# Continuum level density in a microscopic cluster model: Parameters of resonances

K. Arai
*Department of Physics, Niigata University, Niigata 950-2181, Japan*

A. T. Kruppa
*Institute of Nuclear Research of the Hungarian Academy of Sciences, P.O. Box 51, Debrecen H-4001, Hungary*

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The positions and widths of nuclear resonance states of the nuclei $^8$Be, $^5$He, and $^5$Li have been calculated in the microscopic cluster model using a real square integrable basis. The imposition of Gamow or scattering asymptotic boundary conditions onto the wave function is avoided. The approach is based on the notion of the continuum level density. This density is smoothed by the Strutinsky averaging procedure and it is calculated by making use of the eigenvalues of the full and the free Hamiltonian matrices. The continuum level density is connected to the $S$ matrix and has a Breit-Wigner peak around the resonance energy. This approach is compared with the complex scaling method and with the exact calculation of the scattering phase shift. [S0556-2813(99)03112-X]

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## I. INTRODUCTION

The study of resonance properties is one of the most important subjects in nuclear physics [1]. This topic has recently become even more significant since drip line nuclei can be experimentally investigated [2]. The ground states of these nuclei have a small binding energy and most of the excited states lie above the particle threshold. A possible theoretical tool to describe the particle unstable states or resonance bumps in cross sections is the notion of resonance state.

Few methods have been developed for the description of resonance states using bound-state-type technique, for example, the different versions of the $L^2$-stabilization method [3–13] and the complex scaling method [14–22]. The main merit of these approaches is that the sophisticated models and computer codes which deal with the bound state problem may be applied to resonance state calculations.

The $L^2$-stabilization method is widely used in atomic physics. This approach is based on box quantization and on the repeated diagonalization of the Hamiltonian using a larger and larger box size, at such a region where the resonance wave function is localized. The stabilization diagram (the energy versus the box size) shows flat behavior around the resonance energy. Recently Mandelshtam *et al.* [5] have developed a new version of the stabilization method. The density of states is calculated which has a peak around the resonance energy. The resonance parameters, the position $E_r$ and the width $\Gamma_r$, can be derived from the density of states.

Recently we have generalized [23] the stabilization method of Mandelshtam, Ravuri, and Taylor employing the notion of the continuum level density (CLD). The basic definition of the CLD utilizes the Green operators of the interacting and the free systems. We showed that an approximate CLD, smoothed by the Strutinsky averaging procedure, can be determined without box quantization or without the imposition of the scattering asymptotic boundary condition onto the wave function. The smoothed CLD can be calculated by making use of an arbitrary real square integrable basis. The CLD is connected to the scattering $S$ matrix and gives a Breit-Wigner-type peak around the resonance energy and so the resonance parameters can be determined.

In this paper we present how to apply the CLD method in the framework of the microscopic cluster model and we demonstrate that our approach works also in the presence of the long-range Coulomb interaction. We have applied the CLD method to the well-known two-cluster states of the light nuclei $^8$Be, $^5$He, and $^5$Li. These nuclei are described by the microscopic two-cluster models $\alpha+\alpha$, $\alpha+n$, and $\alpha+p$, respectively. The smoothed approximate CLD is calculated by expanding the relative motion wave function of the clusters on a finite number of shifted Gaussians. We have found that the approximate smoothed CLD is consistent with the exact density derived from the scattering phase shift. The parameters of the low-lying resonance states predicted by the CLD method are compared with two other methods, namely, the complex scaling approach and scattering phase shift calculation.

In Sec. II the microscopic cluster model is presented; in Sec. III we outline the calculation of the approximate CLD. The results for $^8$Be, $^5$He, and $^5$Li are shown in Sec. IV and a summary is given in Sec. V.

## II. MICROSCOPIC CLUSTER MODEL

The models considered here are the microscopic cluster models of the type of the resonating group method (RGM) [24,25]. The main characteristics of this nuclear model are the following. The nucleons are assumed to be arranged in clusters and all nucleons are treated explicitly. The wave function is constructed so as to satisfy the Pauli principle exactly. The model is free from spurious center-of-mass motion, and has good total angular momentum and parity. The nucleons are assumed to interact via an effective nucleon-nucleon interaction. The cluster intrinsic wave functions are taken to be simple shell-model wave functions built up from 0$s$ harmonic-oscillator states. The relative wave function between the clusters is expanded over some basis functions. All

 

symmetries are kept by the trial function itself (''projection before variation'').

For the trial wave function we take the following ansatz:

$$\Psi_{JM}=\sum_k C_k\psi_k=\sum_k C_k\mathcal{A}\{[\Phi_S\Gamma_L(s_k,\boldsymbol{\rho})]_{JM}\}, \quad (1)$$

where the symbol $\mathcal{A}$ is the intercluster antisymmetrizer and $J$ and $M$ are the total angular momentum and its $z$ component, respectively. The function $\Phi_S$ is the product of the cluster intrinsic wave functions:

$$\begin{aligned}\Phi_{SM_S}&=[\Phi_\alpha\Phi_\alpha]_{SM_S} \text{ for the } \alpha+\alpha \text{ system,}\\ &=[\Phi_\alpha\Phi_n]_{SM_S} \text{ for the } \alpha+n \text{ system,}\\ &=[\Phi_\alpha\Phi_p]_{SM_S} \text{ for the } \alpha+p \text{ system.}\end{aligned} \quad (2)$$

The total spin $S$ is 0 for the system $\alpha+\alpha$ and 1/2 for the system $\alpha+n$ or the $\alpha+p$. $\Phi_\alpha$ is the $\alpha$-cluster intrinsic wave function which is described by the 0$s$ harmonic-oscillator shell model with a single size parameter $\nu_\alpha$. $\Phi_n$ and $\Phi_p$ are the spin-isospin function of the neutron and proton, respectively.

The wave function of the intercluster motion is approximated by a liner combination of the shifted Gaussian basis

$$\begin{aligned}\Gamma_{LM}(s_k,\boldsymbol{\rho})&=4\pi\left(\frac{\pi}{4\gamma\rho s_k}\right)^{1/2}\left(\frac{2\gamma}{\pi}\right)^{3/4}\\ &\quad\times e^{-\gamma(\rho^2+s_k^2)}\ I_{L+1/2}(2\gamma\rho s_k)Y_{LM}(\hat{\boldsymbol{\rho}})\\ &=\int d\hat{\mathbf{s}}_k\left(\frac{2\gamma}{\pi}\right)^{3/4}e^{-\gamma(\boldsymbol{\rho}-\mathbf{s}_k)^2}Y_{LM}(\hat{\mathbf{s}}_k),\end{aligned} \quad (3)$$

where $\boldsymbol{\rho}$ is the relative coordinate between the two clusters and $L$, $M$ are the cluster-relative orbital angular momentum and its $z$ component, respectively. $I_{L+1/2}(x)$ denotes the modified Bessel function. In the conventional generator coordinate method (GCM) for computational convenience the parameter $\gamma$ in Eq. (3) is set to be $\gamma=A_1A_2\nu_\alpha/(A_1+A_2)$, where $A_1$ and $A_2$ are the mass numbers of the clusters. In our ansatz this $\gamma$ parameter is chosen to be independent of $\nu_\alpha$. In this way our model becomes Tohsaki-Suzuki's new GCM [26].

