F spin as a partial symmetry

A. Leviatan[1,2,3] and J. N. Ginocchio[2,3]
[1]*Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel*
[2]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
[3]*Institute for Nuclear Theory, University of Washington, Box 351550, Seattle, Washington 98195*
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We use the empirical evidence that *F*-spin multiplets exist in nuclei for only selected states as an indication that *F* spin can be regarded as a partial symmetry. We show that there is a class of non-*F*-scalar IBM-2 Hamiltonians with partial *F*-spin symmetry, which reproduce the known systematics of collective bands in nuclei. These Hamiltonians predict that the scissors states have good *F*-spin and form *F*-spin multiplets, which is supported by the existing data.

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The interacting boson model (IBM-2) [1–3] describes collective low-lying states in even-even nuclei in terms of monople (s_ρ) and quadrupole (d_ρ) proton ($\rho=\pi$) and neutron ($\rho=\nu$) bosons. A microscopic, shell-model-based interpretation of the model [2,3] suggests that the number of bosons of each type (N_ρ) is fixed and is taken as the sum of valence proton and neutron particle and hole pairs counted from the nearest closed shell. The proton-neutron degrees of freedom are naturally reflected in the IBM-2 via an SU(2) *F*-spin algebra [2] with generators $\hat{F}_+ = s^\dagger_\pi s_\nu + d^\dagger_\pi \cdot \tilde{d}_\nu$, $\hat{F}_- = (\hat{F}_+)^\dagger$, $\hat{F}_0 = (\hat{N}_\pi - \hat{N}_\nu)/2$. The basic *F*-spin doublets are $(s^\dagger_\pi, s^\dagger_\nu)$, and $(d^\dagger_{\pi\mu}, d^\dagger_{\nu\mu})$, with *F*-spin projection $+1/2$ ($-1/2$) for proton (neutron) bosons. In a given nucleus, with fixed N_π, N_ν, all states have the same value of $F_0 = (N_\pi - N_\nu)/2$, while the allowed values of the *F*-spin quantum number *F* range from $|F_0|$ to $F_{\max} \equiv (N_\pi + N_\nu)/2 \equiv N/2$ in unit steps. *F*-spin characterizes the π-ν symmetry properties of IBM-2 states. States with maximal *F* spin, $F \equiv F_{\max}$, are fully symmetric and correspond to the IBM-1 states with only one type of bosons [1]. There are several arguments, e.g., the empirical success of IBM-1, the identification of *F*-spin multiplets [4–7] (series of nuclei with constant *F* and varying F_0 with nearly constant excitation energies), and weakness of $M1$ transitions, which lead to the belief that low lying collective states have predominantly $F = F_{\max}$ [8]. States with $F < F_{\max}$, corresponding to ''mixed-symmetry'' states, most notably, the orbital magnetic dipole scissors mode [9], have by now been established experimentally as a general phenomena in deformed even-even nuclei [10].

Various procedures have been proposed to estimate the *F*-spin purity of low-lying states [8]. These involve exploiting the data on $M1$ transitions (which should vanish between pure $F = F_{\max}$ states), extracting the difference in proton-neutron deformations from pion charge exchange [11], using ratios of γ and ground band magnetic moments [12] and the experimental g factors of 2^+_1 states [13], and considering the excitation energy of mixed symmetry states. In the majority of analyses the *F*-spin admixtures in low-lying states are found to be of a few percent ($<10\%$), typically 2-4 % [8]. In spite of its appeal, however, *F* spin cannot be an exact symmetry of the Hamiltonian. The assumption of *F*-spin scalar Hamiltonians is at variance with the microscopic interpretation of the IBM-2, which necessitates different effective interactions between like and unlike nucleons. Furthermore, if *F* spin was a symmetry of the Hamiltonian, then *all* states would have good *F* spin and would be arranged in *F*-spin multiplets. Experimentally this is not the case. As noted in an analysis [5,6] of rare earth nuclei, the ground bands are in *F*-spin multiplets, whereas the vibrational β bands and some γ bands do not form good *F*-spin multiplets. The empirical situation in the deformed Dy-Os region is portrayed in Table I and Fig. 1. From Table I it is seen that, for $F > 13/2$, the energies of the $L = 2^+$ members of the γ bands vary fast in the multiplet and not always monotonically. The variation in the energies of the β bands is large and irregular. Thus both microscopic and empirical arguments rule out *F*-spin invariance of the Hamiltonian. *F* spin can at best be an approximate quantum number which is good only for a selected set of states while other states are mixed. We are thus con-

TABLE I. Energies (in MeV) of 2^+ levels of the ground (g), γ and β bands in *F*-spin multiplets. The mass numbers are $A = 132 + 4F$.

