Effective density-dependent pairing forces in the $T=1$ and $T=0$ channels

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(Received 21 July 1999; published 15 November 1999)

Effective density-dependent pairing forces of zero range are adjusted on gap values in $T=0,1$ channels calculated with the Paris force in symmetric nuclear matter. General discussions on the pairing force are presented. In conjunction with the effective k mass the nuclear pairing force seems to need very little renormalization in the $T=1$ channel. The situation in the $T=0$ channel is also discussed. [S0556-2813(99)00412-4]

PACS number(s): 21.65.+f, 21.30.−x, 21.10.−k, 05.30.Fk

I. INTRODUCTION

The novel availability of exotic nuclei has spurred an immense revival of nuclear structure investigations [1]. Indeed nuclei close to the neutron or proton drip lines may exhibit very unusual features such as pronounced neutron or proton skins [2], and neutron halos [3]. Among many very interesting questions, nuclear pairing has again become on the forefront of theoretical interest. Indeed the existence of neutron halos is due to the pairing force [4,5] and in heavier proton-rich $N\simeq Z$ nuclei the proton-neutron pairing may play an important role [6]. In this work we, therefore, want to address some problems of neutron-neutron and proton-neutron pairing. This concerns for instance considerations of the effective pairing interactions. However, we also will discuss some other aspects of more general character. We will mostly study the infinite matter case.

II. GENERALITIES ON THE NUCLEAR PAIRING FORCES

It is a well established fact that, aside from the exception of magicity, nuclei are superfluid. There can be nn as well as pp pairing whereas pn pairing is less frequent. One of the main questions we will treat here is the effective pairing force. We will do this in the framework of homogeneous nuclear matter at various densities. The limit to finite nuclei can be established through the local density approximation which seems to work very well also for the nuclear pairing problem [7]. Quite generally the equation for the gap Δ in nuclear matter can be written as

$$\Delta_{\mathbf{p}} = -\sum_{\mathbf{k}} v_{\mathbf{pk}} \frac{\Delta_{\mathbf{k}}}{2\sqrt{(\epsilon_k-\epsilon_F)^2+\Delta_{\mathbf{k}}^2}}, \tag{1}$$

where $v_{\mathbf{pk}}$ is the (effective) pairing force, the ϵ_k are the Brueckner-Hartree-Fock single-particle energies and ϵ_F is the Fermi energy. The summation goes over momentum states. In Eq. (1) we did not specify whether we consider the $T=1$ or $T=0$ channels.

The first aspect we want to discuss is what kind of force v_{pk} shall be used in Eq. (1) from a microscopic point of view. The answer to this question is, in principle, very well known since the early days of superconductivity and superfluidity. Since the gap equation can be derived from the Bethe-Salpeter equation for the two-particle many-body Green's function [8,9], the pairing force v_{pk} is built out of the sum of all particle-particle irreducible Feynman graphs [8,9]. To lowest order in the bare interaction it is given by Fig. 1.

In Fig. 1 the dot stands for the bare vertex. The second term represents a ph screening correction to the bare force. The very important point we want to make here is that in no way some type of Bethe-Goldstone or Brueckner G matrix can be used in the gap equation. Since the gap equation is already a kind of in medium two-body Schrödinger equation (see, e.g., Refs. [10,11]) one cannot use a G matrix which in itself is a solution of the in medium two-body problem. Otherwise there is severe double counting. Since the G matrix essentially softens the short-range repulsion one expects that pairing becomes enhanced if used in the gap equation. In the pairing problem everything depends exponentially on the system parameters [12] and this effect can then be quite large. A demonstration is given in Fig. 2, where the nn gap is calculated once with the bare Paris force [13] and once with the corresponding G matrix [14]. One sees that the use of the G matrix enhances the gap value by practically a factor of 2.