The coefficients $C_k$ in Eq. (1) are obtained by application of the Rayleigh-Ritz variational principle. The model wave function in Eq. (1) can be considered as an approximate solution of the eigenvalue problem of the $A$-nucleon Hamiltonian

$$\hat{H}=\sum_{i=1}^{A}\hat{T}_i-\hat{T}_{c.m.}+\sum_{i<j}^{A}\hat{V}_{i,j}, \quad (4)$$

where $\hat{T}_i$ is the kinetic energy of the $i$th nucleon, $\hat{T}_{c.m.}$ is the kinetic energy of the center of mass, and $\hat{V}_{i,j}$ is the effective nucleon-nucleon interaction. In a microscopic two-cluster model this Hamiltonian can be rewritten as

$$\hat{H}=\hat{H}_A^{\rm int}+\hat{H}_B^{\rm int}+\hat{H}^{\rm rel}, \quad (5)$$

where $\hat{H}_A^{\rm int}$ and $\hat{H}_B^{\rm int}$ are the internal Hamiltonian of the clusters $A$ and $B$, respectively. $\hat{H}^{\rm rel}$ is the intercluster Hamiltonian, given by

$$\hat{H}^{\rm rel}=-\frac{\hbar^2}{2\mu}\Delta_\rho+\sum_{i\in A}\sum_{j\in B}\hat{V}_{i,j}, \quad (6)$$

where $\mu$ is the reduced mass.

## III. CONTINUUM LEVEL DENSITY

If a Hamiltonian $\hat{H}$ contains only one dynamical coordinate and has continuous spectrum, then the continuum level density $\Delta(E)$ is defined [27,28] with the help of Green operators,

$$\Delta(E)=-\frac{1}{\pi}\mathrm{Im}\{\mathrm{Tr}[\hat{G}(E+i0)-\hat{G}^0(E+i0)]\}, \quad (7)$$

where $\hat{G}(z)=(z-\hat{H})^{-1}$ is the full Green operator, $\hat{G}^0(z)$ stands for the free Green operator, $\hat{G}^0(z)=(z-\hat{H}_0)^{-1}$, and $H_0$ is the free Hamiltonian (this is defined more precisely later). In Eq. (7), $\mathrm{Tr}[\hat{G}(E+i0)]$ means the trace of the operator $\hat{G}(E+i0)$ and $\mathrm{Im}[z]$ is the imaginary part of $z$. The notation $\hat{G}(E+i0)$ means the limit $\lim_{\varepsilon\to+0}\hat{G}(E+i\varepsilon)$.

It is known that the CLD, in the case of a spherically symmetric Hamiltonian, is proportional to the derivative of the scattering phase shift $\delta(E)$ [27,28]:

$$\Delta(E)=\frac{1}{\pi}\frac{d\delta(E)}{dE}. \quad (8)$$

Since the behavior of the phase shift around a resonance energy is known [29], $\Delta(E)$ can be turned into the following form in the vicinity of a resonance:

$$\Delta(E,E_r,\Gamma_r)=\Delta^r(E,E_r,\Gamma_r)+\Delta^{\rm bg}(E). \quad (9)$$

The background term $\Delta^{\rm bg}(E)$ is a slowly changing function of the energy, and the resonance term has a Breit-Wigner shape:

$$\Delta^r(E,E_r,\Gamma_r)=\frac{1}{\pi}\frac{\Gamma_r/2}{(E-E_r)^2+\Gamma_r^2/4}, \quad (10)$$

where $E_r$ and $\Gamma_r$ are the resonance position and width, respectively. This term produces a sharp peak around $E_r$ in the CLD.

The CLD can be calculated using Eq. (8); it is expressed by the scattering phase shift. The direct application of this equation should mean that the Schrödinger equation with a scattering boundary condition has to be solved. We have shown in Ref. [23] that the CLD, smoothed by the Strutinsky averaging procedure, can be approximated accurately by an arbitrary real square integrable basis without imposing explicitly the scattering asymptotic boundary condition onto

the wave function. If $N$ basis functions are used to solve the eigenvalue problem of the Hamiltonian, then the approximate CLD $\Delta^N(E)$ can be turned into the following form [23]:

$$\Delta^N(E)=\sum_{i=1}^{N}\delta(E-e_i)-\sum_{i=1}^{N}\delta(E-e_i^0), \tag{11}$$

where $\delta(E-e_i)$ is the Dirac $\delta$ function. The eigenvalues of the full Hamiltonian matrix

$$H_{m,n}=\langle\psi_m|\hat{H}|\psi_n\rangle \tag{12}$$

are denoted by $e_i$. In the above equation $\psi_m$ and $\hat{H}$ are defined by Eqs. (1) and (4), respectively.

The quantities $e_i^0$ are the eigenvalues of the free Hamiltonian matrix

$$H^0_{m,n}=\langle\psi^D_m|\hat{H}_0|\psi^D_n\rangle, \tag{13}$$

where the free Hamiltonian $\hat{H}_0$ reads

$$\hat{H}_0=\hat{H}^{\rm int}_A+\hat{H}^{\rm int}_B+\hat{H}^{\rm rel}_0, \tag{14}$$

and $\hat{H}^{\rm rel}_0$ is given by

$$\hat{H}^{\rm rel}_0=-\frac{\hbar^2}{2\mu}\Delta_\rho+\frac{Z_AZ_Be^2}{\rho}. \tag{15}$$

The second term in the above equation is the Coulomb interaction and $Z_A$ and $Z_B$ are the charge numbers of the clusters $A$ and $B$. In the microscopic cluster model the wave function $\psi_m$ can be divided into the terms

$$\psi_m=\psi^D_m+\psi^E_m, \tag{16}$$

where $\psi^D_m$ and $\psi^E_m$ are the direct and exchange terms of $\psi_m$. In Eq. (13) only the direct term should be used.

Since Dirac $\delta$ functions appear in Eq. (11), this expression is not convenient to be considered. The approximate CLD is smoothed by the Strutinsky procedure [30]. A smoothed CLD can be introduced by the folding integral

$$\overline{\Delta^N}(E)=\frac{1}{\Gamma}\int_0^\infty w\left(\frac{E'-E}{\Gamma}\right)\Delta^N(E')dE', \tag{17}$$

where $\Gamma$ is the range parameter of the smoothing. Here we take the Lorentzian shape for the weight function, $w(x)=1/[2\pi(x^2+1/4)]$. After the Strutinsky smoothing procedure the smoothed approximate CLD is expressed as the difference of the two densities

$$\overline{\Delta^N}(E)=\sum_{i=N-N_B}^{N}\frac{1}{\Gamma}w\left(\frac{E-e_i}{\Gamma}\right)-\sum_{i=1}^{N}\frac{1}{\Gamma}w\left(\frac{E-e_i^0}{\Gamma}\right), \tag{18}$$

where $N_B$ ($N_B\leqslant N$) denotes the number of the bound state eigenvalues of the full Hamiltonian matrix. Here we assumed that the eigenvalues are arranged in increasing order of the energy.

In order to compare the approximate CLD with the exact CLD we have to carry out the Strutinsky smoothing of the exact density, too. The exact CLD around the resonance energy has the form of Eq. (9); the smoothed exact CLD has also a similar form

$$\overline{\Delta}(E,E_r,\Gamma_r)=\overline{\Delta}^r(E,E_r,\Gamma_r)+\overline{\Delta}^{\rm bg}(E). \tag{19}$$

The advantage of the Lorentzian shape for the weight function is that the smoothing of the Breit-Wigner shape can be calculated analytically,

$$\begin{aligned}\overline{\Delta}^r(E,E_r,\Gamma_r)=&\frac{\Gamma_r\beta}{2\pi^2}\left\{(E-E_r)^2-\frac{(\Gamma^2-\Gamma_r^2)}{4}\right\}\\&\times\left\{\frac{\pi}{2}+\tan^{-1}\left(\frac{2E}{\Gamma}\right)\right\}\\&+\frac{\Gamma\beta}{2\pi^2}\left\{(E-E_r)^2+\frac{(\Gamma^2-\Gamma_r^2)}{4}\right\}\\&\times\left\{\frac{\pi}{2}+\tan^{-1}\left(\frac{2E_r}{\Gamma_r}\right)\right\}\\&+\frac{\Gamma\Gamma_r}{4\pi^2}(E-E_r)\beta\log\left[\frac{4E^2+\Gamma^2}{4E_r^2+\Gamma_r^2}\right]\end{aligned} \tag{20}$$

and

$$\beta\equiv\{[(E-E_r)^2+(\Gamma-\Gamma_r)^2/4][(E-E_r)^2+(\Gamma+\Gamma_r)^2/4]\}^{-1}. \tag{21}$$

It should be noted that in the limit of $\Gamma\to0$, $\overline{\Delta}^r$ tends to the nonsmoothed density $\Delta^r$.

