Polarization transfer observables for quasielastic proton-nucleus scattering in terms of a complete Lorentz invariant representation of the *NN* scattering matrix

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For the calculation of polarization transfer observables for quasielastic scattering of protons on nuclei, a formalism in the context of the relativistic plane wave impulse approximation is developed, in which the interaction matrix is expanded in terms of a complete set of 44 independent invariant amplitudes. A boson-exchange model is used to predict the 39 amplitudes that were omitted in the formerly used five-term parameterization, the SPVAT (scalar, pseudoscalar, vector, axial-vector, tensor) form of the nucleon-nucleon scattering matrix. Use of the complete set of amplitudes eliminates the arbitrariness of the five-term representation. [S0556-2813(99)05711-8]

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I. INTRODUCTION

Quasielastic scattering of protons on nuclei is an attractive phenomenon for the study of the basic nucleon-nucleon interaction in the nuclear medium because it exhibits the approximate behavior of the scattering of a nucleon on only one nucleon of the target nucleus. Quasielastic scattering has been modeled by the relativistic plane wave impulse approximation (RPWIA) [1], which considers it a single-step process, whereby the projectile interacts with only one nucleon of the target nucleus, while the rest of the nucleons remain inert. The well-known and outstanding success of the original RPWIA was its prediction of the analyzing power for the reactions ^{40}Ca$(\vec{p},\vec{p}\,')$ and ^{208}Pb$(\vec{p},\vec{p}\,')$ at 500 MeV; a case in which all nonrelativistic models failed [2].

In the RPWIA approach, the description of the initial and final free particle in the medium is based on a mean-field theory, as described by Serot and Walecka in Ref. [3]. In the RPWIA model the associated Dirac plane waves have their free nucleon mass decreased by the real part of the average nuclear scalar field to yield an effective nucleon mass. The values of the effective masses serve as an indicator of the nuclear medium effects on the *NN* interaction.

In former theoretical studies of scattering [1,4,5] the nucleon-nucleon scattering matrix $(\hat{F})$ was parameterized in terms of the five Fermi covariants, which is commonly referred to as the SPVAT (scalar, pseudoscalar, vector, axial-vector, tensor) form of $\hat{F}$ or the IA1 model. It should be stressed, however, that even though the SPVAT form gave reasonable results for elastic and quasielastic scattering observables, it is, in principle, not correct, since as was first pointed out in Ref. [6], a five-term representation of the relativistic *NN* scattering matrix is necessarily ambiguous. In addition, Tjon and Wallace [7] have shown that a general Lorentz invariant representation of $\hat{F}$ (referred to as the IA2 model) contains additional terms that cannot be neglected. The IA2 representation of $\hat{F}$ contains, in fact, 44 independent invariant amplitudes, instead of the previously used five, which are consistent with parity and time-reversal invariance, as well as charge symmetry, together with the on-mass-shell condition for the external nucleons. Comparisons to the limited data available, with subsequent and more refined calculations [2,8–10], have also revealed that quasielastic $(\vec{p},\vec{p}\,')$ and $(\vec{p},\vec{n})$ scattering prefer different five-term representations of $\hat{F}$, the $(\vec{p},\vec{n})$ data favor a pseudovector πNN coupling, whereas the $(\vec{p},\vec{p}\,')$ data are consistent with a pseudoscalar term for the πNN vertex. Therefore, the most basic question that has to be addressed is the representation of the *NN* scattering matrix.

In the current application of the RPWIA to quasielastic scattering, the following components play a key role:

(1) The amplitudes in the basic two-nucleon interaction, which are partly determined from free *NN* scattering data and partly from a solution of the Bethe-Salpeter equation employing a meson-exchange model for the *NN* force.

(2) The Lorentz covariant set constructed from the Dirac matrices, which serves as a representation for $\hat{F}$.

(3) The effective nucleon mass for both projectile and target nucleons interacting in the nuclear medium.

In this paper a theoretical formalism is presented for the calculation of polarization transfer observables for quasielastic proton-nucleus scattering using a general Lorentz invariant representation of the *NN* scattering matrix [11]; a systematic survey of the predictive power of the model compared to data will be presented in a future paper. By adhering to the simplifying features of the RPWIA, one can focus on the basic *NN* interaction without introducing additional complications. A complete expansion of $\hat{F}$ allows for a correct incorporation of effective-mass-type medium effects (within the RPWIA framework and within the context of the Walecka model). In Sec. II we briefly review the RPWIA and also discuss the ambiguities of the SPVAT form of $\hat{F}$. In Sec. III the general Lorentz invariant representation of $\hat{F}$ is discussed. Section IV presents the transformation from invariant amplitudes to effective amplitudes while, in Sec. V, expressions for the spin observables are derived in terms of the effective amplitudes. A calculation of complete sets of spin observables, based on the IA2 model, for quasielastic ^{40}Ca$(\vec{p},\vec{p}\,')$ scattering at 500 MeV is presented in Sec. VI. Section VII summarizes the main aspects of this paper.

II. RELATIVISTIC PLANE WAVE IMPULSE APPROXIMATION

Complete sets of spin observables[1] (P, A_y, $D_{l'l}$, $D_{s's}$, $D_{s'l}$, $D_{l's}$, D_{nn}) for quasielastic $(\vec{p},\vec{p}\,')$ and $(\vec{p},\vec{n})$ scattering are calculated within a relativistic framework using the relativistic plane wave impulse approximation (RPWIA) [1]. The RPWIA models quasielastic scattering as a single-step process, whereby the projectile knocks out a single bound nucleon from the nucleus. The rest of the nucleons are assumed to remain inert, but their effect is taken into account in that the free mass of the projectile and target nucleons are shifted to *effective masses,* M_1 and M_2, respectively. In the context of the Walecka model [3] the effective masses can both be calculated microscopically as follows. For the projectile,

$$M_1 = M + \langle S \rangle,$$

where M is the free nucleon mass and $\langle S \rangle$ is the average of the real part of the scalar potential $S(\vec{r})$ over the whole nucleus weighted by the probability distribution of the scattering reaction [8]. The effective mass of the target nucleon is determined from

$$M_2 = M - g_s \langle \phi \rangle,$$

where g_s is the scalar meson coupling constant and $\langle \phi \rangle$ is the average of the scalar field $\phi(\vec{r})$ for the specific nucleus with the averaging done as described above. Values of M_1 and M_2 for specific nuclei and incident laboratory energies can be found in Table II of Ref. [8]. Experimental data seem to suggest that the spin observables are target independent [12,13], and therefore we assume, as a first step, a Fermi-gas approximation for the target nucleus. The RPWIA therefore reduces quasielastic proton-nucleus scattering to a two-body scattering process with Dirac spinors (containing effective nucleon masses) describing the external nucleons. A graphical representation of the scattering process is depicted in Fig. 1.

Referring to Fig. 1, the projectile Dirac spinor is given by

$$U(\vec{p}_1, M_1, s_i) = \left[\frac{E_1^* + M_1}{2M_1}\right]^{1/2} \begin{pmatrix} \phi(s_i) \\ \dfrac{\vec{\sigma}\cdot\vec{p}_1}{E_1^* + M_1}\,\phi(s_i) \end{pmatrix}, \tag{2.1}$$

[1]The spin observables are defined in Sec. V.

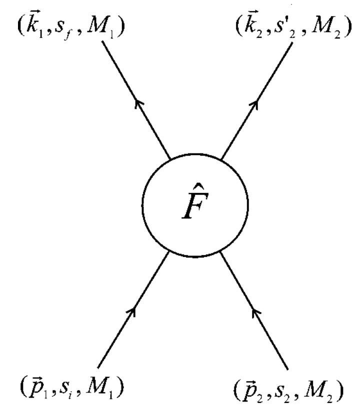

FIG. 1. Two-body scattering process with momentum, mass, and spin labels for the external nucleons. $\hat{F}$ is the 16×16 nucleon-nucleon scattering matrix in the two-nucleon spin space. $\vec{p}_i$ and $\vec{k}_i$ $(i=1,2)$ represent the three-momenta of the particles, respectively. M_1 and M_2 denote the effective masses of the projectile and target nucleons, respectively. s_i, s_2, s_f, and s_2' are the spin four vectors for each particle.

where $E_1^{*^2} = \vec{p}_1^{\,2} + M_1^2$ and the spinor is normalized to $\overline{U}(\vec{p}_1, M_1, s) U(\vec{p}_1, M_1, s') = \delta_{ss'}$. Similar expressions exist for the other three spinors labeled by $\vec{p}_2$, $\vec{k}_1$, and $\vec{k}_2$. The following four-momenta are also defined:

$$p_1^* = (E_1^*, \vec{p}_1), \quad p_2^* = (E_2^*, \vec{p}_2)$$

$$k_1^* = (E_1^{*'}, \vec{k}_1), \quad k_2^* = (E_2^{*'}, \vec{k}_2),$$

where $p_i^{*^2} = M_i^2$ and $k_i^{*^2} = M_i^2$ $(i=1,2)$. For handling the polarization, one requires the spin projection operator,

$$P(\hat{n}) = \frac{1}{2}(I_2 + \vec{\sigma}\cdot\hat{n}), \tag{2.2}$$

for the direction $\hat{n}$ where I_2 is the 2×2 unit matrix. In the basis of Pauli spinors, $\phi(\hat{n})$ for spin direction $\hat{n}$, we have

$$P(\hat{n}) = \phi(\hat{n})\phi^\dagger(\hat{n}). \tag{2.3}$$

Defining

$$\overline{U}(\vec{p}_1, M_1, s_i) = U^\dagger(\vec{p}_1, M_1, s_i)\gamma^0,$$

where the convention of Ref. [14] is used for the gamma matrices, the Lorentz invariant matrix element for the scattering process depicted in Fig. 1 is given by

$$\mathcal{M} = [\overline{U}(\vec{k}_1, M_1, s_f) \otimes \overline{U}(\vec{k}_2, M_2, s_2')]\hat{F}[U(\vec{p}_1, M_1, s_i) \otimes U(\vec{p}_2, M_2, s_2)], \tag{2.4}$$

