Perturbative calculation of the scaled factorial moments in the second-order quark-hadron phase transition within the Ginzburg-Landau description

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The scaled factorial moments F_q are studied for a second-order quark-hadron phase transition within the Ginzburg-Landau description. The role played by the ground state of the system under low temperature is emphasized. After a local shift of the order parameter the fluctuations are around the ground state, and a perturbative calculation for F_q can be carried out. Power scaling between F_q's is shown, and a universal scaling exponent $\nu \simeq 1.75$ is given for the case with weak correlations and weak self-interactions.

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I. INTRODUCTION

It is well known that the ultrarelativistic heavy-ion collision is a unique way to study the vacuum properties of quantum chromodynamics (QCD) in the laboratory. In the collisions the kinetic energies of the colliding particles are converted into thermal ones, and a hot new matter state, quark-gluon plasma (QGP), might be formed. The system will cool with its subsequent expanding and will undergo a phase transition from the deconfined QGP to confined hadrons. Since only the final state particles in the collisions are observable in experiments, one may be asked to search for the signals about the phase transition from only those particles. Since the existence of the phase transition is associated with properties of the nontrivial chromodynamical vacuum, the study of quark-hadron phase transition has been a hot point in both particle physics and nuclear physics for more than a decade. Besides the unique features of QCD the lack of control of the temperature in the phase transition distinguishes the problem from the standard critical phenomena such as ferromagnetism. The nonperturbative nature of the hadronization process in the phase transition precludes at this stage any observable hadronic predictions from first principles, and some approximate models are used. One of the models is the Ginzburg-Landau model which can be used as a framework to calculate various moments of the multiplicity distribution and has been used in the studies of the scaled factorial moments in both first- [1] and second-order [2] phase transitions, the multiplicity difference correlators [3], and the multiplicity distributions in the phase transitions [4].

In the Ginzburg-Landau description of a second-order phase transition, the scaled factorial moments can be expressed as [2]

$$F_q = f_q / f_1^q, \quad f_q = \frac{1}{Z}\int \mathcal{D}\phi \left(\int_\delta dz \left| \phi \right|^2 \right)^q \exp(-F[\phi]), \tag{1}$$

with $Z=\int \mathcal{D}\phi \exp(-F[\phi])$, the free energy functional $F[\phi] = \int dz [a|\phi|^2 + b|\phi|^4 + c|\nabla\phi|^2]$, $a \propto (T - T_C)$ representing the distance from the critical point, and b and c larger than zero. Here $|\phi|^2$ is associated with the hadronic multiplicity density of the system, and $\int_\delta dz$ means integration over a small bin with width δ in the phase space. Similar expressions can be derived for other quantities mentioned above. In all former studies of second-order phase transition the gradient term in the functional $F[\phi]$ is simply taken to be zero, i.e., the field ϕ is regarded as spatially uniform. The spatial integral of the functional over a two-dimensional bin with size δ^2 is then $F[\phi] = \delta^2 (a|\phi^2| + b|\phi|^4)$. This is of course a very crude approximation. The advantage of such an approximation is that it turns the functional integration into a normal one. Thus, the calculation becomes quite easy under the approximation. For $a>0$ the functional takes its minimum at $|\phi|^2 = 0$ corresponding to the quark phase, and for $a<0$ the minimum is at $|\phi|^2 > 0$ corresponding to the hadron phase. In all the studies the interested region is for $a<0$. Numerical results do not show the so-called intermittency behavior, but the F-scaling, $F_q \propto F_2^{\beta_q}$ with universal scaling law $\beta_q = (q-1)^\nu$, is shown to be valid. The exponent ν is called a universal one in the sense that it is insensitive to the values of the parameters in the model and that it is completely determined by the structure of the functional concerned.

The contributions from the gradient term to the moments and to the exponent ν should be evaluated in some way. Once the gradient term is taken into the functional, one is faced with serious difficulty in the calculations, considering the fact that the value of parameter b for the ϕ^4 term can be determined in no way from first principles or from experimental output and may be very large. Even if the parameter b is indeed very small, negative value of a in our interested region also excludes the possibility of performing the usual perturbative calculations. The role played by the gradient term is investigated in [5] and [6]. In [5] ϕ in each bin is still uniform, but the values of ϕ in all neighboring bins are taken to be ϕ_0, field configuration corresponding to the minimum of ''potential'' $V(\phi) \equiv a|\phi|^2 + b|\phi|^4$. So the square of the gradient of ϕ is $\delta^{-2}(\phi - \phi_0)^2$. This approximation also transforms the functional integration into a normal one. Numerical results show that the universal scaling law $\beta_q = (q-1)^\nu$ is still valid and that the exponent ν is almost the same as without the gradient term. In [6] the details of spatial

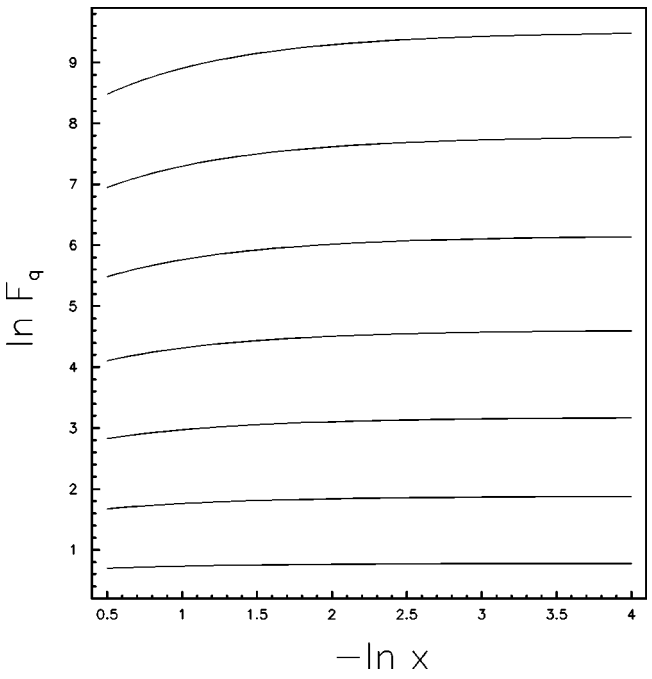

FIG. 1. Dependences of $\ln F_q$ on the bin width $-\ln x$ after the contribution from spatial fluctuations of the phase angle of the field fully taken into account (mode 2). Curves from lower to upper are for q from 2 to 8, respectively.

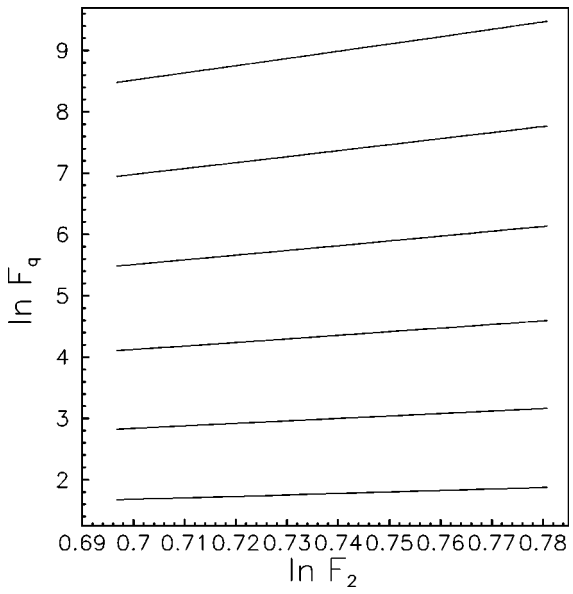

FIG. 2. Scaling behaviors of $\ln F_q$ vs $\ln F_2$ for the same data as in Fig. 1.

fluctuations of ϕ in a bin is simulated by the Ising model for one-component spins s. Each bin is assumed large enough to contain several spin sites. This time, the exponent ν depends on the unknown temperature, and, after averaging over the temperature, ν is still in the range given in [1] and [2].

