Quasideuteron configurations in odd-odd $N=Z$ nuclei

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The isovector $M1$ transitions between low-lying $T=1$ and $T=0$ states in odd-odd $N=Z$ nuclei are analyzed. Simple analytical expressions for $M1$ transition strengths are derived within a single-j-shell approximation for both the $j=l+1/2$ and $j=l-1/2$ cases. The large $B(M1)$ values for the $j=l+1/2$ case are attributed to *quasideuteron configurations*. The $B(M1)$ values for the $j=l-1/2$ case are found to be small due to partial cancellation of spin and orbital parts of the $M1$ matrix element. [S0556-2813(99)02512-1]

I. INTRODUCTION

The structure of self-conjugate nuclei with equal numbers of protons (Z) and neutrons ($N=Z$) is currently attracting a lot of attention. The structure of $N=Z$ nuclei provides a sensitive test for the isospin symmetry [1] of nuclear forces. It is well known that the structure of even-even nuclei with protons and neutrons occupying different shells is determined by Cooper-type pairs with isospin $T=1$ and angular momentum $J=0$ formed by nucleons of the same kind. In nuclei close to the $N=Z$ line, where valence protons and valence neutrons occupy the same shells in addition to the standard pair correlations mentioned above, proton-neutron pair correlations with $T=1$ and with $T=0$ can become important. It means that in addition to the proton-proton and neutron-neutron 0^+ pairs, proton-neutron pairs with different angular momenta can play an important role. Below we will analyze experimental information, which demonstrates the important role of the neutron-proton pairs with $J_T^\pi=1_0^+$ and $J_T^\pi=0_1^+$ in the structure of $N=Z$ nuclei. In $N=Z$ nuclei both kind of states with total isospin quantum numbers $T=0$ and $T=1$ exist. In odd-odd $N=Z$ nuclei the lowest $T=0$ states and $T=1$ states are low-lying (below 4 MeV). This unique phenomenon is in contrast to even-even $N=Z$ nuclei, where the $T=0$ 0_1^+ ground state is lowered and is separated from excited $T=1$ states by a large energy gap. A lot of work, experimental [2–11] and theoretical [12–25], has been carried out recently for the investigation and understanding of the $N=Z$ nuclear structure. One interesting phenomenon is the occurrence of very large magnetic dipole ($M1$) matrix elements between nuclear states along the $N=Z$ line. The $M1$ moments of odd-odd $N=Z$ nuclei have recently been revisited [20] within a simple shell-model approach. Another recent work [21] discusses the interference term between spin and orbital contributions to $M1$ transitions in even-even s-d shell nuclei.

In this paper we focus on isovector $M1$ transitions strength between low-lying states in odd-odd $N=Z$ nuclei which are accessible to γ spectroscopy. Some of the odd-odd $N=Z$ nuclei exhibit very strong isovector $M1$ transitions between the yrast states with quantum numbers $J_T^\pi=0_1^+$ and 1_0^+ (see Table I). In a few nuclei, ^{10}B, ^{22}Na, and ^{26}Al, the strong $M1$ transitions are fragmented among two or three states. Other odd-odd $N=Z$ nuclei ^{14}N, ^{30}P, ^{34}Cl, ^{38}K have considerably weaker $0_1^+ \rightarrow 1_0^+$ transitions, in some cases with almost vanishing $M1$ strengths. The total $B(M1;0_{T=1}^+ \rightarrow 1_{T=0}^+)$ strength between the low-lying states in odd-odd $N=Z$ nuclei depends sensitively on the mass number A and due to the underlying nuclear shell structure do not show a smooth behavior.

Nowadays exact shell-model calculations can be performed for nuclei with mass numbers smaller than about $A \approx 60$ with conventional techniques, e.g., [25]. The shell-model problem can, at least partly, be solved approximately, but with controllable accuracy, for heavier nuclei with newly developed statistical Monte Carlo methods [14,18]. Both numerical techniques are powerful and important methods to describe in detail the structure of light and medium mass nuclei, including $M1$ properties of $N=Z$ nuclei, on a microscopic level. Besides these sophisticated numerical approaches sometimes simple models can yield analytical results, which can help to clarify the underlying physics of a certain phenomenon in an approximate but simple and transparent way. Important examples are the analytical Schmidt values for magnetic moments of odd-mass nuclei. The Schmidt values were obtained in a pure, i.e., noninteracting, shell-model approach considering one nucleon outside the even-even core in a single-j shell orbital. The Schmidt values serve as important benchmarks for the actual values of $M1$ moments found experimentally in odd-A nuclei.

In the next section we will discuss analytical formulas for $T=0 \rightarrow T=1$ isovector $M1$ transition matrix elements in odd-odd $N=Z$ nuclei, which we derive in a simple core +two-nucleon single-j-shell approximation. $M1$ transition matrix elements are found to be large between two-nucleon *quasideuteron configurations*, which we define below. Reduction of $M1$ strengths in other cases will be understood as partial cancellation of spin and orbital parts of $M1$ matrix elements. Analytical expressions will be given, which relate the isovector $M1$ transition strengths in $N=Z$ odd-odd nuclei to magnetic moments in neighboring odd-mass nuclei. In Sec. III we will compare the experimental data to the simple analytical formulas. Good agreement is obtained. Predictions for isovector $M1$ transition strengths in heavier odd-odd $N=Z$ nuclei are done from an extrapolation of our formulas up to ^{82}Nb.

TABLE I. Experimental [29–31] and calculated $B(M1)$ values for odd-odd $N=Z$ nuclei. $M1$ transitions are omitted which carry less than 5% of the total $\Sigma_i\, B(M1;0_1^+\rightarrow 1_i^+)$ strength for a given configuration. In the column ''Free theory'' results of calculations for the free spin g factors are shown and in the column ''Eff. theory,'' for the effective spin g factors $g_s^{\text{eff}}=0.7g_s^{\text{free}}$. The numbers in parenthesis give the experimental errors in the last digits.