where $\hat{F}$ is the 16×16 nucleon-nucleon scattering matrix. The question arises as to what form of $\hat{F}$ is to be used in Eq. (2.4), assuming parity and time-reversal invariance as well as charge symmetry. Once a choice of $\hat{F}$ has been made, analytical expressions for all spin observables, namely the unpolarized double differential cross section, the analyzing power and the polarization transfer observables can be obtained from Eq. (2.4). All previous calculations [1,8–10,15] of spin observables for quasielastic proton-nucleus scattering have parameterized $\hat{F}$ in terms of only the five Fermi covariants:

$$\hat{F}=F_S(I_4\otimes I_4)+F_P(\gamma^5\otimes\gamma^5)+F_V(\gamma^\mu\otimes\gamma_\mu)+F_A(\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu)+F_T(\sigma^{\mu\nu}\otimes\sigma_{\mu\nu}), \tag{2.5}$$

where the latter is commonly called the SPVAT form or IA1 representation of $\hat{F}$. The amplitudes F_L $(L=S,P,V,A,T)$ are obtained by fitting to free NN scattering data [15]. This procedure, however, does not uniquely fix the form of the matrix $\hat{F}$. To see this we note that the pseudoscalar covariant, $PS=\gamma^5\otimes\gamma^5$, has exactly the same matrix elements between positive energy free mass Dirac spinors $(M_1=M_2=M)$ as the pseudovector covariant, $PV=\not{q}\gamma^5/2M\otimes\not{q}\gamma^5/2M$, i.e.,

$$[\overline{U}_1(M)\otimes\overline{U}_2(M)][PV-PS][U_1(M)\otimes U_2(M)]=0.$$

This is called the equivalence theorem [16]. We can therefore replace PS with PV in Eq. (2.5) without altering the amplitudes, F_L. Even though these two representations are equivalent on shell $(p^2=M^2)$, they will give different results when sandwiched between *positive energy Dirac spinors containing an effective nucleon mass*, since then, matrix elements between negative energy states now also enter. This is because the effective mass spinor can always be expanded in a free mass basis:

$$U(\vec{p}_1,M_1,s_i)=\alpha_U U(\vec{p}_1,M,s_i)+\alpha_V V(\vec{p}_1,M,s_i),$$

where V is the negative energy Dirac spinor [14]. There also exists the relation [8],

$$\mathcal{M}_{PV}=\frac{M_1M_2}{M^2}\mathcal{M}_{PS}, \tag{2.6}$$

where $\mathcal{M}_{PS}$ and $\mathcal{M}_{PV}$ are the contribution of the pseudoscalar covariant and pseudovector covariant, respectively, to the invariant matrix element given by Eq. (2.4). Note that in Eq. (2.6), the pseudovector covariant is $PV=\not{q}\gamma^5/2M\otimes\not{q}\gamma^5/2M$, but where $q=p_1^*-k_1^*=k_2^*-p_2^*$, i.e., the momenta are on mass shell with respect to the effective masses, M_1 and M_2. In the equivalence theorem, the momenta must be on mass shell with respect to the free mass. The above equality has been used in Refs. [8–10] to investigate the sensitivity of the spin observables to the difference between using a pseudoscalar covariant or a pseudovector covariant. The ambiguity, which is inherent in any five-term or incomplete representation of $\hat{F}$ (such as the IA1 representation), was first pointed out in Ref. [6]. Tjon and Wallace have developed a general Lorentz invariant representation of $\hat{F}$. The formalism can be found in Refs. [7,11] and is applied to *elastic* proton-nucleus scattering in Refs. [17–19]. We will refer to this as the IA2 representation of $\hat{F}$ and discuss, in the next section, its application to *quasielastic* proton-nucleus scattering.

III. IA2 REPRESENTATION OF $\hat{F}$ APPLIED TO QUASIELASTIC PROTON-NUCLEUS SCATTERING

From Eqs (3.1) and (3.18) in Ref. [19] the IA2 representation of $\hat{F}$ is given by

$$\hat{F}=\sum_{\rho_1\rho_1';\rho_2\rho_2'}\sum_{n=1}^{13}F_n^{\rho_1\rho_1'\rho_2\rho_2'}[\Lambda_{\rho_1'}(\vec{k}_1;M)\otimes\Lambda_{\rho_2'}(\vec{k}_2;M)]K_n[\Lambda_{\rho_1}(\vec{p}_1;M)\otimes\Lambda_{\rho_2}(\vec{p}_2;M)], \tag{3.1}$$

where M refers to the free nucleon mass. Henceforth, the notation

$$\{\rho\}=\rho_1\rho_1';\rho_2\rho_2'$$

will be used. In Eq. (3.1), $F_n^{\{\rho\}}$ $(n=1–13)$ are the invariant amplitudes for each rho-spin sector (which is defined by the rho-spin labels, $\rho_1\rho_1';\rho_2\rho_2'$, where $\rho=\pm$); $\Lambda_\rho(\vec{p},M)$ are covariant projection operators given by

$$\Lambda_\rho(\vec{p},M)=\frac{\rho\not{p}+M}{2M}=\frac{\rho(E\gamma^0-\vec{p}\cdot\vec{\gamma})+M}{2M}, \tag{3.2}$$

where $E^2=\vec{p}^2+M^2$, and K_n $(n=1–13)$ are kinematic covariants constructed from the Dirac matrices:

$$K_1=S=I_4\otimes I_4,$$

$$K_2=P=\gamma^5\otimes\gamma^5,$$

$$K_3=V=\gamma^\mu\otimes\gamma_\mu,$$

$$K_4=A=\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu,$$

$$K_5=T=\sigma^{\mu\nu}\otimes\sigma_{\mu\nu},$$

$$K_6=Q_{11,\mu}(I_4\otimes\gamma^\mu),$$

$$K_7=Q_{22,\mu}(\gamma^\mu\otimes I_4),$$

$$K_8=Q_{11,\mu}(\gamma^5\otimes\gamma^5\gamma^\mu),$$

$$K_9=Q_{22,\mu}(\gamma^5\gamma^\mu\otimes\gamma^5),$$

$$K_{10}=Q_{12,\mu}(I_4\otimes\gamma^\mu)\tilde{S},$$

$$K_{11}=Q_{21,\mu}(\gamma^\mu\otimes I_4)\tilde{S},$$

$$K_{12}=Q_{12,\mu}(\gamma^5\otimes\gamma^5\gamma^\mu)\tilde{S},$$

$$K_{13}=Q_{21,\mu}(\gamma^5\gamma^\mu\otimes\gamma^5)\tilde{S},$$

where

$$Q_{ij,\mu}=\frac{(p_i'+p_j)_\mu}{2M} \quad \text{with} \quad p_1'=k_1 \ \text{and} \ p_2'=k_2.$$

With each combination of rho-spin labels $\{\rho_1\rho_1'\rho_2\rho_2'\}$ is associated a pair (ij) to index a specific rho-spin sector (or subclass); see Table I of [19]. For example, $\{++++\}\equiv(11)$ and $\{+-+-\}\equiv(22)$. Parity and time-reversal invariance, together with charge symmetry and the on-mass-shell condition for external nucleons, lead to $\hat{F}$ being completely specified by 44 independent invariant amplitudes [11]. Five amplitudes in subclass $\hat{F}^{11}$ are completely specified by fitting to physical free NN scattering data and are therefore identical to the SPVAT amplitudes in the IA1 representation of $\hat{F}$. The remaining 39 off-shell amplitudes (contained in subclasses $\hat{F}^{12}$ to $\hat{F}^{44}$) are obtained by solving the Bethe-Salpeter equation in a three-dimensional quasipotential reduction [20,21], with pure pseudovector pion-nucleon coupling, to determine a complete set of helicity amplitudes. The invariant amplitudes are related via matrix equations to the helicity amplitudes [11]. The IA2 representation is a complete and unambiguous expansion of $\hat{F}$, since covariants cannot be added or changed arbitrarily without violating the above-mentioned symmetries. Amplitudes which are solely determined by physical scattering data are isolated in subclass $\hat{F}^{11}$, while the remaining amplitudes are determined by solving a dynamical equation, the Bethe-Salpeter equation using a meson-exchange model for the NN force.

From Eq. (3.1) four cases concerning the combination of projectile and target nucleon masses can be distinguished:

(1) No medium effect ($M_1=M_2=M$): In this case only subclass $\hat{F}^{11}$ will contribute to the invariant scattering amplitude. It is important to note that in this special case the IA2 representation of $\hat{F}$ is equivalent to the SPVAT parameterization of $\hat{F}$. This fact can be used to perform numerical checks on the formalism presented in this paper as is discussed in Sec. VI.

(2) Projectile relativity ($M_1\neq M; M_2=M$): Contributions to the invariant scattering amplitude arise from $\hat{F}^{11},\hat{F}^{21},\hat{F}^{31}$, and $\hat{F}^{41}$ where the latter three subclasses require *at least* projectile relativity for a contribution.

(3) Target relativity ($M_1=M; M_2\neq M$): Contributions to the invariant scattering amplitude arise from $\hat{F}^{11},\hat{F}^{12},\hat{F}^{13}$ and $\hat{F}^{14}$ where the latter three subclasses require *at least* target relativity for a contribution.

(4) Target and projectile relativity ($M_1\neq M; M_2\neq M$): Now all subclasses will contribute to the invariant scattering amplitude but $\hat{F}^{22}$, $\hat{F}^{23}$, $\hat{F}^{24}$, $\hat{F}^{32}$, $\hat{F}^{33}$, $\hat{F}^{34}$, $\hat{F}^{42}$, $\hat{F}^{43}$ and $\hat{F}^{44}$ require *at least* projectile *and* target relativity for a contribution.

