Spin polarizabilities of the nucleon in a large N_c baryon model with dispersion relations

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We calculate the spin polarizabilities of the nucleon by means of the dispersion relations applied to model-independent pion photoproduction amplitudes both in the $N\pi$ and $\Delta\pi$ channels. Here, we follow the idea of the $1/N_c$ expansion. By careful application of dispersion relations we show that the spin polarizabilities coincide with those in heavy baryon chiral perturbation theory, if we take the narrow decay-width limit of the Δ particle. [S0556-2813(99)02311-0]

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I. INTRODUCTION

The electromagnetic polarizabilities are the quantities to represent the response of the nucleon to external electromagnetic fields and reflect its internal structure. Recently, Ragusa showed [1] that there are four independent spin polarizabilities $\gamma_i(i=1,\ldots,4)$ in the amplitude of $\mathcal{O}(\omega^3)$, and that the forward spin polarizability γ_0 is given by $\gamma_0=\gamma_1-\gamma_2-2\gamma_4$. There has not yet been any experiment about the spin polarizabilities.

Theoretical investigations on the spin polarizabilities have been carried out within the framework of the heavy baryon chiral perturbation theory (HBChPT). Bernard *et al.* [2] showed that the value of γ_0 from the leading $N\pi$ loops has an opposite sign to that of the pion photoproduction multipole analysis [3], and that the contribution of the $\Delta(1232)$ pole is inevitable to reduce largely that of the $N\pi$-loop terms. Hemmert *et al.* [4,5] gave the spin polarizabilities γ_1 to γ_4 within a small scale expansion framework, where the Δ is introduced as an independent field, and the mass difference between the nucleon and Δ state is treated as an additional small scale.

In this paper we study the spin polarizabilities $\gamma_1,\ldots,\gamma_4$ by careful application of the dispersion relations to the pion photoproduction amplitudes, which satisfy the low energy theorems and then are model independent. Such model-independent amplitudes can also be constructed in the chiral soliton model [6]. In previous papers, we calculated with use of the dispersion relations the electric and magnetic polarizabilities [6] and the forward spin polarizability γ_0 [7], and found that our calculations well reproduce the results of the $N\pi$-loop and Δ-pole terms calculated in HBChPT, if we take the narrow width limit of the Δ state [7].

The chiral soliton model has flavor-spin symmetry in the large N_c QCD, which implies the large N_c consistency condition on meson-baryon reaction amplitudes [8,9], where N_c is the number of colors. The consistency condition makes the pion production amplitudes with the magnetic dipole interaction vertices finite at high energies and then the dispersion integrals of the magnetic part of the amplitudes converge. In this sense we study the subject from the view point of the $1/N_c$ expansion.

We show in this paper that the results on the spin polarizabilities also reduce to the same ones by HBChPT at the narrow width limit of the Δ state, if we carefully apply the dispersion relations. We argue that the convergent dispersion integral does not necessarily mean no need of the subtraction, and discuss how to remedy the ill-defined dispersion integrals. We discuss the contributions from the interference terms between the electric and magnetic amplitudes in the $N\pi$ and $\Delta\pi$ channels, which appear at the finite Δ width. These are not taken into account in HBChPT, because they are of higher order in the small parameter expansion. Comparison with the multipole analyses [10,11] is given.

In Sec. II we calculate the imaginary parts of Compton scattering amplitude with use of the electric pion photoproduction amplitude in the $N\pi$ channel, and argue a subtle problem in the application of the dispersion relation. The contributions from the magnetic pion photoproduction amplitudes and $\Delta\pi$ channels are discussed in Secs. III and IV, respectively. Numerical results and discussion are given in Sec. V.

II. THE ELECTRIC PION PHOTOPRODUCTION AMPLITUDES AND DISPERSION RELATIONS

We consider the spin-dependent part of Compton scattering amplitude excluding the nucleon-pole terms, which is represented, at the center-of-mass system, as [5]

$$f_{\rm spin}=A_3(\omega,\theta)R_3+A_4(\omega,\theta)R_4+A_5(\omega,\theta)R_5+A_6(\omega,\theta)R_6, \tag{2.1}$$

where R_3, etc., are defined as

$$\begin{aligned} &R_3=i\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}'^*\times\boldsymbol{\epsilon}), \quad R_4=i\boldsymbol{\sigma}\cdot(\boldsymbol{k}'\times\boldsymbol{k})(\boldsymbol{\epsilon}'^*\cdot\boldsymbol{\epsilon}),\\ &R_5=i\boldsymbol{\sigma}\cdot[(\boldsymbol{\epsilon}'^*\times\boldsymbol{k})(\boldsymbol{\epsilon}\cdot\boldsymbol{k}')-(\boldsymbol{\epsilon}\times\boldsymbol{k}')(\boldsymbol{\epsilon}'^*\cdot\boldsymbol{k})], \\ &R_6=i\boldsymbol{\sigma}\cdot[(\boldsymbol{\epsilon}'^*\times\boldsymbol{k}')(\boldsymbol{\epsilon}\cdot\boldsymbol{k}')-(\boldsymbol{\epsilon}\times\boldsymbol{k})(\boldsymbol{\epsilon}'^*\cdot\boldsymbol{k})]. \end{aligned} \tag{2.2}$$

