PHYSICAL REVIEW C, VOLUME 60, 061304

Phase/shape coexistence in ^{152}Sm in the geometric collective model

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(Received 13 August 1999; published 15 November 1999)

It is shown that data on $E2$ transition rates in ^{152}Sm can be reproduced in the geometric collective model and that the calculations support the phase/shape coexistence interpretation of this pivotal nucleus. The coexistence is further supported by microscopic calculations, as is the concept of a sudden onset of deformation in this region. [S0556-2813(99)50512-8]

PACS number(s): 21.60.Ev, 21.10.Ky, 21.10.Re, 27.70.+q

In recent work [1,2], rather startling properties of ^{152}Sm have led to new theoretical interpretations, both of this nucleus [3] and of the way in which deformation evolves in some nuclear transitional regions [4]. These interpretations were originally motivated by the discovery that the $B(E2{:}2_3^+ \rightarrow 0_2^+)$ value was extremely small ($\lesssim$0.05 W.u.) [1,2] and by experiments [2] which showed that a number of other $B(E2)$ values for ^{152}Sm in the literature were incorrect, and which provided new measurements of them.

The nucleus ^{152}Sm has historically proven a challenge for theoretical descriptions. Its level scheme may at first seem suggestive of a traditional picture in which the 0_2^+ state is interpreted as a β-vibrational bandhead and the 2_3^+ state as a γ-vibrational bandhead. The suppressed $2_3^+ \rightarrow 0_2^+$ transition would seem to be consistent with such a picture, since, as a $2_\gamma^+ \rightarrow 0_\beta^+$ transition, it would require the simultaneous destruction of a γ phonon and the creation of a β phonon. However, closer inspection of the level scheme reveals that the level spacings and $B(E2)$ values cannot be the result of simple mixing of pure rotational bands [2,5]. Moreover, the interband transitions cannot be fully explained even when three-band mixing effects are taken into account [6].

A very recent analysis of ^{152}Sm in the interacting boson approximation (IBA) [7] showed that the main features of this nucleus, including most of the known $B(E2)$ values, can be reproduced by calculations within a limited region of IBA parameter space [1]. The IBA results, combined with existing $E0$ and (p,t) experimental measurements [8,9], motivate a picture in which the yrast levels of ^{152}Sm have a moderately deformed (rotational) structure while the low lying nonyrast states are more spherical in structure and constitute vibrational excitations built on the 0_2^+ state.

In this view the 2_2^+ level is a single phonon excitation of the 0_2^+ state and the 4_2^+, 2_3^+, and 0_3^+ states comprise a two-phonon triplet. Additionally, some levels of a three-phonon multiplet were also proposed. This multiplet structure based on the 0_2^+ level is quite evident in the ^{152}Sm level scheme in Fig. 1. Of course, these phonon excitations are not pure. They mix with the deformed yrast states and, in the IBA description, the dominant phonon amplitudes already diminish in importance for some low spin multiphonon states [2]. This type of coexistence shows some common features with the coexistence observed in the case of intruder states which occur, for example, in the Cd and Hg regions. However, here it develops within single proton and neutron major shells, in contrast to the situation for the cross major shell particle-hole excitations that apply to intruder states; it occurs in nuclei with substantial numbers of both valence neutrons and protons, and its evolution is more rapid, being evident only in a narrow span of nuclei.

The coexistence interpretation of ^{152}Sm was recently shown to be closely related to a view [4] of how deformation and collectivity develop in some regions of nuclei in which, instead of a gradual increase in deformation as a function of the number of valence nucleons, nuclei remain anharmonic vibrators, with collectivity increasing with valence nucleon number, until a ''critical'' point where the minimum in the potential energy surface suddenly jumps to a large finite deformation — this structural evolution is more akin to a phase transition than to a gradual shape evolution.