F	Energy	^{A}Dy	${}^{A+4}$Er	${}^{A+8}$Yb	${}^{A+12}$Hf	${}^{A+16}$W	${}^{A+20}$Os
6	$E(2^+_g)$	0.14	0.13	0.12	0.12	0.12	0.14
	$E(2^+_\gamma)$	0.89	0.85	0.86	0.88		0.86
	$E(2^+_\beta)$	0.83	1.01	1.07	1.06		0.74
13/2	$E(2^+_g)$	0.10	0.10	0.10	0.10	0.11	0.13
	$E(2^+_\gamma)$	0.95	0.90	0.93	0.96		
	$E(2^+_\beta)$	1.09	1.17	1.14	0.99		
7	$E(2^+_g)$	0.09	0.09	0.09	0.10	0.11	0.13
	$E(2^+_\gamma)$	0.97	0.86	0.98	1.08		0.87
	$E(2^+_\beta)$	1.35	1.31	1.23	0.95		0.83
15/2	$E(2^+_g)$	0.08	0.08	0.08	0.09	0.11	
	$E(2^+_\gamma)$	0.89	0.79	1.15	1.23	1.11	
	$E(2^+_\beta)$	1.45	1.53	1.14	0.90	1.08	
8	$E(2^+_g)$	0.07	0.08	0.08	0.09		
	$E(2^+_\gamma)$	0.76	0.82	1.47	1.34		
	$E(2^+_\beta)$		1.28	1.12	1.23		
17/2	$E(2^+_g)$	0.08	0.08	0.08			
	$E(2^+_\gamma)$	0.86	0.93	1.63			
	$E(2^+_\beta)$	1.21	0.96	1.56			

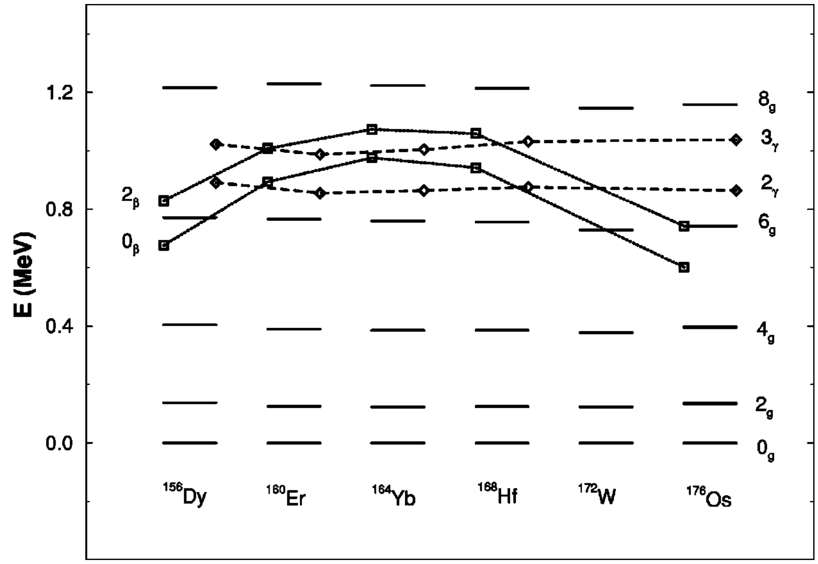

FIG. 1. Experimental levels of the ground γ and β bands in an F-spin multiplet $F=6$ of rare earth nuclei. Levels shown are up to $L=8_g^+$ for the ground band, $L=2_\gamma^+,3_\gamma^+$ for the γ band (diamonds connected by dashed lines) and $L=0_\beta^+,2_\beta^+$ for the β band (squares connected by dotted lines).