Nucleus	Used configuration	$E_x^{\exp}(0_1^+)$ (MeV)	$E_x^{\exp}(1^+)$ (MeV)	$B^{\exp}(M1;0_1^+\rightarrow 1^+)$ (μ_N^2)	$\Sigma_i\, B(M1;0_1^+\rightarrow 1_i^+)$ (μ_N^2)		
					Expt.	Free theory	Eff. theory
Deuteron	$\pi 1s_{1/2}\nu 1s_{1/2}$		0			15.86	7.77
$^6_3\text{Li}_3$	$\pi 1p_{3/2}\nu 1p_{3/2}$	3.562	0	15.4(4)	15.4(4)	12.96	7.34
$^{10}_5\text{B}_5$	$\pi 1p_{3/2}^{-1}\nu 1p_{3/2}^{-1}$	1.740	0.718	7.5(32)	8.1(33)	12.96	7.34
		1.740	2.154	0.59(5)			
$^{14}_7\text{N}_7$	$\pi 1p_{1/2}\nu 1p_{1/2}$	2.312	0	0.05(2)	0.05(2)	0.58	0.13
	$\pi 1p_{3/2}^{-1}\nu 1p_{3/2}^{-1}$	2.312	3.948	$5(^{+3}_{-2})$	$5(^{+3}_{-2})$	12.96	7.34
$^{18}_9\text{F}_9$	$\pi 1d_{5/2}\nu 1d_{5/2}$	1.041	0	20(4)	20(4)	15.18	9.37
$^{22}_{11}\text{Na}_{11}$	$\pi 1d_{5/2}^3\nu 1d_{5/2}^3$	0.657	0.583	5.0(3)	>12.5	15.18	9.37
		0.657	1.936	4.4(10)			
		0.657	3.943	>4.4			
$^{26}_{13}\text{Al}_{13}$	$\pi 1d_{5/2}^{-1}\nu 1d_{5/2}^{-1}$	0.228	1.057	8(2)	9.4(30)	15.18	9.37
		0.228	1.850	0.8(8)			
		0.228	3.723	0.6(2)			
$^{30}_{15}\text{P}_{15}$	$\pi 1d_{3/2}\nu 1d_{3/2}$	0.677	0	1.3(1)	1.3(1)	0.42	0.012
$^{34}_{17}\text{Cl}_{17}$	$\pi 1d_{3/2}\nu 1d_{3/2}$	0	0.461	0.23(2)	0.23(2)	0.42	0.012
$^{38}_{19}\text{K}_{19}$	$\pi 1d_{3/2}^{-1}\nu 1d_{3/2}^{-1}$	0.130	0.460	0.47(4)	1.17(24)	0.42	0.012
		0.130	1.698	0.7(2)			
$^{42}_{21}\text{Sc}_{21}$	$\pi 1f_{7/2}\nu 1f_{7/2}$	0	0.611	11(4)	11(4)	18.23	12.16

II. ANALYTICAL FORMULAS FOR $M1$ TRANSITION STRENGTHS

Attempting to understand the observed data on isovector $0_{T=1}^+\rightarrow 1_{T=0}^+$ $M1$ transition strengths in odd-odd $N=Z$ nuclei we have applied the shell model in the core+*two-nucleon single-j-shell approximation* (2NSj). I.e., we consider an odd-odd $N=Z$ nucleus as an inert $J_T^\pi=0_0^+$ even-even $N=Z$ core with two valence nucleons, one proton and one neutron, in the same shell-model orbital with quantum numbers (nlj).

These two valence nucleons can couple to product states with total angular momentum $J=0,1,\ldots,2j$ and positive parity. The states with even spin quantum numbers have the isospin quantum number $T=1$ and states with odd J have $T=0$. The free one-proton–one-neutron system is the deuteron. In the lowest states of the deuteron, the bound $J_T^\pi=1_0^+$ ground state and the unbound $J_T^\pi=0_1^+$ resonance, both nucleons occupy the $1s_{1/2}$ shell with $j=l+1/2$.

In generalization of the deuteron case we denote the wave functions in the 2NSj approximation as *quasideuteron configurations* (QDC) in the $j=l+1/2$ cases. This is in agreement with the conclusion made in [20]. It is clarified below that $M1$ properties of QDC differ considerably from the case with $j=l-1/2$ due to the interference of orbital and spin parts in the $M1$ matrix elements. In reality the proton-neutron pairs coupled to angular momentum $J^\pi=0^+$ or 1^+ can be distributed with some weights over the several single particle (nlj) orbitals. Experimental indications on this effect

will be also briefly discussed below.

The $M1$ transition operator

$$\mathbf{T}(M1)=\sqrt{\frac{3}{4\pi}}[g_p^l\mathbf{L}_p+g_p^s\mathbf{S}_p+g_n^l\mathbf{L}_n+g_n^s\mathbf{S}_n]\frac{\mu_N}{\hbar} \quad (1)$$

is the sum of the orbital and spin parts for protons and for neutrons. Here $\mathbf{L}_\rho(\mathbf{S}_\rho)$ is the orbital (spin) angular momentum operator for $\rho\in\{p,n\}$, $g_\rho^{l(s)}$ is the orbital (spin) g factor and $\mu_N=e\hbar/2M_pc$ represents the nuclear magneton.