Given the importance of this result it is clearly of great interest to test whether the ^{152}Sm behavior can be reproduced in other models and, if so, whether it also corresponds to a very limited region of parameter space. The two principal phenomenological approaches to collective behavior in nuclei are the IBA and the geometric model of Bohr and Mottelson [10] in which the Hamiltonian is written in terms of the shape coordinates β and γ, and partial derivatives with respect to them. A practical embodiment of the Bohr-Mottelson approach is the geometric collective model (GCM) of Gneuss, Greiner, and colleagues [11,12].

It is therefore the primary purpose of this Rapid Commu-

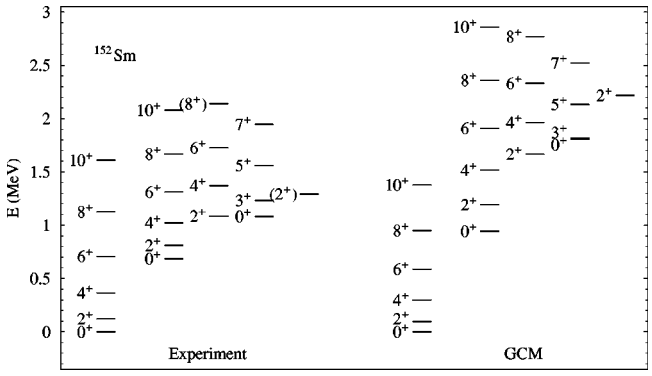

FIG. 1. Energy levels of ^{152}Sm. Left, experiment; right, GCM calculations discussed in the text.

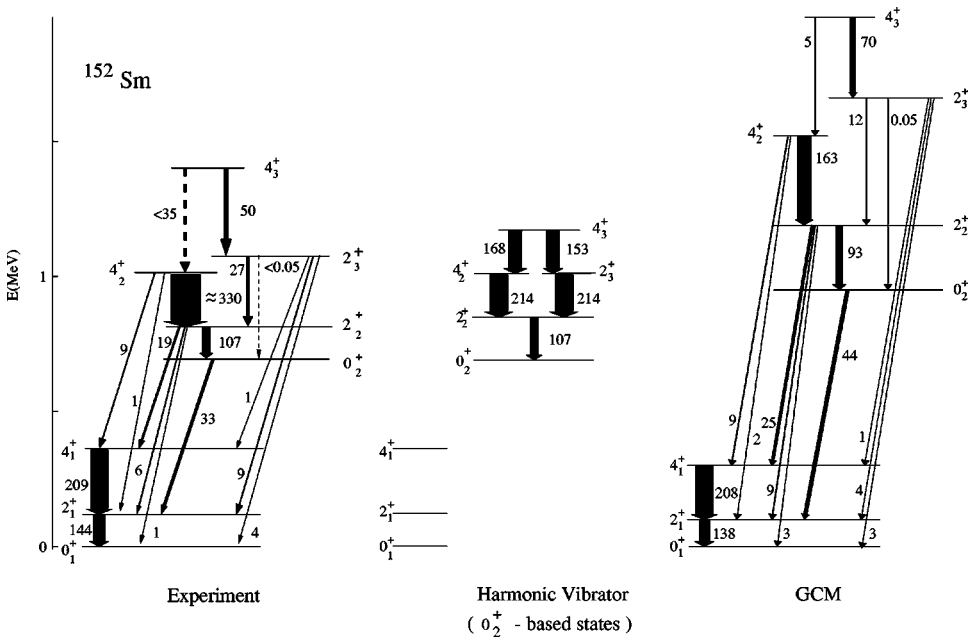

FIG. 2. Comparison of experimental $B(E2)$ values (in W.u.) in ^{152}Sm with the harmonic vibrator predictions for the 0_2^+-based levels and with the present GCM calculations. The thickness of the transition arrows is a guide to the corresponding $B(E2)$ values. The dashed arrows denote upper limits.

nication to study ^{152}Sm with the GCM — to see if the extensive set of data now available [2] in ^{152}Sm can be reproduced and whether it provides evidence for phase/shape coexistence. We note that these data are substantially different in a number of key transition rates from previous values and that this is the first time that the GCM has been applied to the new data. We will also present potential energy surface calculations which show that the hypothesis of Ref. [3], based on a phenomenological model, has a microscopic basis.