Though the simulation in [6] is convincing, it is for lattice with one-component spins. In the Ginzburg-Landau model for a second-order phase transition, the field ϕ is a complex number, or in other words, ϕ has two components. At first glimpse, the simulation in [6] does not correspond to the real problem discussed in the Ginzburg-Landau model, but as will be explained soon in this paper, it relates to the physics in an indirect way.

In [7], we attempted to investigate the universality of the exponent ν, with the spatial fluctuations of the phase angle of the complex field ϕ fully taken into account. As will be shown below, the contribution from spatial fluctuations of the phase angle of the field ϕ can be evaluated in a complete and rigorous way, and the integration over the spatial fluctuations of the phase angle of the field ϕ will reduce the problem to one with a one-component field.

The first observation is that all terms except the gradient one in the functional integral of Eq. (1) depend only on $|\phi|^2$. Then it is convenient to write the two-component field ϕ as a complex number in the form $\phi=\phi_{\mathrm{R}}\exp(i\phi_{\mathrm{I}})$. The spatial fluctuations of the field can be those of the magnitude ϕ_{R} and/or of the phase angle ϕ_{I} (or orientation in an abstract space). The gradient term turns out to be

$$|\nabla\phi|^2=(\nabla\phi_{\mathrm{R}})^2+\phi_{\mathrm{R}}^2(\nabla\phi_{\mathrm{I}})^2. \tag{2}$$

Generally, the phase angle ϕ_{I} can be in any form, and the full contribution from its fluctuations must be evaluated. Fortunately, the integral over ϕ_{I} can be carried out easily since it is of Gaussian form. Then one transforms the twofold functional integral into a onefold one and gets

$$f_q=\frac{\int \mathcal{D}\phi_{\mathrm{R}}\left(\int_\delta dz\,\phi_{\mathrm{R}}^2\right)^q \exp(-F[\phi_{\mathrm{R}}])}{\int \mathcal{D}\phi_{\mathrm{R}}\exp(-F[\phi_{\mathrm{R}}])}, \tag{3}$$

with functional $F[\phi_{\mathrm{R}}]$ exactly the same form as the original $F[\phi]$. The important difference between this expression from Eq. (1) is that the functional integral variable in this new expression is a real function instead of a complex function as in Eq. (1). Then f_q and F_q can be simulated by a one-component field as in Ref. [6].

Now we take the field ϕ_{R} (magnitude of ϕ) uniform, or in other words, the gradient term of ϕ_{R} is omitted. (Calculations based on this approximation will be referred to as mode 2 in this paper.) Based on the work Ref. [6] one can drop off the $\nabla\phi_{\mathrm{R}}$ term, because the problem now is exactly within the scope of Ref. [6], and the conclusions in Ref. [6] encourage us to neglect the spatial fluctuations of ϕ_{R} as long as the universal scaling exponent ν is concerned. Then one gets the factorial moments as functions of variable x

$$f_q=\frac{\int_0^\infty dy\,y^{2q}\exp(xy^2-y^4)}{\int_0^\infty dy\,\exp(xy^2-y^4)}, \tag{4}$$

with $x=a\,\delta^{3/2}/b^{1/4}$. From this expression the scaled factorial moments $\ln F_q$ can be calculated, and the results are shown as functions of $-\ln x$ in Fig. 1 for q from 2 to 8 within the

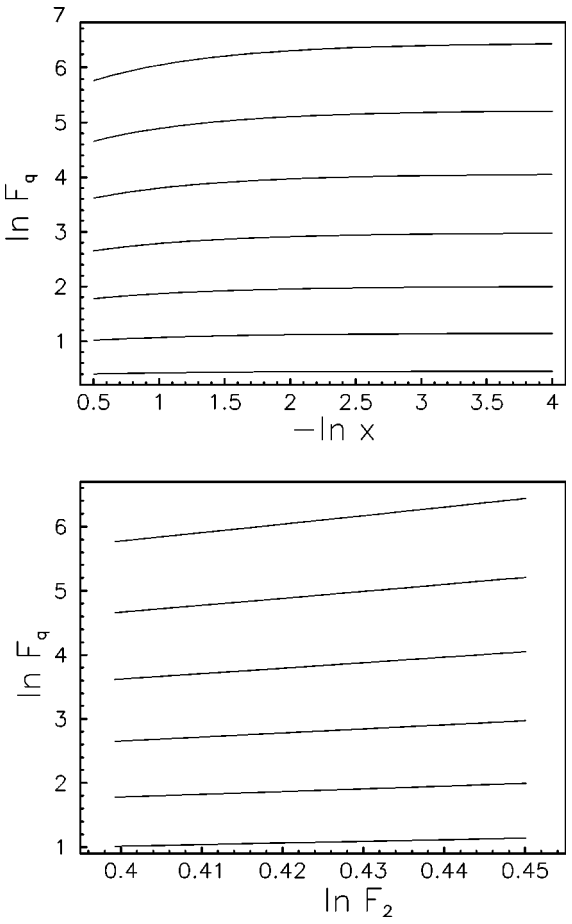

FIG. 3. Upper part: $\ln F_q$ as functions of $-\ln x$ without spatial fluctuations (mode 1); lower part: scaling behaviors between $\ln F_q$ and $\ln F_2$, with the same data as in the upper part.

range $x \in (0.5,4.0)$. One can see clearly that no strict intermittency can be claimed since all F_q approach finite values in the small x limit. So, no intermittency is shown in the phase transition, as shown in former studies. More importantly, the power law can be found between F_q and F_2, as shown in Fig. 2 with the same data as in Fig. 1.

For the convenience of comparison with the former case, we write down the expressions of the scaled factorial moments without spatial fluctuations (mode 1 in this paper), which can be read

$$f_q = \frac{\int_0^\infty dy y^q \exp(xy - y^2)}{\int_0^\infty dy \exp(xy - y^2)}, \tag{5}$$

with $x = a\delta/\sqrt{b}$. Numerical results for $\ln F_q$ in this mode are shown in Fig. 3. In the upper part of the figure $\ln F_q$ are shown as functions of $-\ln x$ for q from 2 to 8 with x in the same interval $x \in (0.5,4.0)$, and in the lower part $\ln F_q$ are shown as functions of $\ln F_2$ with the same data as in upper part. One can see from the upper part of the figure that the general behaviors of $\ln F_q$ as functions of $-\ln x$ are similar to those in Fig. 1, though the definition of x in this case is

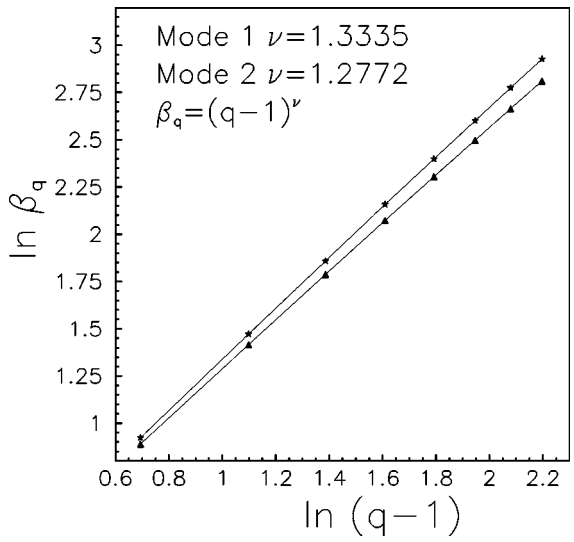

FIG. 4. Scaling behaviors of $\ln \beta_q$ as a function of $\ln(q-1)$ for the two modes.

different from that for Fig. 1. The values of $\ln F_q$ in the two cases are also different. For same value of x, $\ln F_q$ in the former case have larger values. This difference is reasonable if one notices the difference in the definition of variable x. What interests us is the scaling law between F_q and F_2. The power law scaling between F_q and F_2 can be seen obviously in the lower part of Fig. 3, the same as shown in other studies cited in the references.

