Three-body treatment of the $(d,{}^2\mathrm{He})$ reaction on the basis of the adiabatic approximation

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A parameter-free method is proposed for the analysis of the $(d,{}^2\mathrm{He})$ reaction. The three-body final state is treated in the adiabatic approximation. The method is successfully applied to the data at 270 MeV on $^{12}\mathrm{C}$ target. [S0556-2813(99)04611-7]

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I. INTRODUCTION

The $(d,{}^2\mathrm{He})$ reaction at intermediate energies ($E_d > 100$ MeV/nucleon) has become one of the most powerful probes for the study of spin- and isospin-excitations in nuclei [1–6]. It provides information on such modes in the (n,p) channel with an experimentally much more favorable condition than other charge-exchange reactions of the same type. Rich information of polarization observables also makes this reaction notably useful [1,7]. ^{2}He is in reality a pair of protons coupled to the $^1S_0$ state. It is measured by the coincidence detection of two protons emitted in close geometries, but under a large background of uncorrelated protons produced by the deuteron breakup. This difficulty, which was serious at lower energies and effectively hampered the extensive use of the reaction [8], however, has been overcome by the use of a magnetic spectrometer as intermediate-energy deuteron beams become available [9]. The simple one-step reaction mechanism expected at such energies also leads to transparent interpretations of the data.

That the reaction has a three-body final state still gives rise to difficulties in theoretical treatment, although the selectivities on transfered angular momenta make the problem considerably simple to attack. In the previous publication [1], where the data of detailed angular distributions for differential cross sections as well as for vector- and tensor-analyzing powers were presented for the first time, an attempt was made to describe the reaction based on the distorted-wave Born approximation (DWBA) theory. The result was encouraging, as all the data for low-lying discrete levels of the residual nucleus were reasonably well reproduced by the one-step DWBA calculations. But the ^{2}He optical potential generating the final scattering wave function was arbitrarily searched to fit the data, giving rise to ambiguities in interpretation of the result. There is no phenomenological information on the ^{2}He-residual-nucleus effective interaction. Besides, the validity for describing the final scattering state by a local optical potential, as usually done in two-body reactions, is doubtful [10].

A parameter-free method to treat such three-body dynamics was proposed on the basis of the adiabatic (or also known as ''sudden'') approximation [11]. It was successfully applied to the analysis of the (^{3}He, ^{2}He) transfer reaction, but employing the zero-range approximation to simplify the interaction with the transfered nucleon. Its extension to the $(d,{}^2\mathrm{He})$ reaction should be straightforward in principle, yet it has never been done presumably because the complicated charge-exchange form factor is involved. A purpose of this article is to present an explicit formulation for the three-body treatment of the $(d,{}^2\mathrm{He})$ reaction and to apply it to the analysis of the previous data on $^{12}\mathrm{C}$ target at 270 MeV. Since the present method is realistic but acceptably simple, it will hopefully encourage extensive applications of the reaction.

II. FORMULATION OF THE CALCULATION

Let us consider a description of one-step direct reaction with a general form of $A(a,b)B$, where a and b are the bound and scattering states of nucleons 1 and 2, respectively. The spin and isospin of $A(B)$ and $a(b)$ are denoted by $I_A(I_B)$, $T_A(T_B)$, $s_a(s_b)$, and $t_a(t_b)$, respectively. $\boldsymbol{R}$ represents the relative coordinate between the projectile and the target, $\boldsymbol{r}$ the one between the particles 1 and 2, and their corresponding asymptotic wave numbers are denoted by $\boldsymbol{K}$ and $\boldsymbol{k}$, respectively. $\mu_{ij}=m_i m_j/(m_i+m_j)$ is the reduced mass of the particles i and j. The T-matrix element is evaluated in a way analogous to Ref. [11]; the charge-exchange process is treated to first order and the exit-channel wave function is treated in the adiabatic approximation. While polarization observables are subjects of interest here, the adiabatic approximation, as described below, is not capable to include the spin-orbit potential. The incident-channel wave function is therefore generated using the conventional optical potential. It is worth noting that the possibility of projectile breakup, which allows the charge-exchange from the continuum of incident-channel directly to the final scattering state, is discussed for the $(d,{}^2\mathrm{He})$ reaction in Ref. [12], but without the spin-orbit potential.

The three-body wave function $\Psi^{(+)}$ describing the final scattering state satisfies the Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu_{bB}}\nabla^2_{\boldsymbol{R}}+\sum_{i=1,2}V_{iB}(\boldsymbol{R},\boldsymbol{r})+H_{12}-E\right]\Psi^{(+)}(\boldsymbol{K}_f,\boldsymbol{k};\boldsymbol{R},\boldsymbol{r})=0. \tag{1}$$

The adiabatic approximation to solve this equation is to neglect the excitation of the particles 1-2 system [13], i.e., to replace the sub-Hamiltonian H_{12} by ϵ $(=\hbar^2k^2/2\mu_{12})$, relative energy between particles 1 and 2. The condition $E \gg \epsilon$ for the validity of this approximation is well fulfilled in experiments we are concerned with. Also the interaction V_{iB} between particles i and B is approximated by the optical

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potential U_{iB} evaluated at $\frac{1}{2}(E-\epsilon)$, neglecting its spin-orbit component. The sum of potentials is then expanded in multipoles of $\boldsymbol{R}$ and $\boldsymbol{r}$ as

$$U_{1B}\left(\left|\boldsymbol{R}+\frac{\boldsymbol{r}}{2}\right|\right)+U_{2B}\left(\left|\boldsymbol{R}-\frac{\boldsymbol{r}}{2}\right|\right)=4\pi\sum_{\lambda}U_{\lambda}(R,r)[i^{\lambda}Y_{\lambda}(\hat{\boldsymbol{R}}),i^{\lambda}Y_{\lambda}(\hat{\boldsymbol{r}})]_0^0, \quad (2)$$

where the square bracket means a vector coupling, for example, as

$$[i^{L}Y_{L}(\hat{\boldsymbol{R}}),i^{l}Y_{l}(\hat{\boldsymbol{r}})]_J^{\mu}=i^{L+l}\sum_{Mm}\langle LlMm|J\mu\rangle Y_L^M(\hat{\boldsymbol{R}})Y_l^m(\hat{\boldsymbol{r}}).$$