From Eq. (3.1) we see that medium effects can never occur in subclass $\hat{F}^{11}$ due to the accompanying positive energy projection operators. Medium effects in the IA2 representation of $\hat{F}$ arise only due to off-shell amplitudes (which are contained in the subclasses $\hat{F}^{12}$ to $\hat{F}^{44}$). This is in contrast to IA1 where medium effects are included only in subclass $\hat{F}^{11}$. One can now substitute Eq. (3.1) into Eq. (2.4) and proceed from there to calculate the spin observables in terms of $|\mathcal{M}|^2$, which is directly related to the invariant amplitudes $F_n^{\{\rho\}}$. We will, however, not follow this direct approach due to the following reasons.

(1) Following the standard procedure (see Ref. [14] for example) one finds that $|\mathcal{M}|^2$ contains traces over at least eight gamma matrices. The number of gamma matrices increase as the covariants become more complicated. Since the number of terms generated by such a trace is given by $[N!/(N/2)!2^{N/2}]$ (where N refers to the number of gamma matrices), and since there is a double sum over the rho-spin sectors, a very large number of terms will occur.

(2) Since we are applying a relativistic formalism to a nuclear physics problem, it might be more instructive to rewrite the NN scattering matrix in a form that is more familiar to traditional nuclear physics. We will therefore follow a similar approach as in Ref. [22] where an *effective t* matrix is derived, which is a 4×4 matrix, but which still contains all the information coming from the relativistic analysis. From Eq. (2.1) we can write

$$U(\vec{p}_1,M_1,s_i)=\left(\frac{E_1^*}{M_1}\right)^{1/2} u^+(\vec{p}_1,M_1)\phi(s_i)$$

where, as a 4×2 matrix,

$$u^+(\vec{p}_1,M_1)=\left(\frac{E_1^*+M_1}{2E_1^*}\right)^{1/2}\begin{pmatrix} I_2 \\ \dfrac{\vec{\sigma}\cdot\vec{p}_1}{E_1^*+M_1}\end{pmatrix}. \tag{3.3}$$

Similarly,

$$\overline{U}(\vec{p}_1,M_1,s_i)=\left(\frac{E_1^*}{M_1}\right)^{1/2}\phi^\dagger(s_i)\overline{u}^+(\vec{p}_1,M_1)$$

where, as a 2×4 matrix,

$$\overline{u}^+(\vec{p}_1,M_1)=u^{+\dagger}(\vec{p}_1,M_1)\gamma^0.$$

$u^\rho(\vec{p})$ (where $\rho=\pm$) contains no reference to the spin and is normalized to

$$u^{\rho'^\dagger}(\rho'\vec{p})u^\rho(\rho\vec{p})=\delta_{\rho'\rho}.$$

In terms of u^+ the invariant matrix element [Eq. (2.4)] is given by

$$\mathcal{M}=\left(\frac{E_1^*E_2^*E_1^{*'}E_2^{*'}}{M_1^2M_2^2}\right)^{1/2}[\phi^\dagger(s_f)\overline{u^+}(\vec{k}_1,M_1)$$
$$\otimes\phi^\dagger(s_2')\overline{u^+}(\vec{k}_2,M_2)]\hat{F}[\phi(s_i)u^+(\vec{p}_1,M_1)$$
$$\otimes\phi(s_2)u^+(\vec{p}_2,M_2)].$$

Use of the identity $AC\otimes BD=(A\otimes B)(C\otimes D)$, where $\otimes$ refers to the usual Kronecker product, leads to the expression

$$\mathcal{M}=\left(\frac{E_1^*E_2^*E_1^{*'}E_2^{*'}}{M_1^2M_2^2}\right)^{1/2}[\phi^\dagger(s_f)\otimes\phi^\dagger(s_2')][\overline{u^+}(\vec{k}_1,M_1)$$
$$\otimes\overline{u^+}(\vec{k}_2,M_2)]\hat{F}[u^+(\vec{p}_1,M_1)\otimes u^+(\vec{p}_2,M_2)]$$
$$[\phi(s_i)\otimes\phi(s_2)]. \quad (3.4)$$

Defining the *effective t* matrix as

$$\hat{t}=[\overline{u^+}(\vec{k}_1,M_1)\otimes\overline{u^+}(\vec{k}_2,M_2)]\hat{F}[u^+(\vec{p}_1,M_1)$$
$$\otimes u^+(\vec{p}_2,M_2)] \quad (3.5)$$

and $g_1=[[E_1^*E_2^*E_1^{*'}E_2^{*'}]/M_1^2M_2^2]^{1/2}$, Eq. (3.4) becomes

$$\mathcal{M}=g_1[\phi^\dagger(s_f)\otimes\phi^\dagger(s_2')]\hat{t}[\phi(s_i)\otimes\phi(s_2)]. \quad (3.6)$$

Since $\hat{t}$ is a 4×4 matrix it can be expanded in terms of a basis constructed from the Pauli matrices and the momenta of the scattering process. Define the three-momentum transfer $\vec{q}=\vec{p}_1-\vec{k}_1=\vec{k}_2-\vec{p}_2$, the average momentum $\vec{p}_a=\frac{1}{2}(\vec{p}_1+\vec{k}_1)$, and a vector orthogonal to both $\vec{q}$ and $\vec{p}_a$, $\vec{N}=\vec{q}\times\vec{p}_a=\vec{p}_1\times\vec{k}_1$. Note that

$$\vec{q}\cdot\vec{p}_a=\frac{1}{2}(\vec{p}_1^2-\vec{k}_1^2).$$

For quasielastic scattering $|\vec{p}_1|\neq|\vec{k}_1|$ and, therefore, $\vec{q}$ and $\vec{p}_a$ are not orthogonal, however, $\vec{N}\cdot\vec{q}=\vec{N}\cdot\vec{p}_a=0$. Assuming only *parity invariance*, $\hat{t}$ can be written in terms of a set of eight *linearly independent matrices* in the spin of the two interacting nucleons:

$$\hat{t}=\sum_{n=1}^{8}b_n(\chi_n^{(1)}\otimes\chi_n^{(2)}), \quad (3.7)$$

where

$$\chi_1^{(1)}=I_2,\quad \chi_1^{(2)}=I_2,$$
$$\chi_2^{(1)}=\vec{N}\cdot\vec{\sigma},\quad \chi_2^{(2)}=\vec{N}\cdot\vec{\sigma},$$
$$\chi_3^{(1)}=\vec{N}\cdot\vec{\sigma},\quad \chi_3^{(2)}=I_2,$$
$$\chi_4^{(1)}=I_2,\quad \chi_4^{(2)}=\vec{N}\cdot\vec{\sigma},$$
$$\chi_5^{(1)}=\vec{q}\cdot\vec{\sigma},\quad \chi_5^{(2)}=\vec{q}\cdot\vec{\sigma},$$
$$\chi_6^{(1)}=\vec{p}_a\cdot\vec{\sigma},\quad \chi_6^{(2)}=\vec{p}_a\cdot\vec{\sigma},$$
$$\chi_7^{(1)}=\vec{q}\cdot\vec{\sigma},\quad \chi_7^{(2)}=\vec{p}_a\cdot\vec{\sigma},$$
$$\chi_8^{(1)}=\vec{p}_a\cdot\vec{\sigma},\quad \chi_8^{(2)}=\vec{q}\cdot\vec{\sigma}.$$

In the next section the invariant amplitudes F_n^ρ $(n=1\text{–}13)$ are transformed to a set of eight effective amplitudes b_n $(n=1\text{–}8)$, and expressions for the spin observables are derived in terms of the effective amplitudes.

IV. TRANSFORMATION FROM THE INVARIANT AMPLITUDES TO THE EFFECTIVE AMPLITUDES

Expressions for the effective amplitudes of $\hat{t}$ are now derived. Taking the trace of Eq. (3.7) yields

$$b_1=\frac{1}{4}\mathrm{Tr}[\hat{t}]. \quad (4.1)$$

Multiply Eq. (3.7) with $(\vec{N}\cdot\vec{\sigma}\otimes\vec{N}\cdot\vec{\sigma})$ and take the trace of the resulting equation. Since $\vec{N}\cdot\vec{q}=\vec{N}\cdot\vec{p}_a=0$, there will be no contribution from the last four terms of Eq. (3.7) and, therefore,

$$b_2=\frac{1}{4\vec{N}^4}Y_2(\vec{N},\vec{N}), \quad (4.2)$$

where

$$Y_2(\vec{a},\vec{b})=\mathrm{Tr}[(\vec{a}\cdot\vec{\sigma}\otimes\vec{b}\cdot\vec{\sigma})\hat{t}].$$

Similar arguments lead to

$$b_3=\frac{1}{4\vec{N}^2}\mathrm{Tr}[(\vec{N}\cdot\vec{\sigma}\otimes I_2)\hat{t}] \quad (4.3)$$

and

$$b_4=\frac{1}{4\vec{N}^2}\mathrm{Tr}[(I_2\otimes\vec{N}\cdot\vec{\sigma})\hat{t}]. \quad (4.4)$$