From Fig. 2 and the lower part of Fig. 3, one can get the scaling exponents β_q for the two different modes by fitting the curves. β_q can also be given analytically. One can expand the expressions for $\ln F_q$ in the two modes as power series of x in small x limit, and then one gets the slopes K_q for $\ln F_q$ and $\beta_q = K_q/K_2$. The expressions for K_q for the two modes in this paper are

$$K_q = \frac{\Gamma(q/2+1)}{\Gamma(q/2+1/2)} - q\frac{\Gamma(3/2)}{\Gamma(1)} + (q-1)\frac{\Gamma(1)}{\Gamma(1/2)} \quad \text{for mode 1,}$$

$$K_q = \frac{\Gamma(q/2+3/4)}{\Gamma(q/2+1/4)} - q\frac{\Gamma(5/4)}{\Gamma(3/4)} + (q-1)\frac{\Gamma(3/4)}{\Gamma(1/4)} \quad \text{for mode 2.}$$

One can find only a small difference between the exponents ν from these two expressions. The results are shown in Fig. 4. In mode 1 (without spatial fluctuations) $\nu = 1.3335$, and in mode 2 (with spatial fluctuations of the phase angle of the field ϕ) $\nu = 1.2772$. The exponents obtained from these analytical expressions are very close to the ones from the fitting. The universal exponents ν are also very close to one another and can be regarded as the same within accuracy 4%. Physically, these two modes correspond to different situations. In mode 1 no spatial fluctuation of ϕ is in the problem, but in mode 2 the spatial fluctuations of the phase angle of the complex field ϕ are fully evaluated. Since these two different considerations give very close exponents ν, one can say that the exponent ν is indeed insensitive to the spatial fluctuations of the phase angle.

It is, of course, very interesting to investigate directly the effect of the term $(\nabla\phi_R)^2$ on the moments, which is the main topic in this paper.

Our second observation is that the final state particles are in a finite phase space at any high but finite colliding energy. This means that the fluctuations of the field ϕ should not be uniform since the field must be zero in the region excluded by the conservation laws. Thus there exists a boundary condition for ϕ_R. For convenience, we use ϕ instead of ϕ_R in the following if no confusion will arise. The boundary condition of ϕ is of Dirichlet type in our problem because of the fact that the particle density out of a finite region should be zero. In the following, we only discuss a one-dimensional phase space such as the rapidity, and the boundary condition can, not losing any generality, be written as $\phi(0)=\phi(L)=0$, with L the length of the finite phase space interval. With the gradient term in the functional, the functional integral can only be calculated perturbatively. But there are two important differences from the usual perturbations. The first difference is the finite size of the phase space. The second is the nonpositivity of the coefficient of the Gaussian term in the functional $F[\phi]$. So, some new techniques are needed which will be discussed in this paper.

The organization of the paper is as follows. In Sec. II we discuss the ground state of a finite-size system under various boundary conditions. In Sec. III a new perturbative calculation scheme is proposed with the effect of local spontaneous symmetry breaking taken into account. In Sec. IV we calculate the scaled factorial moments perturbatively. Section V is for our main results and conclusions.

II. LOCAL SPONTANEOUS SYMMETRY BREAKING FOR FINITE-SIZE SYSTEM

Finite-size effects near critical points have been remained over the past two decades to be an important topic of the active research both theoretically and experimentally [8] in condensed matter physics. Nowadays, the experimental sample is usually so pure and so well shielded from perturbing fields that the correlation length can grow up to several thousand angstroms as the critical point is approached. When one or more dimensions of a bulk system are reduced to near or below a certain characteristic length scale, the associated properties are modified reflecting the lower dimensionality. It is believed that finite-size effects are precursors of the critical behavior of the infinite system and can be exploited to extract the limiting behavior. The finite-size scaling behavior plays a central role, as predicted by both the phenomenological [9] and renormalization group [10] theories. Those theories allowed a systematic discussion of the finite-size effects and, consequently, form the cornerstone of our current understanding of the way in which the singularities of an infinite system are modified by the finiteness of the system in some or all of the dimensions. Of course, the exact form of scaling functions cannot be given in those scaling theories.

In 1985, Brézin and Zinn-Justin (BZ) [11] and Rudnick, Guo and Jasnow (RGJ) [12] developed two field-theoretical perturbation theories for the calculation of the finite-size scaling functions within the ϕ^4 model which corresponds to the Ising model. Most applications of these theories to three-dimensional systems have been restricted to T higher than the bulk critical temperature T_C [13] with a few calculations in the region below T_C [14]. However, some limitations exist in the theories of [11] and [12]. As pointed out in the first paper in [15], the theory of BZ is not applicable for $T<T_C$ and the results from RGJ theory are not quantitatively reliable in the same temperature region since the coefficients of the Gaussian terms in the integrals are negative for those temperatures. In [16] the order parameter is expanded into a sum of eigenfunctions of ∇^2 for various boundary conditions. Again, the functional integral is turned out into a product of normal integrals. But the fluctuations can be evaluated only for temperature not too far below the critical point. The authors of [15] tried to avoid the difficulty mathematically, but they failed to account for the origin of the difficulty physically. Although the modified perturbation method in [15] can be used for both $T>T_C$ and $T<T_C$, the calculation is lengthy and can be done only to first order in practice. Since one does not know the exact order of values of higher order terms, theoretical results may have a large uncertainty.

It has not been determined which physical effect causes the failure of direct perturbative calculations of fluctuations for finite-size systems with temperature below T_C. In our opinion, the real origin of the difficulty lies in the lack of knowledge about the spontaneously symmetry breaking for finite-size systems. It is well known that an infinite system will have nonzero mean order parameter ϕ_0, which is called ground state of the system in this paper since it corresponds to minimum of the Hamiltonian H, if the temperature is below the critical one, and everyone knows that the difficulty of negative coefficient of the Gaussian term can be overcome by shifting the order parameter, $\phi\rightarrow\phi+\phi_0$. This phenomenon is known as spontaneous symmetry breaking because of the fact that ϕ_0 does not have the same symmetry as H does. This kind of spontaneous symmetry breaking for an infinite system can be called global since the shift ϕ_0 is the same constant for every point in the space. For a finite-size system, such a simple shift of the order parameter does not work because of the existence of specific boundary conditions for the system. Anyway, fluctuations of the system, in their own sense, should be around a certain ground state which corresponds to the minimum of the Hamiltonian H, and they can be approximated by Gaussian terms in most cases if they are not very large. Thus one sees that the ground state plays a determinative role in the study of fluctuations in the phase transitions at low temperature. For an infinite system, the ground state ϕ_0 is constant and can easily be calculated. But for a finite-size system, the ground state is usually not a constant but depends on the boundary conditions imposed on it; this is understandable. For an infinite system the ground state is determined completely by the self-interactions of the field. In other words, the ground state is dictated only by the ''potential,'' and there is no boundary effect. For a finite-size system, however, the effect of the boundary must be taken into account. For the case with local interactions, the effect is realized through the gradient term. Thus the ground state for a system with finite size is deter-

mined by the gradient term and the ''potential.'' Then the shift of the field at a point depends on the position in the space. So, the spontaneous symmetry breaking for a finite-size system can be called a local one. Therefore, the solution for the ground state is nontrivial but necessary, and one has reason to hope that the difficulty mentioned above for finite-size systems can be overcome once the ground state is known.