Here, $\boldsymbol{\epsilon}(\boldsymbol{\epsilon}')$ and $\boldsymbol{k}(\boldsymbol{k}')$ are the polarization vector and the momentum of the incident (outgoing) photon, respectively,

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and $\boldsymbol{\sigma}$ denotes the Pauli matrix of the nucleon. The independent structure functions $A_i(\omega,\theta)$ with $i=3,\ldots,6$ are functions of the photon energy $\omega(=\omega')$ and the scattering angle θ. We note that A_3 is $\mathcal{O}(\omega^3)$ at low energies, but other A_i's with i being from 4 to 6 behave as $\mathcal{O}(\omega)$ because of the additional ω^2 dependence coming from R_i except for R_3.

We apply the forward dispersion relations to the structure functions $A_i(\omega,0)$ except for the anomalous part coming from the $\pi^0\gamma\gamma$ interaction. The forward spin polarizability $\gamma_0=\gamma_1-\gamma_2-2\gamma_4$ is given as the following dispersion integral of Im $A_3(\omega,0)$:

$$\gamma_0=\frac{2}{\pi}\int_{\omega_{th}}^{\infty}d\omega\,\frac{\text{Im }A_3(\omega,0)}{\omega^4}. \tag{2.3}$$

The other polarizabilities γ_2, γ_3, and γ_4 are given by the dispersion integrals of Im $A_4(\omega,0)$, Im $A_6(\omega,0)$, and Im $A_5(\omega,0)$ divided by ω^2, respectively, instead of Im $A_3(\omega,0)/\omega^4$ in Eq. (2.3). The anomalous terms contribute to γ_1, γ_3, and γ_4.

The imaginary part of the scattering amplitude is calculated from the photoabsorption one of the nucleon using the unitarity condition. In this section we evaluate the contribution of the $\gamma+N\rightarrow\pi+N$ amplitude with the electric interaction, which we call the electric Born amplitude. The amplitude is decomposed into three terms as

$$f_N^a=i\epsilon_{a3b}\tau^b f_N^{(-)}+\tau^a f_N^{(0)}+\delta_{a3}f_N^{(+)}, \tag{2.4}$$

where τ^a's are the isospin matrices.

The electric Born amplitude satisfying the low energy theorem within the static kinematics is written as

$$f_{N,e}^{(-)}=\left(\frac{eG_{NN\pi}}{8\pi M}\right)\left\{i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}+2\frac{i\boldsymbol{\sigma}\cdot(\boldsymbol{k}-\boldsymbol{q})(\boldsymbol{\epsilon}\cdot\boldsymbol{q})}{m_\pi^2-(k-q)^2}\right\}, \tag{2.5}$$

where k and q are the incident photon and the outgoing pion four-momenta, respectively, $\boldsymbol{\epsilon}$ the polarization vector of the incident photon. This amplitude is also obtained in the chiral soliton model, where it is seen from the $1/N_c$ viewpoint: The πNN coupling constant $G_{NN\pi}$ is of $\mathcal{O}(N_c^{3/2})$, the nucleon mass of $\mathcal{O}(N_c)$ and the pion mass and momenta of $\mathcal{O}(1)$. We, then, see that $f_{N,e}^{(-)}$ is of $\mathcal{O}(N_c^{1/2})$, while $f_{N,e}^{(+,0)}$ are of $\mathcal{O}(N_c^{-1/2})$ and behave as $\mathcal{O}(\omega)$, that is, though the latter amplitudes satisfy the low energy theorem, they do not lead to finite results without unitarization, and are discarded in the following.

With use of the unitarity condition we find the imaginary part of the spin-dependent part as

$$\begin{aligned}&\frac{q}{4\pi}\int d\Omega_q\,2f_{N,e}^{(-)\dagger}f_{N,e}^{(-)}\\&=2q\left(\frac{eG_{NN\pi}}{8\pi M}\right)^2\{I(v)R_3+J_1(v,\theta)R_4/m_\pi^2-J_2(v,\theta)R_5/m_\pi^2\\&\quad+J_2(v,\theta)R_6/m_\pi^2+J_3(v,\theta)R_7/m_\pi^4\},\end{aligned} \tag{2.6}$$

