Low-lying continuum structures in ^{8}B and ^{8}Li in a microscopic model

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We search for low-lying resonances in the ^{8}B and ^{8}Li nuclei using a microscopic cluster model and a variational scattering method, which is analytically continued to complex energies. After fine-tuning the nucleon-nucleon interaction to get the known 1^+ state of ^{8}B at the right energy, we reproduce the known spectra of the studied nuclei. In addition, our model predicts a 1^+ state at 1.3 MeV in ^{8}B, relative to the ^{7}Be$+p$ threshold, whose corresponding pair is situated right at the ^{7}Li$+n$ threshold in ^{8}Li. Lacking any experimental evidence for the existence of such states, it is presently uncertain whether these structures really exist or whether they are spurious resonances in our model. We demonstrate that the predicted state in ^{8}B, if it exists, would have important consequences for the understanding of the astrophysically important ^{7}Be$(p,\gamma)^8$B reaction.

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I. INTRODUCTION

The ^{7}Be$(p,\gamma)^8$B and ^{7}Li$(n,\gamma)^8$Li reactions play important roles in nuclear astrophysics. The former process produces ^{8}B in the sun, which is the dominant source of the high-energy solar neutrinos (via the β decay to ^{8}Be) [1], while the latter one is a key reaction in inhomogeneous big bang models [2]. In stellar environments only the very low-energy continuum structures of the participating nuclei are probed. However, terrestrial experiments often cannot study these low-energy processes directly, and have to extrapolate the measured data (e.g., cross sections) from higher energies down to the astrophysically interesting region. Obviously, the extrapolation procedure can be strongly influenced by higher lying structures present in the continuum. For example, the known 1^+ and 3^+ resonances in ^{8}B cause bumps in the ^{7}Be$(p,\gamma)^8$B cross section, which have to be treated properly if one wants to interpret the results of both the radiative capture [3] and Coulomb dissociation [4] measurements correctly.

The aim of the present work is to explore the continuum structure of ^{8}B and ^{8}Li using a microscopic model that has been previously applied to describe the ground states of these nuclei [5] and in the calculation of the nonresonant ^{7}Be$(p,\gamma)^8$B cross section [6,7]. According to the Ref. [8] compilation, there is a resonance in ^{8}B at $E_r=0.63$ MeV (in this paper the energies are given in the center-of-mass frame, relative to the ^{7}Be$+p$ or ^{7}Li$+n$ threshold) and a 3^+ state at 2.18 MeV. Many calculations predict the first state to be a 1^+ resonance, see, e.g., Ref. [9]. In ^{8}Li there is a bound 1^+ state, a 3^+ resonance at 0.22 MeV, and a broad 1^+ state at 1.18 MeV. The Ref. [8] compilation does not give any further low-lying states in ^{8}Li and ^{8}B. However, there are indications for the existence of a 2^- state in ^{8}Li at 1.18 MeV [10], for a 1^+ state of ^{8}B at 2.7 MeV, and for a 1^- or 2^- level in ^{8}B at 2.9 MeV [11].

As far as theoretical models are concerned, the potential model of Ref. [12] seemed to indicate an additional $M1$ (1^+ or 2^+) state in ^{8}B at about 1.4 MeV. However, that calculation was criticized [13] by pointing out that certain model assumptions in Ref. [12], such as the ^{7}Be (g.s.)$+p$ nature of the $M1$ states, were rather unphysical. Signs of new states have also surfaced in microscopic models. For example, in Ref. [7] we noted that our microscopic ^{4}He$+^3$He$+p$ cluster model also seemed to indicate the presence of a 1^+ resonance at around 1.5 MeV.

In order to be able to make a well-founded prediction on the possible existence of new continuum states in ^{8}B (and ^{8}Li) in any model, one should satisfy two important criteria. First, the properties of the known 1^+ state at 0.63 MeV must be well reproduced by the model. In certain approaches, as in the potential model, this is a tough problem. Secondly, the recognition of any possible new resonance should be fairly reliable, even for broad states. Below we present a model which satisfies both conditions and predicts new states in ^{8}B and ^{8}Li.

