Structure of ^{12}Be and ^{12}O ground states

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Consideration of Coulomb energies for ^{12}O and ^{12}Be, combined with other information, strongly suggests that the ^{12}Be(g.s.) contains about 52% of the configuration ^{10}Be(g.s.)$\otimes(2s\frac{1}{2})^2$. [S0556-2813(99)04512-4]

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I. INTRODUCTION

In a recent paper, Barker [1] computes the width of the ground state of ^{12}O, the mirror of ^{12}Be. Involved are the proportions of the two nucleons p^2,s^2,d^2 and the energy of the ^{11}N($\frac{1}{2}^+$) ground state. The configuration admixture is the same as for its mirror ^{12}Be.

A reasonable shell-model calculation of the structure of the lowest 0^+, $T=2$, level for $A=12$ is very difficult. It involves both p- and sd-shell excitations, and hence two major problems: (1) the competition between internal p-shell excitations and components of the type ^{10}Be$\otimes(sd)^2$, and (2) within $(sd)^2$, the competition between $(s\frac{1}{2})^2$ and $(d\frac{5}{2})^2$. The two-body residual interaction within the sd shell has 0^+ diagonal elements that are significantly more attractive for $(d\frac{5}{2})^2$ than for $(s\frac{1}{2})^2$. But for ^{10}Be$+n$, the $s\frac{1}{2}$ single-particle energy is significantly below that for $d\frac{5}{2}$, so that the one-body $(s\frac{1}{2})^2$ Hamiltonian is significantly lower than for $(d\frac{5}{2})^2$. The upshot is that, even within $(sd)^2$ space, small changes in the interaction can drastically alter the ratio of s^2 to d^2 in the lowest state. And, of course, the ratio of s^2 to d^2 greatly affects mixing with the p-shell components. Furthermore, the lowest p-shell 0^+ state [2] is roughly degenerate with the lowest $(sd)^2$ state [3]. Because of this shell-model instability, we seek other avenues for clues to the s^2:d^2:p shell admixture in ^{12}Be(g.s.).

II. EXPERIMENTAL INFORMATION

The experimental information is sparse and ambiguous. The β decay of ^{12}Be, the ^{10}Be(t,p), and the ^{15}N(^{3}He,^{6}He) reactions offers some information.

Suzuki and Otsuko [4] note that the $\log ft$ for the decay of ^{12}Be is about 3.83 while p-shell calculations yield 3.4–3.5. They estimate that the $(p\frac{1}{2})^2$ occupancy is less than 35%. Keller *et al.* [5] searched for decay to the 1^- level of ^{12}B which requires $(2s)^2$ in ^{12}Be. They conclude that the latter is no more than 20%. Barker [1] uses recent half-life data to arrive at 50% for $(p\frac{1}{2})^2$. The β decay has shed no light on the amount of $d(\frac{5}{2})^2$.

In the ^{10}Be$(t,p)^{12}$Be reaction [3], the calculated cross sections for the ground state (g.s.) show negligible differences in shape for p^2, s^2, or d^2 transfer. A shell-model calculation yielded a ratio of 1.38 for s^2/d^2 components in the lowest $0^+(sd)^2$ state. With that s^2/d^2 ratio, the $(sd)^2$ cross sections are 7 times those for Cohen and Kurath's [2] ^{10}Be $+p^2\rightarrow{}^{12}$Be transfer.

Assuming the neutron p shell is filled in ^{15}N, the three-neutron pickup in ^{15}N(^{3}He,^{6}He) should populate the $(p\frac{1}{2})^2$ neutron piece of ^{12}N$(0^+$,2). A measurement at 76 MeV was reported by Robertson [6]; no sign was found of such a level at the expected energy of 12.3 MeV, from the isobaric multiplet mass equation. (A similar attempt at Princeton also failed.) An upper limit was set which was $\frac{1}{4}$ of that for the same reaction to the ^{8}B$(0^+$,2) state, suggesting that the $(p\frac{1}{2})^2$ strength in ^{12}Be might be less than 25%. Of course, with the dominant p^{-1} g.s. of ^{15}N the $(sd)^2$ components of the ^{12}N$(0^+$,2) state could not be populated.

