Mass formulas and thermodynamic treatment in the mass-density-dependent model of strange quark matter

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The previous treatments for strange quark matter in the quark mass-density-dependent model have unreasonable vacuum limits. We provide a method to obtain the quark mass parametrizations and give a self-consistent thermodynamic treatment which includes the MIT bag model as an extreme. In this treatment, strange quark matter in bulk still has the possibility of absolute stability. However, the behavior of the sound velocity is opposite to previous findings.

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I. INTRODUCTION

Since Witten's conjecture that quark matter with strangeness per baryon of order unity might be bound [1], an extensive body of literature has investigated the stability and/or probabilities of strange quark matter (SQM) [2]. Because the application of perturbative quantum chromodynamics (QCD) to strong-coupling domain is unbelievable while the lattice approach is presently limited to the case of zero chemical potential, we have to resort to phenomenological models. One of the most famous models is the MIT bag model with which Farhi and Jaffe find that SQM is absolutely stable around the normal nuclear density for a wide range of parameters [3]. Further investigations have also been carried out by many other authors in the bag model [4–6]. A recent investigation indicates a link of SQM to the study of quark condensates [7] while a more recent work has carefully studied the relation between the charge and critical density of SQM [8].

Chakrabarty *et al.* [9,10] have discussed the limitation of the conventional MIT bag model which assumes that the quarks are asymptotically free within the bag. In order to incorporate the strong interaction between quarks, one way is to fall back on the perturbation theory, which is questionable in the strong-coupling domain. An alternative way is to make the quark masses density dependent. In this nonperturbative treatment, the strong interaction between quarks is mimicked by the proper variation of quark masses with density. There are two questions of crucial importance to this model. One is how to parametrize quark masses, the other concerns thermodynamic treatment. However, the two aspects are not self-consistent in literature presently.

Here are the popularly used parametrizations for quark masses m_q $(q=u,d,s)$:

$$m_{u,d}=\frac{B}{3n_b}, \tag{1}$$

$$m_s=m_{s0}+\frac{B}{3n_b}, \tag{2}$$

where m_{s0} is the s quark current mass, n_b is the baryon number density, B is the famous MIT bag constant. Equation (1) was first used to study light quark matter [11], and later extended to Eq. (2) to investigate strange quark matter [9,10,12].

As for the thermodynamic treatment, there exist two controversial ones in the literature up to now. One expresses the total pressure of SQM as [9,10]

$$P_1=-\Omega, \tag{3}$$

where Ω is the ordinary thermodynamic potential density of SQM [see Eq. (40)]. The other adopts the following expression [12]:

$$P_2=-\Omega+n_b\frac{\partial\Omega}{\partial n_b}. \tag{4}$$

The extra term in Eq. (4) is said to arise from the baryon density dependence of quark masses. This difference leads to significantly different results. Therefore, it is meaningful to take a check of the two thermodynamic treatments.

As is well known, the QCD vacuum is not necessarily empty. To obtain the vacuum properties, let us take the limit $n_b\to 0$ for the two treatments. It is easy to obtain, at zero temperature, the limits

$$\lim_{n_b\to 0} P_1=0, \tag{5}$$

$$\lim_{n_b\to 0} E_1=B, \tag{6}$$

for the first treatment, and the limits

$$\lim_{n_b\to 0} P_2=-B, \tag{7}$$

$$\lim_{n_b\to 0} E_2=2B, \tag{8}$$

for the second treatment. Here E_1 and E_2 are the corresponding energy densities.

According to the fundamental idea of MIT bag model, QCD vacuum has a constant energy density B, the famous bag constant. The mass parametrization (1) is just obtained from this requirement ($\lim_{n_b\to 0} E_1 \to 3m_q n_b$ for flavor-symmetric case) [11]. The constant vacuum energy comes from the fact that QCD vacuum must have a pressure to maintain pressure balance at the bag boundary. Obviously, the first treatment can give the correct vacuum energy and a wrong QCD vacuum pressure. On the contrary, the second treatment leads to the correct QCD vacuum pressure but a wrong vacuum energy. In fact, this is just caused by the ignorance of the QCD vacuum energy which guarantees the pressure balance at the bag boundary.