Expressing the wave function of ejectile b by

$$\psi_{s_bt_b}^{m_b\nu_b}(\boldsymbol{k};\boldsymbol{r})=\frac{4\pi}{kr}\sum_{lms_b'}\chi_l(k;r)\langle ls_b'mm_b-m|s_bm_b\rangle Y_l^m(\hat{\boldsymbol{k}})\times[i^{l}Y_{l}(\hat{\boldsymbol{r}}),\phi_{s_b'}]_{s_b}^{m_b}\phi_{t_b}^{\nu_b}, \quad (3)$$

where $\phi_{s_b'}^{m_b'}$ ($\phi_{t_b}^{\nu_b}$) is the vector coupling of spin (isospin) functions of particles 1 and 2, the adiabatic three-body wave function $\Psi_{\rm ad}^{(+)}$ can be expanded as [14]

$$\begin{aligned}\Psi_{\rm ad}^{(+)}(\boldsymbol{K}_f,\boldsymbol{k};\boldsymbol{R},\boldsymbol{r})&=\frac{4\pi}{K_fR}\sum_{JLlMm}\langle LlMm|JM+m\rangle\langle ls_b'mm_b-m|s_bm_b\rangle\\&\times Y_L^{M*}(\hat{\boldsymbol{K}}_f)\frac{4\pi}{kr}\chi_l(k;r)\phi_{s_b'}^{m_b-m}\phi_{t_b}^{\nu_b}\\&\times\sum_{m'}\langle ls_b'm'm_b-m'|s_bm_b\rangle Y_l^{m'*}(\hat{\boldsymbol{k}})\\&\times\sum_{l'L'}\chi_{L'l';Ll}^{J}(K_f;R,r)[i^{L'}Y_{L'}(\hat{\boldsymbol{R}}),i^{l'}Y_{l'}(\hat{\boldsymbol{r}})]_J^{M+m},\end{aligned} \quad (4)$$

where $K_f=[2\mu_{bB}(E-\epsilon)]^{1/2}/\hbar$. For ^{2}He, $t_b=1$, $s_b=s_b'=0$, and accordingly $l=0$. Due to the absence of spin-orbit interaction, s_b' remains as a constant. From Eqs. (1), (2), and (4), coupled equations which $\chi_{L'l';Ll}^{J}$ should satisfy are deduced to be [15]

$$\begin{aligned}\left[\frac{d^2}{dR^2}+K_f^2-\frac{L(L+1)}{R^2}-\frac{2\mu_{bB}}{\hbar^2}U_0(R,r)\right]\chi_{Ll;Ll}^{J}(K_f;R,r)&=\frac{2\mu_{bB}}{\hbar^2}\hat{L}\hat{l}\sum_{L'l'}i^{L'+l'-L-l}(-)^{L'+l+J}\\&\times\chi_{L'l';Ll}^{J}(K_f;R,r)\sum_{\lambda\neq0}\hat{\lambda}\langle L\lambda00|L'0\rangle\\&\times\langle l\lambda00|l'0\rangle W(LlL'l';J\lambda)U_{\lambda}(R,r),\end{aligned} \quad (5)$$

where $\hat{\lambda}=(2\lambda+1)^{1/2}$ and $W(LlL'l';J\lambda)$ is the Racah coefficient. Since r appears only as a parameter, they are numerically solved at each given r with the boundary condition that

$$\chi_{L'l';Ll}^{J}(K_f;R,r)\overset{R\to\infty}{\to}e^{i\sigma_L}\delta_{L'L}\delta_{l'l}F_L(K_fR)+e^{i\sigma_{L'}}T_{L'l';Ll}^{J}(r)\times[G_{L'}(K_fR)+iF_{L'}(K_fR)], \quad (6)$$

where F_L and G_L are Coulomb wave functions and σ_L is the Coulomb phase shift.

The incident-channel scattering wave function is expanded in an ordinary form as

$$\begin{aligned}\chi_i^{(+)}(\boldsymbol{K}_i;\boldsymbol{R})\psi_{s_at_a}^{m_a\nu_a}(\boldsymbol{r})&=\frac{4\pi}{K_iR}\sum_{JLM}\chi_{JL}(K_i;R)\langle Ls_aMm_a|JM+m_a\rangle\\&\times Y_L^{M*}(\hat{\boldsymbol{K}}_i)[i^{L}Y_{L}(\hat{\boldsymbol{R}}),\psi_{s_a}(\boldsymbol{r})]_J^{M+m_a}.\end{aligned} \quad (7)$$

The bound state wave function of projectile a is given by

$$\psi_{s_at_a}^{m_a\nu_a}(\boldsymbol{r})=\frac{1}{r}\sum_{l_as_a'}u_{l_as_a'}(r)[i^{l_a}Y_{l_a}(\hat{\boldsymbol{r}}),\phi_{s_a'}]_{s_a}^{m_a}\phi_{t_a}^{\nu_a}, \quad (8)$$

where $\phi_{s_a'}^{m_a'}$ ($\phi_{t_a}^{\nu_a}$) is the vector coupling of spin (isospin) functions of particles 1 and 2. For the deuteron, $t_a=0$, $s_a=s_a'=1$, and $l_a=0$, 2.