Following the same reasoning as above, one can also derive a set of four coupled equations relating the amplitudes b_5, b_6, b_7, and b_8. A set of coupled equations arise since the vectors $\vec{q}$ and $\vec{p}_a$ *are not orthogonal for quasielastic scattering*, (i.e., $\vec{q}\cdot\vec{p}_a\neq0$). The solutions are

$$b_5=\frac{r_2{}^2Y_2(\vec{p}_a,\vec{p}_a)-r_2r_3Y_2(\vec{p}_a,\vec{q})-r_2r_3Y_2(\vec{q},\vec{p}_a)+r_3{}^2Y_2(\vec{q},\vec{q})}{M^2(r_1r_3-r_2{}^2)^2}, \tag{4.5}$$

$$b_6=\frac{r_1{}^2Y_2(\vec{p}_a,\vec{p}_a)-r_1r_2Y_2(\vec{p}_a,\vec{q})-r_1r_2Y_2(\vec{q},\vec{p}_a)+r_2{}^2Y_2(\vec{q},\vec{q})}{M^2(r_1r_3-r2_2{}^2)^2}, \tag{4.6}$$

$$b_7=\frac{-r_1r_2Y_2(\vec{p}_a,\vec{p}_a)+r_2{}^2Y_2(\vec{p}_a,\vec{q})+r_1r_3Y_2(\vec{q},\vec{p}_a)-r_2r_3Y_2(\vec{q},\vec{q})}{M^2(r_1r_3-r_2{}^2)^2}, \tag{4.7}$$

$$b_8=\frac{-r_1r_2Y_2(\vec{p}_a,\vec{p}_a)+r_1r_3Y_2(\vec{p}_a,\vec{q})+r_2{}^2Y_2(\vec{q},\vec{p}_a)-r_2r_3Y_2(\vec{q},\vec{q})}{M^2(r_1r_3-r_2{}^2)^2}, \tag{4.8}$$

where

$$r_1=\frac{2\vec{q}^2}{M},$$

$$r_2=\frac{2\vec{p}_a\cdot\vec{q}}{M}, \quad \text{and}$$

$$r_3=\frac{2\vec{p}_a^2}{M}.$$

The next step is to derive an expression for the $\hat{t}$ matrix which is convenient for use in the calculation of the traces which determine the effective amplitudes. Substitution of Eq. (3.1) into Eq. (3.5) leads to

$$\hat{t}=\sum_{\{\rho\}}\sum_{n=1}^{13}F_n^{\{\rho\}}[\overline{\Gamma}_{\rho_1'}(\vec{k}_1,M,M_1)$$
$$\otimes\overline{\Gamma}_{\rho_2'}(\vec{k}_2,M,M_2)]K_n[\Gamma_{\rho_1}(\vec{p}_1,M,M_1)$$
$$\otimes\Gamma_{\rho_2}(\vec{p}_2,M,M_2)], \tag{4.9}$$

where we have introduced the 4×2 Γ matrices defined as

$$\Gamma_\rho(\vec{p},M,M^*)=\Lambda_\rho(\vec{p},M)u^+(\vec{p},M^*). \tag{4.10}$$

In Eq. (4.10) M^* denotes an effective mass and Eq. (3.3) has been generalized to

$$u^+(\vec{p},M^*)=\begin{pmatrix} I_2\phi(\vec{p}_1,M^*) \\ \vec{\sigma}\cdot\vec{p}\chi(\vec{p},M^*)\end{pmatrix}. \tag{4.11}$$

Equation (4.11) reduces to Eq. (3.3) if we set

$$\phi(\vec{p}_1,M_1)=\left(\frac{E^*(\vec{p}_1)+M_1}{2E^*(\vec{p}_1)}\right)^{1/2},$$

where $E^*(\vec{p}_1)=\sqrt{\vec{p}_1^2+M_1^2}$ and

$$\chi(\vec{p}_1,M_1)=\phi(\vec{p}_1,M_1)[E^*(\vec{p}_1)+M_1)]^{-1}.$$

We can obtain an explicit expression for Γ_ρ as follows. From Eq. (4.11) we can write

$$u^+(\vec{p},M^*)=\begin{pmatrix} I_2\phi(\vec{p},M^*) \\ 0\end{pmatrix}+\begin{pmatrix} 0 \\ \vec{\sigma}\cdot\vec{p}\chi(\vec{p},M^*)\end{pmatrix}$$

and, therefore

$$u^+(\vec{p},M^*)=\phi(\vec{p},M^*)(\hat{e}_1\otimes I_2)+\chi(\vec{p},M^*)(\hat{e}_2\otimes\vec{\sigma}\cdot\vec{p}), \tag{4.12}$$

where

$$\hat{e}_1=\begin{pmatrix}1\\0\end{pmatrix} \quad\text{and}\quad \hat{e}_2=\begin{pmatrix}0\\1\end{pmatrix},$$

with $\hat{e}_i^\dagger\hat{e}_j=\delta_{ij}$. To write the ρ-spin projection operator in $(2\times2)\otimes(2\times2)$ form, we recall that

$$\gamma^0=\begin{pmatrix} I_2 & 0 \\ 0 & -I_2\end{pmatrix}=\sigma_3\otimes I_2 \quad\text{and} \tag{4.13}$$

$$\vec{p}\cdot\vec{\gamma}=\begin{pmatrix} 0 & \vec{p}\cdot\vec{\sigma} \\ -\vec{p}\cdot\vec{\sigma} & 0\end{pmatrix}=i\sigma_2\otimes\vec{p}\cdot\vec{\sigma}. \tag{4.14}$$

Substitution of Eqs. (4.13) and (4.14) into Eq. (3.2) leads to

$$\Lambda_{\rho}(\vec{p},M)=\frac{\rho E_{p}}{2M}(\sigma_{3}\otimes I_{2})-\frac{i\rho}{2M}(\sigma_{2}\otimes\vec{p}\cdot\vec{\sigma})+\frac{1}{2}(I_{2}\otimes I_{2}). \tag{4.15}$$

Substitution of Eqs. (4.12) and (4.15) into Eq. (4.10), and using the properties of the Pauli matrices, allows one to write

$$\Gamma_{\rho}(\vec{p},M,M^{*})=\sum_{i=1}^{2} h_{\rho}^{(i)}(\vec{p},M,M^{*})[\hat{e}_{i}\otimes A_{i}(\vec{p})], \tag{4.16}$$

where

$$A_{i}(\vec{p})=\begin{cases} I_{2}; & i=1\\ \vec{p}\cdot\vec{\sigma}; & i=2,\end{cases}$$

with

$$h_{\rho}^{(1)}(\vec{p},M,M^{*})=\frac{\rho E_{p}}{2M}\phi(\vec{p},M^{*})-\frac{\rho\vec{p}^{2}}{2M}\chi(\vec{p},M^{*})+\frac{1}{2}\phi(\vec{p},M^{*}) \quad \text{and}$$

$$h_{\rho}^{(2)}(\vec{p},M,M^{*})=\frac{\rho}{2M}\phi(\vec{p},M^{*})-\frac{\rho E_{p}}{2M}\chi(\vec{p},M^{*})+\frac{1}{2}\chi(\vec{p},M^{*}).$$

Similar steps lead to

$$\overline{\Gamma}_{\rho}(\vec{p},M,M^{*})=\sum_{i=1}^{2} j_{\rho}^{(i)}(\vec{p},M,M^{*})[\hat{e}_{i}^{\dagger}\otimes A_{i}(\vec{p})], \tag{4.17}$$

where

$$j_{\rho}^{(1)}(\vec{p},M,M^{*})=h_{\rho}^{(1)}(\vec{p},M,M^{*}) \quad \text{and}$$

$$j_{\rho}^{(2)}(\vec{p},M,M^{*})=-h_{\rho}^{(2)}(\vec{p},M,M^{*}).$$

To calculate the effective amplitudes, the contribution of each covariant to the trace relations must be determined. $\hat{t}$ is calculated from Eq. (4.9) using the explicit forms of $\Gamma_{\rho_i}(\vec{p},M,M^{*})$ and $\overline{\Gamma}_{\rho_i}(\vec{p},M,M^{*})$ in Eqs. (4.16) and (4.17). Use is then made of Eqs. (4.1)–(4.8) to determine the effective amplitudes, b_1 to b_8. This procedure requires the calculation of traces of a set of matrices t_i, where $i=1$–46. With each covariant is associated a set of t matrices of which the trace must be taken. We wrote a program in the MATHEMATICA computer language to do the required trace algebra. The eight effective amplitudes are linear functions of $F_n^{\{\rho\}}$, i.e.,

$$b_{i}=b_{i}(\{F_{n}^{\{\rho\}}\}). \tag{4.18}$$

The isospin zero (isospin one) effective amplitudes are obtained by substituting the isospin zero (isospin one) invariant amplitudes into Eq. (4.18).

V. EXPRESSIONS FOR SPIN OBSERVABLES IN TERMS OF THE EFFECTIVE AMPLITUDES

In this section expressions are derived for the unpolarized double differential cross section, the analyzing power, and the polarization transfer observables in terms of the effective amplitudes b_n for both $(\vec{p},\vec{p}\,')$ and $(\vec{p},\vec{n})$ scattering. Working in the nucleon-nucleon laboratory frame, the spin in the incident beam direction is described in terms of three orthogonal unit vectors $(\hat{l},\hat{s},\hat{n})$, where $\hat{l}$ is along the beam direction, $\hat{s}$ lies perpendicular and to the side of $\hat{l}$ in the scattering plane, and the normal unit vector is $\hat{n}=\hat{l}\times\hat{s}$. Similarly, the spin of the final beam is described in terms of $(\hat{l}\,',\hat{s}\,',\hat{n})$.

