}-\frac{12}{5}\gamma^p_{E3}-3\gamma^p_{MT}+\hat{k}_2\cdot\hat{k}_1\left(\gamma^p_{M1\nu}+\gamma^p_{E2\nu}+\gamma^p_{ET}+\frac{8}{5}\gamma^p_{M3}\right)+4(\hat{k}_2\cdot\hat{k}_1)^2(\gamma^p_{MT}\\
& +\gamma^p_{E3})\Bigg]+\vec{\sigma}\cdot\hat{k}_2\times\hat{k}_1\hat{\epsilon}_2\cdot\hat{\epsilon}_1\left(\gamma^p_{M1\nu}-\gamma^p_{E2\nu}+\gamma^p_{ET}+\frac{2}{5}\gamma^p_{M3}+\hat{k}_2\cdot\hat{k}_1(4\gamma^p_{MT}-2\gamma^p_{E3})\right)+(\vec{\sigma}\cdot\hat{\epsilon}_2\times\hat{k}_1\hat{\epsilon}_1\cdot\hat{k}_2-\vec{\sigma}\cdot\hat{\epsilon}_1\\
& \times\hat{k}_2\hat{\epsilon}_2\cdot\hat{k}_1)[\gamma^p_{ET}+\gamma^p_{M3}-\gamma^p_{M1\nu}-\hat{k}_2\cdot\hat{k}_1(4\gamma^p_{MT}+\gamma^p_{E3})]+(\vec{\sigma}\cdot\hat{\epsilon}_2\times\hat{k}_2\hat{\epsilon}_1\cdot\hat{k}_2-\vec{\sigma}\cdot\hat{\epsilon}_1\times\hat{k}_1\hat{\epsilon}_2\cdot\hat{k}_1)\\
& \times\left(\frac{7}{5}\gamma^p_{E3}+2\gamma^p_{MT}-\gamma^p_{M2\nu}-3\hat{k}_2\cdot\hat{k}_1\gamma^p_{M3}\right)\Bigg\}
\end{aligned}
\tag{53}
$$

and, comparing with a chiral expansion of the Compton amplitude at $\mathcal{O}(p^3)$, we can read off [14]

$$
\begin{aligned}
&\gamma^p_{E3}=\frac{4K_p}{45\pi m_\pi^3},\quad \gamma^p_{M3}=\frac{4K_p}{45\pi m_\pi^3},\\
&\gamma^p_{ET}=-\frac{13K_p}{45\pi m_\pi^3},\quad \gamma^p_{MT}=-\frac{K_p}{45\pi m_\pi^3},\\
&\gamma^p_{E1\nu}=-\frac{189K_p}{45\pi m_\pi^3},\quad \gamma^p_{M1\nu}=-\frac{9K_p}{45\pi m_\pi^3},\\
&\gamma^p_{E2\nu}=\frac{78K_p}{225\pi m_\pi^3},\quad \gamma^p_{M2\nu}=-\frac{42K_p}{225\pi m_\pi^3}.
\end{aligned}
\tag{54}
$$

As before, there exist pion pole (anomaly) contributions to the higher order spin polarizabilities which must be included when comparing with data:

$$
\begin{aligned}
&{}_a\gamma^p_{E1\nu}=-\frac{72K_p}{\pi g_A m_\pi^3},\quad {}_a\gamma^p_{M1\nu}=\frac{72K_p}{\pi g_A m_\pi^3},\\
&{}_a\gamma^p_{E2\nu}=\frac{368K_p}{5\pi g_A m_\pi^3},\quad {}_a\gamma^p_{M2\nu}=-\frac{368K_p}{5\pi g_A m_\pi^3},\\
&{}_a\gamma^p_{E3}=-\frac{16K_p}{\pi g_A m_\pi^3},\quad {}_a\gamma^p_{M3}=\frac{16K_p}{\pi g_A m_\pi^3},\\
&{}_a\gamma^p_{ET}=\frac{8K_p}{\pi g_A m_\pi^3},\quad {}_a\gamma^p_{MT}=-\frac{8K_p}{\pi g_A m_\pi^3}.
\end{aligned}
\tag{55}
$$