with $R_7=i\boldsymbol{\sigma}\cdot(\boldsymbol{k}'\times\boldsymbol{k})(\boldsymbol{\epsilon}'^*\cdot\boldsymbol{k})(\boldsymbol{\epsilon}\cdot\boldsymbol{k}')$. Here, $I(v)=(1-v^2)/(2v)\ln\{(1+v)/(1-v)\}$ with v the pion velocity defined as $v=q/\omega$ in the static kinematics, and $J_i(v,\theta)$ are functions depending on v and θ. At $\theta=0$ they are given by

$$J_1(v,0)=(1-v^2)\left[\frac{1}{2}-\frac{1-v^2}{4v}\ln\left(\frac{1+v}{1-v}\right)\right], \tag{2.7}$$

$$\begin{aligned}J_2(v,0)&=J_3(v,0)/(1-v^2)\\&=(1-v^2)\left[-\frac{3}{4}+\frac{3-v^2}{8v}\ln\left(\frac{1+v}{1-v}\right)\right].\end{aligned} \tag{2.8}$$

It is known that the spin factor R_7 of the last term in Eq. (2.6) is not independent of the other spin factors; actually, we have

$$R_7=[\omega^4-(\boldsymbol{k}'\cdot\boldsymbol{k})^2]R_3+(\boldsymbol{k}'\cdot\boldsymbol{k})R_5-\omega^2R_6, \tag{2.9}$$

where we have used the relation $\omega=\omega'$. Redistributing the last term in Eq. (2.6) into the first, the third, and the fourth terms in Eq. (2.6), we find that Im $A_5(\omega,0)=$ Im $A_6(\omega,0)=0$ due to the cancellation $J_2(v,0)-J_3(v,0)/(1-v^2)=0$. The naive application of the dispersion integrals, therefore, yields $\gamma_3^{N,e}=\gamma_4^{N,e}=0$, except for the anomalous term. However, we point out that the last term in Eq. (2.6) should not be taken into account as the contribution to the spin polarizabilities: Let us consider the coefficient function of R_7, which we denote temporarily as $A_7(\omega,0)$ at $\theta=0$. Following the argument by Low [12], we see that $A_7(\omega,0)$ has no singularities as $\omega\rightarrow 0$, because we are considering the non-Born terms of the Compton scattering with the minimum excitation energy to be m_π. This means that $A_7(\omega,0)$ behaves as ω or higher at low energies, and then $A_7(\omega,0)\,R_7$ does not contribute to the spin polarizabilities, because R_7 itself is of order of ω^4, while the spin polarizabilities are defined as the coefficients of ω^3 terms of the amplitudes $A_i(\omega,0)R_i$'s. We must, therefore, disregard the $A_7(\omega,0)\,R_7$ term, and neglect it in the application of the dispersion relations. Thus, both Im $A_5(\omega,0)$ and Im $A_6(\omega,0)$ are proportional to $J_2(v)$ and do not vanish.[1] We shall see in the Appendix how this situation is described in the case of HBChPT.

The spin polarizabilities are thus given through the dispersion integrals of the electric part in $A_i(\omega,0)$'s with $i=3,\ldots,6$ in the $N\pi$ channel as

$$\gamma_1^{N,e}=2\gamma_2^{N,e}=4\gamma_3^{N,e}=-4\gamma_4^{N,e}=\frac{e^2G_{NN\pi}^2}{96\pi^3M^2m_\pi^2}, \tag{2.10}$$

which are the same as those of the $N\pi$ loops in HBChPT [5]. Because the proton-neutron difference depends on the amplitude $f_{N,e}^{(0)}$, we cannot predict the difference between them. It

[1] L'vov already argued about this problem and states that the dispersion relation does not work in this case from the high-energy behavior of the relativistic invariant amplitudes with Regge-pole assumption [13].

is also known that the prediction of HBChPT up to chiral order ϵ^3 yields no isospin dependence.

III. CONTRIBUTIONS FROM MAGNETIC BORN AMPLITUDES AND INTERFERENCE TERMS

The magnetic Born amplitude for the $\gamma+N\to\pi+N$ process is written as

$$f_{N,m}^{(\pm)}=\left(\frac{eG_{NN\pi}\mu_V}{16\pi M^2}\right)\{t_1^{(\pm)}P_1(\hat{\boldsymbol{q}},\hat{\boldsymbol{s}})+t_3^{(\pm)}P_3(\hat{\boldsymbol{q}},\hat{\boldsymbol{s}})\}, \quad (3.1)$$

where μ_V is the vector part of the nucleon magnetic moment defined by $(\mu_p-\mu_n)/2$ in units of the nuclear magneton, $P_1(\hat{\boldsymbol{q}},\hat{\boldsymbol{s}})=(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{q}})(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{s}})$ and $P_3(\hat{\boldsymbol{q}},\hat{\boldsymbol{s}})=3(\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{s}})-(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{q}})(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{s}})$ are the P-wave projection operators for the $J=1/2$ and 3/2 states, respectively, and $\hat{\boldsymbol{q}}=\boldsymbol{q}/q$ and $\hat{\boldsymbol{s}}=\boldsymbol{s}/k$ with $\boldsymbol{s}=\boldsymbol{k}\times\boldsymbol{\epsilon}$. $t_i^{(\pm)}$ are given by