It should be pointed out that all perturbation theories mentioned above are based on Fourier decomposition of the order parameter. This method is natural because the decomposition enables one to transform the functional integral into an infinite product of tractable normal integrals. Although such a decomposition has a simple physical explanation which is very fruitful for the understanding of properties of infinite systems and can deduce reliable physical results, as in the case of usual field theories in particle physics, it brings about a great deal of calculations for finite-size systems; this is not surprising. As is well known, quantities complicated in coordinate space may have simple momentum spectra and thus look simple in momentum space, but those obviously nonzero only in a finite range must have puzzling momentum spectra. Therefore, for the study of properties of finite-size systems, calculations in coordinate space might be simpler and more effective. The point here is that one must calculate the complicated functional integral which is very difficult to be evaluated directly.

In this section, we first calculate the ground states for a ϕ^4 model of a second-order phase transition with one-component order parameter under various boundary conditions. All the boundary conditions are useful in the study of condensed matter physics. Then, with the ground states, the Hamiltonian of the system is reexpressed as Gaussian terms and higher order perturbations of a locally shifted order parameter. It is shown that the perturbative calculation can be done with the new Hamiltonian for temperatures far below the bulk critical point.

In the ϕ^4 model for a second-order phase transition in condensed matter physics with a one-component order parameter, the partition function can be expressed as a functional integral of an exponential of the Hamiltonian H of the system

$$Z=\int \mathcal{D}\phi \exp(-H) = \int \mathcal{D}\phi \exp\left\{-\int d^3\mathbf{r}\left[\frac{\gamma}{2}\phi^2+\frac{1}{2}(\nabla\phi)^2+\frac{u}{4!}\phi^4\right]\right\}, \tag{6}$$

in which $\gamma=a'(T-T_C)$, a' and u are temperature dependent positive constants, ϕ is the order-parameter of the system. In the following, we are limited only to a film system with thickness L. Since we are interested only in the temperature region $T<T_C$ or $\gamma<0$, the Hamiltonian H can be standardized by introducing correlation length $\xi=\sqrt{-1/\gamma}$, new order-parameter $\Psi=\phi/\phi_0$, with $\phi_0=\sqrt{-6\gamma/u}$ the vacuum expectation of the order parameter for bulk system, scaled coordinates $\mathbf{r}'=\mathbf{r}/L$, and reduced thickness $l=L/\xi$, into

$$H=\int d^3\mathbf{r}'\frac{L^3\phi_0^2}{\xi^2}\left[\frac{1}{2l^2}(\nabla'\Psi)^2-\frac{1}{2}\Psi^2+\frac{1}{4}\Psi^4\right]. \tag{7}$$

From this expression one can get the equation for the ground state by $\delta H/\delta\Psi=0$. The ground state $\Psi_0(z)$ satisfies

$$\frac{1}{l^2}\frac{d^2\Psi_0}{dz^2}=-\Psi_0+\Psi_0^3. \tag{8}$$

In the equation we have used z instead of z' in the range (0, 1) to denote the coordinate along the thickness direction. Derivatives in other directions do not appear in the equation since any state with nonzero derivatives in other directions does not correspond to the minimum of H. But if the system is fully limited in all directions, the last equation should have ∇^2 in place of d^2/dz^2. In [17] the last equation is solved analytically for Dirichlet boundary conditions $\Psi(0)=\Psi(1)=0$. The exact solution is

$$\Psi_0(z)=\sqrt{\frac{2k}{\sqrt{1+k^2}}}\sin(2zF(k),k), \tag{9}$$

in which k is determined by l through $l=2\sqrt{1+k^2}F(k)$. Here, $F(k)$ is the first kind of complete elliptic integral; $\sin(z,k)$ is an elliptic sine function. Unfortunately, no simple compact solution is found yet for other boundary conditions. One can easily see that the main obstacle comes from the nonlinear term Ψ_0^3 in the second-order differential equation of Ψ_0 in Eq. (8). To find approximate solutions of Ψ_0 for other boundary conditions, the following method can be used. First of all, we replace Ψ_0^3 in Eq. (8) by $\lambda\Psi_0$ and get a solution satisfying the same boundary condition. For Dirichlet boundary conditions, the solution is

$$\Psi_0=A\sin\pi z, \quad \text{with} \quad \lambda=1.0-\pi^2/l^2. \tag{10}$$

The constant A can be determined by requiring the mean square of the deviation caused by the replacement, i.e., the integral $\int_0^1 dz(\Psi_0^3-\lambda\Psi_0)^2$, to be minimum. Thus one gets

$$\Psi_0(z)=\sqrt{\frac{4}{3}\left(1-\frac{\pi^2}{l^2}\right)}\sin\pi z. \tag{11}$$

Now one can see that the requirement of a minimum deviation caused by the replacement is equivalent to retaining $\sin\pi z$ term but neglecting terms with higher frequency in Ψ_0^3. Thus, this approximation is equivalent to the standard functional variation method. The virtue of this method is that it is simpler and can be used in a step-by-step way. As discussed in [17] the ground state is $\Psi_0=0$ if the reduced thickness l of the film is less than π. The existence of minimum reduced thickness of the film implies a shift of the critical temperature for the finite system from the bulk one. The exact solutions and the approximate ones are compared in Fig. 5 for $l/\pi=1.05$, 1.10, 1.15, and 1.20. A very good

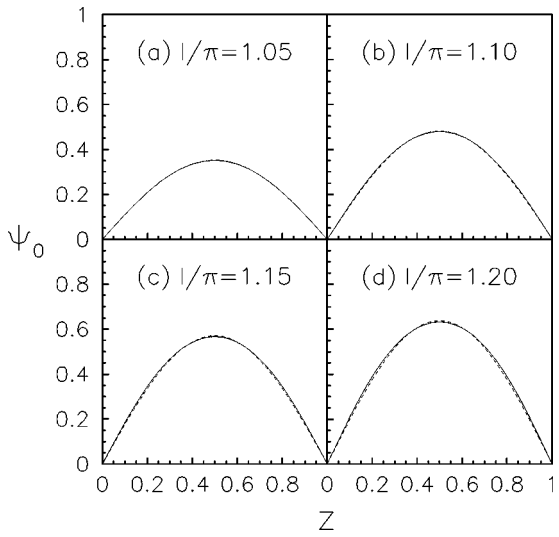

FIG. 5. Comparison between exact solutions and approximate ones for Eq. (8) under Dirichlet boundary conditions for $l/\pi = 1.05$, 1.10, 1.15, and 1.20. The solid curves correspond to exact solutions; the dotted curves are drawn according to Eq. (11).