It should be pointed out that in getting the unreasonable limits (5)–(8), we have used the quark mass formulas (1) and (2). Because these formulas are pure parametrizations without any real support from underlying theories, one may ask if the contradictions can be solved by choosing other parametrizations. According to our present investigation, one should modify the quark mass formulas and thermodynamic treatment simultaneously.

It is the aim of this paper to give a self-consistent treatment which includes the conventional MIT bag model as an extreme. In our new treatment, strange quark matter in bulk still has the possibility of absolute stability. However, the lower density behavior of the sound velocity in SQM is opposite to previous findings.

In the following section, we first derive the new quark mass formulas and describe our thermodynamic treatment, and then in Sec. III, we present our results in studying SQM with this model. Section IV is a short summary.

II. FRAMEWORK

Let us schematically write the QCD Hamiltonian density as

$$H_{\mathrm{QCD}} = H_k + \sum_q m_{q0}\bar{q}q + H_{\mathrm{I}}, \tag{9}$$

where H_k is the kinetic term, m_{q0} is the quark current mass, and H_{I} is the interaction part. The summation goes over all flavors considered.

The basic idea of the quark mass-density-dependent model of strange quark matter is that the system energy can be expressed as the same form with a proper noninteracting system. The strong interaction between quarks is included within the appropriate variation of quark masses with density. To avoid confusion with other mass concepts, we refer such a density-dependent mass to an equivalent mass in this paper. Therefore, if we use the equivalent mass m_q, the system Hamiltonian density should be replaced by a Hamiltonian density of the form

$$H_{\mathrm{eqv}} = H_k + \sum_q m_q\bar{q}q, \tag{10}$$

where m_q is the equivalent mass to be determined. Obviously, we must require that the two Hamiltonian densities H_{eqv} and H_{QCD} have the same eigenenergy for any eigenstate $|\Psi\rangle$, i.e.,

$$\langle\Psi|H_{\mathrm{eqv}}|\Psi\rangle = \langle\Psi|H_{\mathrm{QCD}}|\Psi\rangle. \tag{11}$$

Applying this equality respectively to the state $|n_b\rangle$ with baryon number density n_b and the vacuum state $|0\rangle$, and then taking the difference, one has

$$\langle n_b|H_{\mathrm{eqv}}|n_b\rangle - \langle 0|H_{\mathrm{eqv}}|0\rangle = \langle n_b|H_{\mathrm{QCD}}|n_b\rangle - \langle 0|H_{\mathrm{QCD}}|0\rangle. \tag{12}$$

The simplest and most symmetric solution for the equivalent mass from this equation is

$$m_q = m_{q0} + \frac{\langle H_{\mathrm{I}}\rangle_{n_b} - \langle H_{\mathrm{I}}\rangle_0}{\sum_q [\langle\bar{q}q\rangle_{n_b} - \langle\bar{q}q\rangle_0]} \tag{13}$$

$$\equiv m_{q0} + m_{\mathrm{I}}, \tag{14}$$

where we have used the symbol definitions: $\langle H_{\mathrm{I}}\rangle_{n_b} \equiv \langle n_b|H_{\mathrm{I}}|n_b\rangle$, $\langle H_{\mathrm{I}}\rangle_0 \equiv \langle 0|H_{\mathrm{I}}|0\rangle$, and $\langle\bar{q}q\rangle_{n_b} \equiv \langle n_b|\bar{q}q|n_b\rangle$, $\langle\bar{q}q\rangle_0 \equiv \langle 0|\bar{q}q|0\rangle$.