The target form factor is defined as

$$F_{I_BI_AT_BT_A}^{M_BM_A\nu_B\nu_A}(\boldsymbol{y})=\left\langle I_BM_BT_B\nu_B\left|\sum_{t,p}V_{tp}\right|I_AM_AT_A\nu_A\right\rangle, \quad (9)$$

where $\boldsymbol{y}=\boldsymbol{R}+\boldsymbol{r}/2$ and V_{tp} is the effective interaction between nucleons in target and projectile. Its matrix element with respect to the coupled spin and isospin of particles 1 and 2 can be expressed, by analogy with the point-particle scattering [16], as

$$\begin{aligned}\langle\phi_{s_b'}^{m_b'}\phi_{t_b}^{\nu_b}|F_{I_BI_AT_BT_A}^{M_BM_A\nu_B\nu_A}|\phi_{s_a'}^{m_a'}\phi_{t_a}^{\nu_a}\rangle&=(-)^{s_b'-m_b'}\sum_{ls'jmt}\langle I_AjM_AM_B-M_A|I_BM_B\rangle\\&\times\langle s_a's_b'm_a'-m_b'|s'm_a'-m_b'\rangle\\&\times\langle ls'mm_a'-m_b'|jM_B-M_A\rangle f_{ls'jt}(y)i^{-l}Y_l^{m*}(\hat{\boldsymbol{y}}),\end{aligned} \quad (10)$$

where l, s', j, and t are the orbital angular momentum, spin, total angular momentum, and isospin transfered to the target, respectively. In order to carry out the integration over $\boldsymbol{r}$, the form factor is expanded in multipoles as

$$f_{ls'jt}(y)i^l Y_l^m(\hat{\boldsymbol{y}})=\sum_{L_1L_2} H_{L_1L_2}^{ls'jt}\left(R,\frac{r}{2}\right)[i^{L_1}Y_{L_1}(\hat{\boldsymbol{R}}),i^{L_2}Y_{L_2}(\hat{\boldsymbol{r}})]_l^m. \tag{11}$$

The coefficient of expansion is given by [17]

$$H_{L_1L_2}^{ls'jt}(r_1,r_2)=i^{l-L_1-L_2}\sum_{l_1l_2\lambda}\delta_{l_1+l_2,l}(-)^{l+L_1+l_2}(2\lambda+1)\langle l_1\lambda 00|L_10\rangle\langle l_2\lambda 00|L_20\rangle W(L_1l_1L_2l_2;\lambda l)r_1^{l_1}r_2^{l_2}$$
$$\times\sqrt{\frac{\pi(2l+1)!}{(2l_1)!(2l_2)!}}\int\frac{f_{ls'jt}(y')}{y'^l}P_\lambda(\cos\theta)d(\cos\theta), \tag{12}$$

where $y'=(r_1^2+r_2^2+2r_1r_2\cos\theta)^{1/2}$.

It will be a good approximation to evaluate the T-matrix element in the prior formalism by using $\chi_i^{(+)}\psi_{s_at_a}^{m_a\nu_a}$ and the $l'=l$ component of $\Psi_{\mathrm{ad}}^{(-)}$, which is the projection onto the elastic channel of the final scattering state. Although it appears similar to a simple extension of DWBA, the present method is beyond that as U_{iB} allows the coupling with $l'\neq l$ channels through the dependence of $\chi^J_{L'l';Ll}$ on r. If $s_b'=s_b=0$, that is the case for ^{2}He, the T-matrix element is considerably simplified and can be written explicitly as

$$T(\boldsymbol{k},K_i,\boldsymbol{K}_f)=\langle\Psi_{\mathrm{ad}}^{(-)}(l'=l)|F_{I_BI_AT_BT_A}^{M_BM_A\nu_B\nu_A}|\chi_i^{(+)}\psi_{s_at_a}^{m_a\nu_a}\rangle$$
$$=\frac{4\pi}{kK_iK_f}\sum_j\hat{j}\langle I_AjM_AM_B-M_A|I_BM_B\rangle\sum_{J_iL_iL_fl'l_as_a't}\langle L_is_a0m_a|J_im_a\rangle\langle L_fjM_fM_B-M_A|J_im_a\rangle Y_{L_f}^{M_f}(\hat{\boldsymbol{K}}_f)$$
$$\times i^{L_i-L_f-L'}\hat{L}_f\hat{L}_i\hat{L}'\hat{l}\hat{s}_a\langle L_fL'00|L_i0\rangle W(L_fL'J_is_a;L_ij)W(L'l_ajs_a';ls_a)$$
$$\times\int dr\,\chi_0(k;r)\,u_{l_as_a'}(r)\int dR\,\chi_{L_f0;L_f0}^{L_f}(K_f;R,r)H_{L'l_a}^{ls_a'jt}\left(R,\frac{r}{2}\right)\chi_{J_iL_i}(K_i;R), \tag{13}$$

where $M_f=m_a+M_A-M_B$. Further restrictions of $t=1$, $s_a'=1$, and $l_a=0$, 2 are imposed for the $(d,{}^2\mathrm{He})$ reaction. It is worth noting that the DWBA representation is obtained from Eq. (13) by replacing $\chi_{L_f0;L_f0}^{L_f}(K_f;R,r)$ with $\chi_{J_fL_f}(K_f;R)$, the distorted wave describing the center of mass motion of particles 1 and 2. The difference of the present calculation from the one given in Ref. [1], however, lies not only in the treatment of the final scattering wave function but also in the description of the incident projectile. Contributions from the deuteron D state [$l_a=2$ component in Eq. (13)] were neglected in Ref. [1], which further simplified the calculation.

Finally, attention must be drawn to the specific treatment of the $(d,{}^2\mathrm{He})$ reaction. The triple-differential cross section for the three-body (d,pp) reaction is obtained from

$$\frac{d^3\sigma}{d\Omega_{\boldsymbol{K}_f}d\Omega_{\boldsymbol{k}}d\epsilon}[(d,pp)]=\frac{2\pi\mu_{aA}}{\hbar^2K_i}\rho\frac{1}{(2I_A+1)(2s_a+1)}$$
$$\times\sum_{M_BM_Am_a}[|T(\boldsymbol{k},K_i,\boldsymbol{K}_f)|^2$$
$$+|T(-\boldsymbol{k},K_i,\boldsymbol{K}_f)|^2], \tag{14}$$

where ρ is the phase-space distribution with respect to $\Omega_{\boldsymbol{K}_f}$, $\Omega_{\boldsymbol{k}}$, and ϵ. The summation for $\boldsymbol{k}$ and $-\boldsymbol{k}$, which takes the indistinguishability of two protons into account, results in multiplication of a factor of 2 as p-p is assumed to be in the $^1S_0$ state. The two-body cross section of the $(d,{}^2\mathrm{He})$ reaction is defined in most experimental articles as [8,9]