II. MODEL

We use the same model as in Refs. [5–7]. The eight-body, three-cluster (^{4}He$+^3$He$+p$) wave function of ^{8}B is chosen as

$$\Psi=\sum_{I_7,I,l_2}\sum_{i=1}^{N_7}\mathcal{A}\{[[\Phi^p_{s_2}\Phi^{^7\mathrm{Be},i}_{I_7}]_I\chi^i_{l_2}(\boldsymbol{\rho}_2)]_{JM}\},\qquad(1)$$

where $\mathcal{A}$ is the intercluster antisymmetrizer, $\boldsymbol{\rho}_2$ and l_2 are the relative coordinate and relative angular momentum between ^{7}Be and p, respectively, s_2 and I_7 are the spin of the proton and ^{7}Be, respectively, I is the channel spin, and $[\cdots]$ denotes angular momentum coupling. While Φ^p is a proton spin-isospin eigenstate, the antisymmetrized ground state ($i=1$) and continuum excited distortion states ($i>1$) of ^{7}Be are represented by the wave functions

$$\Phi^{^7\mathrm{Be},i}_{I_7}=\sum_{j=1}^{N_7}c_{ij}\sum_{l_1}\mathcal{A}\{[[\Phi^\alpha\Phi^h]_{s_1}\Gamma^j_{l_1}(\boldsymbol{\rho}_1)]_{I_7M_7}\}.\qquad(2)$$

Here $\mathcal{A}$ is the intercluster antisymmetrizer between α and h, Φ^α and Φ^h are translationally invariant harmonic oscillator shell model states ($\alpha=^4$He, $h=^3$He), $\boldsymbol{\rho}_1$ is the relative coordinate between α and h, l_1 is the α-h relative angular momentum, s_1 is the spin of h, and $\Gamma^j_{l_1}(\boldsymbol{\rho}_1)$ is a Gaussian

function with a width of γ_j. The c_{ij} parameters are determined from a variational principle for the ^{7}Be energy. A similar wave function is used for ^{8}Li within the ^{4}He+^{3}H $+n$ cluster model space. Using Eq. (1) in the eight-nucleon Scrödinger equation, we get an equation for the unknown relative motions χ.

In order to ensure that all existing resonances are recognized in the continuum, we search for the poles of the ^{7}Be $+p$ (^{7}Li+n) scattering matrices. These matrices are generated with the help of the Kohn-Hulthén variational method for scattering processes [14]. In that method the relative motions in Eq. (1) are expanded in terms of square-integrable functions (Gaussians in our case) matched with the correct scattering boundary condition. The resulting variational scattering matrices are analytically continued to the Riemann surface of complex energies, using the methods of Ref. [15]. That is, we generate solutions to the scattering problem with such asymptotic behavior that corresponds to complex energies

$$\chi(\varepsilon,\rho)\rightarrow H^-(k\rho)-\tilde{S}(\varepsilon)H^+(k\rho). \tag{3}$$

Here ε and k are the *complex* energies and wave numbers of the relative motions, and H^- and H^+ are the incoming and outgoing Coulomb functions, respectively. These solutions have no physical meaning except when $\tilde{S}$ is singular, where $\tilde{S}$ coincides with the physical S matrix. We perform this analytic continuation procedure and search for the poles of S. The parameters of the complex-energy poles are related to the resonance parameters (E_r position and Γ width) through the

$$\varepsilon = E_{\mathrm{r}} - i\Gamma/2 \tag{4}$$

equation. We mention that this way of determining resonance parameters is the most closely related to scattering theory [16]. The results obtained can differ from those coming from other methods, especially for broad states [15].

We note that the ^{7}Be+p scattering picture is valid only below the ^{4}He+^{3}He+p three-body breakup threshold, which is at 1.587 MeV experimentally (2.467 MeV in the case of ^{7}Li and ^{4}He+^{3}H+n). Above this energy three-body scattering asymptotics should be used, which is beyond our capacity. Although the continuum-excited distortion states in Eq. (1) can mimic three-body break-up at some approximate level, any states predicted above these energies in our model should be handled with care.

Our approach contains a large and physically motivated part of the eight-body Hilbert space. The next step is to find an effective nucleon-nucleon (NN) interaction that is the most suitable for the description of the problem at hand. As we mentioned, the precise reproduction of the known 1^+ state in ^{8}B (and in ^{8}Li) is a first priority. In most of our previous calculations for ^{8}B [6,7] we used the Minnesota (MN) force. However, it turned out that this interaction gave an incorrect channel spin ratio for the 1^+_1 state of ^{8}Be [17], which is a member of a $T=1$ triplet together with the 1^+ states of ^{8}B and ^{8}Li. This flaw is most probably caused by the fact that the reproduction of this state at the right energy in ^{8}Be requires an exchange mixture parameter value (u) of the MN force which is incompatible with the experimental data in the singlet-odd NN channel ($u>1$). To avoid this problem in the present work, here we apply the modified Hasegawa-Nagata interaction [18], which is also widely used in cluster model calculations.