A. Unpolarized double differential cross section

For the scattering process in Fig. 1 one can write down the following expression for the differential cross section, $d\sigma$ [14]:

$$d\sigma=\frac{1}{|\vec{v}_{1}-\vec{v}_{2}|}\left(\frac{M_{1}}{E_{1}^{*}}\frac{M_{2}}{E_{2}^{*}}\right)\left(\frac{M_{1}}{E_{1}^{*\prime}}\frac{M_{2}}{E_{2}^{*\prime}}\right)\times(2\pi)^{4}\delta(p_{1}^{*}+p_{2}^{*}-k_{1}^{*}-k_{2}^{*})\frac{d^{3}\vec{k}_{1}}{(2\pi)^{3}}\frac{d^{3}\vec{k}_{2}}{(2\pi)^{3}}|\mathcal{M}|^{2}.$$

As we consider the quasielastic scattering at energies much higher than the interaction energies amongst the target nucleons, we assume the latter to be practically noninteracting. Therefore, the momentum distribution of these nucleons can be obtained in a Fermi-gas model. Following the same arguments as in Ref. [1] allows one to write down the following expression for the *double differential cross section:*

$$\frac{d\sigma}{d\Omega_{1}'dE_{1}'}=\frac{|\vec{k}_{1}|E_{1}'}{|\vec{q}|E_{1}^{*\prime}}\int_{p_{min}}^{k_{f}} d|\vec{p}_{2}|d\phi|\vec{p}_{2}|f(p_{1}^{*},p_{2}^{*},k_{f})|\mathcal{M}|^{2}, \tag{5.1}$$

where

$$f(p_1^*,p_2^*,k_f)=\frac{3}{16\pi^3k_f^3}\frac{M_1^2M_2^2}{[(p_1^*\cdot p_2^*-M_1^2M_2^2)^2]^{1/2}} \quad (5.2)$$

and

$$p_{min}=\left|\frac{q}{2}-\frac{\omega^*}{2}\left[1-\frac{4M_2^2}{q_\mu q^\mu}\right]^{1/2}\right|. \quad (5.3)$$

In Eq. (5.3), q^μ is the four-momentum transfer $q^\mu=(\omega^*,\vec{q})$ where

$$\omega^*=E_1^*-E_1^{*'}=E_2^{*'}-E_2 \quad \text{and} \quad \vec{q}=\vec{p}_1-\vec{k}_1=\vec{k}_2-\vec{p}_2.$$

Equation (5.1) is defined to be zero when $|\vec{k}_1|\leqslant k_F$ or $|\vec{k}_2|\leqslant k_F$. This effect is called Pauli blocking. To obtain the Fermi momentum k_f, the required effective density is calculated in an eikonal approximation as shown in Ref. [1]. More refined values of k_f for specific target nuclei can be found in Table II of Ref. [8]. We define the function

$$\Gamma''(\vec{p}_1,\vec{p}_2,\vec{k}_1,\vec{k}_2)=\sum_{s_i,s_f}\sum_{s_2,s_2'}|\mathcal{M}|^2.$$

Substitution of Eq. (3.7) into Eq. (3.6) leads to

$$\mathcal{M}=g_1\sum_{n=1}^{8}b_n[\phi^\dagger(s_f)\chi_n^{(1)}\phi(s_i)][\phi^\dagger(s_2')\chi_n^{(2)}\phi(s_2)]$$

and, therefore, one can write

$$\Gamma''(\vec{p}_1,\vec{p}_2,\vec{k}_1,\vec{k}_2)=g_1^2\sum_{m,n=1}^{8}b_m^*b_n\mathrm{Tr}[\chi_n^{(1)}\chi_m^{(1)}]\mathrm{Tr}[\chi_n^{(2)}\chi_m^{(2)}].$$

An explicit expression for $\Gamma''(\vec{p}_1,\vec{p}_2,\vec{k}_1,\vec{k}_2)$ is given in the Appendix. To obtain the unpolarized double differential cross section, one sums over the initial spin and average over the final spin which leads to

$$\left(\frac{d\sigma}{d\Omega_1'dE_1'}\right)_{unpol}=\frac{|\vec{k}_1|E_1'}{4|\vec{q}|E_1^{*'}}\int_{p_{min}}^{k_f}\times d|\vec{p}_2|d\phi|\vec{p}_2|f(p_1^*,p_2^*,k_f)\times\Gamma''(\vec{p}_1,\cdots\vec{k}_2). \quad (5.4)$$

For $(\vec{p},\vec{p}\,')$ scattering,

$$\left(\frac{d\sigma}{d\Omega_1'dE_1'}\right)_{unpol}^{(p,p')}=\frac{|\vec{k}_1|E_1'}{4|\vec{q}|E_1^{*'}}\int_{p_{min}}^{k_f}d|\vec{p}_2|d\phi|\vec{p}_2|f(p_1^*,p_2^*,k_f)\times Z_{eff}\Gamma''(\vec{p}_1,\cdots\vec{k}_2,\{b_i(I=1)\})+N_{eff}\Gamma''(\vec{p}_1,\cdots\vec{k}_2,\{b_i^{ave}\}),$$

where

$$b_i^{ave}=\frac{1}{2}[b_i(I=0)+b_i(I=1)].$$

For the charge-exchange reaction $(\vec{p},\vec{n})$,

$$\left(\frac{d\sigma}{d\Omega_1'dE_1'}\right)_{unpol}^{(p,n)}=\frac{|\vec{k}_1|E_1'}{4|\vec{q}|E_1^{*'}}\int_{p_{min}}^{k_f}d|\vec{p}_2|d\phi|\vec{p}_2|f(p_1^*,p_2^*,k_f)\times N_{eff}\Gamma''(\vec{p}_1,\ldots\vec{k}_2,\{b_i^{ch-ex}\}),$$

where the charge-exchange amplitudes are defined as

$$b_i^{ch-ex}=\frac{1}{2}[b_i(I=1)-b_i(I=0)].$$

The quantities Z_{eff} and N_{eff} are defined in Ref. [1] and values for specific targets are given in Table II of Ref. [8].

B. Analyzing power

The definition of the analyzing power is given in terms of polarized double differential cross sections as

$$A_y=\frac{\frac{d\sigma}{d\Omega_1'dE_1'}(\hat{s}_f=+\hat{n})-\frac{d\sigma}{d\Omega_1'dE_1'}(\hat{s}_f=-\hat{n})}{\frac{d\sigma}{d\Omega_1'dE_1'}(\hat{s}_f=+\hat{n})+\frac{d\sigma}{d\Omega_1'dE_1'}(\hat{s}_f=-\hat{n})} \quad (5.5)$$

where, for example,

$$\frac{d\sigma}{d\Omega_1'dE_1'}(\hat{s}_f)=\frac{|\vec{k}_1|E_1'}{|\vec{q}|E_1^{*'}}\int_{p_{min}}^{k_f}d|\vec{p}_2|d\phi|\vec{p}_2|f(p_1^*,p_2^*,k_f)\times\frac{1}{2}\tilde{\Gamma}'(\hat{s}_f)$$

is averaged over incident spin directions $\hat{s}_i$, and the target particles' initial and final spin as contained in the factor

$$\tilde{\Gamma}'(\vec{p}_1,\ldots,\vec{k}_2,\hat{s}_f)=\sum_{s_i,s_2,s_2'}|\mathcal{M}|^2$$
$$=\mathrm{Tr}[\chi_m^{(1)}\hat{P}(\hat{s}_f)\chi_n^{(1)}]\mathrm{Tr}[\chi_m^{(2)}\chi_n^{(2)}], \tag{5.6}$$

where use was made of Eqs. (2.2) and (2.3). A calculation of the traces in Eq. (5.6) shows that $\tilde{\Gamma}'(\vec{p}_1,\ldots,\hat{s}_f)$ has the following structure:

$$\tilde{\Gamma}'(\vec{p}_1,\ldots,\hat{s}_f)=f_1(\vec{p}_1,\ldots,\vec{k}_2)+f_2(\vec{p}_1,\ldots,\vec{k}_2)\vec{N}\cdot\hat{s}_f$$
$$+f_3(\vec{p}_1,\ldots,\vec{k}_2)\vec{p}_a\cdot(\vec{q}\times\hat{s}_f). \tag{5.7}$$

Defining the combination function

$$\Gamma'(\vec{p}_1,\ldots,\vec{k}_2,\hat{s}_f)=\tilde{\Gamma}'(\vec{p}_1,\ldots,\vec{k}_2,\hat{s}_f)$$
$$-\tilde{\Gamma}'(\vec{p}_1,\ldots,\vec{k}_2,-\hat{s}_f), \tag{5.8}$$

and using Eq. (5.7) in Eq. (5.8), yields

$$\Gamma'(\hat{s}_f)=2[f_2(\vec{p}_1,\ldots,\vec{k}_2)\vec{N}\cdot\hat{s}_f$$
$$+f_3(\vec{p}_1,\ldots,\vec{k}_2)\vec{p}_a\cdot(\vec{q}\times\hat{s}_f)].$$

The explicit forms of the functions f_2 and f_3 can be inferred from Eq. (A2) in the Appendix. If $\hat{s}_f=\hat{n}$, then

$$\vec{N}\cdot\hat{n}=p_1k_1\sin\theta_L$$

and

$$\vec{p}_a\cdot(\vec{q}\times\hat{n})=-p_1k_1\sin\theta_L.$$

The analyzing power (which is equal to the polarization in the RPWIA model) for the $(\vec{p},\vec{p}\,')$ reaction is given by

$$A_y(\vec{p},\vec{p}\,')=\frac{\int_{p_{min}}^{k_f}d|\vec{p}_2|d\phi|\vec{p}_2|f(p_1^*,\ldots,p_2^*,k_f)(Z_{eff}\Gamma'(\hat{n},\{b_i(I=1)\})+N_{eff}\Gamma'(\hat{n},\{b_i^{ave}\}))}{\int_{p_{min}}^{k_f}d|\vec{p}_2|d\phi|\vec{p}_2|f(k_f)(Z_{eff}\Gamma''(\{b_i(I=1)\})+N_{eff}\Gamma''(\{b_i^{ave}\}))}$$

and the analyzing power for the $(\vec{p},\vec{n})$ reaction is given by

$$A_y(\vec{p},\vec{n})=\frac{\int_{p_{min}}^{k_f}d|\vec{p}_2|d\phi|\vec{p}_2|f(k_f)\Gamma'(\hat{n},\{b_i^{ch-ex}\})}{\int_{p_{min}}^{k_f}d|\vec{p}_2|d\phi|\vec{p}_2|f(k_f)\Gamma''(\{b_i^{ch-ex}\})}. \tag{5.9}$$

Since a $(\vec{p},\vec{n})$ reaction implies that the incident proton could only have scattered off a neutron, we set $Z_{eff}=0$ and, therefore, N_{eff} appears as a common factor in the numerator and denominator and cancels out, which means that N_{eff} does not appear in Eq. (5.9).