A $\Delta T=1$ isovector $M1$ transition, for instance between the $0_{T=1}^+$ and $1_{T=0}^+$ yrast states in odd-odd $N=Z$ nuclei, is generated by the isovector (IV) part of the $M1$ transition operator

$$\mathbf{T}_{\text{IV}}(M1)=\sqrt{\frac{3}{4\pi}}\left[\frac{g_p^l-g_n^l}{2}(\mathbf{L}_p-\mathbf{L}_n)\right.$$
$$\left.+\frac{g_p^s-g_n^s}{2}(\mathbf{S}_p-\mathbf{S}_n)\right]\frac{\mu_N}{\hbar}. \quad (2)$$

This is a consequence of the tensor properties of the $M1$ transition operator in the isospin space. In the simple 2NSj approximation it is possible analytically to derive expressions for the reduced matrix elements (m.e.) of the $M1$ transition operator. For the $M1$ transition m.e. between states with total angular momentum quantum numbers $J=0$ and 1, i.e., $\langle(\pi j\otimes\nu j);J=0\|\mathbf{T}(M1)\|(\pi j\otimes\nu j);J=1\rangle$, one obtains

$$\langle 0^+ \| \mathbf{T}(M1) \| 1^+ \rangle = \sqrt{\frac{3}{4\pi} \frac{j+1}{j}} \left[(g_p^l - g_n^l) l \right.$$
$$\left. + \frac{g_p^s - g_n^s}{2} \right] \mu_N, \quad \text{for} \quad j = l + \frac{1}{2},$$

$$(3)$$

i.e., for QDC, and

$$\langle 0^+ \| \mathbf{T}(M1) \| 1^+ \rangle = \sqrt{\frac{3}{4\pi} \frac{j}{j+1}} \left[(g_p^l - g_n^l)(l+1) \right.$$
$$\left. - \frac{g_p^s - g_n^s}{2} \right] \mu_N, \quad \text{for} \quad j = l - \frac{1}{2}.$$

$$(4)$$

The analysis of Eqs. (3) and (4) results in some interesting conclusions. Comparing Eqs. (3) and (2) one can see that the orbital proton $\langle \mathbf{L}_p \rangle$ and neutron $\langle \mathbf{L}_n \rangle$ nondiagonal m.e. have opposite signs and equal absolute values. This is valid also for the spin proton $\langle \mathbf{S}_p \rangle$ and neutron $\langle \mathbf{S}_n \rangle$ m.e.. Since the $\langle \mathbf{S}_p \rangle$ and $\langle \mathbf{S}_n \rangle$ have opposite signs as well as spin g_p^s and g_n^s factors, the nondiagonal spin part of the total m.e. of the isovector $M1$ transition operator is large for both the $j = l + 1/2$ and the $j = l - 1/2$ cases. The orbital part of the total m.e. increases with increasing l and can be comparable with the spin part for both cases. However, in the case of $j = l + 1/2$, i.e., when the orbital angular momentum and spin of the single particle are aligned, the spin and orbital parts are summed up in phase, which results in large absolute values of the total m.e. and consequently in large values of the reduced $M1$ transition strength $B(M1; 0_1^+ \rightarrow 1_0^+) = \langle 0^+ \| \mathbf{T}(M1) \| 1^+ \rangle^2$. The opposite happens in the $j = l - 1/2$ case—the orbital and spin parts partially cancel and the reduced $M1$ m.e. becomes small. The constructive [Eq. (3)] and destructive [Eq. (4)] interference of the orbital and spin parts plays also an important role in Gamow-Teller transitions and $M1$ γ transitions in even-even $N=Z$ nuclei, as it was recently shown in [21,22].

Other isovector transitions that involve states with the spin J^π quantum number different from $J^\pi = 0^+$ and $J^\pi = 1^+$ ($\Delta T = 1$, $J+1 \rightarrow J$) in odd-odd $N=Z$ nuclei can be sizable in the strength with the $1^+ \rightarrow 0^+$ transition strength. In the 2NSj one can derive the simple relation

$$B(M1; J+1 \rightarrow J)$$

$$= \frac{3(J+1)(2j+2+J)(2j-J)}{4j(j+1)(2J+3)} B(M1; 1^+ \rightarrow 0^+).$$

$$(5)$$

This relation is valid for both the $j = l + 1/2$ and $j = l - 1/2$ cases. The dependence of the ratio $R = B(M1; J+1 \rightarrow J)/B(M1; 1^+ \rightarrow 0^+)$ on the spin quantum number J of the final state for different single j is shown in Fig. 1. As can be seen from Fig. 1 the number of transitions sizable in strength with $1^+ \rightarrow 0^+$ transition increases with increasing j. Since the $B(M1, 0^+ \rightarrow 1^+)$ value is large for the QDC, the $B(M1, J$
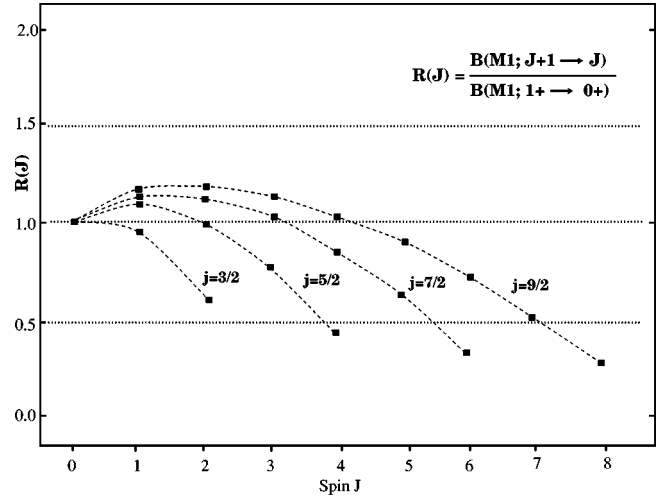

FIG. 1. Ratio R of $M1$ transition strengths between QDC as a function of the total spin quantum number J plotted for different single particle orbitals $j = 3/2, 5/2, 7/2, 9/2$.

$+1 \rightarrow J)$ values can be also large in this case.[1] The QDC states with $J = 0, \ldots, 2j$ form the band of states connected by strong $M1$ transitions that is similar to the ''shears band'' in heavy nuclei [27]. The strong $M1$ transitions caused by the QDC in odd-odd $N=Z$ nuclei and the strong $M1$ transitions related to the ''shears'' mechanism have similar noncollective nature.