TABLE I. The low-energy spectrum of ^{8}B and ^{8}Li. The experimental data are taken from Ref. [8], except for the first 1^+ state of ^{8}B, which is from Filippone [3]. All numbers are in MeV.

		Theory		Experiment	
	J^π	E_r	Γ	E_r	Γ
^{8}B	2^+	-0.215		-0.137	
	1^+	0.632	0.034	0.632±0.01	0.037±0.005
	1^+	1.278	0.564		
	3^+	2.98	0.808	2.183±0.03	0.35±0.04
	1^+	4.33	1.5		
^{8}Li	2^+	-2.021		-2.033	
	1^+	-0.975		-1.052±0.001	
	1^+	0.037	0.006		
	3^+	0.937	0.327	0.222±0.003	0.033±0.006
	1^+	2.29	1.0	1.18	~1.0

We assume a pure Wigner form for the spin-orbit force and fine-tune the Majorana component (while adjusting the Wigner component to keep $W+M=$const) of the medium range part of the central interaction, in order to get the 1^+ state of ^{8}B at 632 keV, its experimental value taken from Filippone *et al.* [3] (note that Ref. [8] gives 637 keV). The required modification in the central force is about 5%. The strength of the short-range spin-orbit interaction is fixed by requiring that the experimental spin-orbit splitting between the $3/2^-$ and $1/2^-$ states of ^{7}Be be reproduced correctly. We use the same interaction for both ^{8}B and ^{8}Li. Our prime target is ^{8}B here, a more precise reproduction of the ^{7}Li states and the fine-tuning of the NN force for the 1^+ resonance of ^{8}Li is not pursued.

III. RESULTS

We show the spectrum of ^{8}B and ^{8}Li coming from our calculation in Table I, together with the experimental numbers. In these calculations we used $l_1=1$ inside the seven-nucleon subsystems in Eq. (2) in all cases, $l_2=1$ in the case of the 1^+ and 2^+ states, and $l_2=1$ and 3 in the case of the 3^+ state. The total spin S took all possible values. We emphasize again that our model is not a fully adequate description for those states that lie above the three-body thresholds. These thresholds are situated at 1.34 and 2.13 MeV, respectively, in our model, compared to the experimental 1.59 and 2.47 MeV. One can see in Table I, that the properties of the 1^+_1 state in ^{8}B are nicely reproduced. This is a significant achievement as other models, most notably the ^{7}Be+p potential model, tend to strongly overestimate the width of this state. We also note that the NN interaction which is fine-tuned to get the 1^+ state at the right position, slightly

TABLE II. Characterization of the calculated 1^+ states shown in Table I by (i) the weights of the orthogonal (L,S) components, (ii) the weights of the orthogonal (I,l_2) components, (iii) the amounts of clustering of the nonorthogonal (I_7,I,l_2) components, and (iv) the weights of the shell-model-like (I_7,j) configurations in the ^{8}B and ^{8}Li wave function. For further details, see the text.

		^{8}B			^{8}Li		
Configuration		1_1^+	1_2^+	1_3^+	1_1^+	1_2^+	1_3^+
(L,S)	(0,1)	0.010	0.001	0.791	0.001	0.0002	0.737
	(1,0)	0.188	0.728	0.049	0.211	0.726	0.077
	(1,1)	0.786	0.207	0.154	0.759	0.237	0.179
	(2,1)	0.016	0.064	0.006	0.029	0.037	0.007
(I,l_2)	(1,1)	0.237	0.888	0.184	0.239	0.870	0.142
	(2,1)	0.465	0.107	0.736	0.417	0.115	0.849
	(0,1)	0.298	0.005	0.080	0.344	0.015	0.009
(I_7,I,l_2)	(3/2,1,1)	0.021	0.890	0.113	0.030	0.887	0.138
	(3/2,2,1)	0.621	0.119	0.816	0.611	0.195	0.853
	(1/2,1,1)	0.438	0.048	0.050	0.474	0.070	0.004
	(1/2,0,1)	0.472	0.017	0.054	0.538	0.051	0.008
(I_7,j)	(3/2,3/2)	0.108	0.409	0.321	0.123	0.406	0.316
	(3/2,1/2)	0.418	0.190	0.317	0.408	0.202	0.317
	(1/2,3/2)	0.418	0.190	0.317	0.408	0.202	0.317
	(1/2,1/2)	0.049	0.206	0.044	0.068	0.191	0.049