$$t_1^{(+)}=2t_1^{(-)}=-\frac{2q}{3M}\,\frac{\Delta}{\omega+\Delta},$$

$$t_3^{(+)}=\frac{q\omega}{2M}\left[-\frac{2\Delta}{\omega^2-\Delta^2+i\Delta\Gamma_\Delta}+\frac{2}{3}\,\frac{\Delta}{\omega(\omega+\Delta)}\right],$$

$$t_3^{(-)}=\frac{q\omega}{2M}\left[\frac{\Delta}{\omega^2-\Delta^2+i\Delta\Gamma_\Delta}-\frac{2}{3}\,\frac{\Delta}{\omega(\omega+\Delta)}\right], \quad (3.2)$$

with Δ being the mass difference of the nucleon and the $\Delta(1232)$. In the above we use the relation $\mu_V^{\Delta N}=-(3/\sqrt{2})\mu_V$ as well as $G_{\Delta N\pi}=-(3/\sqrt{2})G_{NN\pi}$ obtained in the chiral soliton model, which are shared with the large N_c baryon model [8]. Here, in order to avoid the pole at $\omega=\Delta$ on the real axis, we have introduced the finite width of the Δ state given by $\Gamma_\Delta=(1/6\pi)(G_{\Delta N\pi}/2M)^2q^3$. This is the expression given by Kokkedee without relativistic correction [14], and yields 145 MeV with the experimental value of $G_{NN\pi}$ at $q=227$ MeV.

It should here be noted that each of the N- and Δ-pole terms in $f_{N,m}^{(\pm)}$ is of $\mathcal{O}(N_c^{3/2})$ owing to the isovector magnetic moment, $e\mu_V/2M$, of $\mathcal{O}(N_c)$ [8,14], but the sums reduce to $\mathcal{O}(N_c^{1/2})$ due to the cancellation among the N- and Δ-pole terms, that is due to the large N_c consistency condition. This is a remarkable point of the chiral soliton model as well as the large N_c baryon model, because the cancellation makes the amplitudes finite at infinite ω at the same time. The amplitude $f_{N,m}^{(0)}$ is of $\mathcal{O}(N_c^{-1/2})$, where the consistency condition does not work, and is then discarded in the following.

For the magnetic part we obtain

$$\gamma_2^{N,m}=-\gamma_4^{N,m}=\gamma_0^{N,m}, \quad \gamma_1^{N,m}=\gamma_3^{N,m}=0, \quad (3.3)$$

where

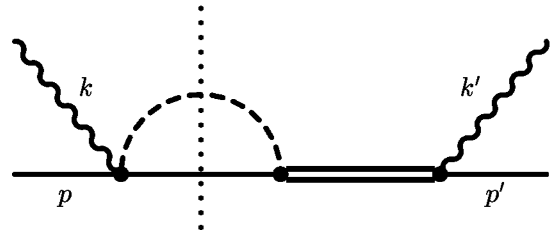

FIG. 1. An example of the diagrams of the interference between the electric and magnetic amplitudes. The solid, double-solid, dashed, and wavy lines denote the nucleon, Δ particle, pion, and photon, respectively. The vertical dotted line denotes the on shell.

$$\gamma_0^{N,m}=-\frac{e^2G_{NN\pi}^2\mu_V^2}{192\pi^3M^4}\int_{\omega_{th}}^{\infty}\frac{d\omega}{\omega^4}q^3\left[\frac{\omega\Delta^2(\omega+8\Delta)}{(\omega^2-\Delta^2)^2+\Delta^2\Gamma_\Delta^2}-\frac{2\Delta^2}{(\omega+\Delta)^2}\right]. \quad (3.4)$$

Since the above integrand contains $(G_{NN\pi}/2M)^2q^3$, the same form as $\Gamma_\Delta(q)$ appears in the numerator, and then we may obtain, at the limit of $\Gamma_\Delta\to 0$,

$$\gamma_0^{N,m}=-\frac{e^2\mu_V^2}{8\pi M^2\Delta^2}, \quad (3.5)$$

which is equal to the results of the Δ-pole contribution in HBChPT [5].