Sometimes Eq. (1) is divided into a low momentum and a high momentum space and the high momentum space is eliminated in renormalizing consistently the bare interaction in the low momentum space [14]. This type of procedure is, of course, perfectly allowed, since it is only a mathematical trick for solving Eq. (1). Unfortunately in nuclear physics it is quite a widespread habit (for decades) (see, for example, [15] and the critiques given in [10]) to use some kind of G matrix in Eq. (1) as, for example, Skyrme forces which are to be considered as a phenomenological representation of a microscopic G matrix. One will object that one of the most successful nuclear nn pairing forces, namely the Gogny force [16] is also to be considered as a G matrix. Things are, however, more subtle there as we now will explain. The first observation is that the Gogny force in the $^1S_0$ channel is of finite range but density independent. Second, one finds when

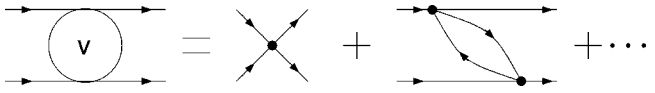

FIG. 1. Schematic representation of the pairing force v_{pk} to lowest order in the bare interaction.

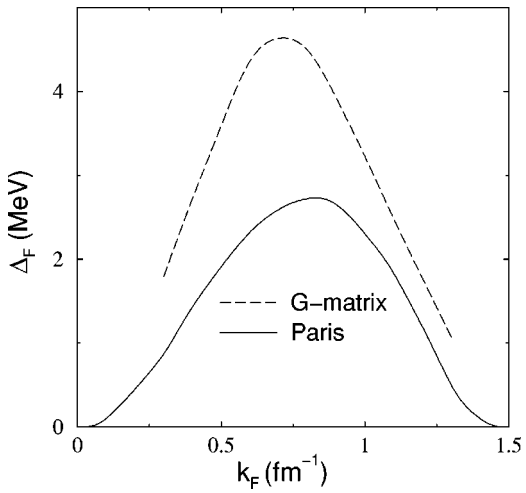

FIG. 2. Pairing gap Δ_F in neutron matter as a function of the Fermi momentum k_F with the Paris force and with the corresponding G matrix.

solving the gap equation with the Gogny force in nuclear matter that it gives results which are very close to the ones obtained with the Paris force or any other realistic bare nucleon-nucleon force. This is demonstrated in Fig. 3 where we compare results of the gap from the D1, D1S, and Paris forces.

We see that D1S is still much closer to Paris than D1. Indeed D1S has been readjusted [17] to give in the first place a lower surface tension than D1 but at the same time to give a smaller even-odd staggering so that it becomes in closer agreement with experiment. It is very surprising that this readjustment brought D1S so close to the bare Paris force. So in the $S=0$ $T=1$ channel the Gogny force acts like a realistic bare force at least in what concerns energies up to the Fermi energy. This conclusion was also found in [4] and is further confirmed by the fact that the scattering length corresponding to D1S, $a_{D1S}=12.16$ fm, is very large and of the same order of magnitude as the experimental value $a=18.5$ fm.

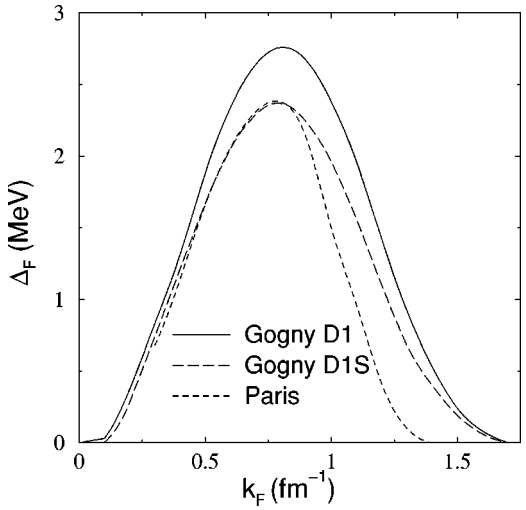

FIG. 3. Pairing gap Δ_F in the $^1S_0$ channel in symmetric nuclear matter calculated with the Gogny forces D1 and D1S, compared with the Paris force results, from Refs. [14,7].

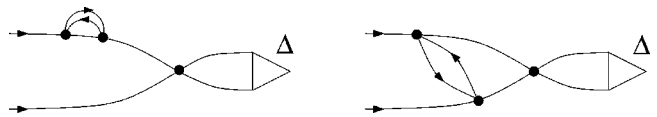

FIG. 4. Schematic representation of the effective mass enhancement and of the screening effect to lowest order in the interaction.