We have then the chiral predictions (in units of $10^{-4}\ \mathrm{fm}^6$)

$$
\begin{aligned}
\mathcal{O}(p^3):\quad &\gamma^p_{E3}=0.11,\quad \gamma^p_{M3}=0.11,\quad \gamma^p_{ET}=-0.37,\\
&\gamma^p_{MT}=-0.03,\\
&\gamma^p_{E1\nu}=-5.05,\quad \gamma^p_{M1\nu}=-0.26,\quad \gamma^p_{E2\nu}=0.45,\\
&\gamma^p_{M2\nu}=-0.24.
\end{aligned}
\tag{56}
$$

Again we note that the size of $\gamma^p_{E1\nu}$ dominates by over an order of magnitude any of the other higher order spin polarizabilities. The modifications arising from inclusion of the $\Delta(1232)$ are found to be

$$\delta\gamma_{E3}^{p}=-\frac{L_p}{540}\left(\frac{\Delta^2+2m_\pi^2}{m_\pi^2(\Delta^2-m_\pi^2)^2}-\frac{3\Delta}{(\Delta^2-m_\pi^2)^{5/2}}\ln R\right),$$

$$\delta\gamma_{M3}^{p}=-\frac{L_p}{540}\left(\frac{\Delta^2+2m_\pi^2}{m_\pi^2(\Delta^2-m_\pi^2)^2}-\frac{3\Delta}{(\Delta^2-m_\pi^2)^{5/2}}\ln R\right),$$

$$\delta\gamma_{ET}^{p}=\frac{L_p}{1080}\left(\frac{2\Delta^4+50\Delta^2m_\pi^2-13m_\pi^4}{m_\pi^2(\Delta^2-m_\pi^2)^3}-\frac{3\Delta\left(\frac{1}{2}\Delta^2-8m_\pi^2\right)}{(\Delta^2-m_\pi^2)^{7/2}}\ln R\right),$$

$$\delta\gamma_{MT}^{p}=\frac{L_p}{2160}\left(\frac{\Delta^2+2m_\pi^2}{m_\pi^2(\Delta^2-m_\pi^2)^2}-\frac{3\Delta}{(\Delta^2-m_\pi^2)^{5/2}}\ln R\right),$$

$$\delta\gamma_{E1\nu}^{p}=\frac{L_p}{720}\left(\frac{-5\Delta^6+26\Delta^4m_\pi^2+693\Delta^2m_\pi^4+126m_\pi^6}{m_\pi^2(\Delta^2-m_\pi^2)^4}-\frac{3\Delta(4\Delta^4+152\Delta^2m_\pi^2+124m_\pi^4)}{(\Delta^2-m_\pi^2)^{9/2}}\ln R\right),$$

$$\delta\gamma_{M1\nu}^{p}=\frac{4}{9}\frac{b_1^2\alpha}{M^2\Delta^4}+\frac{L_p}{1080}\left(\frac{-3\Delta^4+9\Delta^2m_\pi^2-9m_\pi^4}{m_\pi^2(\Delta^2-m_\pi^2)^3}+\frac{27\Delta\left(\frac{3}{2}\Delta^2+m_\pi^2\right)}{(\Delta^2-m_\pi^2)^{7/2}}\ln R\right),$$

$$\delta\gamma_{E2\nu}^{p}=\frac{L_p}{1800}\left(\frac{7\Delta^4-93\Delta^2m_\pi^2+26m_\pi^4}{m_\pi^2(\Delta^2-m_\pi^2)^3}-\frac{3\Delta(27\Delta^2+23m_\pi^2)}{(\Delta^2-m_\pi^2)^{7/2}}\ln R\right),$$

$$\delta\gamma_{M2\nu}^{p}=\frac{L_p}{1800}\left(\frac{17\Delta^4+72\Delta^2m_\pi^2-14m_\pi^4}{m_\pi^2(\Delta^2-m_\pi^2)^3}-\frac{3\Delta(27\Delta^2+2m_\pi^2)}{(\Delta^2-m_\pi^2)^{7/2}}\ln R\right), \tag{57}$$

and numerically this leads to the predictions

$$\mathcal{O}(\epsilon^3):\quad \gamma_{E3}^p=0.11,\quad \gamma_{M3}^p=0.11,\quad \gamma_{ET}^p=-0.28,\quad \gamma_{MT}^p=-0.03,$$
$$\gamma_{E1\nu}^p=-5.16,\quad \gamma_{M1\nu}^p=0.83,\quad \gamma_{E2\nu}^p=0.28,\quad \gamma_{M2\nu}^p=-0.22. \tag{58}$$

