Vanishing Gamow-Teller transition rate for $A = 14$ and the nucleon-nucleon interaction in the medium

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The problem of the near vanishing of the Gamow-Teller transition (GT) in the $A = 14$ system between the lowest $J = 0^+$ $T = 1$ and $J = 1^+$ $T = 0$ states is revisited. The model space is extended from the valence space (p^{-2}) to the valence space plus all $2\hbar\omega$ excitations. The question is addressed as to what features of the effective nucleon-nucleon interaction in the medium are required to obtain the vanishing GT strength in this extended space. It turns out that a combination of a realistic strength of the tensor force combined with a spin-orbit interaction which is enhanced as compared to the free interaction yields a vanishing GT strength. Such an interaction can be derived from a microscopic meson exchange potential if the enhancement of the small component of the Dirac spinors for the nucleons is taken into account. [S0556-2813(99)04112-6]

In this work, we reconsider the old problem of the near-vanishing of the Gamow-Teller transition matrix element in the $A = 14$ system between the $J = 0^+$ $T = 1$ ground state of ^{14}O or ^{14}C and the $J = 1^+$ $T = 0$ ground state of ^{14}N. This is an allowed transition, but somehow the configurations of the initial and final states conspire to make this matrix element nearly vanish. Therefore, the calculation of this transition strength can serve as a very sensitive test for the nucleon-nucleon (NN) interaction in the nuclear medium.

The simplest shell-model configuration consists of two holes in the p shell for both the initial and final states. Using an LS representation, the wave functions can be written as

$$\Psi(J = 0^+, T = 1) = C_i^S |^1S_0\rangle + C_i^P |^3P_0\rangle,$$

$$\Psi(J = 1^+, T = 0) = C_f^S |^3S_1\rangle + C_f^P |^1P_1\rangle + C_f^D |^3D_1\rangle.$$

It was shown analytically by Inglis [1] that it was not possible to get $B(\text{GT})$ to vanish if the two-body interaction consisted of only a central and a spin-orbit interaction. Afterwards Jancovici and Talmi [2] demonstrated that one could get $B(\text{GT})$ to vanish if one also included a two-body tensor interaction.

What happens when we increase the model space by allowing $2\hbar\omega$ configurations? Can we then get $B(\text{GT})$ to vanish *without* a tensor interaction? We have previously performed such larger-space calculations [3,4] but we have not specifically addressed this problem. We used a G-matrix derived from the realistic interaction Nijm II [5,6] which, of course, contains a tensor interaction. The specific result in [3] was that in the small space (p^{-2}) the value of $B[\text{GT}(0^+ 1 \rightarrow 1^+ 0)]$ was 3.967, and that in the large space it was found to be 1.795. This is far from zero, but it is encouraging that higher shell admixtures will reduce $B(\text{GT})$. We will come back to this later.

Zheng and Zamick [7] studied the effects of varying the strengths of the spin-orbit and tensor interactions on $B(\text{GT})$ in the small space p^{-2}. Indeed, the main purpose of this

work will be to extend this study to the large space. The motivation is the following: The interaction to be used in a rather small model space contains large correction terms to account for the renormalization of the NN interaction to this small model space. On the other hand, the NN interaction in larger model spaces requires renormalization only with respect to short-range correlations and, therefore, the G matrix might be an appropriate approximation.

In order to explore the sensitivity of the GT strength on the spin-orbit and tensor interactions, we employ the two-body interaction introduced in [7]