But in the $j = l - 1/2$ case one cannot expect large $B(M1; J+1 \rightarrow J)$ values because they are proportional to the small $B(M1; 1^+ \rightarrow 0^+)$ value by a spin-dependent proportionality factor, which is close to one.

Equations (3) and (4) can be used also to derive within the single-j-shell approximation a unique formula for the $B(M1)$ values for both $j = l + 1/2$ and $j = l - 1/2$ cases in terms of magnetic moments of neighboring odd-A nuclei. In the independent particle model, the magnetic dipole moment μ of a nucleon in an orbital (nlj) is given by the Schmidt values [28]:

$$\mu_\rho \left(j = l + \frac{1}{2} \right) = \left[g_l^\rho l + \frac{g_s^\rho}{2} \right] \mu_N,$$

$$(6)$$

$$\mu_\rho \left(j = l - \frac{1}{2} \right) = \frac{j}{j+1} \left[g_l^\rho (l+1) - \frac{g_s^\rho}{2} \right] \mu_N,$$

$$(7)$$

where $\rho = \pi$ for proton and ν for neutron.

The combination of Eqs. (3),(4) with Eqs. (6),(7) results in a simple relation between the $M1$ strengths of transitions between 2NSj states and magnetic moments of neighboring odd-mass nuclei:

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[1]However, it should be emphasized that configuration mixing can significantly reduce the $B(M1, J+1 \rightarrow J)$ values for $J > 1$ (even if the 1^+ and 0^+ states are predominantly QDC states).

$$B(M1;(\pi j\times\nu j),0^+\to(\pi j\times\nu j),1^+)$$

$$=\frac{3}{4\pi}\frac{j+1}{j}[\mu_\pi(j)-\mu_\nu(j)]^2. \qquad (8)$$

The spin-orbit interference is hidden now in $\mu_\pi(j)$ and $\mu_\nu(j)$ quantities. Expression (8) does not explicitly contain orbital and spin g factors and therefore can help to explore the structure of the yrast 0^+ state and the 1^+ state in odd-odd $N=Z$ nucleus in an alternative way.

In contrast to the isovector $M1$ transitions discussed above, the magnetic dipole moments in odd-odd nuclei are generated by the isoscalar (IS) part

$$\mathbf{T}_{IS}(M1)=\left[\frac{g_p^l+g_n^l}{2}(\mathbf{L}_p+\mathbf{L}_n)+\frac{g_p^s+g_n^s}{2}(\mathbf{S}_p+\mathbf{S}_n)\right]\frac{\mu_N}{\hbar} \qquad (9)$$

of the $M1$ operator. We consider now magnetic dipole moments of odd-odd nuclei in the 2NSj approximation. The expressions for the $M1$ moment in a state with angular momentum quantum number J can be written in the following way:

$$\mu\left(j=l+\frac{1}{2}\right)=\frac{J}{2l+1}\left[(g_p^l+g_n^l)l+\frac{g_p^s+g_n^s}{2}\right]\mu_N, \qquad (10)$$

and

$$\mu_\rho\left(j=l-\frac{1}{2}\right)=\frac{J}{2l+1}\left[(g_p^l+g_n^l)(l+1)-\frac{g_p^s+g_n^s}{2}\right]\mu_N. \qquad (11)$$

The partial cancellation of spin and orbital parts in the $j=l-1/2$ case is also obvious from Eq. (11). However, the μ values are not very sensitive to the spin part due to the small value of the sum of proton and neutron spin g factors.

III. DISCUSSION

In this section we will confront the simple formulas from above with the data. Using Eqs. (3),(4) and free g factors ($g_p^l=1.0$, $g_n^l=0.0$, $g_p^s=5.5857$, and $g_n^s=-3.8263$), we obtain for the $B[M1;(\pi j\otimes\nu j),J^\pi=0^+\to(\pi j\otimes\nu j),J^\pi=1^+]$ values the expressions

$$B(M1;0^+\to1^+)=\frac{3}{4\pi}\frac{j+1}{j}[l+4.706]^2\mu_N^2,$$

$$\text{for}\quad j=l+\frac{1}{2} \qquad (12)$$

and

$$B(M1;0^+\to1^+)=\frac{3}{4\pi}\frac{j}{j+1}[l-3.706]^2\mu_N^2,$$
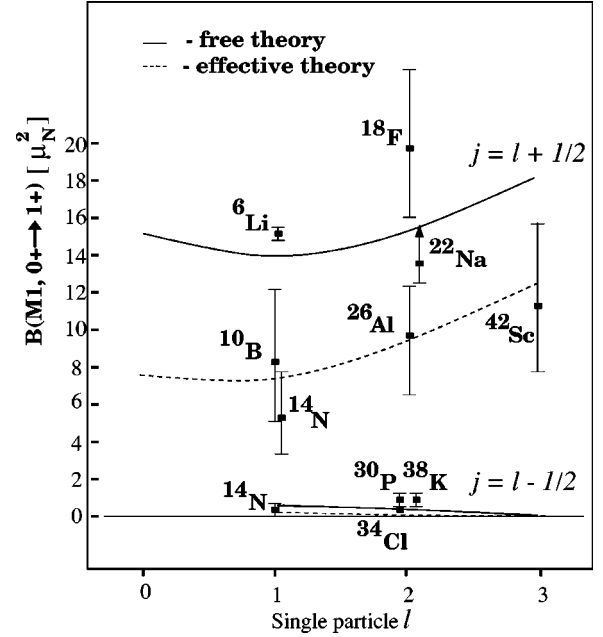
$$\text{for}\quad j=l-\frac{1}{2}. \qquad (13)$$

FIG. 2. The calculated and experimental $\Sigma_i\, B(M1;0_1^+\to1_i^+)$ values (see Table I, columns 6–8) are given as a function of the single-particle orbital angular momentum l. The results of the calculations are shown for the QDC $j=l+1/2$ branch [Eq. (12)] and for the $j=l-1/2$ branch [Eq. (13)]. The full lines correspond to the free theory (free spin g factors) and the dashed lines to the effective theory (with quenching factor $\alpha_q=0.7$). The experimental data for different elements are labeled. The value for ^{22}Na represents a lower limit. One expects the experimental values to lie between or in the vicinity of the two lines for both $j=l+1/2$ and $j=l-1/2$ cases.