The higher order polarizabilities can be extracted from the dispersive results via the relations

$$4\pi\gamma_{E3}^p=-\frac{1}{3M}(a_{4,t}+a_{6,t}+a_{2,t})+\frac{1}{12M^3}a_5,$$

$$4\pi\gamma_{M3}^p=-\frac{1}{3M}(a_{4,t}+a_{6,t}-a_{2,t})-\frac{1}{12M^3}a_5,$$

$$4\pi\gamma_{ET}^p=\frac{1}{6M}(2a_{6,t}+3a_{5,t}-a_{4,t}+a_{2,t})+\frac{1}{48M^3}\times(-6a_3+7a_5+3a_6),$$

$$4\pi\gamma_{MT}^p=\frac{1}{6M}(2a_{6,t}-3a_{5,t}-a_{4,t}-a_{2,t})+\frac{1}{48M^3}\times(-6a_3-7a_5+3a_6),$$

$$4\pi\gamma_{E2\nu}^p=-\frac{1}{2M}(a_{4,\nu}+a_{6,\nu}+a_{2,\nu})+\frac{1}{5M}(6a_{6,t}+5a_{5,t}+a_{4,t}+9a_{2,t})-\frac{1}{40M^3}(-10a_3+90a_4-7a_5+15a_6),$$

$$4\pi\gamma_{M2\nu}^p=-\frac{1}{2M}(a_{4,\nu}+a_{6,\nu}-a_{2,\nu})+\frac{1}{5M}(6a_{6,t}-5a_{5,t}+a_{4,t}-9a_{2,t})-\frac{1}{40M^3}(-10a_3+90a_4+7a_5+15a_6),$$

$$4\pi\gamma_{E1\nu}^p=\frac{1}{2M}(a_{6,\nu}+2a_{5,\nu}-a_{4,\nu}+a_{2,\nu})+\frac{1}{2M}(-2a_{6,t}-5a_{5,t}+a_{4,t}-3a_{2,t})+\frac{1}{16M^3}(-2a_3-36a_4+19a_5+5a_6),$$

$$4\pi\gamma_{M1\nu}^p=\frac{1}{2M}(a_{6,\nu}-2a_{5,\nu}-a_{4,\nu}-a_{2,\nu})+\frac{1}{2M}(-2a_{6,t}+5a_{5,t}+a_{4,t}+3a_{2,t})+\frac{1}{16M^3}(-2a_3-36a_4-19a_5+5a_6), \tag{59}$$

which yields

$$\text{DR:}\quad \gamma_{E3}^p=+0.059+0.00-0.006,\quad \gamma_{M3}^p=+0.088-0.001+0.003,$$
$$\gamma_{ET}^p=-0.15+0.00-0.037,\quad \gamma_{MT}^p=-0.090+0.00-0.008,$$
$$\gamma_{E1\nu}^p=-3.42+0.20+0.15,\quad \gamma_{M1\nu}^p=+2.23+0.06-0.06,$$
$$\gamma_{E2\nu}^p=+1.30+0.01-0.05,\quad \gamma_{M2\nu}^p=-0.60+0.01-0.002. \tag{60}$$

where the meaning of the second and third entries in Eq. (60) is as explained below Eq. (38). Obviously only the dispersive correction coefficients $\gamma^p_{E1\nu}$, $\gamma^p_{M1\nu}$, $\gamma^p_{E2\nu}$, and $\gamma^p_{M2\nu}$ are sizable and are in qualitative agreement with the chiral $\mathcal{O}(\epsilon^3)$ predictions.

V. CONCLUSIONS

Above we have shown how the use of dispersion relations allows extraction of information about higher order polarizabilities of the proton which is not available from direct cross section analysis. We have also seen how such measurements can be confronted with theoretical predictions for such quantities based on quark model and/or chiral perturbative pictures of proton structure. Although a simple harmonic oscillator model contains too small a gap between the ground and excited states and therefore overpredicts both the conventional as well as the higher order polarizabilities, a simple $\mathcal{O}(p^3)$ or $\mathcal{O}(\epsilon)^3$ HBχPT is in basic agreement with the dispersive evaluation, except for a sign problem in the case of $\alpha^p_{E\nu}$. Since general sum rule arguments disagree with the sign of the experimentally extracted term, this is clearly an area which demands additional study. We also presented theoretical predictions for higher order—$\mathcal{O}(\omega^5)$—contributions to the spin polarizabilities, which can in principle be extracted once spin-dependent data become available. Clearly there is a great deal of nucleon structure information contained in such higher order polarizabilities and our paper has just touched the surface.

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