overbinds the 2^+ ground state. This can be explained by the fact that the (^{4}He,^{3}He)p model space is closer to the true wave function of the 2^+ ground state than to that of the 1^+ resonance. It means that while describing the 1^+ state, the NN force has to be made stronger in order to compensate for those missing parts of the Hilbert space that cannot be represented by the (^{4}He,^{3}He)p wave function. However, this modification of the force leads to the slight overbinding in the 2^+ state.

The most surprising result in Table I is the prediction of an additional 1^+ state at 1.28 MeV. Note that although this resonance is situated rather close to our ^{4}He+^{3}He+p three-body threshold, it is not a threshold effect or a similar artifact. By making the NN force slightly stronger, both the energy of the 1_2^+ state and that of ^{7}Be is lowered (while the position of the three-body threshold is unchanged), which makes this state more bound relative to the three-body threshold. The corresponding 1_2^+ state in ^{8}Li is very close to the ^{7}Li+n two-body threshold, therefore its parameters can depend on the details of the model rather strongly.

One can see in Table I that the known 3^+ states appear in our model, although somewhat shifted to higher energies, compared with the experimental situation. In addition, we have a third 1^+ state, although it lies above the three-body threshold in both ^{8}B and ^{8}Li. Whether this state would appear in a realistic model, which contains the correct three-body asymptotics, remains a question.

Turning our attention back to the 1_2^+ states, we note that no experimental evidence has been found so far that would support the existence of such structures [8]. It seems rather unlikely, e.g., that the existence of such a state in ^{8}Li, right at the ^{7}Li+n threshold, could be reconciled with the cross section measurements of Ref. [19]. Also, no indications of such states were found in ^{6}Li$(t,p)^8$Li and ^{9}Be$(d,^3$He$)^8$Li experiments [8]. Nevertheless we feel obliged to report on the existence of these states in our model. Although rather unlikely, these states can be spurious solutions in our model. Certain variational approaches are known to produce such artifacts [20]. Spurious states can also appear if incorrect boundary conditions are used. This would happen here also in the case of ^{8}Li, if we searched for 2^+ states in a three-body bound state calculation. We would find two such three-body bound states, one of them lying above the ^{7}Li+n threshold. Using a wave function which has the correct asymptotic behavior (scattering state in ^{7}Li+n) would result in the disappearance of the second ''state.'' For further details, see Ref. [21]. The situation is different in the present work. We do not know any conceivable reason why spurious states might appear in our description. Another possibility is that the predicted states are situated at significantly higher energies in reality, but for some reason they appear at low energies in our model. In the following we discuss some characteristic features of the newly found resonances, which might help to find them experimentally if they exist, or else find a way to understand them and get rid of them in our model if they turned out to be spurious states.

In Table II we give the probabilities of the various angular momentum channels which build up the wave functions of the 1^+ states of ^{8}B and ^{8}Li in several different forms. In these calculations a three-body bound state approximation was used for the 1^+ resonances, where the relative motions were expanded in terms of square-integrable basis functions. The resulting probabilities characterize the inner parts of the resonance wave functions rather well.

First the weights of the orthogonal channels are given in Table II in the (L,S) and (I,l_2) representations, respectively. Here $[(l_1,l_2)L,(s_1,s_2)S]J$ and $[[(l_1,s_1)I_7,s_2]I,l_2]J$ coupling schemes are assumed, respectively. The third way of

expressing the importance of the various *nonorthogonal* channels is to calculate their amounts of clustering [22]. This quantity gives the probability that the full wave function of a state lies completely in the given subspace (thus the sum of the probabilities is obviously not 1). The fourth way of characterizing the 1^+ states in Table II is to calculate the weights of the shell-model-like $|I_7,j\rangle$ configurations, where $I_7=3/2$ or 1/2 is the spin of ^{7}Be/^{7}Li, while $j=3/2$ or 1/2 comes from the coupling of the orbital momentum (l_2) and spin (s_2) of p/n. We calculated this quantity in the following way. First we expressed the $|I_7,p_j\rangle$ state in terms of the (L,S) components, and then evaluated its square by substituting the (L,S) weights.