C. Polarization transfer observables

The polarization transfer observables are defined in terms of linear combinations of polarized double differential cross sections as follows:

$$D_{i'j}=\frac{\frac{d\sigma}{d\Omega_1'dE_1'}(\hat{s}_i,\hat{s}_f)-\frac{d\sigma}{d\Omega_1'dE_1'}(-\hat{s}_i,\hat{s}_f)-\frac{d\sigma}{d\Omega_1'dE_1'}(\hat{s}_i,-\hat{s}_f)+\frac{d\sigma}{d\Omega_1'dE_1'}(-\hat{s}_i,-\hat{s}_f)}{\frac{d\sigma}{d\Omega_1'dE_1'}(\hat{s}_i,\hat{s}_f)+\frac{d\sigma}{d\Omega_1'dE_1'}(-\hat{s}_i,\hat{s}_f)+\frac{d\sigma}{d\Omega_1'dE_1'}(\hat{s}_i,-\hat{s}_f)+\frac{d\sigma}{d\Omega_1'dE_1'}(-\hat{s}_i,-\hat{s}_f)}. \tag{5.10}$$

In Eq. (5.10) a typical polarized differential cross section is

$$\frac{d\sigma}{d\Omega_1' dE_1'}(\hat{s}_i,\hat{s}_f)=\frac{|\vec{k}_1|E_1'}{|\vec{q}|E_1^{*\prime}}\int_{p_{min}}^{k_f} d|\vec{p}_2|d\phi|\vec{p}_2|f(p_1^*,p_2^*,k_f)\times\frac{1}{2}\tilde{\Gamma}(\hat{s}_i,\hat{s}_f), \tag{5.11}$$

where

$$\begin{aligned}\tilde{\Gamma}(\vec{p}_1,\ldots,\vec{k}_2,\hat{s}_i,\hat{s}_f) &= \sum_{s_2,s_2'}|\mathcal{M}|^2\\ &= g_1^2\sum_{m,n=1}^{8} b_m^* b_n[\mathrm{Tr}(\hat{P}(\hat{s}_i)\chi_m^{(1)}\hat{P}(\hat{s}_f)\chi_n^{(1)})]\\ &\quad\times[\mathrm{Tr}(\chi_m^{(2)}\chi_n^{(2)})]=f_1(\vec{p}_1,\ldots,\vec{k}_2)\\ &\quad+\vec{A}_1\cdot\hat{s}_i+\vec{A}_2\cdot\hat{s}_f+(\hat{s}_i\cdot\vec{A}_3)(\hat{s}_f\cdot\vec{A}_4)\\ &\quad+(\hat{s}_i\cdot\hat{s}_f)(\vec{A}_6\cdot\vec{A}_7)+\hat{s}_i\cdot(\hat{s}_f\times\vec{A}_5)\end{aligned}$$

with $\vec{A}_i$ functions of only the three-momenta, $\vec{p}_1$ to $\vec{k}_2$ of which the explicit form can be inferred from Eq. (A1). Define again, now dictated by the form of Eq. (5.10), a function:

$$\begin{aligned}\Gamma(\vec{p}_1,\ldots,\vec{k}_2,\hat{s}_i,\hat{s}_f)&=\tilde{\Gamma}(\hat{s}_i,\hat{s}_f)-\tilde{\Gamma}(-\hat{s}_i,\hat{s}_f)-\tilde{\Gamma}(\hat{s}_i,-\hat{s}_f)\\&\quad+\tilde{\Gamma}(-\hat{s}_i,-\hat{s}_f)\\&=4[(\hat{s}_i\cdot\vec{A}_3)(\hat{s}_f\cdot\vec{A}_4)+\hat{s}_i\cdot(\hat{s}_f\times\vec{A}_5)\\&\quad+(\hat{s}_i\cdot\hat{s}_f)(\vec{A}_6\cdot\vec{A}_7)].\end{aligned} \tag{5.12}$$

The explicit expression for Γ contains various kinematical parameters which are presented in the first column of Table I. The other columns contain the values of these quantities in the laboratory frame, for each polarization transfer observable; θ refers to the laboratory scattering angle, $p_1=|\vec{p}_1|$ and $k_1=|\vec{k}_1|$. Use of Eqs. (5.11), (5.12), and (5.4) leads to

$$D_{i'j}=\frac{\int_{p_{min}}^{k_f} d|\vec{p}_2|d\phi|\vec{p}_2|f(p_1^*,p_2^*,k_f)4\Gamma(\hat{s}_i,\hat{s}_f)}{\int_{p_{min}}^{k_f} d|\vec{p}_2|d\phi|\vec{p}_2|f(p_1^*,p_2^*,k_f)\Gamma''(\vec{p}_1,\ldots,\vec{k}_2)}. \tag{5.13}$$

The polarization transfer observables for the $(\vec{p},\vec{p}\,')$ reaction are given by

$$D_{i'j}[(\vec{p},\vec{p}\,')]=\frac{\int_{p_{min}}^{k_f} d|\vec{p}_2|d\phi|\vec{p}_2|f(k_f)(4Z_{eff}\Gamma(\hat{s}_i,\hat{s}_f,\{b_i(I=1)\})+4N_{eff}\Gamma(\hat{s}_i,\hat{s}_f,\{b_i^{ave}\}))}{\int_{p_{min}}^{k_f} d|\vec{p}_2|d\phi|\vec{p}_2|f(k_f)(Z_{eff}\Gamma''(\{b_i(I=1)\})+N_{eff}\Gamma''(\{b_i^{ave}\}))},$$

and the corresponding observables for the $(\vec{p},\vec{n})$ reaction are given by

$$D_{i'j}[(\vec{p},\vec{n})]=\frac{\int_{p_{min}}^{k_f} d|\vec{p}_2|d\phi|\vec{p}_2|f(k_f)4\Gamma(\hat{s}_i,\hat{s}_f,\{b_i^{ch-ex}\})}{\int_{p_{min}}^{k_f} d|\vec{p}_2|d\phi|\vec{p}_2|f(k_f)\Gamma''(\{b_i^{ch-ex}\})}. \tag{5.14}$$

Once again, as in Eq. (5.9), the effective number of neutrons does not appear in Eq. (5.14).

Although the primary aim of this paper is to present the theoretical formalism for calculating quasielastic proton-nucleus polarization transfer observables, in the next section we give a brief glimpse of the predictive power of the formalism by applying it to quasielastic ^{40}Ca$(\vec{p},\vec{p}\,')$ scattering at 500 MeV. A systematic study of the predictive power of the model, as well as a comparison to IA1-based predictions, will be presented in a future paper.

VI. RESULTS

Before presenting the results we mention the numerical checks that were performed to verify that the transformation from invariant amplitudes $F_n^{\{\rho\}}$ to effective amplitudes b_n was carried out correctly and that the expressions for the spin observables in terms of the effective amplitudes are indeed correct. For $M_1=M_2=M$ only subclass $\hat{F}^{11}$ contributes to

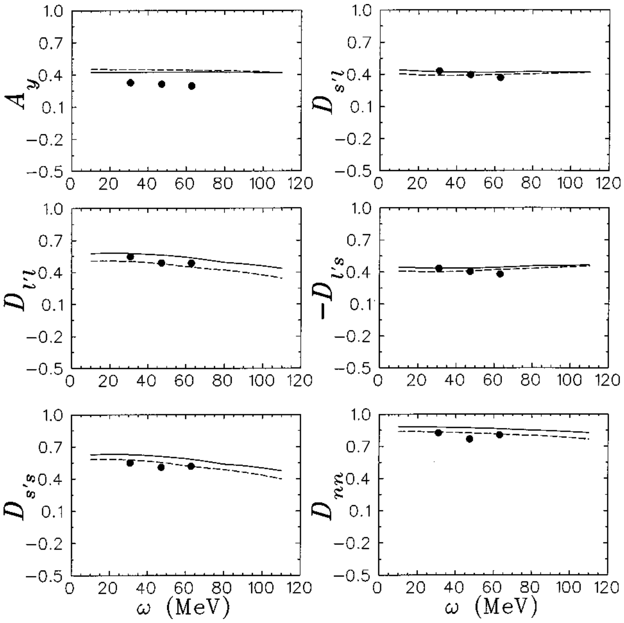

FIG. 2. Spin observables for a range of transferred energy ω over the quasielastic peak for inclusive proton scattering from ^{40}Ca at 500 MeV and $\theta_{lab}=19°$. The centroid of the quasielastic peak is at $\omega\approx 63$ MeV. Data are from Ref. [23]. The solid line represents the IA2 calculation and the dashed line represents the free mass calculation.

the invariant matrix element and the IA2 representation is therefore equivalent to the SPVAT form of $\hat{F}$. We therefore verified that our expressions for the spin observables in terms of the effective amplitudes give exactly the same numerical result as the corresponding expressions in Ref. [1], which contain only the five SPVAT amplitudes. This confirms that the transformation to effective amplitudes has been carried out correctly for only the SPVAT covariants. To verify the transformation for covariants K_6 to K_{13}, we derived expressions for the spin observables directly for each individual covariant K_6 to K_{13}. This involves traces over Dirac matrices (as opposed to the trace algebra involving Pauli matrices presented in this paper) and provides a nontrivial check for the transformation involving covariants K_6 to K_{13}. The fact that two independent ways give numerically the same result for all spin observables confirms the correctness of the transformation to effective amplitudes and the expressions for the spin observables derived in this paper.

The formalism in the previous sections is now applied to quasielastic $^{40}\mathrm{Ca}(\vec{p},\vec{p}\,')$ scattering at an incident laboratory kinetic energy of 500 MeV and a laboratory scattering angle of 19°. In the original calculation of Horowitz and Murdock in Ref. [1], it was found that the use of an effective mass for both the projectile and target nucleons moved the theoretical calculation closer to the data [23] and below the free mass calculation for A_y. This was referred to as the quenching effect in the analyzing power and claimed to be a ''relativistic signature.'' In Ref. [1] the SPVAT parametrization of $\hat{F}$ was used. Figure 2 shows the results employing the IA2 representation of $\hat{F}$. The solid line represents the calculation using an effective mass for the projectile and target nucleons with $M_1/M=0.892$ and $M_2/M=0.817$ taken from Table II in Ref. [8] for ^{40}Ca at $T_{lab}=500$ MeV. The dashed line is the free mass calculation. The data are from Ref. [23]. We note that the quenching effect in A_y is very small compared to Fig. 6 of Ref. [1] over the entire energy range. The result is that the IA2 calculation does not describe A_y, as well as the IA1 calculation of Ref. [1]. For the other observables, the effective mass and the free mass calculation do equally well. This is in contrast to the result in Ref. [1] where the $D_{i'j}$'s only preferred a free mass calculation.