$$V(r) = V_c(r) + x \cdot V_{\text{so}} + y \cdot V_t, \tag{1}$$

where "so" stands for the two-body spin-orbit interaction, t for the tensor interaction, and $V_c(r)$ is a spin-dependent central interaction. Note that this interaction is not only used for the residual interaction of the nucleons in the model space but has also been *employed to determine the single-particle part of the Hamiltonian which is due to the interaction with the respective core*. The parameters x and y were introduced so one could easily vary the strengths of the spin-orbit and tensor interactions. Roughly speaking, $x = 1$, $y = 1$ gives the best fit to a realistic G matrix. However, for ($x = 1$, $y = 1$) the value of $B(\text{GT})$ was too large: $B(\text{GT}) = 2.980$. This is similar to what happened with the realistic Nijm II interaction mentioned above [5]. It was noticed by Zheng and Zamick [7] that one could get $B(\text{GT})$ to vanish in at least two ways: one way is to keep the tensor strength fixed at $y = 1$ and increase the spin-orbit strength parameter from $x = 1$ to $x = 1.4$. Another way was to keep $x = 1$ and decrease the tensor strength by a factor of 2 ($y = 0.5$). All this is in the small space.

As a first step we use the interaction of Zheng and Zamick [7] given in Eq. (1) in a large space ($p^{-2} + 2\hbar\omega$). To see if we can get $B(\text{GT})$ to vanish without any tensor force, we set $y = 0$ and vary x. The results are shown in Fig. 1 where $B(\text{GT})$ is plotted vs x, the strength of the spin-orbit interaction. Starting from $x = 0$, we do indeed see a rapid drop in
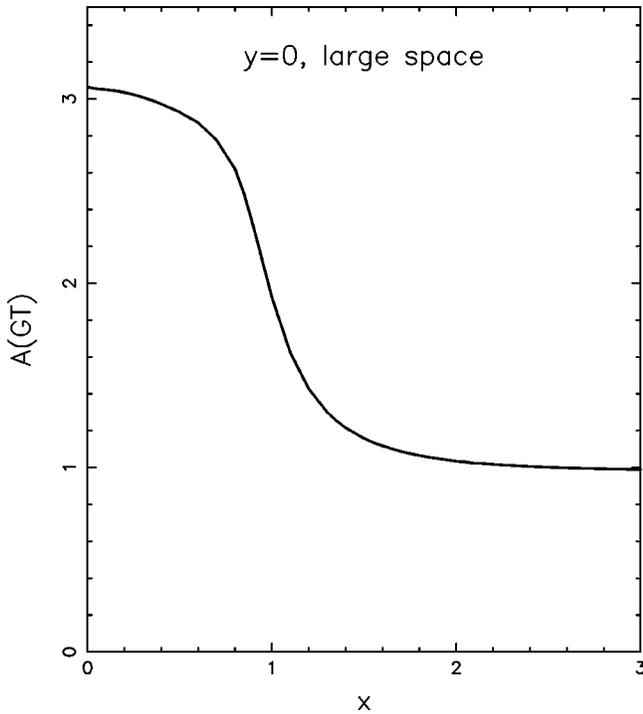
FIG. 1. The Gamow-Teller amplitude $A(\text{GT})$ calculated with the (x,y) interaction of Eq. (1) in large [i.e., $(0+2)$ $\hbar\omega$] space, as a function of x (the spin-orbit strength) and with $y=0$ (no tensor interaction).

$B(\text{GT})$ as x is increased. However, the curve flattens out at around $x=1.5$, and the value of $B(\text{GT})$ is close to unity up to $x=7.5$. Hence it appears that, in our parametrization, one cannot get $B(\text{GT})$ to vanish in the large space $p^{-2}+2\hbar\omega$ without a tensor interaction.

In Fig. 2, we show the *amplitude* $A(\text{GT})$ for $x=1$ as a function of y in the small space (p^{-2}). We note that the amplitude [and hence $B(\text{GT})$] does go to zero, however, it does so not at $y=1$ but rather close to $y=0.5$, about half the full tensor strength. Thus, this figure confirms the early work of Jancovici and Talmi [2] that with a tensor interaction we can get $B(\text{GT})$ to vanish. There is concern, however, that the strength of the tensor interaction needed in this small model space is quenched by a factor of 1.33 or so as compared to the realistic estimate.