Therefore, if quarks are decoupled, they should take the equivalent mass of the form (13) to keep the system energy unchanged. From Eq. (13) we see that the equivalent mass m_q includes two parts: one is the original mass or current mass m_{q0}, the other is the interacting part m_{I}. Because m_{I} equals the ratio of the total interacting part of the energy density and the total relative quark condensate, it is flavor-independent and density dependent. Because of the quark confinement and the asymptotic freedom, i.e.,

$$\lim_{n_b\to 0} m_{\mathrm{I}} = \infty, \tag{15}$$

$$\lim_{n_b\to\infty} m_{\mathrm{I}} = 0, \tag{16}$$

the reasonable form might be

$$m_{\mathrm{I}} = \frac{D}{n_b^z}. \tag{17}$$

Accordingly, we have

$$m_q = m_{q0} + \frac{D}{n_b^z}, \tag{18}$$

where D is a free parameter to be determined by stability arguments. Obviously, $z>0$ for confined particles and $z<0$ for nonconfined particles. In Eqs. (1) and (2), $z=1$. However, just as mentioned in the Introduction, Eqs. (1) and (2) are closely linked to the first thermodynamic treatment, and thus unsuitable for our case. We now discuss the determination of $m_{\rm I}$ which is consistent with our thermodynamic treatment.

Firstly, we express the interacting part of the energy density $\langle H_{\rm I}\rangle \equiv \langle H_{\rm I}\rangle_{n_b} - \langle H_{\rm I}\rangle_0$ [the numerator in Eq. (13)] as

$$\langle H_{\rm I}\rangle = \frac{1}{2V}\int\int_V v(r)(3n_b d\vec{r}_1)(3n_b d\vec{r}_2) \tag{19}$$

$$= 18\pi n_b^2 \int_0^R v(r) r^2\, dr, \tag{20}$$

where $r = |\vec{r}_1 - \vec{r}_2|$, $v(r)$ is the quark-quark interaction, R is the SQM radius, $V = 4/3\pi R^3$ is the volume. The extra factor 1/2 is responsible for double counting.

Because of the following obvious equality:

$$\lim_{n_b\to 0} \frac{\langle \bar{q}q\rangle_{n_b}}{\langle \bar{q}q\rangle_0} = 1, \tag{21}$$

the Taylor series of the relative condensate at zero density has the following general form:

$$\frac{\langle \bar{q}q\rangle_{n_b}}{\langle \bar{q}q\rangle_0} = 1 - \frac{n_b}{\rho_q'} + \text{ higher orders in } n_b + \cdots. \tag{22}$$

If taking it only to first order approximation, we have

$$\sum_q [\langle \bar{q}q\rangle_{n_b} - \langle \bar{q}q\rangle_0] = \sum_q [-\langle \bar{q}q\rangle_0/\rho_q'] n_b \equiv A n_b. \tag{23}$$

Taking the ratio of Eqs. (20) and (23), we get

$$m_{\rm I} = \frac{18\pi}{A} n_b \int_0^R v(r) r^2\, dr. \tag{24}$$

According to the lattice calculation [13] and string model investigation [14], the quark-quark interaction is proportional to the distance, i.e., $v(r) = \alpha r$. We thus have

$$m_{\rm I} = \frac{18\pi\alpha}{A} n_b \frac{R^4}{4} \propto \frac{1}{n_b^{1/3}}. \tag{25}$$

Therefore, we should take in Eq. (18) $z = 1/3$, i.e.,

$$m_q = m_{q0} + \frac{D}{n_b^{1/3}}, \tag{26}$$

where D is a parameter to be determined by stability arguments.

Because the Hamiltonian density $H_{\rm eqv}$ has the same form as that of a system of free particles with equivalent mass m_q, the energy density of SQM can be expressed as

$$E = \sum_{i=u,d,s,e} \frac{g_i}{2\pi^2} \int_0^{p_{f,i}} \sqrt{p^2 + m_i^2}\; p^2\, dp + B, \tag{27}$$

where

$$p_{f,i} = \left(\frac{6}{g_i}\pi^2 n_i\right)^{1/3} \tag{28}$$

is the corresponding Fermi momentum.