approximation can be seen. For larger l, the same approximative method can be used further after shift $\Psi_0=\Psi_0' + \sqrt{4(1-\pi^2/l^2)/3}\sin\pi z$ in Eq. (8). For Neumann boundary conditions, $\Psi_0'(0)=\Psi_0'(1)=0$, the ground state can also be obtained in a similar way. The result is

$$\Psi_0(z)=1.0 \quad \text{for} \quad T<T_c. \tag{12}$$

Then one can consider mixed boundary conditions $\Psi_0(0)=0,\Psi_0'(1)=0$. The first order approximation of the solution for the ground state is

$$\Psi_0(z)=\sqrt{\frac{4}{3}\left(1-\frac{\pi^2}{4l^2}\right)}\sin\frac{\pi z}{2} \quad \text{for} \quad l\geqslant \pi/2. \tag{13}$$

As a final example, we give the ground state for periodic boundary condition $\Psi_0(z)=\Psi_0(1+z)$. The ground state is

$$\Psi_0(z)=1.0 \quad \text{for} \quad T<T_c. \tag{14}$$

Though the ground state for periodic and Neumann boundary conditions are the same the fluctuations of the fields in the two cases are different. It should be pointed out that $-\Psi_0$ is also a ground state of the system. Then the fluctuations of the system can be around either Ψ_0 or $-\Psi_0$. This is the copy for finite-size systems of spontaneous symmetry breaking in ϕ^4 model. The significant difference from the usual spontaneous symmetry breaking is that the ground state is usually not a constant and depends on the boundary conditions, so that we have a local spontaneous symmetry breaking in this paper. With the ground state Ψ_0, one can locally shift the order parameter $\Psi=\Psi'+\Psi_0$; then the Hamiltonian H turns out to be

$$H=H[\Psi_0]+\frac{L^3\phi_0^2}{\xi^2}\int d^3\mathbf{r}\frac{1}{2}\left[\frac{1}{l^2}(\nabla\Psi')^2-\Psi'^2+3\Psi_0^2\Psi'^2 + 2\Psi_0\Psi'^3+\frac{1}{2}\Psi'^4\right]. \tag{15}$$

In this expression, $H[\Psi_0]$ has the same form as $H[\Psi]$ in Eq. (7) with Ψ_0 in place of Ψ. Now the quadratic part of fluctuation Ψ' is positive definite for l larger than a characteristic length, or for temperature enough below the critical point. Then one sees that the new Hamiltonian can be safely used to calculate perturbatively fluctuations at low temperature region for finite-size systems. Then the difficulty of the negative coefficients of the Gaussian terms is avoided after the effects of local spontaneous symmetry breaking are taken into consideration.

III. PERTURBATIVE THEORY FOR A FINITE-SIZE SYSTEM UNDER $T \ll T_C$

From Eq. (15), a new perturbative theory can be developed for a finite-size system with local spontaneous symmetry breaking. First of all, one can introduce for a one-dimensional system a generating functional $Z[J]$

$$Z[J]=\int \mathcal{D}\phi \exp\left(-H+\int dzJ\phi\right). \tag{16}$$

The generalization to more general cases is obvious. Up to an unimportant constant factor, the generating functional for a one-dimensional system can, in a standard way, be written as

$$Z[J]=\exp\left(\lambda_1\int dzJ\Psi_0\right)\exp\left\{-\lambda\int dz\left[\Psi_0\left(\frac{\delta}{\lambda_1\delta J}\right)^3 + \frac{1}{4}\left(\frac{\delta}{\lambda_1\delta J}\right)^4\right]\right\}\exp\left[\frac{1}{2}\frac{\lambda_1^2}{\lambda}\int dzdyJ(z)G(z,y)J(y)\right], \tag{17}$$

with $\lambda_1=L\sqrt{6|\gamma|/u}=L\phi_0$, $\lambda=6L\gamma^2/u$. In the last equation, the Green's function $G(z,y)$ satisfies

$$\left[-\frac{1}{l^2}\frac{d^2}{dz^2}-1+3\Psi_0^2(z)\right]G(z,y)=\delta(z-y). \tag{18}$$

The first factor in the generating functional shows a great difference between present theory and the usual ones in that there exists a nontrivial solution for the classical equation $\delta H/\delta\phi=J$ for $J=0$. For systems with higher dimension $d>1$ the only changes are with L^d in place of L in the expressions for parameters λ and λ_1 and with ∇^2 in place of d^2/dz^2 in last equation. The Green's function $G(z,y)$ describes fluctuations in the full space and determines how the fluctuations at different points are correlated. If one can get the solution for $G(z,y)$ for a higher dimensional system, the fluctuations can be evaluated in the same way as for a one-dimensional system. Thus in the following we do not distinguish one- and higher-dimensional systems, and dz is used to

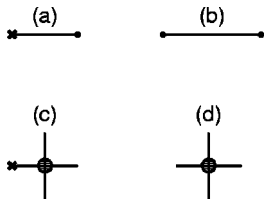

FIG. 6. Feynman diagram representations for (a) the ground state, (b) the propagator, (c) three-line vertex, and (d) four-line vertex.

represent the integral element over a volume in certain space. From Eq. (17), it can be seen that each Green's function G is associated with a factor $1/\lambda$. λ_1 can be regarded as a factor associated with the external source field J. Since the derivative terms in the second factor in Eq. (17) with respect to the external source field J will generate terms with more factors of G in the generating functional, the contribution of them is small if the parameter λ is big enough. Then those terms in the generating functional can be regarded as perturbations. From the expression of λ it is clear that a large λ is equivalent to a small u for fixed L and γ. Thus the condition of a large λ is consistent with that in usual perturbation theory. Then one has all four ingredients diagrammatically represented in Fig. 6 for the perturbative calculations with the Feynman rules: (a) the ground state $\lambda_1\Psi_0(z)$, (b) the Green's function (propagator) $(\lambda_1^2/\lambda)G(z,y)$, (c) three-line vertex $-\lambda/\lambda_1^3\int dz\Psi_0(z)$, and (d) four-line vertex $-\lambda/4\lambda_1^4\int dz$.

Using these ingredients all physical quantities can be calculated. For example, to the first order of the perturbations, one has

$$\langle\Psi(z)\rangle=\Psi_0(z)-\frac{3}{\lambda}\int du\Psi_0(u)G(u,u)G(u,z),$$

$$\langle(\Psi(z)-\Psi_0(z))(\Psi(y)-\Psi_0(y))\rangle = \frac{1}{\lambda}G(z,y)-\frac{3}{\lambda^2}\int duG(z,u)G(u,u)G(u,y).$$

Here, the symbol $\langle\cdots\rangle$ represents the average over the fluctuations; the range of the integral over u is (0, 1).

A most important feature of the perturbation theory is that all the calculations can be done in coordinate space. Once the nontrivial ground state Ψ_0 is known, one can get the Green's function (propagator) $G(x,y)$ from Eq. (18), and other quantities can be obtained from Eq. (17) by directly taking derivatives with respect to the external source field J. This scheme can be used in calculating properties of finite-size systems in condensed matter physics for temperatures $T \ll T_c$.

In the next section we will calculate the scaled factorial moments in a second-order quark-hadron phase transition as an example of the applications of the perturbation theory.

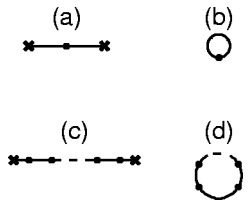

FIG. 7. Connected zero-order diagrams for the contributions to f_q. In the diagrams the number of dots is equal to q and an integral over the coordinate in a range with length δ is implied. So (a) and (b) are for f_1, and (c) and (d) are for f_q with q dots in the diagrams.