In Fig. 3, we repeat the calculations in a large space ($p^{-2}+2\hbar\omega$ excitations). We keep the spin-orbit strength fixed at $x=1$, and we vary y, plotting the amplitude $A(\text{GT})$ as a function of y. We see that Fig. 3 is completely different from Fig. 2. The amplitude never changes sign, and hence $B(\text{GT})$ never goes to zero. The curve is relatively flat from $y=0$ to $y=1.5$. Does this mean that the old ideas are wrong and that one cannot get $B(\text{GT})$ to go to zero even with a tensor interaction?

Before we jump to such a conclusion, let us repeat the calculation but with a stronger spin-orbit interaction. Now we keep x fixed at 1.5 rather than 1, and we calculate $A(\text{GT})$ as a function of y. The results of these large-space calculations, which are shown in Fig. 4, are qualitatively similar to the small-space results for $x=1$. The amplitude does change
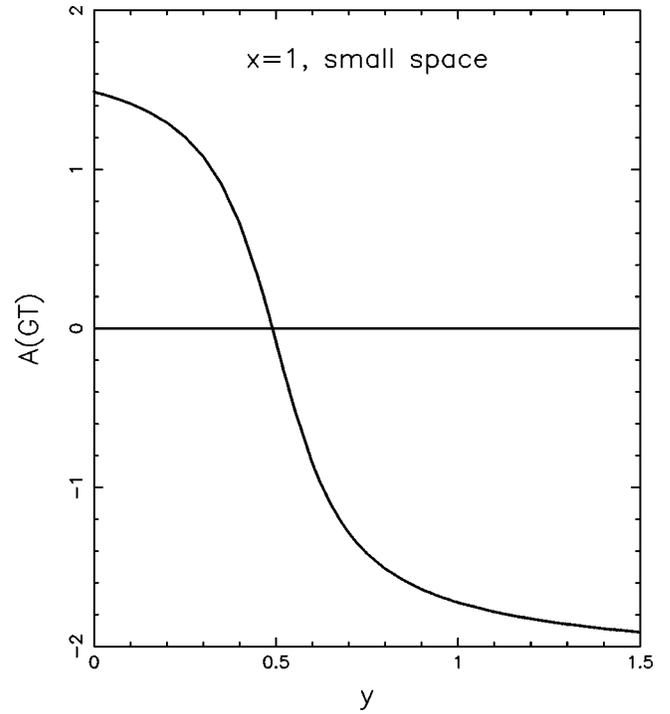

FIG. 2. $A(\text{GT})$ calculated with a spin-orbit strength of $x=1$ (free-space value), as a function of the tensor strength y in small space ($0\,\hbar\omega$).

sign, and $B(\text{GT})$ vanishes near $y=0.75$. With a larger spin-orbit interaction, we regain in a large space the results that were previously obtained in a small space with the ''free'' spin-orbit interaction. The tensor interaction strength y in the
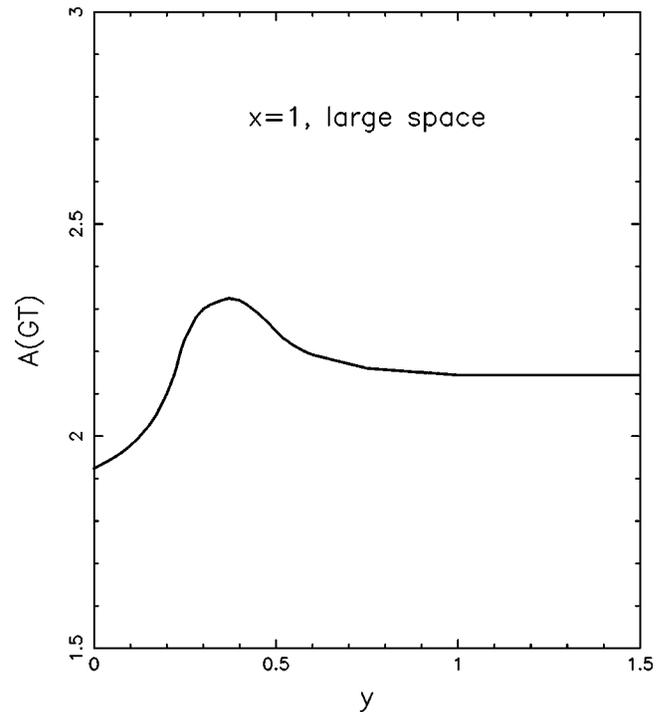