As is usually done, we here assume that the SQM consists of u, d, and s quarks, and electrons (neutrinos enter and leave the system freely). The degeneracy factor g_i is 6 for quarks and 2 for electrons. The electron mass m_e is equal to 0.511 MeV. In order to include the strong interaction between quarks, the quark masses m_q $(q = u,d,s)$ should be replaced with the expression (13) or (26). The extra term B comes from the pressure balance condition, and its physical meaning is still the vacuum energy density or vacuum pressure just as in the MIT bag model. The corresponding pressure is

$$P = \sum_{i=u,d,s,e} \mu_i n_i - E, \tag{29}$$

where μ_i is the chemical potential for particle type i. Because it is equal to the Fermi energy at zero temperature, we have

$$\mu_i = \sqrt{p_{f,i}^2 + m_i^2}. \tag{30}$$

Equation (29) is equivalent to

$$P = -\Omega - B. \tag{31}$$

The second term $-B$ is responsible for pressure balance. Such an extra term is necessary even in the nonrelativistic treatment of SQM [15].

It is clear that the above thermodynamic treatment will approach the conventional MIT bag model if one casts away the interacting part $m_{\rm I}$ of the equivalent mass m_q. It can be proved, from Eqs. (26), (27), and (29), that we have the following correct vacuum limits:

$$\lim_{n_b\to 0} E = B, \tag{32}$$

$$\lim_{n_b\to 0} P = -B. \tag{33}$$

Therefore, the physical meaning of B is the same as that in the conventional bag model. We take $B^{1/4} = 144$ MeV in our present calculation.

III. PROPERTIES OF STRANGE QUARK MATTER

Following previous authors [3], we assume the SQM to be a Fermi gas mixture of u,d,s quarks and electrons with chemical equilibrium maintained by the weak interactions: $d,s \leftrightarrow u + e + \bar{\nu}_e, s + u \leftrightarrow u + d$. For a given baryon number density n_b and total electric charge density Q, the chemical potentials μ_u, μ_d, μ_s, and μ_e are determined by the following equations [8]:

$$\mu_d = \mu_s \equiv \mu, \tag{34}$$

$$\mu_u + \mu_e = \mu, \tag{35}$$

$$\tfrac{1}{3}(n_u + n_d + n_s) = n_b, \tag{36}$$

$$\tfrac{2}{3}n_u - \tfrac{1}{3}n_d - \tfrac{1}{3}n_s - n_e = Q, \tag{37}$$

where the particle number density n_i is related to the corresponding chemical potential μ_i by

$$n_i = \frac{g_i}{6\pi^2}(\mu_i^2 - m_i^2)^{3/2}, \tag{38}$$

which is derived from the relation

$$n_i = -\frac{\partial \Omega_i}{\partial \mu_i}, \tag{39}$$

with

$$\Omega_i = -\frac{g_i}{48\pi^2}\left[\mu_i(\mu_i^2 - m_i^2)^{1/2}(2\mu_i^2 - 5m_i^2) + 3m_i^4 \ln\frac{\mu_i + \sqrt{\mu_i^2 - m_i^2}}{m_i}\right]. \tag{40}$$

In order to include the strong interaction between quarks, the quark masses m_u, m_d, and m_s in the above equations are to be replaced with the density-dependent expression (26) while the electron mass m_e is negligible (0.511 MeV).

For the bulk SQM with weak equilibrium, the previous investigations got a slightly positive charge. Our recent study demonstrates that negative charges could lower the critical density. However, too much negative charge can make it impossible to maintain flavor equilibrium. Therefore, the charge of SQM is not allowed to shift too far away from zero at both positive and negative directions. For this and our methodological purpose, we only consider neutral SQM in this paper, i.e., $Q=0$ in Eq. (37).

Since the baryonic matter is known to exist in the hadronic phase, we must require D to be such that the *ud* system is unbound. This constrains D to be bigger than $(47\ \text{MeV})^2$, i.e., at $P=0$, $E/n_b > 930$ in order not to contradict standard nuclear physics. On the other hand, we are interested in the possibility that SQM might be absolutely stable, i.e., at $P=0$, $E/n_b < 930$, which gives an upper bound $(128\ \text{MeV})^2$. We take $D^{1/2}$ to be 50, 80, and 110 MeV, respectively.

Because the light quark current masses are very small, their value uncertainties are not important. So we take the fixed central values $m_{u0} = 5$ MeV and $m_{d0} = 10$ MeV in our calculation. As for s quarks, we take 150, 120, and 90 MeV, corresponding respectively to $D^{1/2} = 50$, 80, and 110 MeV.