Table I lists the reported [1,3–9] percentage populations for the three components and the ratio s^2/d^2. Results of structure calculations are included. The wide variation is discouraging, but excluding Ref. [7], the indication is that the p strength is less than 35% and the ratio of s^2/d^2 averages to 1.30. We have calculated Coulomb shifts across the $(0^+$,2) quintet to estimate the $(s\frac{1}{2})^2$ strength.

III. POTENTIAL MODEL CALCULATIONS

The procedure is to use a Woods-Saxon nuclear potential, plus a uniform sphere Coulomb potential, to compute energies of a nucleon plus the appropriate $A=11$, $T=\frac{3}{2}$ nucleus. Of relevance here is a $2s\frac{1}{2}$, $1p\frac{1}{2}$, or $1d\frac{5}{2}$ nucleon coupled to $\frac{1}{2}^+$, $\frac{1}{2}^-$, and $\frac{5}{2}^+$ core states, to make 0^+, $T=2$. This proce-

TABLE I. Two-neutron occupancy (%) for ^{12}Be(g.s.) as ^{10}Be $+2n$.

Ref.	$(1p)^2$	$(2s)^2$	$(1d)^2$	$(2s)^2/(1d)^2$	Source
[6]	25				(^{3}He,^{6}He)
[8]	38	32	29	1.10	Calc.
[3]				1.38	Calc.
[5]		20			β decay
[7]	74	8	18	0.39	Calc.
[9]				1.42	Calc.
[4]	35				β decay
[1]	50				β decay

TABLE II. Calculated and experimental excitation energies (MeV) in ^{12}B, ^{12}C, and ^{12}O for pure configurations and a mixture thereof.

	^{12}B	^{12}C	^{12}O
Expt.	12.75 (0.5)	27.60 (0.02)	0 (0.04)
$(1p)^2$	12.81	27.77	0.53
$(2s)^2$	12.71	27.56	−0.52
$(1d)^2$	12.80	27.75	0.63
M [a]	12.75	27.65	−0.03

[a] M is $0.55(2s^2)+0.45[(1p)^2+(1d)^2]$.

dure has been used successfully for $T=2$ quintets with $A=16$ [11] and 20 [12]. In $A=16$, the competition is between d^2 and s^2, with no p^2 component. The result [11] is an s^2 percentage of $(45\pm4)\%$, to be compared with a value of 53% computed [13] in a two-body shell model—using global two-body residual matrix elements, but ''local'' single-particle energies. In $A=20$, there is no evidence for any appreciable s contribution for the 19th and 20th nucleons [12].

With the usual parameters $r_0=1.25$ and $a=0.65$ fm for the Woods-Saxon well, the well depth V_0 is chosen to correctly bind a $1p$, $2s$, or $1d$ neutron to the $\frac{1}{2}^-$, $\frac{1}{2}^+$, and $\frac{5}{2}^+$ levels of ^{11}Be. The consequential energies for ^{12}B, ^{12}C, and ^{12}O and the experimental values are listed in Table II. The ^{12}O values were found using calculated [10] values 1.60, 2.48, and 3.90 MeV for the $\frac{1}{2}^+$, $\frac{1}{2}^-$, and $\frac{5}{2}^+$ levels of the ^{11}N core. Thus, this is really equivalent to coupling two nucleons to ^{10}C.

As expected, the pure $(2s)^2$ predictions are low while the $(1p\frac{1}{2})^2$ and $(1d)^2$ values are high and about the same. Thus an approximately equal admixture of $(2s)^2$ and either $(1p)^2$ or $(1d)^2$ would match the experimental values.

Because the p^2 and d^2 results are so similar, we can lump them together to get a one-parameter description of the Coulomb energies, viz., the amount of the $(2s\frac{1}{2})^2$ component. The result is $0.55(2s\frac{1}{2})^2+0.45$ (average of p^2 and d^2), whose energies are listed as the last entry in Table II.