The GCM Hamiltonian is given [11] by

$$H = T + V, \tag{1}$$

where $T = 1/B_2(\pi\times\pi)^{(0)} + P_3/3\{(\pi\times\alpha)^{(2)}\times\pi\}^{(0)}$ and the potential V is defined, as follows, in terms of the Bohr-Mottelson shape variables β and γ:

$$V = C_2\frac{1}{\sqrt{5}}\beta^2 - C_3\sqrt{\frac{2}{35}}\beta^3\cos 3\gamma + C_4\frac{1}{5}\beta^4 - C_5\sqrt{\frac{2}{175}}\beta^5\cos 3\gamma + C_6\frac{2}{35}\beta^6\cos^2 3\gamma + D_6\frac{1}{5\sqrt{5}}\beta^6. \tag{2}$$

The complexity of this Hamiltonian (eight parameters — six in V and two in T) has historically limited the applications of the GCM. Recently, however, we discussed [13] a simplified approach to the GCM, in terms of the three-parameter potential

$$V = C_2\frac{1}{\sqrt{5}}\beta^2 - C_3\sqrt{\frac{2}{35}}\beta^3\cos 3\gamma + C_4\frac{1}{5}\beta^4 \tag{3}$$

and with the kinetic energy truncated to the first (harmonic) term. With this approach, the idealized paradigms of structure — vibrator, γ-soft, and deformed rotor nuclei — are trivially obtained [13].

We use this simplified approach here. In scanning possible parameters it becomes apparent that, as with the IBA, the GCM seldom gives a small $B(E2{:}2_3^+\rightarrow 0_2^+)$ value. Indeed, the low value of this observable in ^{152}Sm constrains the GCM parameters to a narrowly defined family. Fine tuning the calculations within this family gives a fit to the empirical ^{152}Sm level scheme that we show in Figs. 2 and 3. Figure 2 shows energies and absolute $B(E2)$ values for levels of known lifetimes while Fig. 3 gives energies and relative $B(E2)$ values for those levels for which the lifetimes are not known. The parameter values for this calculation are $B_2 = 61\times10^{-42}$ MeV s^2, $C_2 = -51.5$ MeV, $C_3 = 541.5$ MeV, and $C_4 = 1793$ MeV.

The agreement is quite good. In fact, if anything, the GCM is in slightly better agreement with the data than the IBA although any assessment of the relative merits of the IBA and GCM calculations is subtle and involves an understanding of the different roles of the parameters in the two models and of the structure of the energy surfaces. The GCM calculations reproduce experimental $B(E2)$ values ranging over nearly a factor of 10^4.

The quite different $R_{4/2}$ values for the yrast states and the ''0_2^+-yrast'' states are reflected in the calculations, although $R_{4/2}$ for the 0_2^+-based levels is somewhat lower than the empirical value and most of the energy spacings in the 0_2^+ family are too large. The $B(E2)$ values within this family are quite sensitive to the phonon structure. Figure 2 is designed in the same format as the companion figure with IBA predictions of Ref. [2]. The middle panel shows the values expected *if* the 0_2^+-based levels behaved as a harmonic vibrator. While we describe these states as a coexisting vibratorlike phase in ^{152}Sm, the $E2$ data clearly deviate strongly from the vibrator predictions. The $2_3^+\rightarrow 2_2^+$ transition should be 214 W.u. in the pure vibrator but is measured to be 27 W.u. Likewise, the $4_3^+\rightarrow 4_2^+$ and $4_3^+\rightarrow 2_3^+$ transitions would be $\sim$150 W.u. in the vibrator but, experimentally, one is <35 W.u. and the other is 50 W.u.