VII. SUMMARY

We have presented a theoretical formalism to calculate polarization transfer observables for quasielastic proton-nucleus scattering using a general Lorentz invariant representation of the nucleon-nucleon scattering matrix. In this way we avoid the ambiguities that are inherent in the previously used five-term representation (the SPVAT form) of $\hat{F}$. In the process we have derived an effective t matrix, which is a 4×4 matrix and, therefore, more familiar to nuclear physics, but which still contains all the information coming from the relativistic analysis. This necessitates the transformation from the 44 invariant amplitudes, to a set of eight effective amplitudes as well as the derivation of new expressions for the spin observables in terms of the effective amplitudes. Staying within the framework of the relativistic plane wave impulse approximation (with its many simplifying features)[2] and using a general Lorentz invariant representation of $\hat{F}$ allows us to do an investigation of M^*-type medium effects via quasielastic proton-nucleus scattering. The first application of the formalism to the reaction $^{40}\mathrm{Ca}(\vec{p},\vec{p}\,')$ at $T_{lab}=500$ MeV and $\theta_{lab}=19°$ shows that the IA2 representation of $\hat{F}$ does not lead to such strong medium effects in any of the spin observables, in contrast to the results in Ref. [1], where the medium effect was most noticeable in A_y. There it was also found that the use of an effective mass for the projectile and target nucleons lead to the theoretical calculation being closer to the data than the free mass calculation. The IA2 representation is consistent with data, however, in that it predicts little medium effect in any of the spin observables, even though the prediction of A_y is now a little poorer than before. In a subsequent paper, a systematic study of spin observables, using the IA2 representation of $\hat{F}$, will be presented for both quasielastic $(\vec{p},\vec{p}\,')$ and $(\vec{p},\vec{n})$ data.

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[2]All of which are motivated by experimental data on the spin observables.

TABLE I. Expressions for kinematical quantities containing $\hat{s}_i$ and/or $\hat{s}_f$ for each nonzero polarization transfer observable.

Kinematical quantity	$D_{l'l}$	$D_{s's}$	D_{nn}	$D_{s'l}$	$D_{l's}$
$\vec{q}\cdot\hat{s}_i$	$p_1-k_1\cos\theta$	$-k_1\sin\theta$	0	$p_1-k_1\cos\theta$	$-k_1\sin\theta$
$\vec{q}\cdot\hat{s}_f$	$p_1\cos\theta-k_1$	$-p_1\sin\theta$	0	$-p_1\sin\theta$	$p_1\cos\theta-k_1$
$\vec{p}_a\cdot\hat{s}_i$	$\frac{1}{2}(p_1+k_1\cos\theta)$	$\frac{1}{2}k_1\sin\theta$	0	$\frac{1}{2}(p_1+k_1\cos\theta)$	$\frac{1}{2}k_1\sin\theta$
$\vec{p}_a\cdot\hat{s}_f$	$\frac{1}{2}(p_1\cos\theta+k_1)$	$-\frac{1}{2}p_1\sin\theta$	0	$-\frac{1}{2}p_1\sin\theta$	$\frac{1}{2}(p_1\cos\theta+k_1)$
$\vec{N}\cdot\hat{s}_i$	0	0	$p_1k_1\sin\theta$	0	0
$\vec{N}\cdot\hat{s}_f$	0	0	$p_1k_1\sin\theta$	0	0
$\hat{s}_i\cdot\hat{s}_f$	$\cos\theta$	$\cos\theta$	1	$-\sin\theta$	$\sin\theta$
$\vec{N}\cdot(\hat{s}_i\times\hat{s}_f)$	$p_1k_1\sin^2\theta$	$p_1k_1\sin^2\theta$	0	$p_1k_1\cos\theta\sin\theta$	$-p_1k_1\cos\theta\sin\theta$

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APPENDIX: EXPLICIT EXPRESSIONS FOR SPIN OBSERVABLES IN TERMS OF EFFECTIVE AMPLITUDES a_i

In this appendix we present explicit expressions for the quantities Γ'', Γ', and Γ in terms of the effective amplitudes a_i, which are related as follows to the effective amplitudes b_i:

$$b_1=a_1,$$

$$b_2=\frac{a_2}{m^4},$$

$$b_i=\frac{i}{m^2}a_i \quad \text{for} \quad i=3,4 \quad \text{and}$$

$$b_i=\frac{1}{m^2}a_i \quad \text{for} \quad i=5,6,7,8,$$

where m denotes the free nucleon mass.

$$\begin{aligned}
\frac{1}{g_1^2}\Gamma''(\vec{p}_1,\vec{p}_2,\vec{k}_1,\vec{k}_2)={}&4\mathrm{Im}(a_1)^2+4\mathrm{Re}(a_1)^2+(\vec{N}\cdot\vec{N})^2\left(\frac{4\mathrm{Im}(a_2)^2}{m^8}+\frac{4\mathrm{Re}(a_2)^2}{m^8}\right)\\
&+\vec{N}\cdot\vec{N}\left(\frac{4\mathrm{Im}(a_3)^2}{m^4}+\frac{4\mathrm{Re}(a_3)^2}{m^4}+\frac{4\mathrm{Im}(a_4)^2}{m^4}+\frac{4\mathrm{Re}(a_4)^2}{m^4}\right)+\left(\frac{4\mathrm{Im}(a_6)^2}{m^4}+\frac{4\mathrm{Re}(a_6)^2}{m^4}\right)(\vec{p}_a\cdot\vec{p}_a)^2\\
&+\left(\frac{8\mathrm{Im}(a_6)\mathrm{Im}(a_7)}{m^4}+\frac{8\mathrm{Re}(a_6)\mathrm{Re}(a_7)}{m^4}+\frac{8\mathrm{Im}(a_6)\mathrm{Im}(a_8)}{m^4}+\frac{8\mathrm{Re}(a_6)\mathrm{Re}(a_8)}{m^4}\right)\vec{p}_a\cdot\vec{p}_a\vec{p}_a\cdot\vec{q}\\
&+\left(\frac{8\mathrm{Im}(a_5)\mathrm{Im}(a_6)}{m^4}+\frac{8\mathrm{Re}(a_5)\mathrm{Re}(a_6)}{m^4}+\frac{8\mathrm{Im}(a_7)\mathrm{Im}(a_8)}{m^4}+\frac{8\mathrm{Re}(a_7)\mathrm{Re}(a_8)}{m^4}\right)(\vec{p}_a\cdot\vec{q})^2\\
&+\left(\frac{8\mathrm{Im}(a_5)\mathrm{Im}(a_7)}{m^4}+\frac{8\mathrm{Re}(a_5)\mathrm{Re}(a_7)}{m^4}+\frac{8\mathrm{Im}(a_5)\mathrm{Im}(a_8)}{m^4}+\frac{8\mathrm{Re}(a_5)\mathrm{Re}(a_8)}{m^4}\right)\vec{p}_a\cdot\vec{q}\vec{q}\cdot\vec{q}\\
&+\left(\frac{4\mathrm{Im}(a_5)^2}{m^4}+\frac{4\mathrm{Re}(a_5)^2}{m^4}\right)(\vec{q}\cdot\vec{q})^2+\left(\frac{4\mathrm{Im}(a_7)^2}{m^4}+\frac{4\mathrm{Re}(a_7)^2}{m^4}+\frac{4\mathrm{Im}(a_8)^2}{m^4}\right.\\
&\left.+\frac{4\mathrm{Re}(a_8)^2}{m^4}\right)\vec{p}_a\cdot\vec{p}_a\vec{q}\cdot\vec{q},
\end{aligned} \tag{A1}$$

$$\frac{1}{g_1^2}\Gamma'(\vec{p}_1,\ldots,\vec{k}_2,\hat{s}_f)=\left(\frac{-4\mathrm{Re}(a_2)\mathrm{Im}(a_4)\vec{N}\cdot\vec{N}}{m^6}+\frac{4\mathrm{Im}(a_2)\mathrm{Re}(a_4)\vec{N}\cdot\vec{N}}{m^6}-\frac{4\mathrm{Re}(a_1)\mathrm{Im}(a_3)}{m^2}+\frac{4\mathrm{Im}(a_1)\mathrm{Re}(a_3)}{m^2}\right)\vec{N}\cdot\hat{s}_f$$
$$+\vec{p}_a\cdot(\vec{q}\times\hat{s}_f)\left(\frac{4\mathrm{Re}(a_6)\mathrm{Im}(a_7)\vec{p}_a\cdot\vec{p}_a}{m^4}-\frac{4\mathrm{Im}(a_6)\mathrm{Re}(a_7)\vec{p}_a\cdot\vec{p}_a}{m^4}-\frac{4\mathrm{Re}(a_5)\mathrm{Im}(a_6)\vec{p}_a\cdot\vec{q}}{m^4}\right.$$
$$+\frac{4\mathrm{Im}(a_5)\mathrm{Re}(a_6)\vec{p}_a\cdot\vec{q}}{m^4}-\frac{4\mathrm{Re}(a_7)\mathrm{Im}(a_8)\vec{p}_a\cdot\vec{q}}{m^4}$$
$$\left.+\frac{4\mathrm{Im}(a_7)\mathrm{Re}(a_8)\vec{p}_a\cdot\vec{q}}{m^4}-\frac{4\mathrm{Re}(a_5)\mathrm{Im}(a_8)\vec{q}\cdot\vec{q}}{m^4}+\frac{4\mathrm{Im}(a_5)\mathrm{Re}(a_8)\vec{q}\cdot\vec{q}}{m^4}\right), \tag{A2}$$