IV. THE SCALED FACTORIAL MOMENTS IN THE GINZBURG-LANDAU MODEL

Now we turn to the calculation of the scaled factorial moments F_q in Eq. (1) in a second-order quark-hadron phase transition within the Ginzburg-Landau description. In this description, the free energy functional $F[\phi]$ is in place of the Hamiltonian H in last two sections. After integrating over the phase angle of the field the functional remains in the same form with a real ϕ_R in place of the complex ϕ as discussed before. Although there are very important differences between normal phase transitions in condensed matter physics and a quark-hadron one, the mathematical form in the Ginzburg-Landau description for them is the same. In the Ginzburg-Landau description for a quark-hadron phase transition, the integral variable z is not in coordinate space but represents a collection of measurable quantities such as rapidity, azimuthal angle, etc. In the following, z is identified to the rapidity. For such a one-dimensional system, the local spontaneous symmetry breaking is also given as in Sec. II. A generating functional can also be introduced in the same way as in the last section. The only changes are the expressions for the parameter λ, λ_1, and l. Here we only mention the expression for l. In the present case, the correlation length is $\xi=\sqrt{c/|a|}$, so $l=L\sqrt{|a|/c}$. The parameter c has a simple physical meaning. From the free energy functional one sees that the correlation between fields at different points is realized by means of the gradient term. If c is small there is weak correlation between the fields at different points. Thus the effective length l can be used to measure the strength of the correlations for fixed L and $|a|$. For a system at fixed temperature c is small if there is weak correlation, and vice versa. When $l\to\infty$, one may expect that the influence of correlation can be neglected and that the effect of boundary condition can be neglected. In the calculation of the scaled factorial moments, the factor λ_1 will be cancelled. So λ_1 can be taken to be 1.0 in present calculations. For any parameter l the scaled factorial moments can be rewritten from Eq. (1) as

$$F_q=f_q/f_1^q, \quad f_q=\prod_{i=1}^{q}\int_\delta dz_i\frac{\delta^2}{\delta J^2(z_i)}\frac{Z[J]}{Z[0]}. \tag{19}$$

In this expression, $\int_\delta dz$ represents an integral over a range of length δ. In our calculation, the integral range is chosen around the center of the interval (0, 1), or in other words, in the range $(1/2\text{-}\delta/2,1/2+\delta/2)$. As discussed in the second to

last paragraph in Sec. I, the boundary condition for our case is of Dirichlet type. So the ground state Ψ_0 is given by Eq. (9) and $G(z,y)$ is calculated from Eq. (18).

A. Zero order approximation for f_q

We first calculate the scaled factorial moments F_q in a second-order quark-hadron phase transition at the zero order (or tree-level) approximation to the functional (17). At this level the second factor in Eq. (17) gives a factor 1.0. In the expressions of f_q there are contributions from q-particle correlations represented diagrammatically by connected diagrams in Fig. 7 and the contributions from fewer particle correlations which can be represented by products of disconnected diagrams. We denote f_q^c the contributions to f_q from connected diagrams which give the contribution from the pure q-particle correlations to f_q. Then the factorial moments f_q at tree level can be written as

$$
\begin{aligned}
f_1^{\text{tree}} &= f_1^c, \\
f_2^{\text{tree}} &= (f_1^c)^2 + f_2^c, \\
f_3^{\text{tree}} &= (f_1^c)^3 + 3 f_1^c f_2^c + f_3^c, \\
f_4^{\text{tree}} &= (f_1^c)^4 + 6(f_1^c)^2 f_2^c + 4 f_1^c f_3^c + 3(f_2^c)^2 + f_4^c, \\
f_5^{\text{tree}} &= (f_1^c)^5 + 10(f_1^c)^3 f_2^c + 10(f_1^c)^2 f_3^c + 5 f_1^c f_4^c + 10 f_2^c f_3^c + f_5^c, \\
f_6^{\text{tree}} &= (f_1^c)^6 + 15(f_1^c)^4 f_2^c + 20(f_1^c)^3 f_3^c + 15(f_1^c)^2 f_4^c + 6 f_1^c f_5^c + 10(f_3^c)^2 + 15(f_2^c)^3 + 15 f_2^c f_4^c + 60 f_1^c f_2^c f_3^c + f_6^c, \\
&\cdots .
\end{aligned}
\tag{20}
$$

For the connected contributions to f_q^{tree}, there are only two topologically different diagrams, as shown in Fig. 7. For the first type of diagram with two crosses representing the ground state, the number of identical terms is $N_q^1 = 2^{q-1} q!$. The factor $q!$ comes from the exchange symmetry of the q particles, 2^q from the two lines from each point representing a particle, and a factor 1/2 from the identities of terms with reversal order of the q points. For the second type of diagrams with no cross, the number is $N_q^2 = N_q^1/q = 2^{(q-1)}(q-1)!$. To calculate the diagrams, it would be useful to define

$$
g_i(z,y) = \int_\delta dx_1 dx_2 \cdots dx_i G(z,x_1) G(x_1,x_2) \cdots G(x_i,y), \tag{21}
$$

which satisfies a recursive relation

$$
g_i(z,y) = \int_\delta du\, g_{i-1}(z,u) G(u,y) = \int_\delta dz G(z,u) g_{i-1}(u,y). \tag{22}
$$

Then the contribution from each connected diagram for f_q can be written as

$$
\text{first diagram:} \quad \left(\frac{1}{\lambda}\right)^{q-1} \int_\delta dz\, dy \Psi_0(z) g_{q-2}(z,y) \Psi_0(y),
$$

$$
\text{second diagram:} \quad \left(\frac{1}{\lambda}\right)^{q} \int_\delta dz g_{q-1}(z,z).
$$

So that

$$
f_q^c = \frac{2^q q!}{\lambda^q} \left[\frac{\lambda}{2} \int_\delta dz dy \Psi_0(z) g_{q-2}(z,y) \Psi_0(y) + \frac{1}{2} \int_\delta dz g_{q-1}(z,z) \right]. \tag{23}
$$

B. First order approximation for f_q

Now we discuss f_q at the first order (one-loop level) approximation of the second factor in the functional of Eq. (17). At this approximation, the factor from the second term of the equation is

$$Z_1[J]=1-\frac{1}{\lambda}\int dz\left\{\Psi_0(z)\left[3G(z,z)(GJ)_z+\frac{1}{\lambda}(GJ)_z^3\right]+\frac{1}{4}\left[3G^2(z,z)+\frac{6}{\lambda}G(z,z)(GJ)_z^2+\frac{(GJ)_z^4}{\lambda^2}\right]\right\},$$

in which $(GJ)_z\equiv\int duG(z,u)J(u)$. From the functional at this approximation

$$Z[J]=Z_1[J]\exp\left(\int dzJ\Psi_0\right)\exp\left[\frac{1}{2\lambda}\int dzdyJ(z)G(z,y)J(y)\right]$$

the factorial moments f_q can be directly calculated by using Eq. (19). There are many terms contributing to f_q, among which the most interesting terms are those represented by connected diagrams in Fig. 8 with one bulb which is the vertex for the perturbative interactions. The sum of the contributions from the diagrams to f_q will be denoted by f_q^{loop} in this paper. Then up to the first order approximation of the generating functional, the factorial moments f_q are