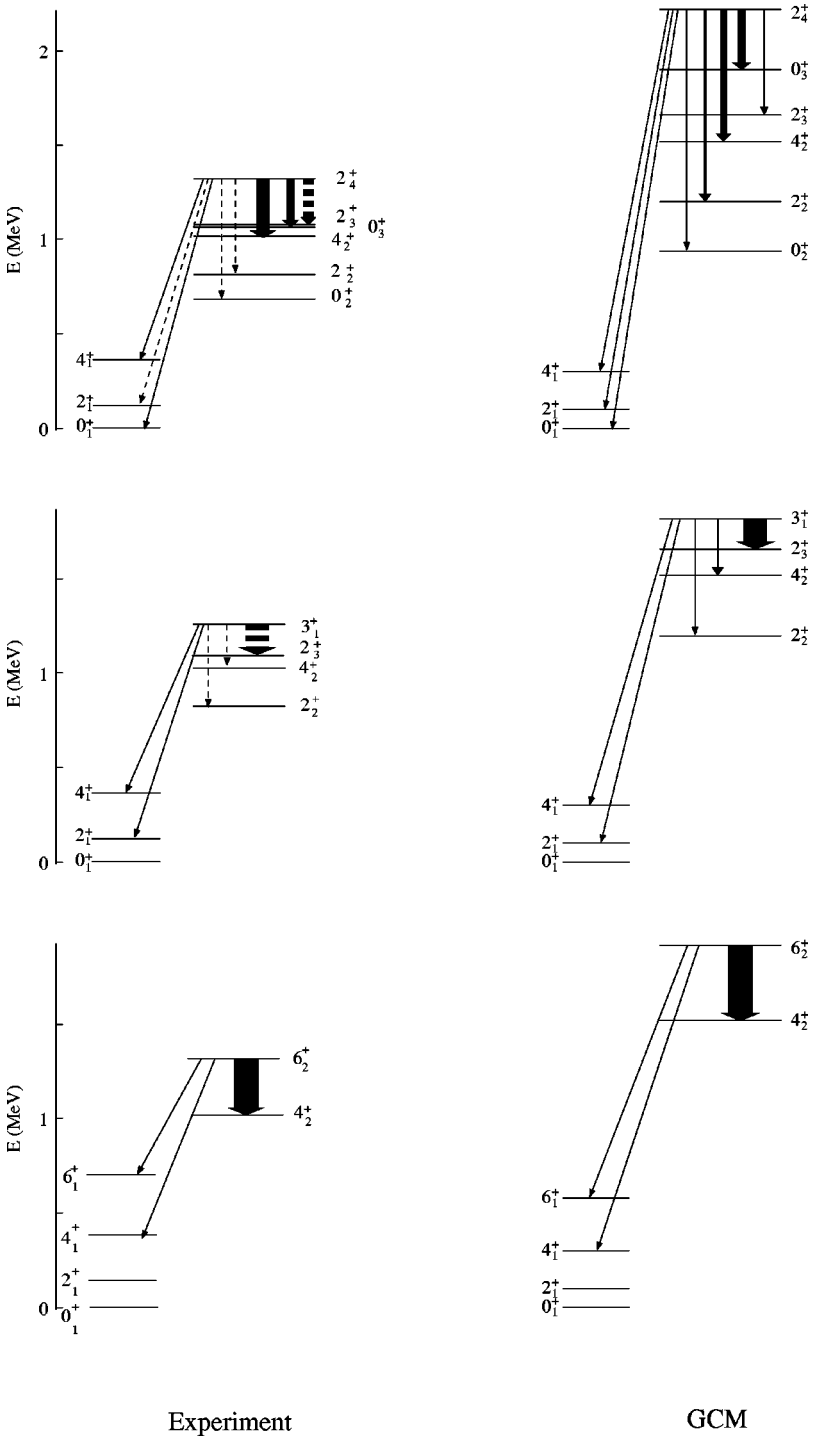

FIG. 3. Comparison of experimental and GCM relative $B(E2)$ values for states with unknown absolute $B(E2)$ values. The dashed transition arrows are upper limits due either to unknown $E2/M1$ mixing ratios or to nonobservation of the transitions.