The reason why the Gogny force acts like a free force in the nn pairing channel in spite of the fact that it has been adjusted to the G matrix from the Sprung-Tourreil force [18] can only be guessed: probably for this force in that channel the Pauli blocking is so efficient that in the G matrix equation, $G=v+v(Q/e)G$, the second term on the right-hand side is suppressed. On the other hand, the question remains why experiment apparently demands a pairing force very close to the bare one. This is true at least in the $T=1$ channel. For the $T=0$ channel much less investigations have been performed and it is unclear whether a bare force can be used as well. One reason which can be advanced to explain the validity of the bare force is a possible cancellation between screening effects and effective mass enhancement. Graphically these two possibly opposing effects are shown to lowest order in the interaction in Fig. 4.

In this respect it should be mentioned that the Hartree-Fock-Bogoliubov (HFB) calculations with the Gogny force are performed with the so-called k mass $m^*<m$. However, one knows that the corresponding level density close to the Fermi energy is much too small. Including E-mass corrections such as the one shown in Fig. 4 brings the effective mass at the Fermi level back to the bare mass or even overshoots it. For consistency the screening of the bare force also shown in Fig. 4 must be included. Larger effective masses enhance pairing while screening probably weakens it so that the net effect could be the bare force. To investigate such effects, extreme care must be taken that both contributions of Fig. 4 are treated on the same footing. Since, as already mentioned, pairing depends exponentially on the system parameters, the slightest imbalance (for example, in treating both graphs of Fig. 4 in slightly different approximations) may cause strong erroneous results. One way to treat things consistently could be to use the Gorkov equations [19] and develop the normal and abnormal parts of the mass operator matrix to second-order Born approximation and solve the corresponding gap equation numerically. In medium effects similar to the ones shown in Fig. 4 have been included in the past to the pairing problem in one way or the other [20]. Practically all calculations resulted in an important reduction of $\Delta=\Delta(k_F)$ compared to the values shown in Fig. 3. It can be deduced from the study in [7] that a reduction of pairing in infinite matter obtained with the Gogny force in a global way, i.e., for all values $0\leqslant k_F\leqslant 1.4$ fm^{-1}, inevitably leads also to a reduction of pairing in the finite nuclei of the same proportions (this fact can be understood via the local-density approximation which as mentioned already, on average, yields comparable results to quantal calculations [7,21]). It, therefore, can be concluded that, e.g., a reduction of the Δ values in Fig. 3 by a factor of 2 (a scenario often encountered

in the calculations of references mentioned above) will fail to reproduce experimental gap values of nuclei when the underlying theory is applied to finite nuclei.

Concluding these general considerations we want to say that in the absence of any necessity stemming from experimental facts it is probably safe to treat nuclear pairing in conventional mean-field theory with the bare nucleon-nucleon potential as this is indicated from the microscopic theory and as apparently is needed to reproduce experimental facts in the $T=1$ channel. Using this philosophy one arrives naturally for $T=0$ np pairing at much stronger gap values [11] since the NN force is strongest in this channel. We will give some more details about this in the next section and also discuss how the bare interaction in the gap equation can be replaced by an equivalent density-dependent zero range force such as they have become quite popular recently in the nuclear structure problem.

III. EFFECTIVE DENSITY-DEPENDENT ZERO RANGE PAIRING FORCES

In the last section we gave arguments that, at least as a first guess, it is indicated to use as the pairing force the bare nucleon-nucleon potential. We here want to develop arguments that this strategy is not necessarily orthogonal to the popular employment of density-dependent zero range forces with a cutoff. Such arguments have first been developed by Bertsch and Esbensen [4] and we here want to refine these arguments, on the one hand, and on the other hand, extend them also to $T=0$ np pairing.

A qualitative argument why a density independent finite range force in the gap equation [Eq. (1)] can be replaced by a density-dependent zero range one with a cutoff, goes as follows. For s-wave pairing only the angle averaged matrix element $\tilde{v}_{pk}$ enters the gap equation $\Delta_p=\Sigma_k\tilde{v}_{pk}\kappa_k$, where $\kappa_k=\Delta_k/2E_k$ is the abnormal density and

$$E_k=\sqrt{(\epsilon_k-\epsilon_F)^2+\Delta_k^2} \tag{2}$$

is the quasiparticle energy. The former is very peaked at $k=k_F$ with a peak width of the order $\Delta=\Delta_{k_F}$. Since anyway in pairing problems only the gap values at $k\simeq k_F$ matters, we see that for Δ_{k_F} only the value of the matrix element $\tilde{v}_{k_Fk_F}$ plays a significant role. In the Gogny force, this matrix element as a function of k_F is shown in Fig. 5. Since a δ force is a constant in k space, one has to weight the δ force with a k_F, i.e., a density-dependent factor similar to $\tilde{v}_{k_Fk_F}$ in order to recover the essential pairing features of the original finite range force. The only thing we have to add is a cutoff value, otherwise the gap equation would diverge. Bertsch and Esbensen [4], therefore, proposed the following density-dependent zero range force:

$$v(\mathbf{r}_1,\mathbf{r}_2)=v_0\left[1-\eta\left(\frac{\rho\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)}{\rho_0}\right)^{\alpha}\right]\delta(\mathbf{r}_1-\mathbf{r}_2), \tag{3}$$

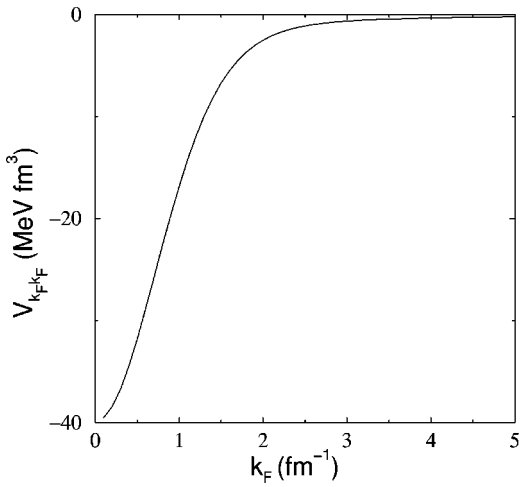

FIG. 5. $v_{k_Fk_F}$ vs k_F in the $S=0$ $T=1$ channel for the Gogny D1 force.

where v_0,η,α are adjustable parameters and ρ_0 is the saturation density. In the gap equation (1), Eq. (3) must be supplemented with a cutoff value ϵ_C which thus constitutes a fourth parameter. However, at zero density the cutoff and v_0 must be chosen such that the scattering length a is reproduced. For Eq. (3) one obtains the relation

$$v_0=-\frac{\hbar^2}{m}\,2\pi^2\frac{1}{\pi/2a+k_C} \tag{4}$$

where $k_C^2/2m=\epsilon_C$. The neutron-neutron scattering length is very large ($a=18.5$ fm) and we take in Eq. (4) the limit $a\rightarrow\infty$ that leads to the relation for v_0 also used in [4]. One finally remains with three adjustable parameters (η,α,ϵ_C) and the gap equation reads

$$1=-\frac{v_0}{\pi^2}\left[1-\eta\left(\frac{\rho}{\rho_0}\right)^{\alpha}\right]\times\left(\frac{m^*(\rho)}{2\hbar^2}\right)^{3/2}\int_0^{\epsilon_C}d\epsilon\sqrt{\frac{\epsilon}{(\epsilon-\epsilon_F)^2+\Delta^2}}. \tag{5}$$

With respect to [4] we considered also a density-dependent effective mass. Since finite nuclei calculations are performed with such an effective mass one must account for it when adjusting a δ force which later shall be used in BCS or HFB calculations. For the effective mass we take the one corresponding to the Gogny force,

$$\left(\frac{m^*(\rho)}{m}\right)^{-1}=1+\frac{m}{2\hbar^2}\frac{k_F}{\sqrt{\pi}}\sum_{c=1}^{2}\left[W_c+2(B_c-H_c)-4M_c\right]\mu_c^3e^{-x_c}\left[\frac{\cosh(x_c)}{x_c}-\frac{\sinh(x_c)}{x_c^2}\right], \tag{6}$$

with $x_c=k_F^2\mu_c^2/2$, and the coefficients W_c,B_c,H_c,M_c,μ_c corresponding to the Gogny force D1 [8,16].