$$\begin{aligned}
f_1&=f_1^{\text{tree}}+f_1^{\text{loop}},\\
f_2&=f_2^{\text{tree}}+2f_1^{\text{tree}}f_1^{\text{loop}}+f_2^{\text{loop}},\\
f_3&=f_3^{\text{tree}}+3f_2^{\text{tree}}f_1^{\text{loop}}+3f_1^{\text{tree}}f_2^{\text{loop}}+f_3^{\text{loop}},\\
f_4&=f_4^{\text{tree}}+4f_3^{\text{tree}}f_1^{\text{loop}}+6f_2^{\text{tree}}f_2^{\text{loop}}+f_1^{\text{tree}}f_3^{\text{loop}}+f_4^{\text{loop}},\\
f_5&=f_5^{\text{tree}}+5f_4^{\text{tree}}f_1^{\text{loop}}+10f_3^{\text{tree}}f_2^{\text{loop}}+10f_2^{\text{tree}}f_3^{\text{loop}}+5f_1^{\text{tree}}f_4^{\text{loop}}+f_5^{\text{loop}},\\
f_6&=f_6^{\text{tree}}+6f_5^{\text{tree}}f_1^{\text{loop}}+15f_4^{\text{tree}}f_2^{\text{loop}}+20f_3^{\text{tree}}f_3^{\text{loop}}+15f_2^{\text{tree}}f_4^{\text{loop}}+6f_1^{\text{tree}}f_5^{\text{loop}}+f_6^{\text{loop}},\\
&\cdots.
\end{aligned}\tag{24}$$

For the perturbative calculation to have high accuracy, we choose the parameter λ large enough to guarantee the following two conditions: (1) $\int dz|\langle\Psi(z)\rangle-\Psi_0(z)|$, the integral of absolute deviation of the mean value of the order parameter Ψ from Ψ_0 is not larger than 0.05; (2) $|Z[0]-1|$ is no more than 0.05. These two conditions ensure the contributions from higher order terms from the second-factor in Eq. (17) can be safely neglected. So our calculations are limited to only the first order approximation. Of course, higher order approximation can be made without difficulty in principle, only with more diagrams drawn and evaluated. In numerical calculation, δ is chosen for $-\ln\delta$ in the range (1,4).

V. MAIN RESULTS AND DISCUSSIONS

As discussed in the last section, we choose the parameter λ to be a large number to ensure the small influence of the perturbations. A large λ corresponds to a small correlation function $G(x,y)/\lambda$. So in the Ginzburg-Landau model for a second-order phase transition under some choice of the parameters, there is weak correlation between the fields at different points together with weak self-interactions. Due to the choice of a large λ ($\lambda_1=1.0$) the ground state Ψ_0 (whose square is the hadronic density at the state) will play a dominant role in Eq. (17) for large enough l, and the Gaussian and higher order fluctuations can only bring about some small corrections to the generating functional. Then with the choice of λ we are dealing with a case with small fluctuations. Because of the large value of λ the first term in the brackets of Eq. (23) plays an important role if the ground state Ψ_0 is obviously nonzero for larger parameter l. Since the powers before the brackets of Eq. (23) will be cancelled, the order of the ratios $f_q^c/(f_1^c)^q$ is $\lambda^{-(q-1)}$, thus very small. $f_q^{\text{loop}}/(f_1^c)^q$ have the order λ^{-q}, even smaller. Then the scaled factorial moments F_q are very close to 1.0. This expectation is confirmed in numerical calculations. Numerical results show that $\ln F_q$, though very small, have quite complicated behaviors. They increase for $-\ln\delta$ within (1.5, 2.5) and then decrease with the increase of $-\ln\delta$, as shown in Fig. 9 for parameter $l=2.63\pi$. Thus there is no intermittency in the phase transition. For other choices of $l/\pi\gg1$ similar results can be obtained. A more important and more interesting phenomenon is the power scaling between F_q and F_2, $F_q\propto F_2^{\beta_q}$, which can be expected from the similar behaviors of $\ln F_q$ in Fig. 9 and are shown in Fig. 10 with the same data as in Fig. 9. β_q can be obtained easily from a linear fitting to the curves in Fig. 10. As in former studies of F_q in Refs. [1,2] in the phase transitions, β_q satisfies a universal scaling law

$$\beta_q=(q-1)^\nu,\quad\text{with}\quad\nu=1.7539\quad\text{for}\quad l/\pi=2.63,\tag{25}$$

as shown in Fig. 11. In this case the universal exponent ν depends only on the value of parameter l which is a function of parameters $|a|$ for the temperature and c for the correlation strength. The dependence of the exponent ν on temperature is consistent with Ref. [6]. But the exponent ν is very

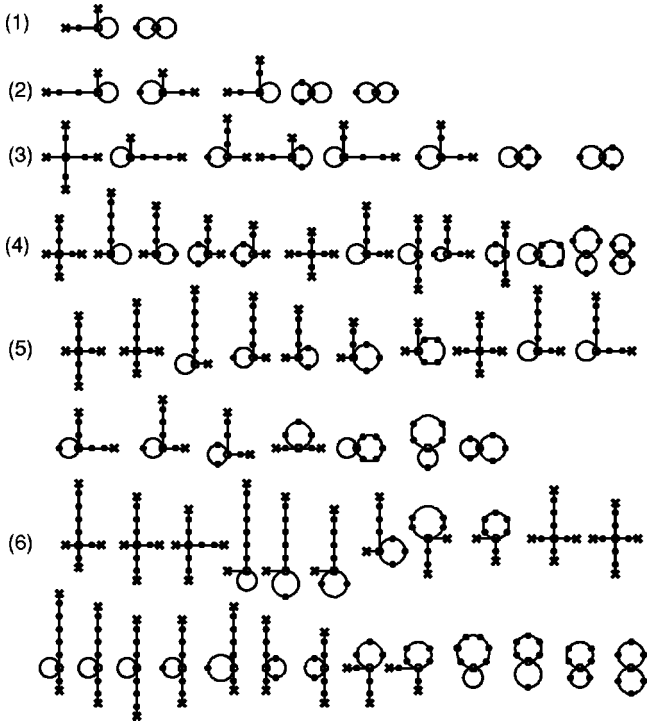

FIG. 8. Connected first-order diagrams for the contributions to f_q for $q=1$, 2, 3, 4, 5, 6, respectively.

different from those exponents given in former studies. The discrepancy is caused from the different assumptions made in former studies and in the present one. In former studies, the effect of the gradient term is neglected, but the ϕ^4 term (which describes the self-interaction) is emphasized. In those studies, the factorial moments f_q can be written as

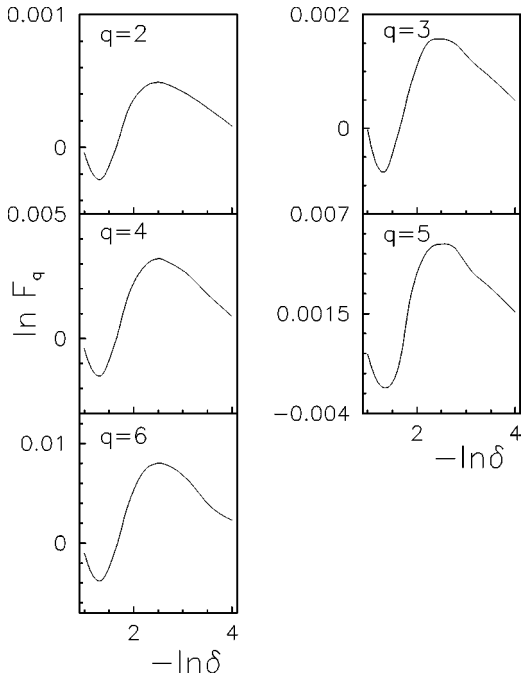

FIG. 9. Dependences of the scaled factorial moments $\ln F_q$ on the bin width $-\ln\delta$ from 1.0 to 4.0 for parameter $l=2.63\pi$ for $q=2$, 3, 4, 5, and 6.