$$\frac{d\sigma}{d\Omega}[(d,{}^2\mathrm{He})]=\frac{1}{2}\int_{\epsilon_{\min}}^{\epsilon_{\max}}d\epsilon\int_{4\pi}d\Omega_{\boldsymbol{k}}\frac{d^3\sigma}{d\Omega_{\boldsymbol{K}_f}d\Omega_{\boldsymbol{k}}d\epsilon}[(d,pp)], \tag{15}$$

while analyzing powers are averaged over ϵ weighting with the cross section. The factor $\frac{1}{2}$ in Eq. (15) arises again from the indistinguishability of two protons, which, in theoretical calculations, cancels out with the above mentioned factor 2. The integration limits are chosen in most modern literatures [1–3] as $\epsilon_{\min}=0$ and $\epsilon_{\max}=1$ MeV to keep contributions from p-p partial waves higher than $^1S_0$ negligibly small. Contributions from higher partial-waves as a function of ϵ are discussed in Refs. [18,19], though only for the $^1\mathrm{H}(d,pp)n$ reaction on the basis of the plane-wave impulse approximation (PWIA). According to their analyses, such contributions are of order of a few percent if $\epsilon_{\max}=1$ MeV, but grow to be significant as ϵ increases. In some literatures for the $(d,{}^2\mathrm{He})$ reaction, however, the integration limits are chosen to be different, e.g., $\epsilon_{\max}=5$ MeV in Ref. [4], or even not specified [5,6]. One must therefore be careful in comparing the magnitude of the cross section between different experiments, as well as in discussing the spin-flip strength.

TABLE I. Optical-potential parameters used for the calculation.

	$E_{\rm lab}$ (MeV)	V_R (MeV)	r_R (fm)	a_R (fm)	W_V (MeV)	W_D (MeV)	r_I (fm)	a_I (fm)	V_{LS} (MeV)	r_{LS} (fm)	a_{LS} (fm)	r_C (fm)	Ref.
$d+^{12}$C	270	27.7	1.22	0.86	40.6	0.0	1.10	0.892	3.81	0.71	0.68	1.3	[20]
$p+^{12}$C	134	16.54	1.230	0.784	10.26	0.0	1.313	0.540				1.394	[21]

III. CALCULATIONS AND THE RESULTS

The incident-channel distorted wave is generated by the optical potential with the standard form

$$U(R)=-V_R f(r_R,a_R;R)-iW_V f(r_I,a_I;R) + i4a_I W_D \frac{d}{dR} f(r_I,a_I;R)+2\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{R} V_{LS} \times \frac{d}{dR} f(r_{LS},a_{LS};R)\boldsymbol{L}\cdot\boldsymbol{s}+U_{\rm Coul}(R), \tag{16}$$

where

$$f(r_i,a_i;R)=\left[1+\exp\left(\frac{R-r_i A^{1/3}}{a_i}\right)\right]^{-1}.$$

The same form excluding the spin-orbit component is used for U_{pB} in describing the final-state scattering wave. The Coulomb part, however, is not expanded in multipoles of r so that the boundary condition [Eq. (6)] is satisfied at a reasonable size of R. Since the ejectile consists of two protons, the deviation caused by this becomes sizable only in the region of $r \geqslant 2R$, where contributions to the T-matrix element are very small. Thus the Coulomb interaction is approximated to act on the center of mass of the p-p system. The potential parameters used in the present calculation are listed in Table I. It should be noted that the parameters for deuteron have been obtained from the new data on elastic scattering [20] and differ from those used in Ref. [1]. The proton potential on (excited) ^{12}B target is approximated by that on ^{12}C target. The adiabatic coupled-channels equations [Eq. (5)] are solved taking relative ^1S and ^1D waves into account.

Both the deuteron and ^{2}He wave functions are calculated by using the Reid softcore potential [22]. The choice of two-nucleon force, however, has negligibly small influence on the result. The target form factors are calculated by using ordinary and well established parameters. The effective two-body interaction is taken from the central and tensor parts of the t-matrix parametrization at 140 MeV by Franey and Love [23]. The effect of the single-nucleon knock-on exchange (SNKE) is included by a short-range approximation for the central part [24]. Since only the $s=t=1$ components are involved in the reaction, the interaction is given in an explicit form as

$$V(\boldsymbol{r}_{ij})=[V_{11}^0(r_{ij})\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j+V_{11}^2(r_{ij})S_{12}]\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j, \tag{17}$$

$$V_{11}^0(r)=3.5Y(r/1.4)+(257.05+132.37i)Y(r/0.4) -(631.79+470.63i)Y(r/0.25) -(44.38+3.53i)\delta(r),$$

$$V_{11}^2(r)=(54.12+16.52i)r^2Y(r/0.7) -(302.1+162.0i)r^2Y(r/0.55) +(1706.9+976.4i)r^2Y(r/0.4) -(9194.1+5643.6i)r^2Y(r/0.25),$$

where $Y(x)=e^{-x}/x$ and $\boldsymbol{\sigma}$, $\boldsymbol{\tau}$, and S_{12} are the spin, isospin, and tensor operators, respectively. It is expanded in multipoles to calculate $f_{ls'jt}$. The spectroscopic amplitudes are obtained from the wave functions of Cohen and Kurath [25] and of Millener and Kurath [26] for the positive- and negative-parity final states, respectively. Their values are listed in Table II. The single-particle wave functions are calculated using Woods-Saxon potentials with $r_0=1.25$ fm, $a=0.65$ fm, and $V_{LS}=6$ MeV by varying the well depth so that the $p_{3/2}$ proton is bound by 15.96 MeV and the $p_{1/2}$ neutron by 3.37 MeV.