The $B(M1)$ values from Eqs. (12),(13) are plotted with solid curves in Fig. 2 as functions of the single-particle orbital angular momentum l. The deuteron belongs to the $j=l+1/2$ branch. Only the spin part would contribute to the $B(M1)$ value if the $J^\pi=0_{T=1}^+$ state of the deuteron were bound. The calculated $B(M1)$ value for a modeled bound state is very large: $B[M1;(\pi s_{1/2}\nu s_{1/2}),J=0^+\to(\pi s_{1/2}\nu s_{1/2}),J=1^+]=15.86\mu_N^2$. Other $B(M1)$ values calculated from Eqs. (12),(13) assuming reasonable single-particle orbitals are given in Table I together with the corresponding experimental $B(M1)$ values.

For $j=l+1/2$ cases Eq. (12) yields large $B(M1)$ values sizable with the $B(M1;0^+\to1^+)$ for the deuteron. Therefore, the large $B(M1;0^+\to1^+)$ values in odd-odd $N=Z$ nuclei can be considered an indication of the QDC. The strong $M1$ transitions in ^{6}Li and ^{18}F are the best examples for transitions between QDC. In ^{22}Na a large lower limit for the total $B(M1;0^+\to1^+)$ value is obtained from summing up the $M1$ strengths from three transition fragments (see Table I). This value agrees with the estimate for a $B(M1)$ value between QDC, too.

For the four odd-odd $N=Z$ nuclei ^{10}B, ^{14}N, ^{26}Al, and ^{42}Sc large $B(M1)$ values are known, as well. These values are, however, smaller by a factor of about 2 than the corresponding QDC estimates with free g factors. Deviations of the data from the simple expressions (12),(13) can be attrib-

uted to configuration mixing [26], which are neglected in the simple 2NSj.

Configuration mixing can be taken into account to a certain extent by using quenched spin g factors $g_s = \alpha_q g_s^{\text{free}}$ with a quenching factor α_q. We have computed effective $B(M1)$ values with $\alpha_q = 0.7$ for both cases $j = l+1/2$ and $j = l-1/2$ from Eqs. (12),(13).

The results are included in Table I and plotted with dashed curves in Fig. 2. The effective $B(M1)$ values agree with the data from ^{10}B, ^{14}N, ^{26}Al, and ^{42}Sc. These agreements with the estimates using quenched g factors indicate that a precise quantitative understanding requires larger scale shell-model calculations. The main mechanism is, however, understood already in the simple 2NSj.

Small experimental isovector $B(M1)$ values were found in ^{14}N, ^{30}P, ^{34}Cl, and ^{38}K. Small $B(M1)$ values are calculated in the 2NSj for the $j = l-1/2$ case, regardless whether free g factors or quenched g factors are used. The best examples for an isovector $M1$ transition between the 0_1^+ state and the 1_1^+ state, which almost vanishes due to the cancellation of the orbital and the spin parts, are observed in ^{34}Cl and ^{38}K. This cancellation can reduce the $B(M1)$ value by a factor of more than 20 in comparison to the large *quasideuteron* $M1$ transitions. Such a drastic difference between *quasideuteron* transitions and *nonquasideuteron* transitions can be qualitatively well understood within the 2NSj approximation.

Particularly interesting cases are ^{14}N and ^{30}P. In ^{14}N two kinds of transitions coexist at low energy. The transition from the 0_1^+ state to the 1_1^+ ground state is very weak and can be related to the $j = l-1/2$ ($p_{1/2}$ shell) case. The next 1_2^+ state at 3.948 MeV is also low-lying and connected with the 0_1^+ state by a strong $M1$ transition. This transition can be considered as a *quasideuteron* transition ($j = l+1/2$ case) in the $p_{3/2}$ shell. It means that both $\pi(1p_{1/2}^1) \times \nu(1p_{1/2}^1)$ and $\pi(1p_{3/2}^{-1}) \times \nu(1p_{3/2}^{-1})$ components must be present in the 0_1^+ wave function with amplitudes smaller than one. This fact may explain why the experimental $B(M1;0_1^+ \to 1_2^+)$ transition strength is smaller than the estimated one. Similarly one can explain the fact that the experimental $B(M1)$ value for the $0_1^+ \to 1_1^+$ transition in ^{30}P is larger than the calculated value for the supposed $\pi(1d_{3/2}^1) \times \nu(1d_{3/2}^1)$ $j = l-1/2$ configuration of the 0_1^+ and 1_1^+ states. A small fragment of the *quasideuteron* $[\pi(2s_{1/2}^1) \times \nu(2s_{1/2}^1); 0^+] \to [\pi(2s_{1/2}^1) \times \nu(2s_{1/2}^1); 1^+]$ transition enhances the $0_1^+ \to 1_1^+$ transition.

Let us now discuss the relation (8) between isovector $M1$ transitions in the 2NSj and the Schmidt values for magnetic dipole moments, which is an exact equation in the simple single-j-shell approximation. As it was discussed above $B(M1)$ values in odd-odd $N=Z$ nuclei can differ from the pure 2NSj estimates due to configuration mixing. This could be partly taken into account by using quenched spin g factors. On the other hand, configuration mixing can lead also to a deviation of $M1$ moments in odd-A nuclei from the Schmidt values, which were used to eliminate the g factors in Eq. (8). It is, therefore, very interesting to investigate to what extent Eq. (8) can be used to predict isovector $B(M1)$ values

in odd-odd $N=Z$ nuclei from magnetic dipole moments in neighboring odd-A nuclei. For this purpose we replace $\mu_\pi(j)$ and $\mu_\nu(j)$ in Eq. (8) with the corresponding experimental values. The experimental μ_π and μ_ν values are the magnetic moments of the ground states $J^\pi = j^\pi$ in the neighboring odd-proton and odd-neutron nuclei, respectively. Comparing the $B(M1;0_1^+ \to 1_1^+)$ value calculated in this way with the corresponding experimental value we can conclude about the structure of the 0^+ and 1^+ states.