The smoothed CLD has also a peak around a resonance energy. The resonance parameters $E_r$ and $\Gamma_r$ are determined by minimizing the quantity

$$\sum_{i=1}^{n}[\overline{\Delta}^N(\epsilon_i)-\overline{\Delta}(\epsilon_i,E_r,\Gamma_r)]^2, \tag{22}$$

where $n$ points $\epsilon_i$ ($i=1,\ldots,n$) are taken equidistantly at the region of $E_{\rm peak}-d<\epsilon_i<E_{\rm peak}+d$. Here $E_{\rm peak}$ is the peak position of the smoothed approximate CLD and $2d$ is the size of the interval of the fitting. The background term $\overline{\Delta}^{\rm bg}(E)$ is described by a second order polynomial in $E$.

To check the validity of the approximate calculation of the CLD in the microscopic cluster model, we have also calculated the scattering phase shift solving the Schrödinger equation exactly with scattering asymptotic boundary condition. We derived the smoothed exact CLD by means of Eq. (8). The $R$-matrix method [31] is chosen for the scattering calculation. The internal region of wave function is approximated by a superposition of centered Gaussian basis.

The resonance parameters can be determined from the scattering phase shift. Two different methods are considered. In the first method it is assumed that the derivative of the scattering phase shift, around the resonance energy, has the form of the sum of a Breit-Wigner term and a background

term (second order polynomial in $E$); see Eqs. (8) and (9). The resonance parameters are determined from a fitting procedure. The first method is denoted ''phase shift (a)'' in this paper. In the second approach the resonance position $E_r$ is defined as the inflection point of the phase shift $\delta(E)$ (the peak position of $d\delta/dE$) and the resonance width is calculated as $\Gamma_r = 2/(d\delta/dE)_{E=E_r}$ [32]. The second method is denoted by ''phase shift (b).'' We mention that the second method (b) is similar to the first one (a) but the background term is neglected (a pure Breit-Wigner form is assumed for $d\delta/dE$).

We have also used the complex scaling method (CSM) to determine the resonance parameters. This method is widely used to deal with resonance states in atomic and nuclear physics [14–22]. In the complex scaling method, the Hamiltonian $\hat{H}$ is transformed as

$$\hat{H}_\theta = \hat{U}(\theta)\hat{H}\hat{U}(\theta)^{-1}, \tag{23}$$

where $\hat{U}(\theta)$ is the unbounded nonunitary scaling operator which acts on a function $f(\mathbf{r})$ as

$$\hat{U}(\theta)f(\mathbf{r}) = \exp(3i\theta/2)f(\mathbf{r}\exp(i\theta)). \tag{24}$$

The Schrödinger equation $\hat{H}|\Psi\rangle = E|\Psi\rangle$ is rewritten as

$$\hat{H}_\theta|\Psi_\theta\rangle = E_\theta|\Psi_\theta\rangle. \tag{25}$$

The ABC theorem states [33] that the bound state eigenvalues of the Hamiltonian $\hat{H}$ are also eigenvalues of $\hat{H}_\theta$. A resonance state is obtained as a complex eigenvalue $E_\theta = E_r - i\Gamma_r/2$ of the non-self-adjoint Hamiltonian $\hat{H}_\theta$. The resonance eigenvalue does not depend on $\theta$ if $\frac{1}{2}\tan^{-1}[\Gamma_r/2E_r] < \theta < \pi/2$. The wave functions of the resonance and bound states are square integrable in the CSM. When the complex scaling method is applied to the microscopic cluster model, the new GCM method of Tohsaki-Suzuki [26] is used.

## IV. RESULTS AND DISCUSSION

In this section we show our results for the nuclei $^8$Be, $^5$Li, and $^5$He using the microscopic cluster models $\alpha+\alpha$, $\alpha+p$, and $\alpha+n$. First we have checked the approximate calculation of the CLD without the presence of the Coulomb force.

For $^8$Be the effective nucleon-nucleon interaction and other parameters of the cluster model are taken from Ref. [16]. The effective nucleon-nucleon interaction is Volkov No. 1 with Majorana exchange parameter $m=0.6$ [34]. The spin-orbit force can be neglected because it does not contribute to the energy in our model space. The size parameter of the $\alpha$-particle is set to be $\nu_\alpha (=\beta_\alpha/2) = 0.235$ fm$^{-2}$. The binding energy of the $\alpha$ particle is $E_\alpha = -27.450$ MeV. The parameter $\gamma$ in the new GCM basis [see Eq. (3)], is taken to

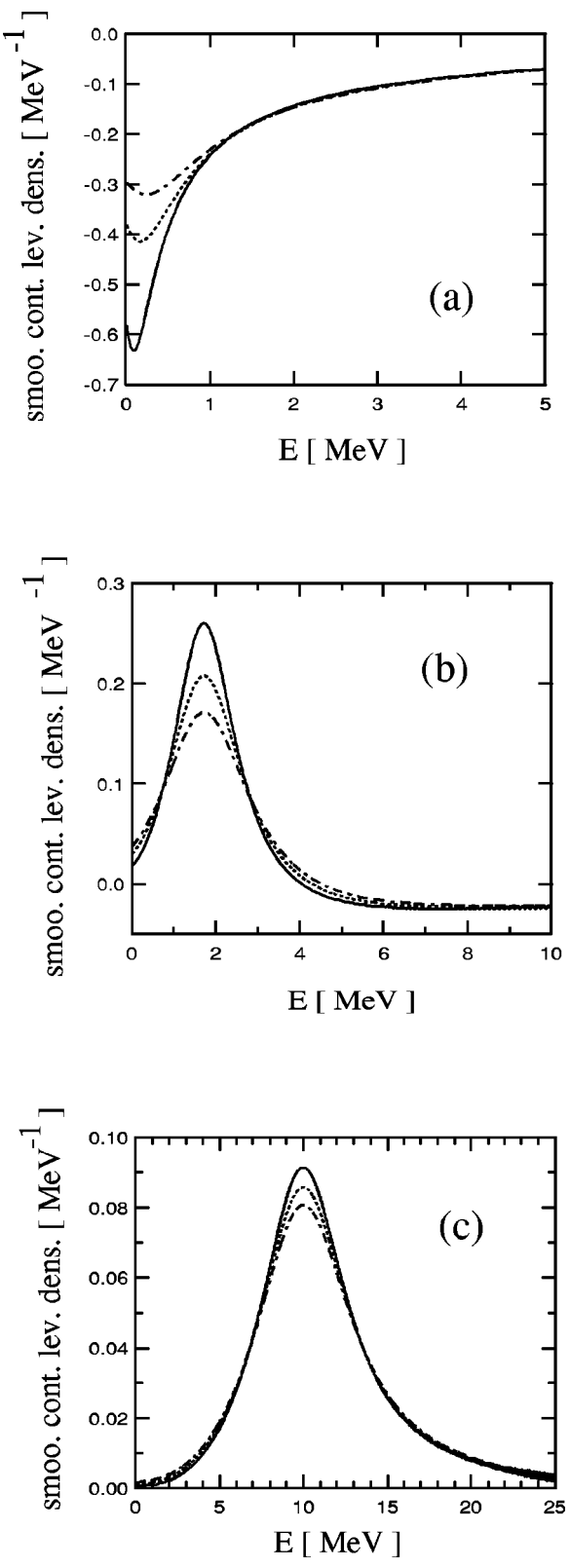


FIG. 1. The smoothed continuum level densities of the $\alpha+\alpha$ system for the states $J^\pi = 0^+$ (a), $2^+$ (b), and $4^+$ (c). The effective nucleon-nucleon interaction is Volkov No. 1($m=0.6$) without Coulomb force. The two-$\alpha$-particles energy is $-54.90$ MeV ($\nu_\alpha = 0.235$ fm$^{-2}$). The solid, dotted, and solid-dotted lines denote the results for smoothing with $\Gamma = 0.4$, 0.8, and 1.2 MeV, respectively.