The amplitude of $u_{l_a s_a'}(r)H_{L'l_a}^{ls'jt}(R,r/2)$, the product of radial components of the deuteron wave function and the expanded form factor [Eqs. (8) and (12)], is presented in Fig. 1 for low-lying discrete levels of ^{12}B but only for $l_a=0$ (deuteron S state) and accordingly $L'=l$. Noting that $\psi_{s_b t_b}$ [Eq. (3)] is a moderate function of r in the region of $\epsilon<1$ MeV, if the range of above function with respect to r is sufficiently short, the three-body wave function $\chi_{L'l';Ll}^J(K;R,r)$ may be replaced effectively by a distorted wave $\chi_{JL}(K;R)$ dependent only on R, as originally discussed for the (d,p) reaction by Johnson and Soper [13]. Figure 1 shows that the amplitude of $u_{0s_a'}H_{l0}^{ls'jt}$ indeed reaches a maximum at $r\sim1.5$ fm and diminishes with increasing r irrespective of both the residual state and the transfered orbital angular momentum l, but varying rather slowly in comparison with K which becomes as large as 4 fm^{-1} at 270 MeV. Consequently the use of an effective distorted wave for the final state can lead to a poor approximation, besides there is practically no way to deduce such a function. The presence of $H_{L'l_a}^{ls'jt}$, however, imposes some restrictions on the reaction as described below.

The calculated angular distributions of the cross sections and the analyzing powers are shown in Figs. 2 and 3, respectively. They are in reasonably good agreement with the data previously presented in Ref. [1]. Although an overall nor-

TABLE II. Spectroscopic amplitudes in neutron-proton representation $(\hat{j}^{-1}\langle I_B T_B \nu_B || [a^\dagger_{j_2}, \tilde{a}_{j_1}]^j || I_A T_A \nu_A\rangle$, $\tilde{a}_{j\mu}=(-)^{j+\mu}a_{j-\mu}$, Edmond's convention for the reduced matrix elements; radial wave functions positive at the origin together with $[i^l Y_l, \phi_s]^j)$ for ^{12}B excited states.

$(I_B)^\pi_n$	E_x (MeV)	$(j_1 j_2)$					
		$0p_{3/2}0p_{3/2}$	$0p_{3/2}0p_{1/2}$	$0p_{1/2}0p_{3/2}$	$0p_{1/2}0p_{1/2}$		
1^+_1	0.	0.0825	0.6907	0.3270	0.0733		
2^+_1	0.95	0.0476	0.6884	−0.1126			
		$0p_{3/2}1s_{1/2}$	$0p_{3/2}0d_{5/2}$	$0p_{3/2}0d_{3/2}$	$0p_{1/2}1s_{1/2}$	$0p_{1/2}0d_{5/2}$	$0p_{1/2}0d_{3/2}$
1^-_1	2.62	0.7285	0.1277	−0.0835	−0.0924		0.0077
2^-_2	4.46	−0.2481	0.6032	−0.0958		−0.2788	−0.0688
4^-_1	4.52		0.8129				

malization factor of 0.7 is required to fit the cross section, relative factors for the 2^+_1, 2^-_2, and 4^-_1 states with respect to the ground state are consistent with those obtained in the analyses of intermediate-energy (p,n) reactions [27–29], indicating that they come from the improper description of the nuclear structure. A notable feature is the very small contributions from the deuteron D state, which are shown by the difference between the dashed and solid curves for each state. Furthermore, effects of coupling with D waves for the p-p system are exceedingly small. Calculations without the

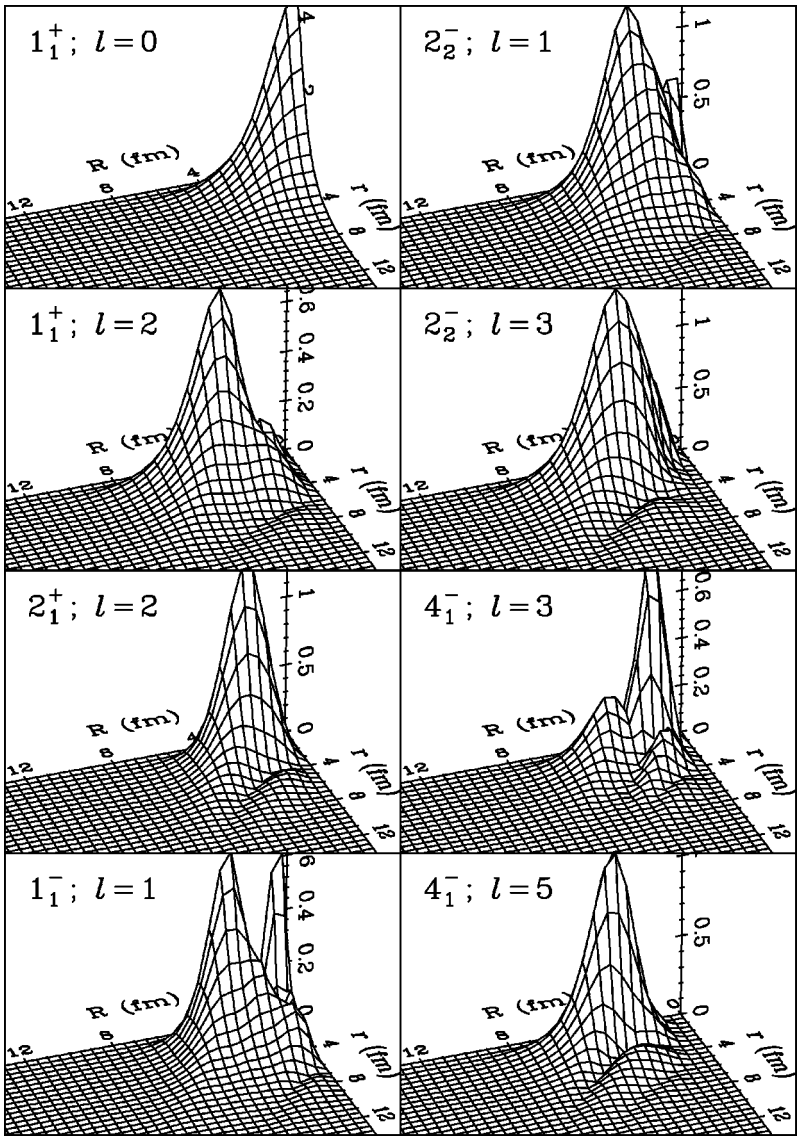

FIG. 1. Amplitude of the product of the deuteron wave function and the expanded form factor $|u_{l_a s'_a}(r) H^{l1 I_B 1}_{L' l_a}(R, r/2)|$, for some $^{12}\mathrm{C}[0^+;\mathrm{gnd}] \rightarrow {}^{12}\mathrm{B}[I^\pi_B]$ transitions, but only for $l_a=0$ (deuteron S state) and accordingly $L'=l$.