FIG. 3. Same as Fig. 2 except here the calculation is done in large space [$(0+2)$ $\hbar\omega$].
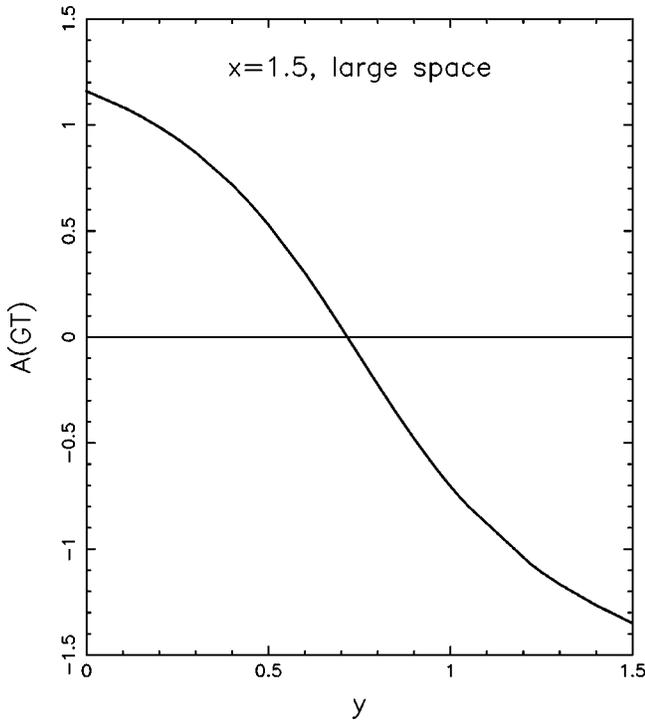
FIG. 4. $A(GT)$ calculated with an enhanced spin-orbit strength of $x=1.5$, as a function of the tensor strength y in large space $[(0+2)\ \hbar\omega]$.

TABLE I. $B(GT)$ for ^{14}C ($J=0^+$ $T=1$) $\rightarrow ^{14}N$ ($J=1^+$ $T=0$) with the Bonn-A interaction.

Space	m_D/m	E(MeV)	B(GT)
$0\ \hbar\omega$	1	1.701	5.294
$0\ \hbar\omega$	0.75	1.045	0.1530
$0\ \hbar\omega$	0.60	1.172	0.0978
$(0+2)\ \hbar\omega$	1	1.825	2.335
$(0+2)\ \hbar\omega$	0.75	1.426	0.1275
$(0+2)\ \hbar\omega$	0.60	1.316	0.0018

The shell-model calculations of $B(GT)$ with Nijm II [5,6] did not include this relativistic feature. Therefore, in the context of the (x,y) interaction, increasing the spin-orbit term by putting $x=1.5$ can be interpreted as a way to simulate the relativistic enhancement of the spin-orbit effects. Since the spin-orbit interaction is inversely proportional to m_D/m a choice of $m_D/m=2/3$, which is a rather realistic one, would be sufficient to increase the spin-orbit interaction by 50% (from $x=1$ to $x=1.5$).

Whether one should use a weaker tensor interaction inside a nucleus is more controversial. Brown and Rho [12] argue that inside a nucleus the masses of all mesons except the pion are less than in free space. Thus, the exchange of a ρ meson between two nucleons would lead to a longer-range repulsion, thus canceling more of the attractive contribution due to one-pion exchange. This would yield a net weaker tensor interaction. However, some of the present authors [3,4] have proposed an alternate picture of why the tensor interaction appears to be weaker inside a nucleus relative to free space. They call this the ''self-weakening'' mechanism. Basically, the idea is that if one introduces higher-shell admixtures perturbatively into valence-space calculations, this will make the tensor interaction appear to be weaker. In the latter picture, the tensor anomaly is explained by doing better nuclear structure calculations.