As an example, let us consider the nucleus ^{42}Sc. This nucleus has one neutron and one proton occupying the $1f_{7/2}$ shell above the even-even ^{40}Ca core. The experimental magnetic dipole moments of the $J^\pi = 7/2^-$ ground states in the neighboring nuclei ^{41}Sc and ^{41}Ca are 5.535 μ_N and -1.595 μ_N, respectively. Substituting these values in Eq. (8) we get for ^{42}Sc $B(M1;0^+ \to 1^+) = 15.62$ μ_N^2. Comparing this value with the experimental value of 11(4) μ_N^2, we can conclude that the wave functions of the 0_1^+ ground state and the excited 1_1^+ state in ^{42}Sc are dominated by the $(7/2_\pi^-) \times (7/2_\nu^-)$ component, where the $J_\rho^\pi = 7/2_\pi^-$ and the $J_\rho^\pi = 7/2_\nu^-$ states are the ground states of ^{41}Sc and ^{41}Ca, respectively.

The $B(M1)$ values estimated from $M1$ moments in neighboring odd-A nuclei and the corresponding experimental data are shown in Table II for other odd-odd $N=Z$ nuclei. Here, we consider for all nuclei only the lowest $0^+ \to 1^+$ transitions. The estimated $B(M1)$ value for the ^{14}N nucleus is larger than the experimental one. This supports our schematic explanation of the mixing of the QDC states with the states formed by a proton-neutron pair in the $j = l-1/2$ orbital. The results for the nucleus ^{30}P are also interesting. They can be interpreted in the following way: The 0_1^+ state and the 1_1^+ state contain a large $(1/2_\pi^+) \times (1/2_\nu^+)$ component. But the $1/2_\pi^+$ ground state of the nucleus ^{29}P and the $1/2_\nu^+$ ground state of ^{29}Si cannot be pure $2s_{1/2}$ states because the corresponding magnetic moments differ very much from the Schmidt values. These states should have more complicated structures that involve at least the $d_{3/2}$ and $d_{5/2}$ orbitals. It explains partially why there are no low-lying pure QDC states in ^{30}P. The estimated $B(M1)$ values for other nuclei are in rough agreement with the data. We conclude that the structures of the 0_1^+ and the 1_1^+ states can often be well approximated by the direct product $(J_p^\pi) \times (J_n^\pi)$ of the ground states of the corresponding (see Table II) odd-Z and odd-N nuclei.

We do not discuss here the nuclear magnetic dipole moments. This has been done recently [20]. We would like only to note that the isoscalar $M1$ moments are less sensitive to the spin part of the $M1$ m.e. [see Eqs. (10),(11)] than the isovector $M1$ transitions. This well known fact is due to the relatively small value of the isoscalar spin g factor $(g_p^s + g_n^s)/2$ (0.88 for free spin g factors) in comparison to the isovector value $(g_p^s - g_n^s)/2$ (4.706 for free spin g factors). Therefore, the constructive or destructive interference between the spin part and the orbital part for the cases with $j = l+1/2$ and with $j = l-1/2$ are less pronounced for the isos-

TABLE II. Experimental [32,33] magnetic dipole moments of odd-N and odd-Z nuclei, experimental [29–31] and estimated [see Eq. (8)] $B(M1)$ values for the transitions between the lowest 0_1^+ and 1_1^+ states in odd-odd $N=Z$ nuclei (see also Table I, column 5). The numbers in parentheses give the experimental errors in the last digits. If no number in parentheses is given, the experimental relative error is smaller than 10^{-3}.

Odd-Z			Odd-N			Odd-odd	$B(M1;0_1^+\to1_1^+)$ (μ_N^2)	
Nucleus	J_p^π	$\mu_p^{exp}(\mu_N)$	Nucleus	J_n^π	$\mu_n^{exp}(\mu_N)$	Nucleus	Expt.	Eq. (8)
$^{11}_5B_6$	$3/2^-$	2.689	$^{11}_6C_5$	$3/2^-$	$-0.964(1)$	$^{10}_5B_5$	7.5(32)	5.32
$^{13}_7N_6$	$1/2^-$	-0.322	$^{13}_6C_7$	$1/2^-$	0.702	$^{14}_7N_7$	0.05(2)	0.75
$^{17}_9F_8$	$5/2^+$	4.722	$^{17}_8O_9$	$5/2^+$	-1.894	$^{18}_9F_9$	20(4)	14.65
$^{21}_{11}Na_{10}$	$3/2^+$	2.386	$^{21}_{10}Ne_{11}$	$3/2^+$	-0.662	$^{22}_{11}Na_{11}$	5.0(3)	3.70
$^{25}_{13}Al_{12}$	$5/2^+$	3.646	$^{25}_{12}Mg_{13}$	$5/2^+$	-0.855	$^{26}_{13}Al_{13}$	8(2)	6.78
$^{29}_{15}P_{14}$	$1/2^+$	1.235	$^{29}_{14}Si_{15}$	$1/2^+$	-0.555	$^{30}_{15}P_{15}$	1.3(1)	2.33
$^{33}_{17}Cl_{16}$	$3/2^+$	0.752	$^{33}_{16}S_{17}$	$3/2^+$	0.644	$^{34}_{17}Cl_{17}$	0.23(2)	0.005
$^{37}_{19}K_{18}$	$3/2^+$	0.203	$^{37}_{18}Ar_{19}$	$3/2^+$	1.145	$^{38}_{19}K_{19}$	0.47(4)	0.35
$^{41}_{21}Sc_{20}$	$7/2^-$	5.535	$^{41}_{20}Ca_{21}$	$7/2^-$	-1.595	$^{42}_{21}Sc_{21}$	11(4)	15.62
$^{49}_{23}V_{26}$	$7/2^-$	4.47(5)	$^{45}_{22}Ti_{23}$	$7/2^-$	$-0.095(2)$	$^{46}_{23}V_{23}$		6.40
$^{51}_{25}Mn_{26}$	$5/2^-$	3.568	$^{47}_{22}Ti_{25}$	$5/2^-$	-0.788	$^{50}_{25}Mn_{25}$		6.34
$^{55}_{27}Co_{28}$	$7/2^-$	4.822(3)	$^{47}_{20}Ca_{27}$	$7/2^-$	$-1.380(24)$	$^{54}_{27}Co_{27}$		11.82
$^{61}_{29}Cu_{32}$	$3/2^-$	2.14(4)	$^{57}_{28}Ni_{29}$	$3/2^-$	$-0.798(1)$	$^{58}_{29}Cu_{29}$		3.44
$^{89}_{41}Nb_{48}$	$9/2^+$	6.216(5)	$^{89}_{40}Zr_{49}$	$9/2^+$	$-1.076(20)$	$^{82}_{41}Nb_{41}$		15.52