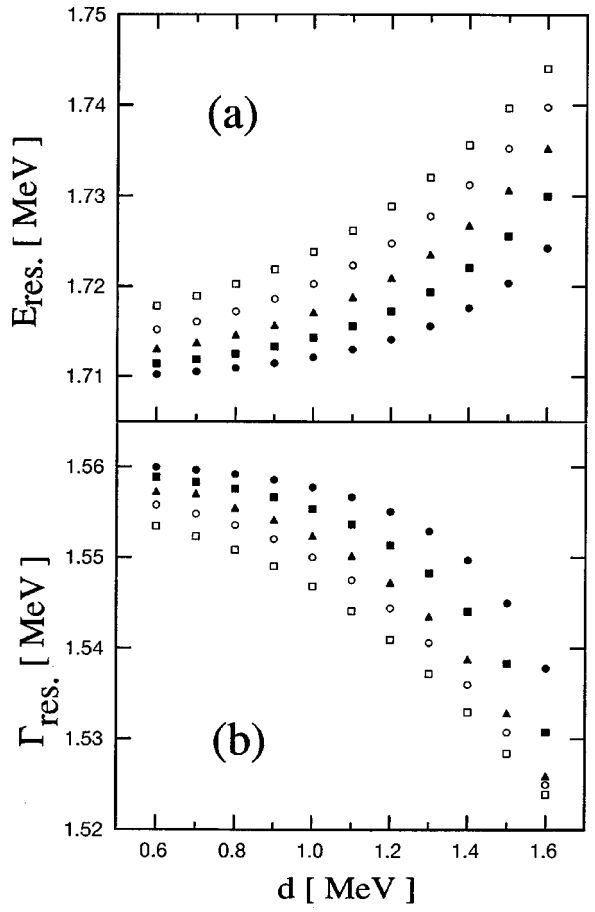


FIG. 2. The resonance energy $E_r$ [MeV] and the width $\Gamma_r$ [MeV] in the CLD method by the fitting procedure with various values for $d$ [MeV] for the state $J^\pi=2^+$ without Coulomb force. Other parameters of the calculation are given in Fig. 1. The solid circle, solid square, solid triangle, open circle, and open square denote the results for smoothing with $\Gamma=0.4$, 0.6, 0.8, 1.0, and 1.2 MeV, respectively.

be $0.4$ fm$^{-2}$ and the generator coordinates $s_k$ are distributed equidistantly with step size $\Delta s_k=1.0$ fm from 1.0 fm up to $1.0+\Delta s_k(N-1)$ fm.

Figure 1 shows the approximate smoothed CLD with smoothing parameters $\Gamma=0.4$ MeV, 0.8 MeV, and 1.2 MeV. The densities of the $2^+$ and the $4^+$ states have an apparent peak around 1.7 MeV and 10 MeV, respectively. The dependence of the peak position on the smoothing parameter $\Gamma$ is weak, but the peak becomes more pronounced (sharper) as the $\Gamma$ parameter decreases. The calculated smoothed CLD hardly depends on the choice of the GCM parameters $\Delta s_k$ and $\gamma$ if they are chosen in reasonable ranges. Without the Coulomb force the $J^\pi=0^+$ ground state of the $^8$Be is bound ($E_B=-0.45$ MeV). The smoothed CLD has no resonance peak in this partial wave.

It should be noted that the peak position of the density, $E_{\rm peak}$ does not agree with the resonance position $E_r$, because the background term $\Delta^{\rm bg}$ in Eq. (9) influences it. The resonance parameters can be determined by employing Eq. (22). We have done this fitting procedure with various intervals $d$. Figure 2 shows the $d$ dependence of the resonance energy $E_r$ and width $\Gamma_r$ for the state $J^\pi=2^+$. This figure shows that the resonance parameters converge as the size $d$ of the interval for the fitting decreases. The $d$ dependence of the resonance parameters become weak for smaller smoothing parameter $\Gamma$. We adopted the ''$d$'' converged values (for the smallest $\Gamma$) as the parameters of the resonance.

Figure 3 displays the convergence behavior of the smoothed approximate CLD for the state $2^+$ as the function of the dimension of the basis $N$. The smoothing parameter is fixed at $\Gamma=0.4$ MeV. The energy spacing between the nearest neighbor eigenvalues becomes larger as the basis dimension $N$ decreases or the energy $E$ increases. In these cases the individual eigenvalues appear as peaks in the smoothed CLD. If we choose a sufficiently large $\Gamma$ parameter, then these peaks washed out. However, the accuracy for the resonance parameters deteriorates [23]. Therefore the smoothing parameter $\Gamma$ should be picked out as small as possible in order to determine the resonance parameters accurately with a finite number of basis functions. Once the dimension of the basis becomes large enough to wash out the peaks of the individual eigenvalues, the form of the approximate CLD hardly depends on the basis dimension $N$. The basis dimensions used for the states $J^\pi=0^+$, $2^+$, and $4^+$ are $N=500$, 500, and 800, respectively. These dimensions are obviously much larger ones than what we used in the complex scaling method or in the variational method for the scattering calculation. Our approximate way of the calculation of the CLD, for the purpose of the determination of resonance parameters, has slow convergence.

In order to check the validity of our approach for the calculation of the CLD we have also calculated the scattering phase shift $\delta(E)$ using an exact scattering asymptotic boundary condition. Figure 4 shows the energy derivative of scattering phase shift for the states $J^\pi=0^+$, $2^+$, and $4^+$. These exact densities give the peak position almost at the same energy as the smoothed approximate CLD. We did not plot the smoothed exact and the smoothed approximate densities on the very same figure, because the two densities are indistinguishable from each other on the scale of the figure. This statement is true if the Strutinsky smoothing procedure is done with the same weight function for both the exact and the approximate calculations. Of course the exact and approximate smoothed densities are in good agreement with each other if $\Gamma$ is sufficiently large to wash out the peaks of individual eigenvalues in the smoothed approximate CLD. It should be stressed that the good agreement that we have found holds not only around the resonance peak but also at nonresonant energies. For the partial wave $0^+$ there is no resonance peak in the densities, but the curves of the exact and approximate densities show a good overall agreement. These observations confirm that our approximate method can describe well not only the resonance densities but also the nonresonant continuum densities of the microscopic cluster model.