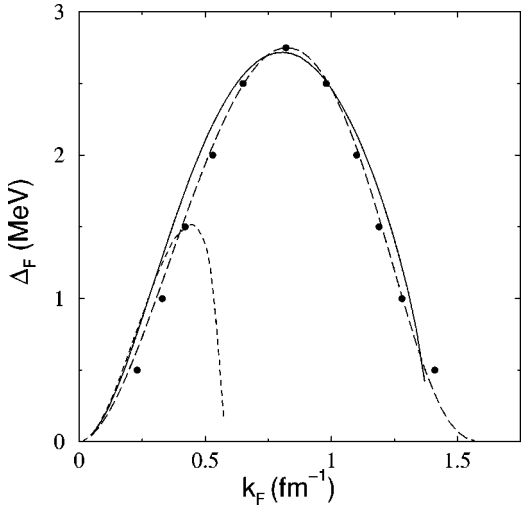

FIG. 6. $T=1$ pairing gap in nuclear matter. The dots are the results of a Hartree-Fock calculation using the Gogny force. The continuous (a) and dashed (b) curves are the results obtained with the effective pairing interaction in Eq. (5) with (a) effective mass m^*/m as in Eq. (6) and (b) $m^*/m=1$ (see text). The dotted line corresponds to the pairing force in Ref. [22] (see text).

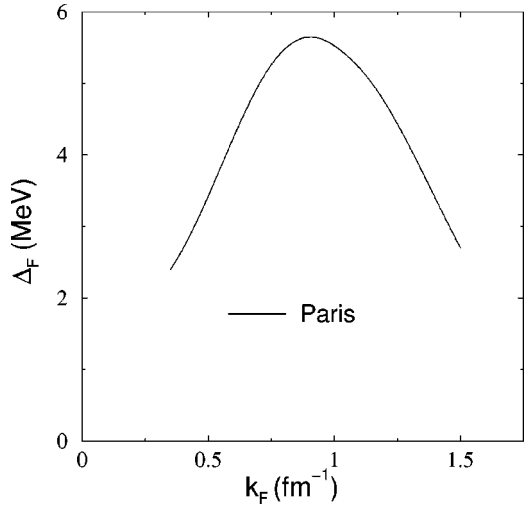

FIG. 7. Pairing gap versus Fermi momentum for symmetric nuclear matter in the $T=0$ channel from the Paris potential.

In Fig. 6 we show the fit to $\Delta(k_F)$ in the isovector channel obtained from Eq. (5) with $\epsilon_C=60$ MeV, $\eta=0.45$, $\alpha=0.47$. Also shown is the fit corresponding to the bare mass (i.e., $m^*/m=1$) with $\epsilon_C=60$ MeV, $\eta=0.70$, $\alpha=0.45$, as in Ref. [4]. In both cases, the corresponding v_0 value is $v_0=481$ MeV fm^3. We see that the fits are good for values of k_F up to the saturation value $k_F=1.35$ fm^{-1}. A density-dependent δ force has also been used for $T=1$ pairing in finite nuclei in the context of the HFB [22] and in the context of relativistic Hartree-Bogoliubov [23]. The strength used there is, however, larger. If we use the pairing force in Ref. [22] with $V_0=700$ MeV fm^3, we get the dotted line curve shown in Fig. 6 that corresponds to the following parameters in our notation: $v_0=1400$ MeV fm^3; $\epsilon_C=7$ MeV; $\eta=1$ MeV and $\alpha=1$ MeV.

For finite nuclei, the force (3) can be used in BCS approximation

$$\Delta_i=-\sum_{k,\epsilon_k\leqslant\epsilon_C}\langle i\bar{i}|v|k\bar{k}\rangle\frac{\Delta_k}{2E_k} \tag{7}$$

or in the HFB approach where the gap equation has the form (7) in the canonical basis. We want to point out that the cutoff has to be counted relative to the bottom of the single-particle well and not from its edge.

IV. PROTON-NEUTRON PAIRING IN THE $T=0$ CHANNEL

In this section we want to extend our considerations to $n-p$ pairing in the $T=0$, i.e., in the deuteron channel. As we suggested earlier, as a first guess one should investigate the gap equation with the bare force. The gap equation in homogeneous symmetric nuclear matter has recently been solved for the $T=0$ channel [11] using the bare Paris force with single-particle energies obtained in Brueckner-Hartree-Fock approximation. Since in the deuteron channel ($T=0$, $S=1$, $L=0,2$) we have a mixture of s and d waves involving the tensor force, the net outcome is more attraction leading to the deuteron bound state in free space. This increased attraction then takes over to the gap equation (which in the zero density limit turns into the Schrödinger equation for the deuteron, see [11,24]) and, not unexpectedly (remember the exponential dependence), the gap values in the $T=0$ channel as a function of k_F are much stronger reaching values more than a factor of 2 larger than in the $T=1$ channel. This is shown in Fig. 7 (Ref. [11]).