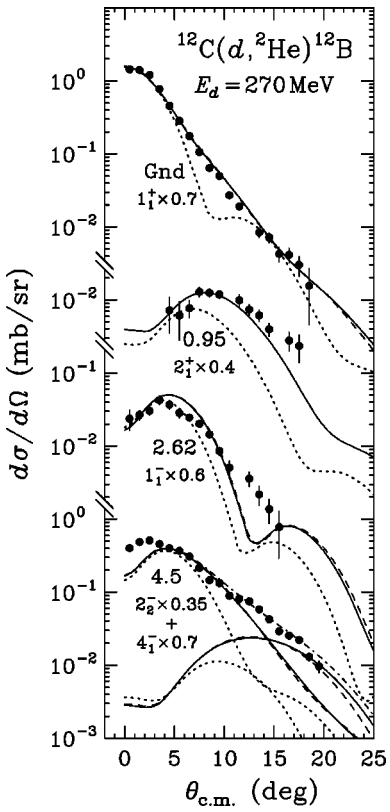

FIG. 2. Angular distributions of the cross section for the ^{12}C(d,^{2}He)^{12}B reaction. The results of calculations are presented normalized by the factors indicated in the figure. The solid curves represent the full calculations while the dotted and dashed curves represent the results without including the tensor interaction and the deuteron D-state, respectively. The sum of 2_2^- and 4_1^- is shown by the dot-dashed curve.

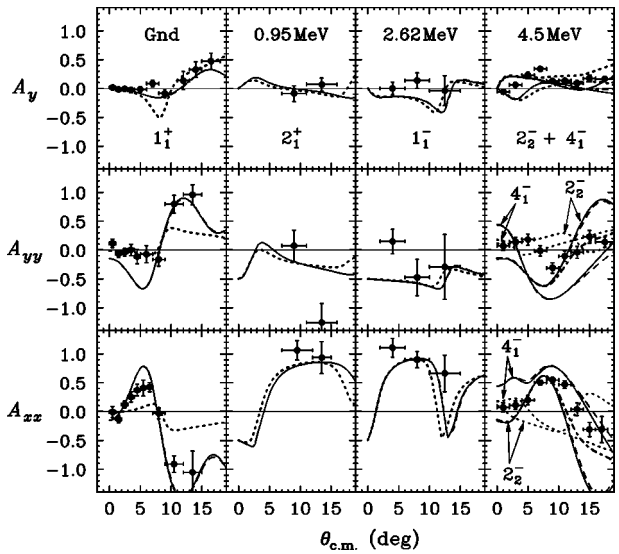

FIG. 3. Angular distributions of the vector- and tensor-analyzing powers A_y, A_{yy}, and A_{xx}, for the ^{12}C(d,^{2}He)^{12}B reaction. For the curves, see the caption of Fig. 2.

coupling, if they are drawn in the figures, are not distinguishable from the solid curves. It is somewhat unexpected since the D-wave breakup effect is known to be important for the deuteron elastic scattering [30]. Indeed $\Psi_{\rm ad}^{(+)}$ suffers nonnegligible effects from the D-wave coupling, but mainly at relatively large r. Those effects are then strongly hindered by the presence of short-range function $H_{L'l_a}^{ls'jt}$ and hardly appear at least in the forward-angle region where the data are available. The situation is similar for the (d,p) reaction where the D-wave breakup effect was reported to be relatively small [31].

The tensor component of the effective interaction [Eq. (17)], on the other hand, plays a definitely important role as in other charge-exchange reactions at similar energies. Their effects are shown by the difference between the dotted and solid curves in the figures. The data, particularly rich structures of A_{yy} and A_{xx} for unnatural-parity states, are never reproduced without the tensor interaction. But effects on analyzing powers for 2_1^+ and 1_1^- (natural-parity) states are, in contrast, very small. This is expected from the consideration in PWIA for 0^+ targets [7]. Analyzing powers for natural-parity states where $l=j$ components alone contribute are reduced to be quotients of simple functions of the interaction because of the strong selectivities of the reaction. Particularly A_{xx} becomes totally independent of the two-body interaction and of the form factor, giving the theoretically maximum value of $+1$. Likewise, for 0^- which exceptionally allows only $l=1$, $A_{yy}=+1$ and $A_{xx}=-2$. Although the problem is simplified in PWIA, it is worth noting that, from the parity conservation, $A_{yy}=+1$ for 0^- and $A_{yy}(0°)=A_{xx}(0°)=-0.5$ for natural-parity states irrespective of the reaction mechanism.

Figure 4(a) shows the calculated triple-differential cross sections plotted as a function of ϵ for the ^{12}C($d,pp[^1S_0]$)^{12}B[1^+;gnd] reaction at various scattering angles. For comparison, the prediction of the Migdal-Watson formalism [32] and the ϵ dependence solely determined by the overlap of spatial components of projectile wave functions, $|\langle\psi_b|\psi_a\rangle|^2\rho$, are presented by dashed and dotted curves, respectively. The shape of spectrum is almost independent of scattering angle at $\theta \geqslant 15°$ and is in good agreement with the Migdal-Watson formalism. As the scattering angle decreases, however, the distribution is relatively enhanced at small ϵ, becoming close to $|\langle\psi_b|\psi_a\rangle|^2\rho$ at very forward angles. $\chi^J_{Ll;Ll}(K;R,r)$ has little influence on the ϵ distribution since it depends on ϵ only through the energy conservation. The projectile wave functions therefore must be primarily responsible for this feature. The behavior of ϵ distribution is readily understood if the reaction is considered in the plane-wave limit. The integration over $\boldsymbol{r}$ is separated in this case, omitting explicit reference to the spin and isospin, to be $\langle\psi_b(\boldsymbol{k};\boldsymbol{r})|e^{i\boldsymbol{q}\cdot\boldsymbol{r}}|\psi_a(\boldsymbol{r})\rangle$, where $\boldsymbol{q}$ is the momentum transfer. At forward angles where $q\sim 0$, it is reduced to be $\langle\psi_b|\psi_a\rangle$, while at backward angles, due to cancellation by rapidly oscillating $e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$, the cross section reflects the ϵ dependence of short-range part of ψ_b and accordingly agrees with the Migdal-Watson formalism.