The interference term between the electric and magnetic terms is also calculated similarly. As an example we show a diagram for the interference part in Fig. 1. This kind of diagrams is not taken into account in HBChPT, because they are of higher order. Contrary, the baryon propagator in the $1/N_c$ expansion is the same order as the pion propagator, so that such terms are not of higher order. The interference term in the $N\pi$ channel is calculated to be

$$\gamma_2^{N,i}=-\frac{e^2G_{NN\pi}^2\mu_V}{16\pi^3M^3}\int_{\omega_{th}}^{\infty}\frac{d\omega}{\omega^3}q\left[[v^2-1+I(v)]\times\frac{\omega\Delta(\omega^2-\Delta^2)}{(\omega^2-\Delta^2)^2+\Delta^2\Gamma_\Delta^2}-\left(\frac{2}{3}v^2-1+I(v)\right)\frac{\Delta}{\omega+\Delta}\right],$$

$$\gamma_4^{N,i}=-\frac{e^2G_{NN\pi}^2\mu_V}{64\pi^3M^3}\int_{\omega_{th}}^{\infty}\frac{d\omega}{\omega^3}q[1-I(v)]\times\left[\frac{\omega\Delta(\omega^2-\Delta^2)}{(\omega^2-\Delta^2)^2+\Delta^2\Gamma_\Delta^2}-\frac{2\Delta}{\omega+\Delta}\right], \quad (3.6)$$

and $\gamma_1^{N,i}=\gamma_3^{N,i}=0$.

IV. CONTRIBUTIONS FROM $\Delta\pi$ CHANNEL

In this section we examine the $\gamma+N\to\pi+\Delta$ contribution. The amplitude is also decomposed into three terms as

$$f_\Delta^a=i\epsilon_{a3b}\mathcal{T}^bf_\Delta^{(-)}+\mathcal{T}^af_\Delta^{(0)}+\mathcal{T}_{a3}^+f_\Delta^{(+)}, \quad (4.1)$$

TABLE I. Calculated spin polarizabilities of the nucleon in the large N_c baryon model. For a comparison those in HBChPT are also shown. Parameters are taken to be empirical ones, except for the $N\Delta$ transition parameters predicted in the soliton model. In the result of the large N_c model E, M, and I denote the contributions of the electric, magnetic, and interference parts, respectively, and the values at the narrow-width limit of the Δ particle are given in the parentheses. For the results of HBChPT the numbers in the parentheses are the values with the old estimation of the $\pi N\Delta$ and $\gamma N\Delta$ couplings. All values are in units of 10^{-4} fm^4.

	Large N_c model							HBChPT [5]			
	$N\pi$ channel			$\Delta\pi$ channel							
	E	M	I	E	M	I	Sum.	$N\pi$ loop	Δ pole	$\Delta\pi$ loop	Sum.
γ_1	5.1	0.0	0.0	−0.4	0.0	0.0	4.7	4.56	0	−0.21	4.35
										(−0.4)	(4.1)
γ_2	2.5	−2.5	−0.1	−0.4	0.1	0.5	0.1	2.28	−2.40	−0.23	−0.35
		(−4.0)	(0.0)				(−1.3)		(−4.0)	(−0.5)	(−2.2)
γ_3	1.3	0.0	0.0	−0.2	0.0	0.0	1.1	1.14	0	−0.12	1.02
										(−0.2)	(0.9)
γ_4	−1.3	2.5	1.2	0.2	−0.1	−0.1	2.5	−1.14	2.40	0.12	1.38
		(4.0)	(0.0)				(2.8)		(4.0)	(0.2)	(3.1)
γ_0	5.1	−2.5	−2.4	−0.4	0.1	−0.3	−0.4	4.5	−2.4	−0.2	2.0
		(−4.0)	(0.0)				(0.5)		(−4.0)	(−0.4)	(0.1)

where $\mathcal{T}^a$ is the transition isospin matrix from N to Δ, and $\mathcal{T}^+_{a3}=\mathcal{T}^a\frac{1}{2}\tau^3+\frac{1}{2}\mathcal{T}^3_{\Delta\Delta}\mathcal{T}^a$. The electric part is obtained by replacing $\boldsymbol{\sigma}$ and $G_{NN\pi}$ in Eq. (2.5) by the transition spin operator $\boldsymbol{S}_{\Delta N}$ and $G_{\Delta N\pi}$, respectively. The magnetic part is given by

$$f^{(-)}_{\Delta,m}=\left(\frac{eG_{\Delta N\pi}\mu_V}{16\pi M^2}\right)\left\{-\frac{(\boldsymbol{S}_{\Delta N}\cdot\boldsymbol{q})(\boldsymbol{\sigma}\cdot\boldsymbol{s})}{\omega}-\frac{4}{5}\frac{(\boldsymbol{S}_{\Delta\Delta}\cdot\boldsymbol{q})(\boldsymbol{S}_{\Delta N}\cdot\boldsymbol{s})}{\omega_q}+2\frac{(\boldsymbol{S}_{\Delta N}\cdot\boldsymbol{s})(\boldsymbol{\sigma}\cdot\boldsymbol{q})}{\omega_q}-\frac{1}{5}\frac{(\boldsymbol{S}_{\Delta\Delta}\cdot\boldsymbol{s})(\boldsymbol{S}_{\Delta N}\cdot\boldsymbol{q})}{\omega}\right\},$$