Table I shows the resonance parameters determined by the smoothed approximate CLD together with the other three methods mentioned earlier, the scattering phase shift methods (a) and (b) and the CSM. The CSM treats the asymptotic boundary condition rigorously and it is able to find these

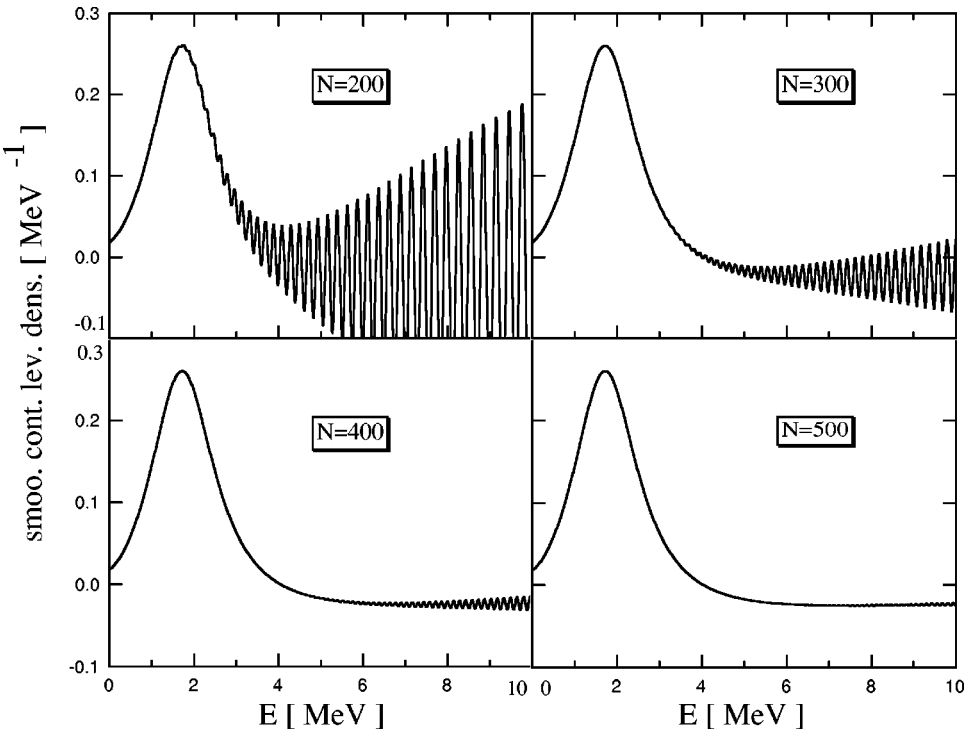


FIG. 3. The smoothed approximate continuum level density for basis dimensions $N=200$, 300, 400, 500 with smoothing $\Gamma=0.4$ MeV. Other parameters of the calculation are given in Fig. 1.

nuclear resonance states accurately [16]. For the states $2^+$ and $4^+$ the resonance parameters calculated with the approximate CLD give identical results with the CSM. The scattering phase shift method (a) and (b) give slightly different results. The results of method (a) which takes into account the effect of the background term for $d\delta/dE$ agrees with the methods of the approximate CLD and the CSM. However the phase shift method (b) gives rather different values especially for the resonance widths.

In general $\Gamma_r$ depends on the energy $E$ [32] but we take $\Gamma_r$ to be independent of $E$. The failure of method (b) probably comes from the fact that both the background term and the energy dependence of $\Gamma_r$ are neglected. We did not take into account the energy dependence of $\Gamma_r$ because this dependence makes the Strutinsky smoothing of the exact CLD more complicated and for a fair comparison we used the same parametrization of the phase shift both in the CLD method and in the scattering calculation.

The resonance width of the complex scaling method is independent of the parametrization of the phase shift since this method gives the pole of the $S$ matrix. The very good agreement of the CLD and complex scaling methods justifies the neglect of the energy dependence of $\Gamma_r$ in the parametrization of the phase shift. We conclude that the background term must be taken into account in the determination of the resonance parameters. The results of scattering phase shift method (a) is more reliable than method (b).

The good agreements found between methods CLD, CSM, and phase shift method (a) show that our approximate way of the calculation of the CLD is a good tool to determine nuclear resonance states. We have found this in spite of the fact that these states have large widths $\Gamma_r \approx 1.6$ MeV and $\Gamma_r \approx 6.0$ MeV for states $2^+$ and $4^+$, respectively.

Next the Coulomb force is added to the nucleon-nucleon interaction. The addition of Coulomb force changes the binding energy of the $\alpha$ particle to $E_\alpha = -26.663$ MeV. Because the nucleon-nucleon force has a short range, previously the free Hamiltonian $H_0^{\rm rel}$ contained only the kinetic energy term. However in the presence the Coulomb force, which has a long range, a corresponding Coulomb term is also added to $H_0^{\rm rel}$, see Eq. (15).

Figure 5 shows the smoothed approximate CLD for the states $0^+$, $2^+$, and $4^+$. The dimension of the basis in the calculation for the states $J^\pi = 0^+$, $2^+$, and $4^+$ is $N=500$, 800, and 800, respectively. The $0^+$, $2^+$, and $4^+$ densities have a resonance peak around 0.6 MeV, 3.1 MeV, and 12 MeV, respectively. The smoothed exact CLDs derived from the scattering phase shift calculation are compared with the approximate ones. The two densities agree with each other very well. We mention once more that the smoothed exact and the smoothed approximate densities are indistinguishable from each other in Figs. 1, 5, 7, and 8. These findings show that our method is a good approximation for the calculation of the CLD even if the long-range Coulomb force is present. Table II shows the resonance parameters for these states with our method and the results of the CSM. The scattering phase shift calculations are also displayed. The resonance parameters determined by each method, except phase shift method (b), are in good agreement with each other. This is true not only for the sharp resonance state $0^+$ but also for the broad resonance states $2^+$ and $4^+$. Phase shift method (b) give fairly good results for the resonance energy $E_r$, but it overestimates the resonance width $\Gamma_r$.

We have also done calculations for the nuclei $^5$He and $^5$Li using the microscopic cluster models $\alpha+n$ and $\alpha+p$. Here the effective nucleon-nucleon interaction is replaced by the Minnesota potential of Thompson, LeMere, and Tang