calar $M1$ moments than for the isovector $B(M1)$ values. The μ values for the $j=l+1/2$ and the $j=l-1/2$ branches differ by approximately a factor of 2 for $j=1/2$ and become almost equal at large j values. This is in an agreement with experiment. It is more difficult to see the difference between the two branches studying the magnetic dipole moments in odd-odd $N=Z$ nuclei. This difference is easier to observe by analyzing the $B(M1)$ values regardless of their large experimental errors. Therefore, isovector $B(M1)$ values can be a more sensitive tool for the investigation of quasideuteron configurations.

IV. CONCLUSIONS

Isovector magnetic dipole transitions between low-lying 0^+ and 1^+ states in odd-odd $N=Z$ nuclei were studied within the simple core+two-nucleon single-j-shell model. Analytical expressions for isovector $B(M1)$ values in odd-odd $N=Z$ nuclei were derived. Low-lying states in odd-odd $N=Z$ nuclei with a proton-neutron pair in a $j=l+1/2$ shell were considered as quasideuteron configurations. These cases are characterized by large $B(M1)$ transition strengths caused by coherent contributions of the orbital and spin parts to the total strength. The large $B(M1)$ values were interpreted as direct indications of QDC in the states which are connected by these strong transitions. Incoherent contribution of the spin and orbital parts to the total transition strength for the states formed by the proton-neutron pair in the $j=l-1/2$ shell strongly reduces the $B(M1)$ values. The $B(M1)$ values for the low-lying states in odd-odd nuclei can be predicted knowing only the single-particle j quantum number for the orbital occupied with the proton-neutron pair. Low-lying QDC states can be expected in the $1f_{7/2}$ nuclei

^{46}V, ^{50}Mn, and ^{54}Co. Weaker transitions can be expected in ^{58}Cu nucleus ($2p_{3/2}$ shell). In the odd-odd $N=Z$ nuclei where the $1f_{5/2}$ and $2p_{1/2}$ valence orbitals configurations become dominant for low-lying states one cannot expect to observe strong $M1$ transitions between these low-lying states. Only in the nuclei where the $1g_{9/2}$ shell plays an important role (for example ^{82}Nb nucleus), can one expect again the low-lying QDC states connected by strong $M1$ transitions (see Table II). It is very interesting to identify the QDC in the heavier odd-odd $N=Z$ nuclei and to check how well they fit into the picture. Recent experiments on low-lying states in the odd-odd $N=Z$ nuclei ^{46}V [10] and ^{54}Co [11] already give some preliminary indications[2] about the existence of QDC in these nuclei with the $f_{7/2}$ shell being the valence shell. We have related the $B(M1)$ values in odd-odd $N=Z$ nuclei with the magnetic moments of neighboring odd-A nuclei. The established simple connection can provide additional information on the structure of the $0_1^+,T=1$ state and the $1_1^+,T=0$ state.

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[2]The measured branching ratios and multipole mixing ratios for some transitions and isospin symmetry with the neighboring nuclei require strong $M1$ transitions.

[1] W. Heisenberg, Z. Phys. **77**, 1 (1932).

[2] C. E. Svensson, S. M. Lenzi, D. R. Napoli, A. Poves, C. A. Ur, D. Bazzacco, F. Brandolini, J. A. Cameron, G. de Angelis, A. Gadea, D. S. Haslip, S. Lunardi, E. E. Maqueda, G. Martínez-Pinedo, M. A. Nagarajan, C. Rossi-Alvarez, A. Vitturi, and J. C. Waddington, Phys. Rev. C **58**, R2621 (1998).

[3] S. Skoda, B. Fiedler, F. Becker, J. Eberth, S. Freund, T. Steinhardt, O. Stuch, O. Thelen, H. G. Thomas, L. Käubler, J. Reif, H. Schnare, R. Schwengner, T. Servene, G. Winter, V. Fischer, A. Jungclaus, D. Kast, K. P. Lieb, C. Teich, C. Ender, T. Härtlein, F. Kock, D. Schwalm, and P. Baumann, Phys. Rev. C **58**, R5 (1998).

[4] G. de Angelis, C. Fahlander, A. Gadea, E. Farnea, W. Gelletly, A. Aprahamian, A. Axelsson, D. Bazzacco, F. Becker, P. G. Bizzeti, A. Bizzeti-Sona, F. Brandolini, D. de Acuña, M. De Poli, J. Eberth, D. Foltescu, S. Lenzi, S. Lunardi, T. Martinez, D. R. Napoli, P. Pavan, C. M. Petrache, C. Rossi-Alvarez, D. Rudolph, B. Rubio, S. Skoda, P. Spolaore, G. Thomas, C. Ur, M. Weiszflog, and R. Wyss, Nucl. Phys. **A630**, 426 (1998).