$$f^{(+)}_{\Delta,m}=\left(\frac{eG_{\Delta N\pi}\mu_V}{16\pi M^2}\right)\left\{-\frac{(\boldsymbol{S}_{\Delta N}\cdot\boldsymbol{q})(\boldsymbol{\sigma}\cdot\boldsymbol{s})}{\omega}-\frac{1}{5}\frac{(\boldsymbol{S}_{\Delta\Delta}\cdot\boldsymbol{q})(\boldsymbol{S}_{\Delta N}\cdot\boldsymbol{s})}{\omega_q}+\frac{(\boldsymbol{S}_{\Delta N}\cdot\boldsymbol{s})(\boldsymbol{\sigma}\cdot\boldsymbol{q})}{\omega_q}+\frac{1}{5}\frac{(\boldsymbol{S}_{\Delta\Delta}\cdot\boldsymbol{s})(\boldsymbol{S}_{\Delta N}\cdot\boldsymbol{q})}{\omega}\right\},\qquad(4.2)$$

where $\boldsymbol{S}_{\Delta\Delta}$ is the spin matrix for the Δ state, and ω_q is the energy of pion. The above amplitudes are also finite at infinite energy owing to the cancellation between the N and Δ pole terms. The results are as follows. The electric part is given by

$$\gamma_1^{\Delta,e}=-\frac{e^2G^2_{\Delta N\pi}}{864\pi^3M^2}\left[\frac{\Delta^2+2m_\pi^2}{(\Delta^2-m_\pi^2)^2}-\frac{3m_\pi^2\Delta\ln R}{(\Delta^2-m_\pi^2)^{5/2}}\right],$$

$$\gamma_2^{\Delta,e}=2\gamma_3^{\Delta,e}=-2\gamma_4^{\Delta,e}=-\frac{e^2G^2_{\Delta N\pi}}{864\pi^3M^2}\left[\frac{1}{\Delta^2-m_\pi^2}-\frac{\Delta\ln R}{(\Delta^2-m_\pi^2)^{3/2}}\right],\qquad(4.3)$$

with $R=\Delta/m_\pi+\sqrt{\Delta^2/m_\pi^2-1}$. The results are the same as the results of the $\Delta\pi$ loops in HBChPT. For the magnetic terms we obtain $\gamma_1^{\Delta,m}=\gamma_3^{\Delta,m}=0$, and

$$\gamma_2^{\Delta,m}=-\gamma_4^{\Delta,m}=-\frac{e^2G^2_{\Delta N\pi}\mu_V^2m_\pi^3}{432\pi^3M^4\Delta^3}\left[\frac{\Delta(24m_\pi^4-20m_\pi^2\Delta^2-\Delta^4)}{6m_\pi^3(\Delta^2-m_\pi^2)}+2\pi+\frac{(8m_\pi^4-12m_\pi^2\Delta^2+3\Delta^4)\ln R}{2m_\pi(\Delta^2-m_\pi^2)^{3/2}}\right].\qquad(4.4)$$

The interference part yields $\gamma_1^{\Delta,i}=\gamma_3^{\Delta,i}=0$, and

$$\gamma_2^{\Delta,i}=\frac{e^2G^2_{\Delta N\pi}\mu_V\Delta}{72\pi^3M^3}\int_{\omega_{th}}^{\infty}\frac{d\omega}{\omega^3}\left[\frac{1-I(v)}{2b}+\frac{1}{b^2}\left(\frac{v^2}{3}-\frac{1-I(v)}{2}\right)\right],$$

$$\gamma_4^{\Delta,i}=-\frac{e^2G^2_{\Delta N\pi}\mu_V\Delta}{144\pi^3M^3}\int_{\omega_{th}}^{\infty}\frac{d\omega}{\omega^3}\frac{1-I(v)}{2b},\qquad(4.5)$$

where $b=\omega/\omega_q$ with ω_q the energy of pion. The magnetic and interference parts in Eqs. (4.4) and (4.5) are not taken into account in HBChPT.

V. RESULTS AND DISCUSSION

Numerical results of the polarizabilities calculated by means of the dispersion relations are given in Table I, where empirical values of the constants in the formulas are used in order to compare the results with those by HBChPT; namely, $f_\pi = 93$ MeV, $M = 939$ MeV, $\Delta = 293$ MeV, $G_{\pi NN} = 13.5$, and $m_\pi = 138$ MeV, provided that the relative sizes of $G_{\Delta N\pi}$ and $\mu_{\Delta N}$ are taken so as to satisfy the large N_c consistency condition. The results of HBChPT [5] are also given in the table. In the results of HBChPT we give two cases for the Δ pole and $\Delta\pi$-loop terms: the upper ones are obtained using the $\pi N\Delta$ and $\gamma N\Delta$ coupling constants determined by the ''small scale expansion'' itself, while the lower in the parentheses are by a tree-level relativistic analysis.