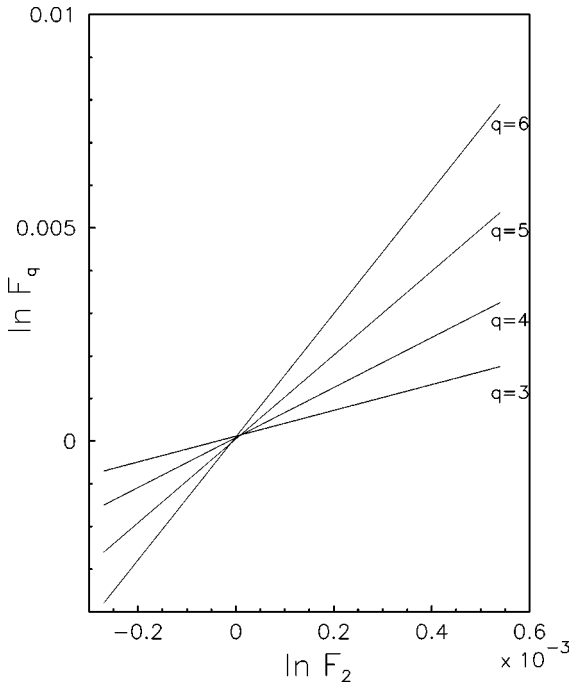

FIG. 10. Power scaling between F_q's with the same data as in Fig. 5.

$$f_q=\int_0^\infty dy y^q \exp(xy-y^2) \Big/ \int_0^\infty dy \exp(xy-y^2), \tag{26}$$

in which x is a parameter representing the bin width. From this expression one can discover that the ϕ^4 term, corresponding to the $-y^2$ term in the exponentials, is very impor-

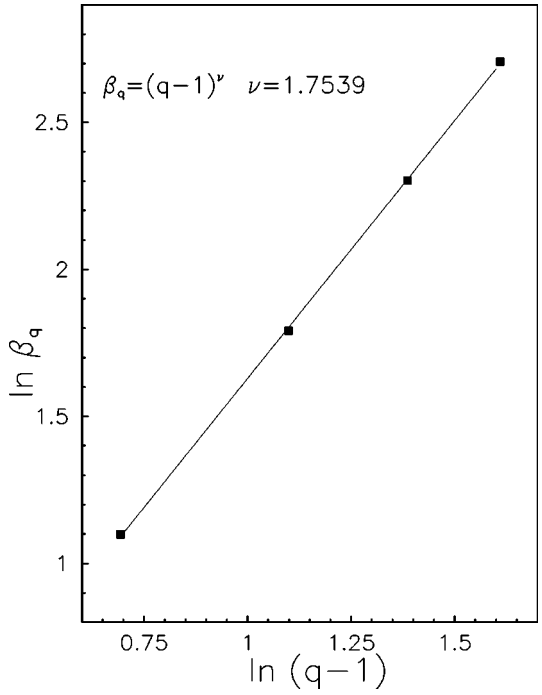

FIG. 11. Universcal scaling between β_q and $(q-1)$ for parameter $l=2.63\pi$.

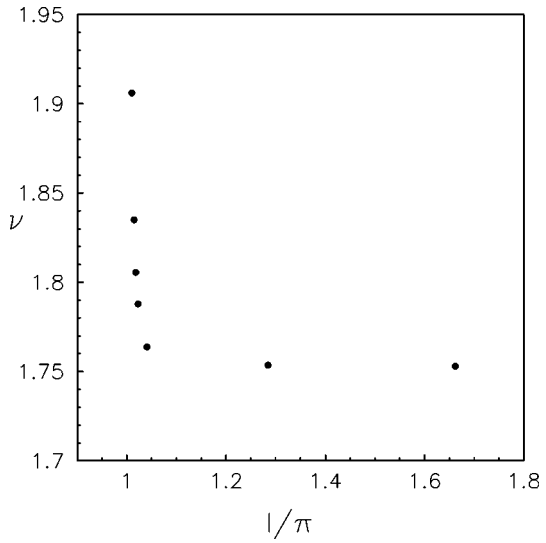

FIG. 12. Dependence of the universal exponent ν on parameter l/π.

tant and cannot be treated as perturbation for any parameter x. It is the term that makes the integrals finite. In present calculations, the role played by the ϕ^4 term is much less important. Its function is to provide a nontrivial ground state Ψ_0 around which are the fluctuations. Then that term is treated as a small perturbation and is very weak indeed with our choice of parameter λ. In former studies the fluctuations are around $\phi_0=0$. Then the discrepancy between the present study and former ones can be understood because they belong to different physical regimes. Former studies are in the nonperturbative regime with trivial ground state, but the present study is in the perturbative one with a nontrivial ground state.

The dependence of the universal exponent ν on the parameter l is also studied for $l/\pi>1$ in which there exists a nontrivial ground state. The result is shown in Fig. 12. For long correlation length (l/π a little larger than 1.0) the fluctuations in neighboring bins are correlated. For these l the values of Ψ_0 are also small, so the two terms in the bracket in Eq. (19) may have comparable contributions to f_q. In this region ν is quite large (about 2). With the increase of l the correlation between the fluctuations in neighboring bins becomes weaker and weaker, and the exponent ν decreases first rapidly and then slowly. When $l/\pi>2.5$ ν approaches a constant, about 1.75. The constant can be anticipated by considering a case with the very weak correlations among particles more than 2 [considering the factor $1/\lambda$ accompanied with the Green's function $G(x,y)$]. Then if only the effects of a weak two-particle correlation are considered, one has

$$f_q=(f_1^c)^q+C_q^2(f_1^c)^{q-2}f_2^c,$$

and then

$$F_q=1.0+C_q^2 f_2^c/(f_1^c)^2.$$

Here C_m^n are the binomial coefficients. Since the ratio $f_2^c/(f_1^c)^2$ is assumed to be very small one gets $\ln F_q \simeq C_q^2 f_2^c/(f_1^c)^2$, so the linear relation between $\ln F_q$'s can be verified, and one can get

$$\beta_q=C_q^2=q(q-1)/2.$$

From this expression, one gets the exponent $\nu=1.7550$, very close to the one obtained in this paper.

As a summary, the spatial correlation of the fluctuations in a second-order quark-hadron phase transition is considered in this paper within the Ginzburg-Landau description. We deal with a case with finite phase space and with negative coefficient of the Gaussian term in the functional. Because of the finite size of the space, calculations in usual space are simpler and more effective. Due to the negative coefficient of the Gaussian term in the functional a local spontaneous symmetry breaking (or nontrivial ground state) exists for a finite size system. We emphasize the importance of the ground state of the system, which is a version of spontaneous symmetry breaking for finite-size systems. Then a new perturbation scheme is developed which is expected to be applicable in the low temperature region in the ϕ^4 model for second-order phase transitions in condensed matter physics. Then as an example of the applications, the scaled factorial moments F_q in a second-order quark-hadron phase transition are calculated perturbatively. Power scaling laws between F_q's are shown and a universal exponent ν is given.

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