This feature of ϵ spectrum has not been confirmed experimentally since experimental apparatus used at intermediate energies has sensitivity only in small ϵ region [1–6] except

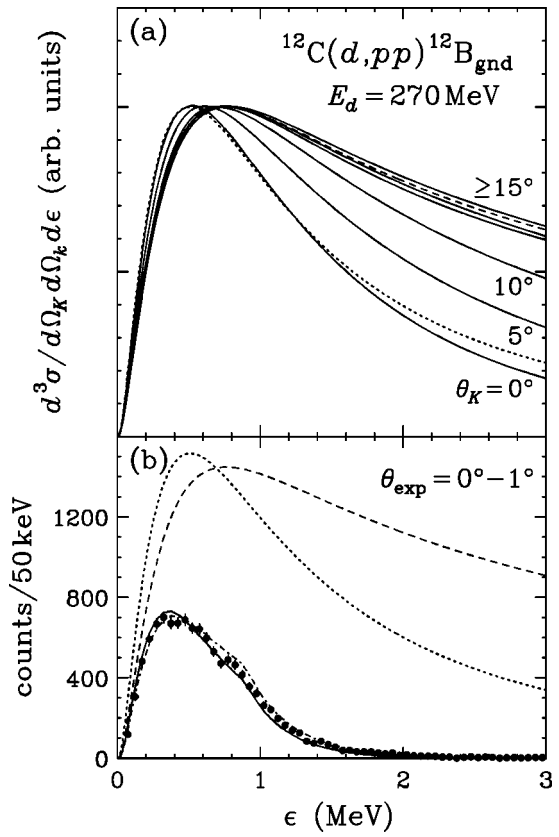

FIG. 4. (a) Calculated ϵ dependence of the cross section for the $^{12}\mathrm{C}(d,pp[^1S_0])^{12}\mathrm{B}[1_1^+;\mathrm{gnd}]$ reaction at scattering angles with an interval of 5° (solid curves). The amplitudes of the cross section are normalized at their maximum. The dotted curve represents $|\langle\psi_b|\psi_a\rangle|^2\rho$ (see text), and the dashed curve represents the prediction of the Migdal-Watson formalism [32]. (b) Experimental ϵ dependence of the spectrum at $\theta=0°$. Predictions of the Migdal-Watson formalism and of $|\langle\psi_b|\psi_a\rangle|^2\rho$ are shown normalized to the data by dot-dashed and solid curves, respectively, taking p-p detection efficiency of the detector system into account. The ratio between the dotted (dashed) and solid (dot-dashed) curves represents the detection efficiency of this system [9].

for the scintillator array used in Ref. [18], the tolerance of which for count rate, however, was too low to allow measurements on targets other than hydrogen. The situation for ordinary magnetic spectrometers is illustrated in Fig. 4(b), where a typical ϵ spectrum at $\theta=0°$ is compared with the predictions based on the Migdal-Watson formalism (solid curve) and on $|\langle\psi_b|\psi_a\rangle|^2\rho$ (dot-dashed curve) taking instrumental p-p detection efficiency into account [9]. The small difference between them substantially inhibits detailed discussion on the ϵ distribution. Besides, contributions from higher partial-waves which may not be neglected at large ϵ must be taken into consideration.

Still the ϵ dependence of the cross section is a subject for validity of the present method since the adiabatic approximation neglects the excitation of p-p system while the $(d,{}^2\mathrm{He})$ cross section is defined by introducing the integration limits on ϵ [Eq. (15)]. For the exit-channel ^{2}He scattering, the coupled descritized continuum channels (CDCC) method will obviously give a better description which should be compared with the present one. Though such a calculation is beyond the scope of this article, a rough estimate will be obtained from a similar reaction, i.e., the deuteron breakup. The agreement between the adiabatic approximation and CDCC for the deuteron breakup is known to be good if the incident energy is sufficiently large (>80 MeV) [30]. Also the energy produced in the charge exchange process is expected to be small and have little influence on the ϵ distribution, since the elementary $^1\mathrm{H}(d,{}^2\mathrm{He})n$ reaction is well described up to $\epsilon=8$ MeV by the PWIA [18] which also neglects the excitation in p-p system.

IV. SUMMARY AND DISCUSSION

A treatment of the $(d,{}^2\mathrm{He})$ reaction describing the three-body final state in the adiabatic approximation has been presented. Calculations are carried out taking the coupling of S and D waves of the p-p system as well as contributions from the deuteron D state into account. Both effects from the D-wave coupling and the deuteron D state are found to be very small at least in the forward-angle region, due to the short-range nature of the form factor. Noting that the reaction inherently shows steep angular distributions because of the broadly spread projectile wave functions and thus measurements are feasible only at forward angles, those effects are safely neglected in analyses of the experimental data, which serves to save computing time considerably. The tensor component of the effective interaction, on the other hand, plays an important role even in this angular region. Though it is known to give good results for nucleon-induced reactions in this energy region, the treatment of SNKE may be a remaining subject; effects for the central interaction are included by the short-range approximation, while those for the tensor interaction are neglected. It is worth noting that, in Ref. [33], both components of SNKE were calculated exactly for a heavy-ion charge exchange reaction and contributions from the tensor component were found to be negligibly small.

The results of the present calculation are in reasonably good agreement with the data at 270 MeV on the ^{12}C target, though an overall normalization factor of 0.7 is required to fit the cross section. Nearly the same normalization factor (0.6) was needed also in the DWBA calculation given in Ref. [1], even after searching for the ^{2}He optical potential. While the absolute value of the cross section is a subject of various models introduced in the calculation, the origin of this factor is most likely to be ascribed to ambiguities in estimation of two-proton tracking efficiency for the detector system [9]. A systematic study in this direction is in progress with targets such as ^{6}Li, ^{9}Be, ^{13}C and sd-shell self-conjugate nuclei, the transition strengths of which are known from β decay or (p,n) reaction as well as shell-model calculations. The degree of fit to the experimental data, including analyzing powers, is nearly the same as that of the DWBA calculation. But the advantage of the present method is that the calculation is performed in a parameter-free manner, i.e., without introducing the unphysical ^{2}He optical potential. This allows applications to analyses of data for heavier targets with neutron excess, where the role of (n,p)-type reaction becomes really important but discrete and known levels are not available for

references to examine the reaction mechanism.