We note that, although the electric part of the polarizabilities is the same as the $N\pi$ and $\Delta\pi$ loops in HBChPT, the numerical values are slightly different, because we did not use the Goldberger-Treiman relation. For the $\Delta\pi$ loops the values in the parentheses are corresponding to the electric part for the $\Delta\pi$ channel. The results of the magnetic part of the $N\pi$ channel with the narrow-width limit shown in the parentheses, are the same as those of the Δ poles in HBChPT. The finite-width effect of the Δ particle reduces the magnetic contribution in the same as for the magnetic polarizability β [7]. In HBChPT the numerical results with the parameters which are determined by the ''small scale expansion'' are similar to the ones with the finite width.

As shown in Sec. III, no interference part of the electric and magnetic amplitudes is calculated in HBChPT, because these terms are of higher orders. We see that the interference part contributes to γ_2 and γ_4, and that their values are small in γ_2, but considerably large in γ_4. For the forward spin polarizability γ_0 the contribution of the magnetic part becomes smaller by the effect of the finite width of the Δ state, but that of the interference part becomes large; as a result, the sum of them is nearly the same as that at the narrow-width limit of the Δ state.

The electric part of the $\Delta\pi$ channel is rather small in agreement with the result for the $\Delta\pi$ loops in HBChPT. The magnetic and the interference parts in the $\Delta\pi$ channel are almost negligible small. It is expected that the contributions of the $\Delta\pi$ channel are small compared with those of the $N\pi$ channel because of the factor ω^4 in the denominator of the dispersion relation. We infer that the effect of the higher resonances other than the Δ state is very small.

The anomalous part is also calculated within the chiral soliton model, which turns out to be the same form as the conventional one [15] and given as follows:

$$\gamma_1^{\rm anom} = -2\gamma_3^{\rm anom} = 2\gamma_4^{\rm anom} = -\frac{e^2 G_{NN\pi}}{16\pi^3 M f_\pi m_\pi^2}\tau_3, \quad (5.1)$$

and $\gamma_2^{\rm anom} = 0$. The numerical values are

$$\gamma_1^{\rm anom} = -2\gamma_3^{\rm anom} = 2\gamma_4^{\rm anom} = -22.8\tau_3 \quad (5.2)$$

in units of 10^{-4} fm^4 for the parameters.

In Table II we show the calculated results of the spin polarizabilities and compare those with the results of HBChPT [5] and of the multipole analysis, where HDT and SAID refer to [10] and [11], respectively. We see good agreement with the results of HBChPT, but the value of γ_4 is large compared with that of HBChPT, and seems to be close to the results of the multipole analysis. This is due to the effect of the interference part between the electric and magnetic amplitudes. As a result, the forward spin polarizability γ_0 is also close to those of the multipole analysis. We cannot evaluate the proton and neutron difference, since we have discarded the amplitude $f^{(0)}$ as it is of higher orders.

TABLE II. Spin polarizabilities of the nucleon in the large N_c baryon model compared with the results in HBChPT and of the multipole analysis. In the parentheses the values at the narrow-width limit of the Δ particle are given in the former one. All values are in unit of 10^{-4} fm^4.

	Large N_c model	HBChPT [5]	Multipole analysis HDT [10]		SAID [11]	
			p	n	p	n
γ_1	4.7	4.4	5.1	6.1	3.1	6.3
γ_2	0.1	−0.3	−1.1	−0.8	−0.8	−0.9
	(−1.3)					
γ_3	1.1	1.1	−0.6	−0.6	0.3	−0.7
γ_4	2.5	1.3	3.4	3.4	2.7	3.8
	(2.8)					
γ_0	−0.1	2.0	−0.6	−0.2	−1.5	−0.4
	(0.5)					

In conclusion we have calculated the spin polarizabilities of the nucleon, where the dispersion relation was used with the imaginary part of the Compton scattering amplitudes constructed from the pion photoproduction amplitudes through the unitarity condition. The form of these amplitudes is model independent, but we imply the large N_c consistency condition on the coupling constants. It may be said, therefore, that our approach is a large N_c chiral perturbation theory. We have shown that the electric and magnetic parts agree with the results of the $N\pi$ loops, Δ poles, and $\Delta\pi$ loops calculated in HBChPT. The numerical results are also similar with each other and qualitatively agree with those of the multipole analysis. The interference term of the electric and magnetic amplitudes in the $N\pi$ channel, which is not considered in HBChPT as higher orders, is, however, large especially in γ_4, and the resulting values of the polarizabilities are closer to those of the multipole analysis. The next-to-leading-order calculation with the $f^{(0)}$ amplitudes is necessary to evaluate the difference between the proton and neutron.