fronted with a situation of having ''special states'' endowed with a good symmetry which does not arise from invariance of the Hamiltonian. These are precisely the characteristics of a ''partial symmetry'' for which a nonscalar Hamiltonian produces a subset of special (at times solvable) states with good symmetry. Such a symmetry notion [14] was recently applied to nuclei [15], to molecules [16], and to the study of mixed systems with coexisting regularity and chaos [17]. Previously determined [11] non-F-scalar Hamiltonians were shown to have solvable ground bands with good F spin. It is the purpose of this paper to analyze in detail these Hamiltonians and to show that their partial F-spin symmetry reproduces the known systematics of ground and excited bands. In particular, we find F-spin multiplets in only selected bands, and observe common collective signatures for the ground and scissors bands in deformed nuclei, e.g., the same F-spin purity and equal moments of inertia. We further test a prediction for the existence of F-spin multiplets of scissors states.

The ground band in the IBM-2 is represented by an intrinsic state which is a product of a proton condensate and a rotated neutron condensate with N_π and N_ν bosons, respectively [18]. It depends on the quadrupole deformations $\beta_\rho,\gamma_\rho,(\rho=\pi,\nu)$ of the proton-neutron equilibrium shapes and on the relative orientation angles Ω between them. For $\beta_\rho>0$, the intrinsic state is deformed and members of the rotational ground-state band are obtained from it by projection. It has been shown in Ref. [11] that the intrinsic state will have a well defined F spin, $F=F_{\max}$, when the proton-neutron shapes are aligned and with equal deformations. The conditions $(\beta_\pi=\beta_\nu,\gamma_\pi=\gamma_\nu,\Omega=0)$ are weaker than the conditions for F-spin invariance, which makes it possible for a non-F-scalar IBM-2 Hamiltonian to have an equilibrium intrinsic state with pure F spin. Since the angular momentum projection operator is an F-spin scalar, the projected states of good L will also have good $F=F_{\max}$. A non-F-spin scalar Hamiltonian which has the above equilibrium condensate as an eigenstate is therefore guaranteed to have a ground band with good F-spin symmetry. Such explicit construction of an IBM-2 Hamiltonian with partial F-spin symmetry was presented in Ref. [11] for the most likely situation, namely, aligned axially symmetric (prolate) deformed shapes $(\beta_\rho=\beta,\gamma_\rho=\Omega=0)$. In this case, the equilibrium deformed intrinsic state for the ground band with $F=F_{\max}$ has the form

$$|c;K=0\rangle\equiv|N_\pi,N_\nu\rangle=(N_\pi!N_\nu!)^{-1/2}(b^\dagger_{c,\pi})^{N_\pi}(b^\dagger_{c,\nu})^{N_\nu}|0\rangle,$$

$$b^\dagger_{c,\rho}=(1+\beta^2)^{-1/2}(s^\dagger_\rho+\beta d^\dagger_{\rho,0}), \tag{1}$$

where K denotes the angular momentum projection on the symmetry axis. The relevant IBM-2 Hamiltonian with partial F-spin symmetry can be transcribed in the form

$$H=\sum_i\sum_{L=0,2}A_i^{(L)}R^\dagger_{i,L}\cdot\tilde{R}_{i,L}+\sum_{L=1,2,3}B^{(L)}W^\dagger_L\cdot\tilde{W}_L$$
$$+C^{(2)}[R^\dagger_{(\pi\nu),2}\cdot\tilde{W}_2+\text{H.c.}], \tag{2}$$

where H.c. means Hermitian conjugate and the dot implies a scalar product. The $R^\dagger_{i,L}$ $(L=0,2)$ are boson pairs with $F=1$ and $(F_0=1,0,-1)\leftrightarrow[i=\pi,(\pi\nu),\nu]$, and $W^\dagger_L(L=1,2,3)$ are F-spin scalar $(F=0)$ boson pairs defined as

$$R^\dagger_{\rho,0}=d^\dagger_\rho\cdot d^\dagger_\rho-\beta^2(s^\dagger_\rho)^2,\quad R^\dagger_{(\pi\nu),0}=\sqrt{2}(d^\dagger_\pi\cdot d^\dagger_\nu-\beta^2 s^\dagger_\pi s^\dagger_\nu),$$