$$\frac{1}{4g_1^2}\Gamma(\vec{p}_1,\ldots,\vec{k}_2,\hat{s}_i,\hat{s}_f)=\left(\frac{2\mathrm{Im}(a_2)^2\vec{N}\cdot\vec{N}}{m^8}+\frac{2\mathrm{Re}(a_2)^2\vec{N}\cdot\vec{N}}{m^8}+\frac{2\mathrm{Im}(a_3)^2}{m^4}+\frac{2\mathrm{Re}(a_3)^2}{m^4}\right)\vec{N}\cdot\hat{s}_f\vec{N}\cdot\hat{s}_i$$
$$+\left(\frac{2\mathrm{Im}(a_2)\mathrm{Im}(a_4)\vec{N}\cdot\vec{N}}{m^6}+\frac{2\mathrm{Re}(a_2)\mathrm{Re}(a_4)\vec{N}\cdot\vec{N}}{m^6}-\frac{2\mathrm{Im}(a_1)\mathrm{Im}(a_3)}{m^2}-\frac{2\mathrm{Re}(a_1)\mathrm{Re}(a_3)}{m^2}\right)$$
$$\times\vec{N}\cdot(\hat{s}_i\times\hat{s}_f)+\vec{p}_a\cdot\hat{s}_f\vec{p}_a\cdot\hat{s}_i\left(\frac{2\mathrm{Im}(a_6)^2\vec{p}_a\cdot\vec{p}_a}{m^4}+\frac{2\mathrm{Re}(a_6)^2\vec{p}_a\cdot\vec{p}_a}{m^4}+\frac{4\mathrm{Im}(a_6)\mathrm{Im}(a_8)\vec{p}_a\cdot\vec{q}}{m^4}\right.$$
$$\left.+\frac{4\mathrm{Re}(a_6)\mathrm{Re}(a_8)\vec{p}_a\cdot\vec{q}}{m^4}+\frac{2\mathrm{Im}(a_8)^2\vec{q}\cdot\vec{q}}{m^4}+\frac{2\mathrm{Re}(a_8)^2\vec{q}\cdot\vec{q}}{m^4}\right)+\vec{p}_a\cdot\hat{s}_i\left(\frac{2\mathrm{Im}(a_6)\mathrm{Im}(a_7)\vec{p}_a\cdot\vec{p}_a}{m^4}\right.$$
$$+\frac{2\mathrm{Re}(a_6)\mathrm{Re}(a_7)\vec{p}_a\cdot\vec{p}_a}{m^4}+\frac{2\mathrm{Im}(a_5)\mathrm{Im}(a_6)\vec{p}_a\cdot\vec{q}}{m^4}+\frac{2\mathrm{Re}(a_5)\mathrm{Re}(a_6)\vec{p}_a\cdot\vec{q}}{m^4}+\frac{2\mathrm{Im}(a_7)\mathrm{Im}(a_8)\vec{p}_a\cdot\vec{q}}{m^4}$$
$$\left.+\frac{2\mathrm{Re}(a_7)\mathrm{Re}(a_8)\vec{p}_a\cdot\vec{q}}{m^4}+\frac{2\mathrm{Im}(a_5)\mathrm{Im}(a_8)\vec{q}\cdot\vec{q}}{m^4}+\frac{2\mathrm{Re}(a_5)\mathrm{Re}(a_8)\vec{q}\cdot\vec{q}}{m^4}\right)\vec{q}\cdot\hat{s}_f$$
$$+\vec{p}_a\cdot\hat{s}_f\left(\frac{2\mathrm{Im}(a_6)\mathrm{Im}(a_7)\vec{p}_a\cdot\vec{p}_a}{m^4}+\frac{2\mathrm{Re}(a_6)\mathrm{Re}(a_7)\vec{p}_a\cdot\vec{p}_a}{m^4}+\frac{2\mathrm{Im}(a_5)\mathrm{Im}(a_6)\vec{p}_a\cdot\vec{q}}{m^4}\right.$$
$$+\frac{2\mathrm{Re}(a_5)\mathrm{Re}(a_6)\vec{p}_a\cdot\vec{q}}{m^4}+\frac{2\mathrm{Im}(a_7)\mathrm{Im}(a_8)\vec{p}_a\cdot\vec{q}}{m^4}+\frac{2\mathrm{Re}(a_7)\mathrm{Re}(a_8)\vec{p}_a\cdot\vec{q}}{m^4}+\frac{2\mathrm{Im}(a_5)\mathrm{Im}(a_8)\vec{q}\cdot\vec{q}}{m^4}$$
$$\left.+\frac{2\mathrm{Re}(a_5)\mathrm{Re}(a_8)\vec{q}\cdot\vec{q}}{m^4}\right)\vec{q}\cdot\hat{s}_i+\left(\frac{2\mathrm{Im}(a_7)^2\vec{p}_a\cdot\vec{p}_a}{m^4}+\frac{2\mathrm{Re}(a_7)^2\vec{p}_a\cdot\vec{p}_a}{m^4}\right.$$
$$\left.+\frac{4\mathrm{Im}(a_5)\mathrm{Im}(a_7)\vec{p}_a\cdot\vec{q}}{m^4}+\frac{4\mathrm{Re}(a_5)\mathrm{Re}(a_7)\vec{p}_a\cdot\vec{q}}{m^4}+\frac{2\mathrm{Im}(a_5)^2\vec{q}\cdot\vec{q}}{m^4}+\frac{2\mathrm{Re}(a_5)^2\vec{q}\cdot\vec{q}}{m^4}\right)\vec{q}\cdot\hat{s}_f\vec{q}\cdot\hat{s}_i$$
$$+\left(\mathrm{Im}(a_1)^2+\mathrm{Re}(a_1)^2-\frac{\mathrm{Im}(a_2)^2(\vec{N}\cdot\vec{N})^2}{m^8}-\frac{\mathrm{Re}(a_2)^2(\vec{N}\cdot\vec{N})^2}{m^8}-\frac{\mathrm{Im}(a_3)^2\vec{N}\cdot\vec{N}}{m^4}-\frac{\mathrm{Re}(a_3)^2\vec{N}\cdot\vec{N}}{m^4}\right.$$
$$+\frac{\mathrm{Im}(a_4)^2\vec{N}\cdot\vec{N}}{m^4}+\frac{\mathrm{Re}(a_4)^2\vec{N}\cdot\vec{N}}{m^4}-\frac{\mathrm{Im}(a_6)^2(\vec{p}_a\cdot\vec{p}_a)^2}{m^4}-\frac{\mathrm{Re}(a_6)^2(\vec{p}_a\cdot\vec{p}_a)^2}{m^4}$$
$$-\frac{2\mathrm{Im}(a_6)\mathrm{Im}(a_7)\vec{p}_a\cdot\vec{p}_a\vec{p}_a\cdot\vec{q}}{m^4}-\frac{2\mathrm{Re}(a_6)\mathrm{Re}(a_7)\vec{p}_a\cdot\vec{p}_a\vec{p}_a\cdot\vec{q}}{m^4}-\frac{2\mathrm{Im}(a_6)\mathrm{Im}(a_8)\vec{p}_a\cdot\vec{p}_a\vec{p}_a\cdot\vec{q}}{m^4}$$
$$-\frac{2\mathrm{Re}(a_6)\mathrm{Re}(a_8)\vec{p}_a\cdot\vec{p}_a\vec{p}_a\cdot\vec{q}}{m^4}-\frac{2\mathrm{Im}(a_5)\mathrm{Im}(a_6)(\vec{p}_a\cdot\vec{q})^2}{m^4}-\frac{2\mathrm{Re}(a_5)\mathrm{Re}(a_6)(\vec{p}_a\cdot\vec{q})^2}{m^4}$$
$$-\frac{2\mathrm{Im}(a_7)\mathrm{Im}(a_8)(\vec{p}_a\cdot\vec{q})^2}{m^4}-\frac{2\mathrm{Re}(a_7)\mathrm{Re}(a_8)(\vec{p}_a\cdot\vec{q})^2}{m^4}-\frac{\mathrm{Im}(a_7)^2\vec{p}_a\cdot\vec{p}_a\vec{q}\cdot\vec{q}}{m^4}-\frac{\mathrm{Re}(a_7)^2\vec{p}_a\cdot\vec{p}_a\vec{q}\cdot\vec{q}}{m^4}$$

$$-\frac{\mathrm{Im}(a_8)^2\vec{p}_a\cdot\vec{p}_a\vec{q}\cdot\vec{q}}{m^4}-\frac{\mathrm{Re}(a_8)^2\vec{p}_a\cdot\vec{p}_a\vec{q}\cdot\vec{q}}{m^4}-\frac{2\,\mathrm{Im}(a_5)\mathrm{Im}(a_7)\vec{p}_a\cdot\vec{q}\vec{q}\cdot\vec{q}}{m^4}$$

$$-\frac{2\,\mathrm{Re}(a_5)\mathrm{Re}(a_7)\vec{p}_a\cdot\vec{q}\vec{q}\cdot\vec{q}}{m^4}-\frac{2\,\mathrm{Im}(a_5)\mathrm{Im}(a_8)\vec{p}_a\cdot\vec{q}\vec{q}\cdot\vec{q}}{m^4}-\frac{2\,\mathrm{Re}(a_5)\mathrm{Re}(a_8)\vec{p}_a\cdot\vec{q}\vec{q}\cdot\vec{q}}{m^4}$$

$$-\frac{\mathrm{Im}(a_5)^2(\vec{q}\cdot\vec{q})^2}{m^4}-\frac{\mathrm{Re}(a_5)^2(\vec{q}\cdot\vec{q})^2}{m^4}\Bigg)\hat{s}_i\cdot\hat{s}_f. \tag{A3}$$

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