APPENDIX: DISPERSION RELATION FOR $N\pi$ ELECTRIC PART

The relevant structure function of the forward scattering amplitude, $A_5(\omega,\theta=0)$, which is equal to $-A_6(\omega,\theta=0)$, in HBChPT [15] is given by two terms as follows:

$$A_{51}(\omega,0)=-\frac{e^2g_A^2}{32\pi^3f_\pi^2m_\pi}\int_0^1 dx\,\frac{(1-x)^2\sin^{-1}ux}{\sqrt{1-u^2x^2}} \quad \text{(A1)}$$

and

$$A_{52}(\omega,0)=\frac{e^2g_A^2}{32\pi^3f_\pi^2m_\pi}u^2\int_0^1 dx\,\frac{x(1-x)^3}{3(1-u^2x^2)^{3/2}}(\sin^{-1}ux+xu\sqrt{1-u^2x^2}), \quad \text{(A2)}$$

where $u=\omega/m_\pi$, and $1/(4\pi)$ is multiplied so as to fit our definition of A_i. The integration over x can be carried out and leads to

$$A_{51}(\omega,0)=-\frac{e^2g_A^2}{32\pi^3f_\pi^2m_\pi}\left(\frac{u}{12}+\frac{u^3}{90}+\mathcal{O}(u^5)\right) \quad \text{(A3)}$$

and

$$A_{52}(\omega,0)=\frac{e^2g_A^2}{32\pi^3f_\pi^2m_\pi}\left(\frac{u^3}{90}+\mathcal{O}(u^5)\right), \quad \text{(A4)}$$

where the u^3 and the higher order terms are the same except for the signs in both functions. The sum is exactly given by

$$A_5(\omega,0)=A_{51}(\omega,0)+A_{52}(\omega,0)=-\frac{e^2g_A^2}{32\pi^3f_\pi^2m_\pi}\frac{u}{12}. \quad \text{(A5)}$$

This shows that there is no branch cut on the real axis in the complex ω plane; namely, Im $A_5(\omega,0)=0$, but each of $A_{51}(\omega,0)$ and $A_{52}(\omega,0)$ has the same imaginary part except for the signs. The analytic continuation to $\omega>m_\pi$ gives

$$\begin{aligned}\text{Im } A_{52}(\omega,0)&=-\text{Im } A_{51}(\omega,0)\\&=\frac{e^2g_A^2}{32\pi^3f_\pi^2m_\pi}\frac{\pi}{8u^3}\Bigg(-6u\sqrt{u^2-1}\\&\quad+(1+2u^2)\ln u+\sqrt{\frac{u^2-1}{u-\sqrt{u^2-1}}}\Bigg).\end{aligned} \quad \text{(A6)}$$

The fact that Im $A_5(\omega,0)=0$ is the same as the situation that the imaginary part of $A_5(\omega,0)$ vanishes after the redistribution of the last term of Eq. (2.6) as we encountered in Sec. II. Indeed, we can see with use of the Goldberger-Treiman relation that Eq. (A6) is equal to the last term in Eq. (2.6) multiplied by ω^2 coming from the identity, Eq. (2.9).

Now, let us consider the dispersion relations for $A_{51}(\omega,0)$ and $A_{52}(\omega,0)$. We have for $A_{51}(\omega,0)$

$$\text{Re } \frac{A_{51}(\omega,0)}{\omega}\bigg|_{\omega=0}=\frac{2}{\pi}\text{P}\int_{m_\pi}^{\omega_{\max}}d\omega'\frac{\text{Im } A_{51}(\omega',0)}{\omega'^2}+\frac{1}{\pi}\text{Im }\int_C d\omega'\frac{A_{51}(\omega',0)}{\omega'^2}, \quad \text{(A7)}$$

where the second term denotes the semicircle integral with the radius $\omega_{\max}$. The same is valid for $A_{52}(\omega,0)$. From Eq. (A3) the left-hand side (LHS) in Eq. (A7) is

$$-\frac{e^2g_A^2}{32\pi^3f_\pi^2}\frac{1}{12m_\pi^2}.$$

In the right-hand side (RHS) the dispersion integral with the imaginary part in Eq. (A6) is found to be equal to the LHS with the vanishing semicircle integral. On the other hand, for the dispersion relation of $A_{52}(\omega,0)$, the fact that the LHS is zero is realized as the cancellation between the second (semicircle) integral and the first conventional one in the RHS. Therefore, if we consider the dispersion relation of the sum $A_5(\omega,0)$ with Im $A_5(\omega,0)=0$, it is necessary to calculate the semicircle integral, which is generally very difficult. However, when we notice the fact that Re $A_{52}(\omega,0)$ is of $\mathcal{O}(\omega^3)$ and higher at low energies, as seen in Eq. (A4), the amplitude $A_{52}(\omega,0)$ does not participate in determining the coefficient of the amplitude at ω^3. Thus, we are allowed to consider only the conventional dispersion relation in Eq. (A7), where the semicircle integral vanishes because of its asymptotic behavior. This is in complete agreement with the prescription we adopt in Sec. II.

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