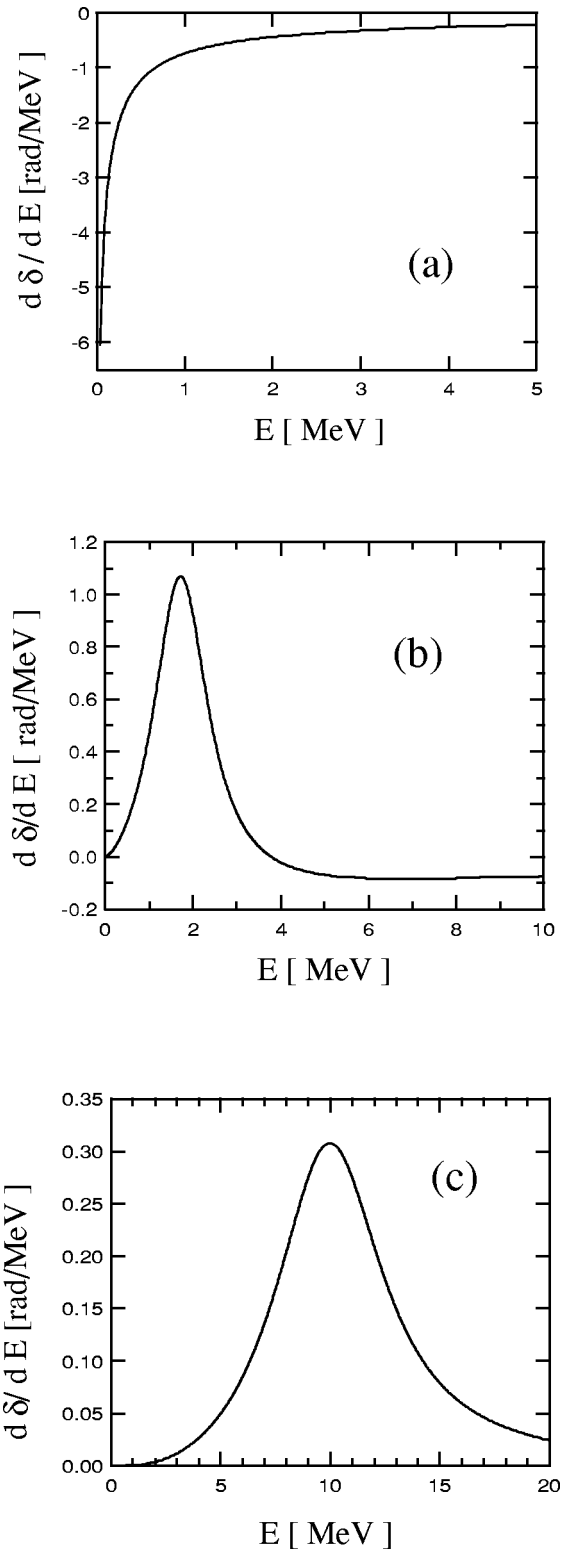


FIG. 4. The energy derivative of the scattering phase shift for the states $J^\pi = 0^+$ (a), $2^+$ (b), and $4^+$ (c) without Coulomb force. Other parameters of the calculation are given in Fig. 1.

[35] with the spin-orbit force of Reichstein and Tang [36]. The exchange mixture parameter $u$ in the Minnesota potential is set to be $u=0.98$ and force set number IV is used for spin-orbit force. The size parameter of the $\alpha$ particle is also changed to $\nu_\beta = 0.303$ fm$^{-2}$ which minimizes the $\alpha$-particle binding energy in the $(0s)^4$ shell model configuration. The binding energy of the $\alpha$ particle is $E_\alpha = -24.687$ MeV. This force and parameter set reproduces well the low-energy $s$- and $p$-wave $\alpha+n$ and $\alpha+p$ phase shifts (see Fig. 6).

TABLE I. The resonance energy $E_r$ [MeV] and width $\Gamma_r$ [MeV] of $^8$Be derived from the smoothed continuum level density (CLD), the complex scaling method (CSM), and scattering phase shift methods (a) and (b). The effective nucleon-nucleon force is Volkov No. 1 ($m=0.6$) without Coulomb force. The two-$\alpha$-particle energy is $-54.90$ MeV ($\nu_\alpha = 0.235$ fm$^{-2}$). The $0^+$ ground state is bound at $-0.45$ MeV.

| $J^\pi$ | $2^+$ | | $4^+$ | |
|---|---|---|---|---|
| | $E_r$ | $\Gamma_r$ | $E_r$ | $\Gamma_r$ |
| CLD | 1.71 | 1.56 | 9.93 | 6.04 |
| CSM | 1.71 | 1.56 | 9.93 | 6.04 |
| Phase shift (a) | 1.71 | 1.56 | 9.93 | 6.04 |
| Phase shift (b) | 1.72 | 1.87 | 9.98 | 6.50 |

In Figs. 7 and 8 we show the smoothed approximate CLD of the states $\frac{3}{2}^-$, $\frac{1}{2}^-$, and $\frac{1}{2}^+$ for the systems $\alpha+n$ and $\alpha+p$. The parameters $\gamma$ and $\Delta s_k$ of the GCM basis are the same as for the nuclei $^8$Be. The basis dimension is $N=800$, 1000, and 800 for the states $J^\pi = \frac{3}{2}^-$, $\frac{1}{2}^-$, and $\frac{1}{2}^+$, respectively. It is known that the nuclei $^5$He and $^5$Li have no $\frac{1}{2}^+$ resonance states in the low-energy region. In accordance with this the calculated smoothed CLD has no peak. The densities of the $\frac{3}{2}^-$ state have a resonance peak around 0.8 MeV for $^5$He and 1.6 MeV for $^5$Li. It is known that the resonance states $\frac{1}{2}^-$ of $^5$He and $^5$Li are broad resonances. The calculated smoothed approximate CLDs has a distinct broad peak around 2 MeV for $^5$He and 3 MeV for $^5$Li, respectively. The smoothed exact CLDs derived from the phase shift calculations are in good agreement with the approximate densities for all the states considered $\frac{1}{2}^+$, $\frac{3}{2}^-$, and $\frac{1}{2}^-$. Table III shows the derived resonance parameters for the states $\frac{3}{2}^-$ and $\frac{1}{2}^-$. The results of the approximate smoothed CLD agree with phase shift method (a), but phase shift method (b) gives slightly different results; especially the widths of the broad resonance states $\frac{1}{2}^-$ are overestimated. In Table III our results are compared with the results of the analytically continued $S$ matrix at complex energy by Csótó and Hale [37]. In their RGM of $\alpha+N$ the $\alpha$-particle wave function is described by superposition of $(0s)^4$ shell model functions with different size parameters. However, the $\alpha+N$ scattering phase shifts hardly differ from ours. Our resonance parameters calculated by the CLD method and phase shift method (a) agree well with their results.

The present approximate method, based on the continuum level density, works nicely to calculate two-cluster resonance states. However, as mentioned before, the basis dimension of the present method is much larger than the basis size of the variational scattering calculation. This is a defect of the present method. The reason for the large basis size is the following. We can calculate only the smoothed CLD, and as

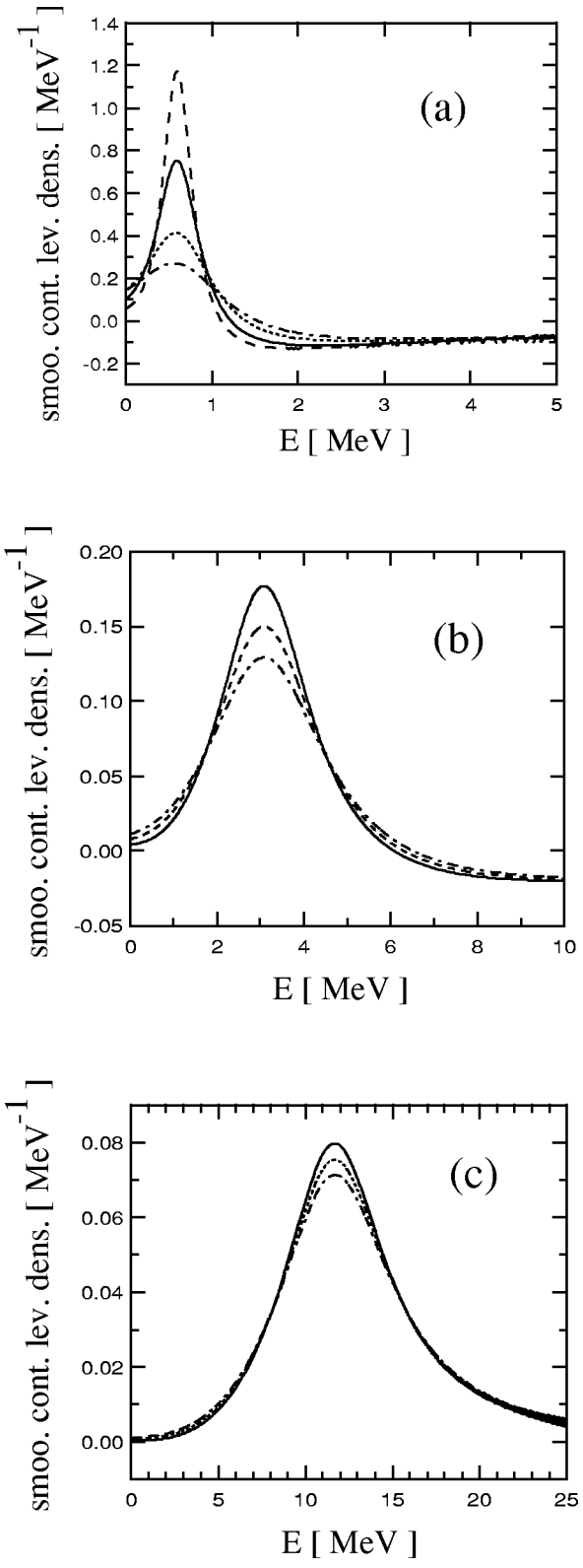


FIG. 5. The smoothed approximate continuum level density of the system $\alpha+\alpha$ for the states $J^\pi=0^+$ (a), $2^+$ (b), and $4^+$ (c). The effective nucleon-nucleon interaction is Volkov No. 1 ($m=0.6$) with Coulomb force. The two-$\alpha$-particles energy is $-53.33$ MeV ($\nu_\alpha=0.235$ fm$^{-2}$). The dashed, solid, dotted, and solid-dotted lines denote the result with $\Gamma=0.2$, 0.4, 0.8, and 1.2 MeV, respectively.