As one can see in Table II, the 1_1^+ state receives its main contribution from the $I=2$ channel spin components, while 1_2^+ is dominated by the $I=1$ channel spin. In addition, as shown by the amounts of clustering, the 1_1^+ state receives large contributions from configurations which contain a ^{7}Be in its excited state ($I_7=1/2$), while in 1_2^+ the role of the excited ^{7}Be core is negligible.

The newly found 1_2^+ states, if they exist, would have important consequences for the radiative capture reactions $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ and $^7\mathrm{Li}(n,\gamma)^8\mathrm{Li}$. We do not discuss these consequences in detail, just mention one interesting example. In order to be able to reliably extrapolate the experimental $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ cross section down to stellar energies, the precise knowledge of the nonresonant $E1$ cross section is necessary. The 1_2^+ resonance in the vicinity of 1.3 MeV, if it exists but is not recognized, would change the average slope of the extracted $E1$ cross section. In order to show this effect, we calculated the $M1$ cross section using the above model. Due to technical simplifications, the asymptotic behavior of the 2^+ bound state could only be correctly described up to $15-20$ fm. Note that this is an acceptable approximation (in contrast to the $E1$ cross section, where it would be totally unphysical) at least in the resonance regions, where the scattering wave functions are large at small radii.

The resulting $M1$ cross section, coming from the 1^+ initial scattering states (that is, no 3^+ state is included), is shown in Fig. 1 (solid line). Also shown in Fig. 1 is the $E1+M1$ cross section (dotted curve) which we get by adding our $M1$ result to an $E1$ curve (dot-dashed line) which has the energy dependence of Refs. [6,7] and is fitted (by eye) to the low-energy Filippone data [3]. One can see that, although the 1_2^+ bump predicted by our model is much too strong, such a structure in this energy region is not *a priori* ruled out by the data. A broader and smaller bump might be consistent with the experiments. Therefore, if such a state really exists, then it is weaker than our model prediction in Table I. We note that the biggest contribution to the 1_2^+ bump in Fig. 1 comes from the initial scattering states where the spin of ^{7}Be is $I_7=3/2$ and the channel spin is $I=1$.

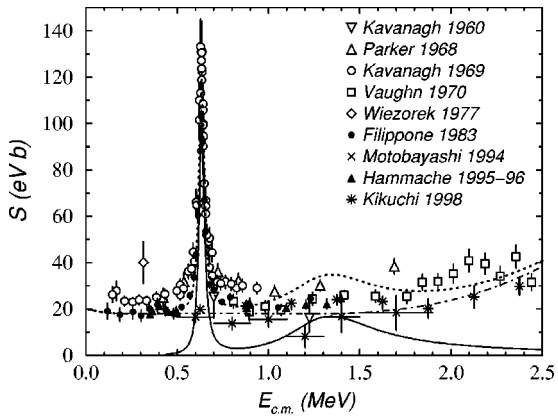

FIG. 1. Astrophysical S factor for the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reaction. The solid curve shows the $M1$ cross section coming from the 1^+ initial states in the present model. The dot-dashed curve is an $E1$ cross section whose energy dependence is taken from Refs. [6,7] and its absolute normalization is fitted (by eye) to the low-energy Filippone data. The dotted curve is the sum of the two. Also shown are the experimental data [3,4] with $\sigma_{dp}=157$ mb normalization, where it applies.

As one can observe, the average slope of the dotted curve in Fig. 1 is rather different from that of the $E1$ cross section in the $0.8<E<1.5$ MeV region. This means that if the 1_2^+ resonance really existed in ^{8}B, then it would make the separation of the measured cross section into $E1$ and $M1$ components more difficult than currently believed.

IV. CONCLUSIONS

In summary, we have searched for resonances in the ^{8}B and ^{8}Li continua, using a microscopic three-cluster model. We used a method, the analytic continuation of the scattering matrix to complex energies, which is well suited to find any resonances, even broad structures, if they exist. The properties of the known 1^+ states in ^{8}B and ^{8}Li are well reproduced in our model. In addition, we have found new 1^+ resonances. We have shown that the state predicted at 1.3 MeV in ^{8}B would have important consequences in the understanding of the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reaction.

As currently there is no experimental indication for the existence of the predicted states, we cannot be sure whether these structures are real or not. In order to be able to confirm or refute the existence of the predicted states, more theoretical and experimental work would be needed.

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