For a given n_b, we first solve for μ_i $(i=u,d,s,e)$ from the equation group (34)–(37), and then calculate the energy density and pressure of SQM from Eqs. (27) and (29):

$$E = \sum_i \frac{g_i m_i^4 x_i^3}{6\pi^2} F(x_i) + B = \sum_i m_i n_i F(x_i) + B, \tag{41}$$

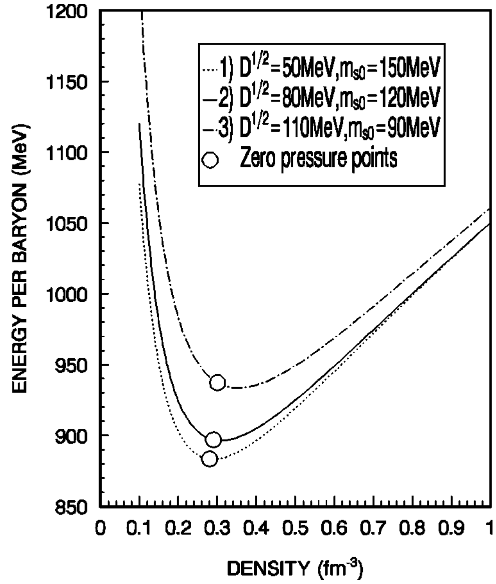

FIG. 1. The energy per baryon vs baryon number density for different parameters. The zero pressure density occurs at the points marked with a circle "○."

$$P = \sum_i \frac{g_i m_i^4 x_i^5}{6\pi^2} G(x_i) - B = \sum_i m_i n_i x_i^2 G(x_i) - B, \tag{42}$$

where the summation goes over u, d, s, and e, and

$$x_i \equiv \frac{p_{f,i}}{m_i} = \frac{\sqrt{\mu_i^2 - m_i^2}}{m_i} \tag{43}$$

is the ratio of the Fermi momentum to the mass that related to particle type i. With the hyperbolic sine function $\mathrm{sh}^{-1}(x) \equiv \ln(x + \sqrt{x^2+1})$, the functions $F(x)$ and $G(x)$ are defined as

$$F(x) \equiv \frac{3}{8}[x\sqrt{x^2+1}(2x^2+1) - \mathrm{sh}^{-1}(x)]/x^3, \tag{44}$$

$$G(x) \equiv \frac{1}{8}[x\sqrt{x^2+1}(2x^2-3) + 3\,\mathrm{sh}^{-1}(x)]/x^5, \tag{45}$$

which have the limit properties

$$\lim_{x\to 0} F(x) = 1, \tag{46}$$

$$\lim_{x\to 0} G(x) = \frac{1}{5}. \tag{47}$$

Therefore, we have the correct limits (32) and (33).

In Fig. 1, we give the energy per baryon vs baryon number density for the three pairs of parameters. We see that SQM is absolutely stable for the first two parameter groups, while metastable for the third group. The points marked with a circle "○" are the zero pressure points where the pressure within SQM is zero. Because of the density dependence of quark masses, the zero pressure density is generally not that

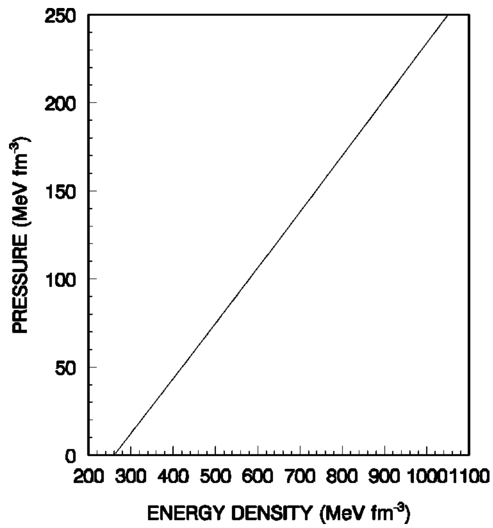

FIG. 2. The equation of state for parameter group $D^{1/2}=80$ MeV and $m_{s0}=120$ MeV. It asymptotically approaches to the ultrarelativistic case as expected.

corresponding to the minimum energy per baryon (as in the usual case), but nearly the case in the first two parameter groups.