TABLE II. The resonance energy $E_r$ [MeV] and width $\Gamma_r$ [MeV] of $^8$Be derived from the smoothed continuum level densities (CLD), the complex scaling method (CSM), and scattering phase shift methods (a) and (b). The effective nucleon-nucleon force is Volkov No. 1 ($m=0.6$) with Coulomb force. The two-$\alpha$-particle energy is $-53.33$ MeV ($\nu_\alpha=0.235$ fm$^{-2}$).

| $J^\pi$ | $0^+$ | | $2^+$ | | $4^+$ | |
|---|---|---|---|---|---|---|
| | $E_r$ | $\Gamma_r$ | $E_r$ | $\Gamma_r$ | $E_r$ | $\Gamma_r$ |
| CLD | 0.60 | 0.25 | 3.07 | 2.38 | 11.60 | 6.99 |
| CSM | 0.59 | 0.24 | 3.06 | 2.38 | 11.60 | 6.99 |
| Phase shift (a) | 0.60 | 0.25 | 3.07 | 2.38 | 11.60 | 7.00 |
| Phase shift (b) | 0.59 | 0.28 | 3.08 | 2.98 | 11.67 | 7.52 |

small a smoothing parameter as possible has to be used in order to get the resonance parameters accurately. It can be seen in Fig. 3, that the lack of a large basis causes oscillations in the smoothed density and these prevent an accurate calculation of the resonance parameters. Although we need a large basis but after the calculation of the matrix elements

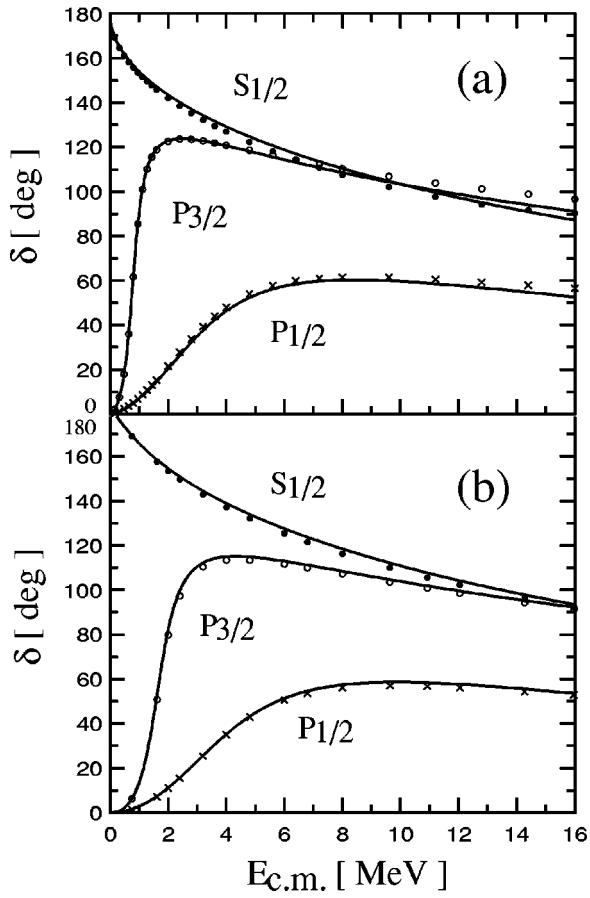


FIG. 6. The scattering phase shift of the states $\frac{3}{2}^-$, $\frac{1}{2}^-$, and $\frac{1}{2}^+$ for the nuclei $^5$He (a) and $^5$Li (b). The effective nucleon-nucleon interaction is the Minnesota potential ($u=0.98$) with Reichstein-Tang spin-orbit force (set No. IV) and the Coulomb force is taken into account. The $\alpha$-particle energy is $-24.69$ MeV ($\nu_\alpha=0.303$ fm$^{-2}$). The experimental data are taken from Ref. [38] for the $\alpha+n$ and from Ref. [39] for the $\alpha+p$ scattering.

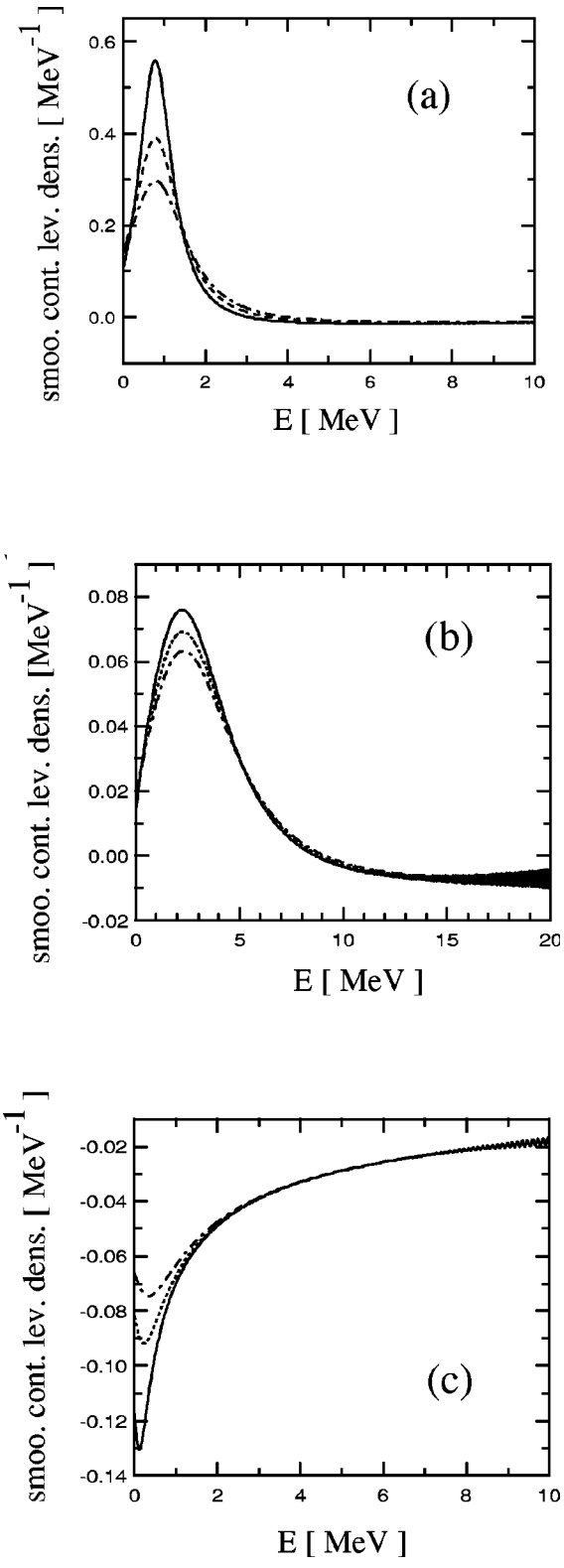


FIG. 7. The smoothed approximate continuum level density of the system $\alpha+n$ for the states $J^\pi=\frac{3}{2}^-$ (a), $\frac{1}{2}^-$ (b), and $\frac{1}{2}^+$ (c). The solid, dotted, and solid-dotted lines give the results for smoothing with $\Gamma=0.4$, 0.8, and 1.2 MeV, respectively. Other parameters of the calculation are given in Fig. 6.