The use of the bare force in the $T=0$ channel may, however, be more questionable than in the $T=1$ channel. This

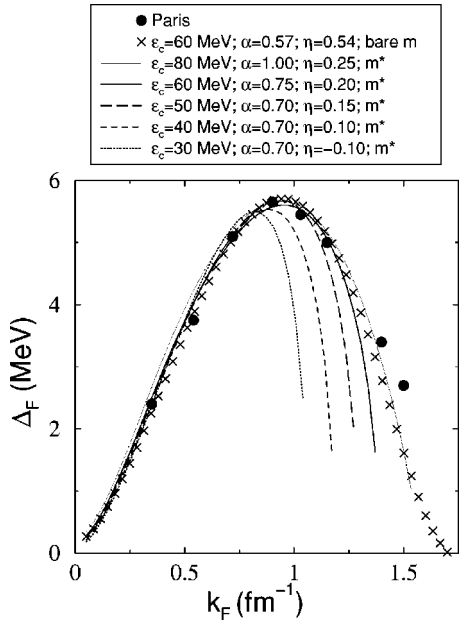

FIG. 8. $T=0$ pairing gap in nuclear matter. The dots are the results [11] obtained from the Paris potential. The various curves correspond to fits with Eq. (5), using different parameters.

stems from the implication of the *d* wave, i.e., the tensor force. The latter seems to be more affected by medium effects than the *s*-wave part and therefore certainly great care must be employed in this channel. In particular, it has been shown in [25] that higher shell admixtures make the tensor force appear weaker in the valence space. Again the possible balance of the two graphs of Fig. 4 should thoroughly be investigated with respect to *s*- and *d*-wave contributions. We do not exclude the possibility that the tensor force is largely screened in the medium and thus the enhancement of the $T=0$ gap values may be brought back closer to values to which we are used in the $T=1$ case. However, without having detailed investigations at hand, we here stick to our working hypothesis and base our considerations on the bare force scenario. In this sense it may be interesting to also adjust, like we have done it for the $T=1$ case, a density-dependent δ force to the calculation with the Paris force shown in Fig. 7. In principle, in this case, the parameter v_0 should be chosen such that the deuteron binding energy is reproduced in free space. We, however, found that with this condition the cutoff parameter must be chosen very large rendering this force not very practicable in actual calculations. We, therefore, adopted the strategy to also vary within very narrow limits the parameter v_0 what may slightly degrade the gap values at very low densities but significantly improves them at the higher densities. In Fig. 8 we show such an adjustment using various cutoffs. The value of v_0 used for the fits in Fig. 8 is

$$v_0 = -1.05 \frac{\hbar^2}{m} \frac{2\pi^2}{k_C}.$$

These fits should be useful for finite nuclei calculations.

V. CONCLUDING REMARKS

In this paper we critically reviewed the use of effective nuclear pairing forces. We argued that a Bethe-Goldstone or Brueckner *G* matrix must not be used in the gap equation. As a first guess, not knowing anything better, the free nucleon-nucleon force may be tried in the gap equation. At least in the traditional $T=1$ channel this prescription seems to work remarkably well, since the best phenomenological force, namely the Gogny force, acts very nearly like a free force in the $T=1$ pairing channel. We then advocated that the same strategy should be adopted in the $T=0$ channel. We pointed out that the situation may, however, be slightly more subtle there because it is the action of the tensor force which makes the $T=0$ channel more attractive than the $T=1$ one. The tensor part of the nuclear interaction is, however, a very delicate subject and it may well be that it is more affected by screening than the rest of the force. In the second part of the work we demonstrated that the use of density-dependent zero range forces in the pairing channel may not be orthogonal to the use of finite-range density-independent forces. Following Bertsch and Esbensen [4], we give parametrizations of density-dependent δ forces which reproduce the gap values in both $T=0$ and $T=1$ channels very well over the whole range of relevant nuclear matter densities. Such forces, augmented by a cutoff, should then also be useful for calculations in finite nuclei.

ACKNOWLEDGMENTS

We acknowledge useful discussions with M. Baldo, V. Lombardo, and N. Vinh Mau. This work was supported by DGICYT-IN2P3 agreement and by DGICYT (Spain) under Contract No. PB95/0123.

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