$$R^\dagger_{\rho,2}=\sqrt{2}\beta s^\dagger_\rho d^\dagger_\rho+\sqrt{7}(d^\dagger_\rho d^\dagger_\rho)^{(2)},$$

$$R^\dagger_{(\pi\nu),2}=\beta(s^\dagger_\pi d^\dagger_\nu+s^\dagger_\nu d^\dagger_\pi)+\sqrt{14}(d^\dagger_\pi d^\dagger_\nu)^{(2)}, \tag{3}$$

$$W^\dagger_L=(d^\dagger_\pi d^\dagger_\nu)^{(L)}\ (L=1,3),\quad W^\dagger_2=s^\dagger_\pi d^\dagger_\nu-s^\dagger_\nu d^\dagger_\pi$$

with $\rho=\pi,\nu$ and $\tilde{R}_{i,L,\mu}=(-1)^{\mu}R_{i,L,-\mu}$, $\tilde{W}_{L,\mu}=(-1)^{\mu}W_{L,-\mu}$. The pair operators satisfy $R_{i,L,\mu}|c\rangle=W_{L,\mu}|c\rangle=0$ and consequently, the condensate is a zero energy eigenstate of H for *any* choice of parameters $A_i^{(L)},B^{(L)},C^{(2)}$, and any *any* N_π,N_ν. When $A_i^{(L)},B^{(L)},A_{\pi\nu}^{(2)}B^{(2)}-(C^{(2)})^2\geq 0$, the above Hamiltonian is positive-definite and hence $|c\rangle$ is its exact ground state with $F=F_{\max}$. H, however, is an F-spin scalar only when $A_\pi^{(L)}=A_\nu^{(L)}=A_{\pi\nu}^{(L)}$ $(L=0,2)$ and $C^{(2)}=0$. We thus have a non-F-spin scalar Hamiltonian with a solvable (degenerate) ground band with $F=F_{\max}$. The degeneracy can be lifted by adding to the Hamiltonian (F-spin scalar) SO(3) rotation terms which produce $L(L+1)$ type splitting but do not affect the wave functions. States in other bands can be mixed with respect to F-spin, hence the F-spin symmetry of H is partial. H trivially commutes with $\hat{F}_0$ but not with $\hat{F}_\pm$. However, $[H,\hat{F}_\pm]|c\rangle=0$ does hold and therefore H will yield F-spin multiplets for members of ground bands. On the other hand, states in other bands can have F-spin admixtures and are not compelled to form F-spin multiplets. These features which arise from the partial F-spin symmetry of the Hamiltonian are in line with the empirical situation as discussed above and as depicted in Table I and Fig. 1. It should be noted that the partial F-spin symmetry of H holds for any choice of parameters in Eq. (2). In particular, one can incorporate realistic shell-model based constraints, by choosing the $A_\rho^{(2)}$ $(\rho=\pi,\nu)$ terms (representing seniority-changing interactions between like nucleons), to be small. For the special choice $A_i^{(2)}=C^{(2)}=0$ and $B^{(1)}=B^{(3)}$, H of Eq. (2) becomes SO(5) scalar which commutes, therefore, with the SO(5) projection operator and hence produces F-spin multiplets with good SO(5) symmetry. Such multiplets were reported in the Yb-Os region of γ-soft nuclei [7].

The same conditions $(\beta_\rho=\beta,\gamma_\rho=\Omega=0)$ which resulted in $F=F_{\max}$ for the condensate of Eq. (1), ensure also $F=F_{\max}-1$ for the intrinsic state representing the scissors band

$$|\mathrm{sc};K=1\rangle=\Gamma_{\mathrm{sc}}^\dagger|N_\pi-1,N_\nu-1\rangle,$$

$$\Gamma_{\mathrm{sc}}^\dagger=b_{c,\pi}^\dagger d_{\nu,1}^\dagger-d_{\pi,1}^\dagger b_{c,\nu}^\dagger. \tag{4}$$

Here $\Gamma_{\mathrm{sc}}^\dagger$ is a $F=0$ deformed boson pair whose action on the condensate with $(N-2)$ bosons produces the scissors mode excitation. Furthermore, the scissors intrinsic state (4) is an exact eigenstate of the following Hamiltonian, obtained from Eq. (2) for the special choice $C^{(2)}=0$ and $B^{(1)}=B^{(3)}=2B^{(2)}\equiv 2B$

$$H'=\sum_i\sum_{L=0,2}A_i^{(L)}R_{i,L}^\dagger\cdot\tilde{R}_{i,L}+B\hat{\mathcal{M}}_{\pi\nu}. \tag{5}$$