It is of interest to evaluate contributions from higher partial waves than $^1S_0$ in p-p relative energy spectra since it has never been done on nuclear (nonhydrogen) targets while experimental study on such targets is definitely difficult as shown in the previous section. In the present formalism, calculation of $^1D_2$ component is straightforward though it is rather involved than that for $^1S_0$ [Eq. (13)]. Before doing that, however, contributions from ^3P waves which should become important at lower ϵ than $^1D_2$ must be evaluated. This requires inclusion of the spin-orbit component in U_{pB} and accordingly a reformulation of the present approximation. A treatment of spin-orbit potential in coupled-channels approach is discussed for the elastic scattering by Tanifuji and Iseri [34], and a similar technique may be applied to the present problem. It will also be possible in this case to treat the incident-channel wave function in the adiabatic approximation and to include effects of the deuteron breakup which may be important as discussed in Ref. [12]. The presence of spin-orbit part in U_{pB}, however, allows transitions to the spin-singlet states of p-p system without spin transfer to the target, which makes the problem much more complicated. Such investigations are therefore reserved for future work.

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[1] H. Okamura *et al.*, Phys. Lett. B **345**, 1 (1995).
[2] H. M. Xu *et al.*, Phys. Rev. C **52**, R1161 (1995); **54**, 3266 (1996).
[3] T. Niizeki *et al.*, Nucl. Phys. **A577**, 37c (1994).
[4] T. Inomata *et al.*, Phys. Rev. C **57**, 3153 (1998).
[5] H. Ohnuma *et al.*, Phys. Rev. C **47**, 648 (1993).
[6] C. Ellegaard *et al.*, Phys. Rev. Lett. **59**, 974 (1987); Phys. Lett. B **231**, 365 (1989).
[7] D. V. Bugg and C. Wilkin, Nucl. Phys. **A467**, 575 (1987); C. Wilkin and D. V. Bugg, Phys. Lett. **154B**, 243 (1985).
[8] T. Motobayashi *et al.*, Nucl. Instrum. Methods Phys. Res. A **271**, 491 (1988).
[9] H. Okamura *et al.*, Nucl. Instrum. Methods Phys. Res. A **406**, 78 (1998).
[10] N. Austern, Phys. Rev. C **30**, 1130 (1984).
[11] M. Yahiro, J. A. Tostevin, and R. C. Johnson, Phys. Rev. Lett. **62**, 133 (1989).
[12] S. Rugmai, J. S. Al-Khalili, R. C. Johnson, and J. A. Tostevin, Phys. Rev. C **60**, 027002 (1999).
[13] R. C. Johnson and P. J. R. Soper, Phys. Rev. C **1**, 976 (1970).
[14] M. A. Nagarajan, I. J. Thompson, and R. C. Johnson, Nucl. Phys. **A385**, 525 (1982).
[15] H. Amakawa and T. Tamura, Phys. Rev. C **26**, 904 (1982).
[16] G. R. Satchler, Nucl. Phys. **55**, 1 (1964).
[17] N. Austern, R. M. Drisko, E. C. Halbert, and G. R. Satchler, Phys. Rev. **133**, B3 (1964).
[18] S. Kox *et al.*, Nucl. Phys. **A556**, 621 (1993).
[19] J. Carbonell, M. B. Barbaro, and C. Wilkin, Nucl. Phys. **A529**, 653 (1991).
[20] T. Ohnishi *et al.*, RIKEN Report No. 29, 1996 (unpublished), p. 56; in *Proceedings of the 12th International Symposium on High-Energy Spin Physics, Amsterdam, 1996*, edited by C. W. de Jager, T. J. Ketel, P. J. Mulders, J. E. J. Oberski, and M. Oskam-Tamboezer (World Scientific, Singapore, 1997), p. 486; Y. Satou (private communication).
[21] W. Bauhoff *et al.*, Nucl. Phys. **A410**, 180 (1983).
[22] R. V. Reid, Jr., Ann. Phys. (N.Y.) **50**, 411 (1968).
[23] M. A. Franey and W. G. Love, Phys. Rev. C **31**, 488 (1985).
[24] M. Golin, F. Petrovich, and D. Robson, Phys. Lett. **64B**, 253 (1976).
[25] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).
[26] D. J. Millener and D. Kurath, Nucl. Phys. **A255**, 315 (1975).
[27] J. Rapaport *et al.*, Phys. Rev. C **24**, 335 (1981).
[28] C. Gaarde *et al.*, Nucl. Phys. **A422**, 189 (1984).
[29] X. Yang *et al.*, Phys. Rev. C **48**, 1158 (1993).
[30] M. Yahiro, Y. Iseri, H. Kameyama, M. Kamimura, and M. Kawai, Suppl. Prog. Theor. Phys. **89**, 32 (1986); N. Austern, Y. Iseri, M. Kamimura, M. Kawai, G. Rawitscher, and M. Yahiro, Phys. Rep. **154**, 125 (1987), and references therein.
[31] G. H. Rawitscher, Phys. Rev. C **11**, 1152 (1975).
[32] K. M. Watson, Phys. Rev. **88**, 1163 (1952); A. B. Migdal, Zh. Éksp. Teor. Fiz. **28**, 3 (1955) [Sov. Phys. JETP **1**, 2 (1955)].
[33] A. Etchegoyen, E. D. Izquierdo, and M. C. Etchegoyen, Phys. Lett. B **231**, 224 (1989).
[34] M. Tanifuji and Y. Iseri, Prog. Theor. Phys. **87**, 247 (1992); Y. Iseri and M. Tanifuji, Phys. Lett. B **354**, 183 (1995).