Alternately, we can use the ^{12}O Coulomb energy calculations for the three configurations, together with unit normalization $[\alpha^2(s^2)+\beta^2(d^2)+\gamma^2(p\text{ shell})=1]$, to obtain any two of the three intensities in terms of the other one. When we do this, we find $\gamma^2<0.495$, and $\beta^2=0$, with $\alpha^2=0.505$ at that limit. When $\gamma^2=0$, β^2 is 0.45, giving $\alpha^2=0.55$. For any value of γ^2, $\alpha^2/\beta^2>1.21$. The striking point is that as γ^2 varies from 0 to 0.495, α^2 changes only from 0.55 to 0.50. Putting in the 40 keV uncertainty in ^{12}O(g.s.) mass gives $0.50\pm.035\leqslant\alpha^2\leqslant0.55\pm.035$.

With the $(sd)^2$ Hamiltonian of Ref. [13] and ^{11}Be single-particle energies for $s\frac{1}{2}$ and $d\frac{5}{2}$, the lowest $(sd)^2 0^+$ state has 78% s^2, 22% d^2. If the physical g.s. arises from mixing of this state with the CK p-shell g.s. [ignoring the second $(sd)^2 0^+$], then our result of $0.50\leqslant\alpha^2\leqslant0.55$ implies $|\psi(\text{g.s.})|^2=(0.64\text{–}0.70)(sd)^2_{0_1^+}$ and hence (0.30–0.36) p shell. For the remainder of this paper, we adopt a mixture of 67%, giving 52% s^2, 15% d^2, and 33% p shell. If the experimental state in ^{12}Be at 2.70 MeV is the other 0^+ level arising from mixing $(sd)^2$ and CK, the observed separation and the intensities above provide a value of $V=1.27$ MeV for the residual matrix element mixing the two. This seems quite a reasonable value. With this value of V, the initial separation between $(sd)^2$ and CK would have been 0.916 MeV, very close to the relative separation calculated: 0.211 MeV for $(sd)^2$, 1.095 MeV for CK. Of course, both would have to be too low by about 0.7 MeV in order for the *absolute* energies to correspond.

Barker [1] obtained what he termed as upper limits on the ^{12}O(g.s.) width for sequential proton decay through ^{11}N(g.s.). With the latter energy at 1.5 MeV, his upper limits were in the range 40–50 keV. It would seem he should then multiply these upper limits by the product of spectroscopic factors $S_1S_2/2$ for the two decays. He takes $S_2=0.40$ (20% s^2); S_1 is known experimentally to be 0.77. Thus, he would have $\Gamma_{\text{TOT}}(2p)=12\text{–}15$ keV, although he quotes ''less than 100 keV,'' and again ''less than about 100 keV.'' If his upper limits correspond to $S_1S_2=2$, then even with our 52% s^2, S_1S_2 is still only 0.80—still giving a very small ^{12}O width, if Barker's convolution is correct. These widths suggest that perhap simultaneous $2p$ (i.e., ^{2}He) decay could be important.

With this admixture we compute the decay widths for $^{12}\text{O}\rightarrow{}^{10}\text{C}+$ two protons. The diproton (^{2}He) energy is 1.78 MeV. For $(1p^2)$ the diproton cluster has one node and $L=0$, while for $(2s)^2$ or $(1d)^2$, there are two nodes and $L=0$. The results for the ''single-particle'' width $\Gamma(^2\text{He})$ are 163 keV and 227 keV, respectively. Our proposed admixture suggests a width of about 340 keV for *simultaneous* $2p$ decay.

Experimental results for the decay have been reported by Kekelis *et al.* [14] to be 400 ± 250 keV and by Kryger *et al.* [15], more recently, to be 578 ± 205 keV. Barker's [1] width of less than 100 keV, found from R-matrix formulas for *sequential* decay, is much smaller than any of these. Ours is not inconsistent with the experimental value. We suggest the need for a new experiment to look for simultaneous two-proton decay.

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