The resulting equation of state is plotted in Fig. 2. Because it is insensitive to parameters, we have only chosen one parameter pair: $D=(80\ \text{MeV})^2$ and $m_{s0}=120$ MeV.

In Fig. 3, we show the sound velocity c of SQM with a dot-dashed line, which is obtained from

$$c=\left|\frac{dP}{dE}\right|^{1/2}. \tag{48}$$

Because the interacting part of the quark masses is negligible at higher densities, it asymptotically tends to the ultrarelativistic value $1/\sqrt{3}$ as in the bag model (solid line). Simultaneously given is that calculated by the same method as in Ref. [12] with $C=90\ \text{MeV fm}^{-3}$ and $m_{s0}=80$ MeV (dotted line). Obviously, the lower density behavior of the sound velocity in our model is opposite to that in the previous calculation.

It is interesting to note that if one considers the thermodynamic relation $P=-\partial(\Omega V)/\partial V$ as being more fundamental than $P=-\Omega$ (as done in Ref. [12]), Eqs. (41) and (42) should be replaced with

$$E=\sum_i m_i n_i F(x_i)+\sum_i m_i n_i f(x_i)+B, \tag{49}$$

$$P=\sum_i m_i n_i x_i^2 G(x_i)-\sum_i m_i n_i f(x_i)-B, \tag{50}$$

where

$$f(x_i)\equiv-\frac{3}{2}\frac{n_b}{m_i}\frac{dm_i}{dn_b}\left[x_i\sqrt{x_i^2+1}-\text{sh}^{-1}(x_i)\right]\Big/ x_i^3, \tag{51}$$

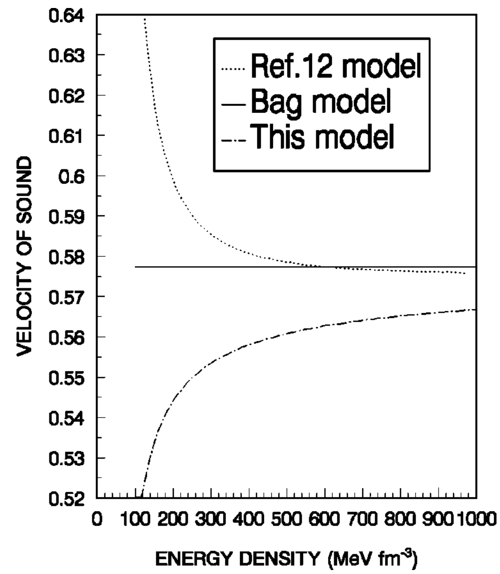

FIG. 3. The sound velocity vs energy density. The dot-dashed line is calculated with the method in this paper, while the dotted line is calculated with the same method in Ref. [12]. Their lower density behavior is obviously opposite. The full line is the ultrarelativistic case.

which has the limit property

$$\lim_{n_b\to 0} f(x_i)=z. \tag{52}$$

However, the modification does not change the properties of SQM significantly this time. For the same parameters, the line in Fig. 1 will move upward slightly while in Fig. 2 and Fig. 3 it will move a little downward. This is because the contribution from the extra term $\Sigma_i m_i n_i f(x_i)$ (arising from the density dependence of the quark masses) is positive to energy and negative to pressure. But the gross features of SQM are still the same.

IV. SUMMARY

We have presented a new version of the quark mass-density-dependent model for SQM. We first note that the previous treatments have unreasonable vacuum limits. Then we provide a practical method to derive the quark mass formulas. In our thermodynamic treatment, the conventional bag model is included as an extreme, and the vacuum still has a constant energy density corresponding to a constant pressure B. In this new treatment, SQM also has the possibility of absolute stability for a wide range of parameters. A noticeable feature is that the sound velocity is smaller than the ultrarelativistic case at lower densities, contrary to the previous finding.

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