The last term in Eq. (5) is the Majorana operator [1], related to the total F-spin operator by $\hat{\mathcal{M}}_{\pi\nu}=[\hat{N}(\hat{N}+2)/4-\hat{F}^2]$, with eigenvalues $k(N-k+1)$ for states with $F=F_{\max}-k$. The Hamiltonian H' is non-F-scalar but is rotational invariant. If we add to it an SO(3) rotation term $H'+\lambda\hat{L}^2$ $(\hat{L}=\hat{L}_\pi+\hat{L}_\nu)$, the resulting Hamiltonian will have a subset of *solvable* states which form the $K=0$ ground band $(L=0,2,4,\ldots)$ with $F=F_{\max}$, and the $K=1$ scissors band $(L=1,2,3\ldots)$ with $F=F_{\max}-1$. The resulting spectrum is

$$E_g(L)=\lambda\,L(L+1),\quad (F=F_{\max}),$$

$$E_{\mathrm{sc}}(L)=BN+\lambda L(L+1),\quad (F=F_{\max}-1), \tag{6}$$

where the Majorana coefficient B may depend on the boson numbers and deformation [8,19,20]. It follows that for such Hamiltonians, with partial F-spin symmetry, both the ground and scissors band have good F spin and have the same moment of inertia. The latter derived property is in agreement with the conclusions of a recent comprehensive analysis of the scissors mode in heavy even-even nuclei [19], which concluded that, within the experimental precisions $(\sim 10\%)$, the moment of inertia of the scissors mode are the same as that of the ground band. It is the partial F-spin symmetry of the Hamiltonian (5) which is responsible for the common signatures of collectivity in these two bands.

TABLE II. The ratio $R=\Sigma B(M1)\uparrow/(C_{F,F_0})^2$ for members of F-spin multiplets. Here $\Sigma B(M1)\uparrow$ denotes summed $M1$ strength to the scissors mode and $C_{F,F_0}=(F,F_0;1,0|F-1,F_0)$. Data taken from Refs. [21,22].

Nucleus	F	F_0	$\Sigma B(M1)\uparrow[\mu_N^2]$	$(C_{F,F_0})^2$	R
^{148}Nd	4	1	0.78 (0.07)	5/12	1.87 (0.17)
^{148}Sm		2	0.43 (0.12)	1/3	1.29 (0.36)
^{150}Nd	9/2	1/2	1.61 (0.09)	4/9	3.62 (0.20)
^{150}Sm		3/2	0.92 (0.06)	2/5	2.30 (0.15)
^{154}Sm	11/2	1/2	2.18 (0.12)	5/11	4.80 (0.26)
^{154}Gd		3/2	2.60 (0.50)	14/33	6.13 (1.18)
^{160}Gd	7	0	2.97 (0.12)	7/15	6.36 (0.26)
^{160}Dy		1	2.42 (0.18)	16/35	5.29 (0.39)
^{162}Dy	15/2	1/2	2.49 (0.13)	7/15	5.34 (0.28)
^{166}Er		−1/2	2.67 (0.19)	7/15	5.72 (0.41)
^{164}Dy	8	0	3.18 (0.15)	8/17	6.76 (0.32)
^{168}Er		−1	3.30 (0.12)	63/136	7.12 (0.26)
^{172}Yb		−2	1.94 (0.22)[a]	15/34	4.40 (0.50)
^{170}Er	17/2	−3/2	2.63 (0.16)	70/153	5.75 (0.35)
^{174}Yb		−5/2	2.70 (0.31)	66/153	6.26 (0.72)

[a]The low value of $\Sigma B(M1)\uparrow$ for ^{172}Yb has been attributed to experimental deficiencies [10].