Of course, the two mechanisms are not mutually exclusive. In the present calculation, the self-weakening mechanism manifests itself in the fact that, in the $A=14$ β decay, we need $y\simeq0.5$ in the small space, but when higher-shell admixtures are introduced we find that $y\simeq0.75$.

In Table I, we depart from our phenomenological (x,y) interaction and show results with the relativistic Bonn-A G-matrix elements. We consider the cases where m_D/m is equal to 1, 0.75, and 0.60, and we perform the calculations in the small and large spaces. In this table, the $B(GT)$'s are shown alongside with the energy of the lowest ($J=0^+$ $T=1$) state in ^{14}C relative to that of the ground state ($J=1^+$ $T=0$) of ^{14}N.

With $m_D/m=1$, we get very close to the nonrelativistic matrix elements. The results for $B(GT)$ are very far from zero, consistent with what we obtained with the (x,y) interaction with $x=1$, $y=1$ as well as with the previously mentioned Nijm II interaction. In the small space, we get $B(GT)=5.294$, and in the large space 2.335, but at least we get closer to zero in the large space.

As we decrease the Dirac effective mass, we get results

large space calculation is closer to the free-space value. All of this may seem somehow *ad hoc*, but as we shall see next, it fits in well with modern ideas about medium modifications of the NN interaction inside the nucleus.

Relativistic mean-field studies within the framework of the so-called quantum-hadro-dynamics or ''Dirac phenomenology'' of Serot and Walecka [8] demonstrated that the structure of the nucleon self-energy leads to an enhancement of the small component of the Dirac spinors for the nucleons inside the nuclear medium as compared to the Dirac spinors for the free nucleon. This enhancement can be parametrized in terms of an effective Dirac mass m_D for the nucleon. The enhancement of the small component corresponds to a reduction of the Dirac mass m_D as compared to the free nucleon mass m. This reduced Dirac mass yields an enhancement of the spin-orbit splitting in the single-particle spectrum.

It was shown by Zheng *et al.* [9,10] that the Dirac phenomenology yields non-negligible effects in nuclear structure calculations. They demonstrated in particular that the enhancement of the spin-orbit splitting just discussed is reproduced in nuclear structure calculations using relativistic NN interactions. The experimental data for the spin-orbit splitting of one-hole states are only reproduced if the reduction of Dirac mass m_D predicted in Dirac-Brueckner-Hartree-Fock calculations [11] is taken into account in calculating the G-matrix elements of the one-boson-exchange interaction (OBE). If this relativistic feature is ignored, the spin-orbit splitting comes out too small. It should be noted that whereas in the relativistic Hartree-Fock method of Ref. [8] there are no pions in the theory, this is not the case in the OBE G-matrix calculations of Ref. [11].

closer and closer to zero. Finally, for $m_D/m=0.6$, we get $B(\text{GT})=0.098$ in the small space and 0.0018 in the large space. This is gratifying. The main reason for this success is of course that by decreasing m_D/m we increase the spin-orbit splitting. Furthermore the Bonn-A potential seems to be very appropriate since it contains a tensor force which is weaker than in other realistic NN interactions [13]. It should be reemphasized that in this work, all the single-particle energies are calculated with the *same* interaction that is used between the valence particles or holes. We feel this is the only way one can truly test the correctness of a given interaction or the G matrix derived therefrom.

It is amusing to note that in order to get $B(\text{GT})$ to vanish for $A=14$ one has to bring out all the artillery. When we allow higher-shell admixtures, we are at first dismayed that—as shown in Fig. 3—we cannot get $B(\text{GT})$ to vanish no matter what the strength of the tensor interaction is. However, if we increase the strength of the spin-orbit interaction, the ideas of Inglis [1] and Jancovici and Talmi [2] are revalidated. Furthermore, justification for this phenomenological step is afforded by the more fundamental Dirac phenomenology approach of Serot and Walecka [8] and Müther, Machleidt, and Brockman [11].

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