[5] D. Rudolph, C. J. Gross, J. A. Sheikh, D. D. Warner, I. G. Bearden, R. A. Cunningham, D. Foltescu, W. Gelletly, F. Hannachi, A. Harder, T. D. Johnson, A. Jungclaus, M. K. Kabadiyski, D. Kast, K. P. Lieb, H. A. Roth, T. Shizuma, J. Simpson, O. Skeppstedt, B. J. Varley, and M. Weiszflog, Phys. Rev. Lett. **76**, 376 (1996).

[6] D. Rudolph, C. Baktash, W. Satula, J. Dobaczewski, W. Nazarewicz, M. J. Brinkman, M. Delvin, H.-Q. Jin, D. R. La Fosse, I. L. Riedinger, D. G. Sarantites, and C.-H. Yu, Nucl. Phys. **A630**, 417c (1998).

[7] J. Terasaki, R. Wyss, and P. H. Heenen, Phys. Lett. B **437**, 1 (1998).

[8] S. M. Vincent, P. H. Regan, D. D. Warner, R. A. Bark, D. Blumenthal, M. P. Carpenter, C. N. Davids, W. Gelletly, R. V. F. Janssens, C. D. O'Leary, C. J. Lister, J. Simpson, D. Seweryniak, T. Saitoh, J. Schwartz, S. Törmänen, O. Juillet, F. Nowacki, and P. Van Isacker, Phys. Lett. B **437**, 264 (1998).

[9] S. M. Lenzi, C. A. Ur, D. R. Napoli, D. Bazzacco, F. Brandolini, J. A. Cameron, G. de Angelis, M. de Poli, E. Farnea, A. Gadea, S. Lunardi, G. Martinez-Pinedo, A. Poves, C. Rossi-Alvarez, H. Somacal, and C. E. Svensson, Nuovo Cimento A **111**, 739 (1998).

[10] C. Friessner, N. Pietralla, A. Schmidt, I. Schneider, Y. Utsuno, T. Otsuka, and P. von Brentano, Phys. Rev. C **60**, 011304 (1999).

[11] I. Schneider, A. F. Lisetskiy, C. Friessner, R. V. Jolos, N. Pietralla, A. Schmidt, and P. von Brentano, Phys. Rev. C (submitted).

[12] H. H. Wolter, A. Faessler, and P. U. Sauer, Nucl. Phys. **A167**, 108 (1971).

[13] A. Petrovici, K. W. Schmid, and Amand Faessler, Nucl. Phys. **A647**, 197 (1999).

[14] K. Langanke, D. J. Dean, S. E. Koonin, and P. B. Radha, Nucl. Phys. **A613**, 253 (1997).

[15] P. Van Isacker and D. D. Warner, Phys. Rev. Lett. **78**, 3266 (1997).

[16] W. Satula and R. Wyss, Phys. Lett. B **393**, 1 (1997).

[17] W. Satula, J. Dobaczewski, and W. Nazarewicz, Phys. Rev. Lett. **81**, 3599 (1998).

[18] T. Otsuka, Michio Honma, and Takahiro Mizusaki, Phys. Rev. Lett. **81**, 1588 (1998).

[19] W.-T. Chou, J.-Y. Zhang, R. F. Casten, and D. S. Brenner, Phys. Lett. B **255**, 487 (1991).

[20] Y. Ronen and S. Shlomo, Phys. Rev. C **58**, 884 (1998).

[21] M. S. Fayache, L. Zamick, Y. Y. Sharon, and Y. Durga Devi, nucl-th/9906071.

[22] Y. Fujita, H. Akimune, I. Daito, M. Fujiwara, M. N. Harakeh, T. Inomata, J. Jänecke, K. Katori, C. Lüttge, S. Nakayama, P. von Neumann-Cosel, A. Richter, A. Tamii, M. Tanaka, H. Toyokawa, H. Ueno, and M. Yosoi, Phys. Rev. C **55**, 1137 (1997), and references therein.

[23] H. Liu and L. Zamick, Phys. Rev. C **36**, 2064 (1987).

[24] L. Zamick and D. C. Zheng, Phys. Rev. C **37**, 1675 (1988).

[25] A. Poves and G. Martinez-Pinedo, Phys. Lett. B **430**, 203 (1998).

[26] Akito Arima and Hisashi Horie, Prog. Theor. Phys. **12**, 4 (1954).

[27] R. M. Clark, R. Krüken, S. J. Asztalos, J. A. Becker, B. Busse, S. Chmel, M. A. Deleplanque, R. M. Diamond, P. Fallon, D. Jenkins, K. Hauschild, I. M. Hibbert, H. Hübel, I. Y. Lee, A. O. Macchiavelli, R. W. MacLeod, G. Schmid, F. S. Stephens, U. J. van Severen, K. Vetter, R. Wadsworth, and S. Wan, Phys. Lett. B **440**, 251 (1998), and references therein.

[28] P. J. Brussard and P. W. M. Glaudemans, *Shell-Model Applications in Nuclear Spectroscopy* (North-Holland, Amsterdam, 1977).

[29] P. M. Endt, At. Data Nucl. Data Tables **55**, 192 (1993).

[30] P. M. Endt, Nucl. Phys. **A521**, 1 (1990).

[31] F. Ajzenberg-Selove, Nucl. Phys. **A523**, 1 (1991).

[32] P. Raghavan, At. Data Nucl. Data Tables **42**, 203 (1989).

[33] B. Hinfurtner, G. Seewald, E. Hagan, E. Zech, and I. S. Towner, Nucl. Phys. **A620**, 317 (1997).