The Hamiltonian H' of Eq. (5) is not F-spin invariant, however, $[H',\vec{F}]|c;K=0\rangle=[H',\vec{F}]|\mathrm{sc};K=1\rangle=0$. This implies that members of both the ground and scissors bands are expected to form F-spin multiplets. For ground bands such structures have been empirically established [4–7]. The prediction for F-spin multiplets of scissors states requires further elaboration. Although the mean energy of the scissors mode is at about 3 MeV [20], the observed fragmentation of the $M1$ strength among several 1^+ states prohibits, unlike ground bands, the use of nearly constant excitation energies as a criteria to identify F-spin multiplets of scissors states. Instead, a more sensitive test of this suggestion comes from the summed ground to scissors $B(M1)$ strength. The IBM-2 $M1$ operator $(\hat{L}_\pi - \hat{L}_\nu)$ is an F-spin vector $(F=1,F_0=0)$. Its matrix element between the ground state $[L=0_g^+,(F=F_{\rm max},F_0)]$ and scissors state $[L=1_{\rm sc}^+,(F'=F-1,F_0)]$ is proportional to an F-spin Clebsch Gordan coefficient $C_{F,F_0}=(F,F_0;1,0|F-1,F_0)$ times a reduced matrix element. It follows that the ratio $B(M1;0_g^+\rightarrow 1_{\rm sc}^+)/(C_{F,F_0})^2$ does not depend on F_0 and should be a constant in a given F-spin multiplet. In Table II we list *all* F-spin partners for which the summed $B(M1)$ strength to the scissors mode has been measured to date [21,22]. It is seen that within the experimental errors, the above ratio is fairly constant. The most noticeable discrepancy for ^{172}Yb $(F=8)$, arises from its measured low value of summed $B(M1)$ strength. The latter should be regarded as a lower limit due to experimental deficiencies (large background and strong fragmentation [10]). These observations strengthen the contention of high F-spin purity and formation of F-spin multiplets of scissors states.

As noted in [5,6] and shown in Table I and Fig. 1, for nuclei with $F=6,\ 6.5$, also members of the γ bands display constant excitation energies and seem to form good F-spin multiplets. This empirical observation has a natural explanation within the family of Hamiltonians with partial F-spin symmetry. For the choice $\beta=\sqrt{2}$ and $A_\pi^{(2)}=A_\nu^{(2)}=A_{\pi\nu}^{(2)}$ in Eq. (5), H' will have both F-spin and SU(3) partial symmetries. In such circumstances, the ground $(K=0)$, scissors $(K=1)$, symmetric-γ $(K=2)$, and antisymmetric-γ $(K=2)$ bands are solvable and have good SU(3) and F-spin symmetries: $[(\lambda,\mu),F]=[(2N,0),F_{\rm max}]$, $[(2N-2,1),F=F_{\rm max}-1]$, $[(2N-4,2),F=F_{\rm max}]$, and $[(2N-4,2),F=F_{\rm max}-1]$, respectively. The intrinsic states for the symmetric-γ or antisymmetric-γ bands are obtained by F-spin coupling the $F=1$ pair $R^\dagger_{i,2,\mu=2}$ to the $(F=F_{\rm max}-1)$ condensate $|N_\pi-1,N_\nu-1\rangle$ with $(N-2)$ bosons to form a N-boson intrinsic state with $F=F_{\rm max}$ or $F=F_{\rm max}-1$. Since, in this case, the commutator $[H',\vec{F}]$ vanishes when it acts on the solvable intrinsic states, the projected states are ensured to have good F spin and form F-spin multiplets. At the same time, since the Hamiltonian is not F-spin scalar, the β bands can have F-spin admixtures and need not form F-spin multiplets.

In summary, we have examined in detail IBM-2 Hamiltonians with partial F-spin symmetry. The latter are not F-spin scalars, yet have a subset of solvable eigenstates with good F-spin symmetry. In particular, the corresponding ground bands form F-spin multiplets with $F=F_{\rm max}$, but excited bands can be mixed, which is in line with the empirically observed F-spin multiplets [4–7]. A class of IBM-2 Hamiltonians with partial F-spin symmetry predict the occurrence of F-spin multiplets of scissors states, with a moment of inertia equal to that of the ground band. This prediction is in agreement with recent analyses of the empirical systematics of excitation energy and $M1$ strength of the scissors mode in even-even nuclei [19,20]. All the above findings illuminate the potential useful role of F-spin (and other) partial symmetries in nuclear spectroscopy and motivate their further study.

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