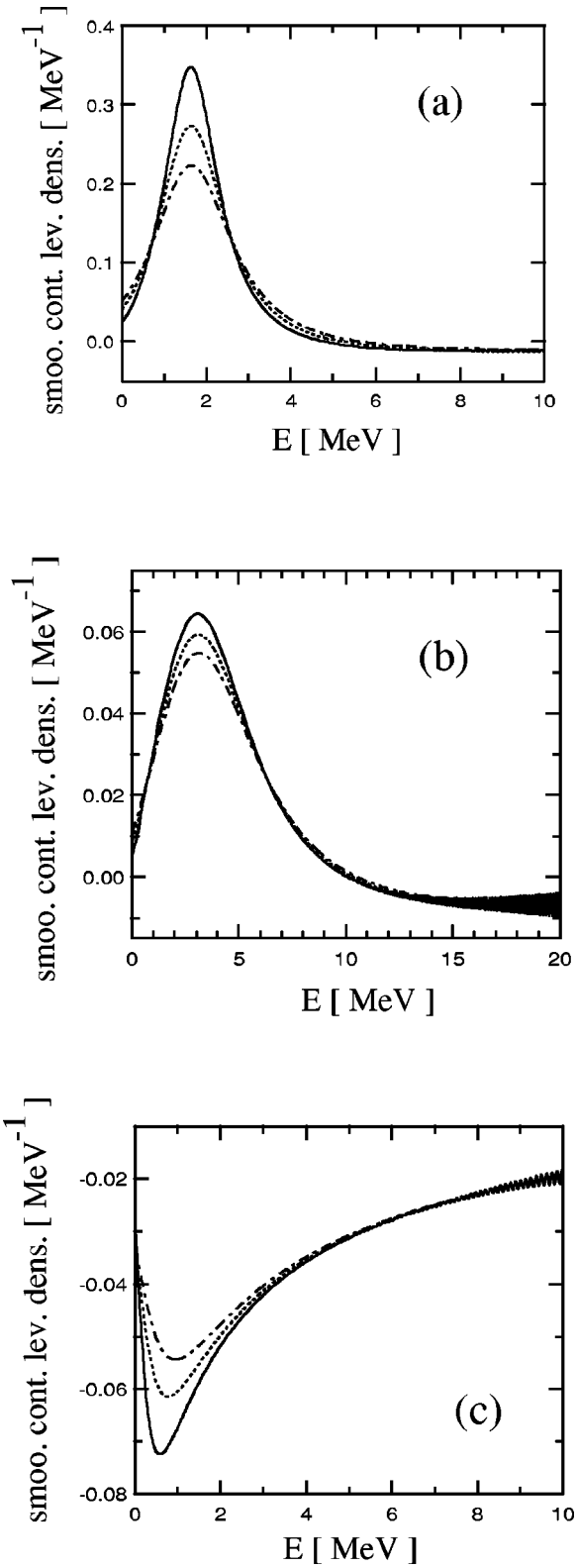


FIG. 8. The smoothed approximate continuum level density of system $\alpha+p$ for the states $J^\pi=\frac{3}{2}^-$ (a), $\frac{1}{2}^-$ (b), and $\frac{1}{2}^+$ (c). The solid, dotted and solid-dotted lines give the results for smoothing with $\Gamma=0.4$, 0.8, and 1.2 MeV, respectively. Other parameters of the calculation are given in Fig. 6.

TABLE III. The resonance energy $E_r$ [MeV] and width $\Gamma_r$ [MeV] of $^5$He and $^5$Li derived from the smoothed continuum level density (CLD) and phase shift methods (a) and (b). The effective nucleon-nucleon forces is the Minnesota potential ($u=0.98$) with Reichstein-Tang spin-orbit force (set No. IV) and the Coulomb force is included. The $\alpha$-particle energy is $-24.69$ MeV ($\nu_\alpha = 0.303$ fm$^{-2}$).

| | $^5$He | | | | $^5$Li | | | |
|---|---|---|---|---|---|---|---|---|
| | $\frac{3}{2}^-$ | | $\frac{1}{2}^-$ | | $\frac{3}{2}^-$ | | $\frac{1}{2}^-$ | |
| | $E_r$ | $\Gamma_r$ | $E_r$ | $\Gamma_r$ | $E_r$ | $\Gamma_r$ | $E_r$ | $\Gamma_r$ |
| CLD | 0.78 | 0.64 | 2.01 | 5.42 | 1.63 | 1.24 | 2.83 | 6.30 |
| Phase shift (a) | 0.78 | 0.64 | 1.98 | 5.45 | 1.63 | 1.24 | 2.82 | 6.32 |
| Phase shift (b) | 0.78 | 0.68 | 2.20 | 7.58 | 1.63 | 1.35 | 3.05 | 9.07 |
| $S$-matrix, RGM [37] | 0.76 | 0.63 | 1.89 | 5.20 | 1.67 | 1.33 | 2.70 | 6.25 |

and two matrix diagonalizations, there is no need for more complicated calculations to get the smoothed CLD.

The present bound-state-type technique avoids imposing scattering or Gamow asymptotic boundary conditions. This is a great advantage of the CLD method since such a bound-state-type technique can be easily extended to more complicated systems. For example, recently three-body resonance states have been discussed in a neutron halo nuclei [21,22]. Since the asymptotic form of a three-body resonance state is not well known, bound-state-type techniques are easy tools to discuss three-body resonance states. We have encouraging results for the extension of the CLD method to the three-body problem and this will be treated in a separate paper.

## V. SUMMARY

We have applied the continuum level density method to determine the resonance states of the nuclei $^8$Be, $^5$He, and $^5$Li. The nuclei $^8$Be, $^5$He, and $^5$Li are described with the microscopic two-cluster model $\alpha+\alpha$, $\alpha+n$, and $\alpha+p$, respectively. Shifted Gaussian bases are used to describe the relative motion wave function of the clusters. The continuum level density is calculated by generalizing the the box-quantized $L^2$-stabilization method. The continuum level density, smoothed by the Strutinsky averaging procedure, can be calculated approximately by making use of the eigenvalues of the full and free Hamiltonian matrices. These eigenvalues are determined using a finite number of real square integrable basis functions. The smoothed approximate continuum level density is found to be in good agreement with the smoothed exact density which is derived from the scattering phase shift.

The continuum level density is connected to the scattering $S$ matrix and has a Breit-Wigner peak around a resonance energy. The smoothed approximate continuum level density has also a resonance peak and then the resonance parameters can be determined with a parameter fitting procedure. The calculated resonance parameters are compared with results of the complex scaling method and scattering phase shift calculation. Very good agreement is found not only for sharp resonances but also for rather broad resonances. We numerically demonstrated that the continuum level density method works also in the presence of the Coulomb force.

It should be emphasized that the present method uses only a square integrable basis to locate resonances but it avoids complex particle coordinates which leads to a non-self-adjoint Hamiltonian in the complex scaling method. In our approach we do not impose a scattering or Gamow asymptotic form for the wave function which makes the calculation more difficult. The box quantization of the standard $L^2$-stabilization technique is also released and in our approach any real square integrable basis can be used to calculate the continuum level density.

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