The GCM calculations (Fig. 2, right) mirror the experimental situation much better than the simple harmonic vibrator does. A number of $B(E2)$ values for transitions that are allowed in the harmonic vibrator are reduced by factors of 2 – 10 in the direction of (albeit sometimes not reaching) the experimental values. We will see below that, as with the IBA wave functions of Ref. [2], part of the explanation for these differences from the predictions with pure vibrator wave functions is a reduction in the dominance of a single vibrator component for some of the 0_2^+-nonyrast states (e.g., 0_3^+, 4_3^+, 2_4^+, etc.) compared to the 0_2^+-yrast levels.

Particularly interesting are the predictions for the crossover transitions between the 0_2^+-based levels and the ground-state band. The strict picture of independent coexisting structures would have led to forbidden crossover transitions. These transitions arise from mixing of the two structures as was demonstrated in Ref. [2] (see also Fig. 4 below). Intriguingly, some of these crossover transitions reach collective magnitudes (20–30 W.u.), while others remain quite weak (<1 up to ~ 10 W.u.). The reproduction of this wide range of values is therefore a challenge for theoretical descriptions. In fact, both features are predicted correctly by the GCM. In

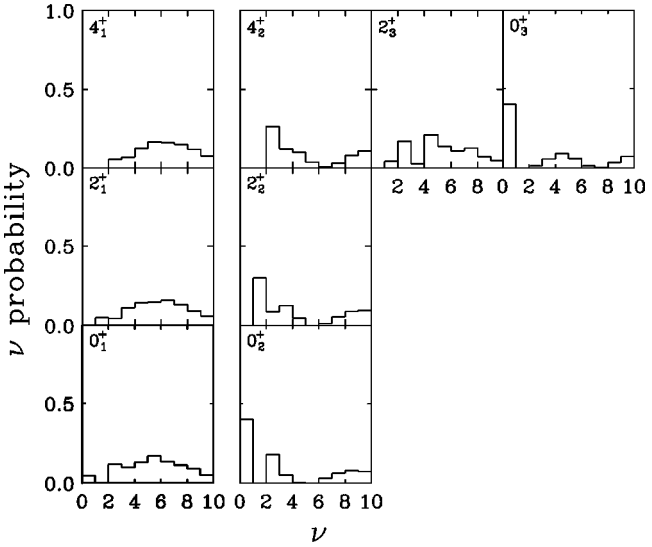

FIG. 4. Probability distributions of the GCM states in ^{152}Sm for the yrast levels and the lowest few 0_2^+-based states.

addition, the predictions of relative $B(E2)$ values for levels with unknown lifetimes, shown in Fig. 3, are also in generally good agreement with the experimental branching ratios. In all of these comparisons, the cases of good agreement and disagreement with the data are similar in the IBA and GCM calculations. We note that a proper fit with the full GCM Hamiltonian of Eqs. (1) and (2) gives good agreement with the data similar to that of our simplified Hamiltonian.

In Refs. [2,3], an important argument for two coexisting sets of structurally different states was the phonon structure of the wave functions. We can also inspect this in the present calculations since the GCM wave functions can be expanded in a quadrupole phonon basis. The phonon expansion in the GCM is analogous to the *d*-boson basis of the IBA, although the microscopic nature of the phonon is conceived differently in these two models.

In the GCM the phonon basis is not unique; the basis states are defined as the eigenfunctions of a five-dimensional harmonic oscillator Hamiltonian

$$H=\frac{1}{B_2'}\left[(\pi\times\pi)^{(0)}+C_2'B_2'\frac{1}{\sqrt{5}}\beta^2\right], \tag{4}$$

containing two parameters B_2' and C_2'. The resulting set of basis functions is dependent upon the Hamiltonian parameters only as the product $C_2'B_2'$. The solutions [14] for the β-dependent part of the wave function depend upon β only via the combination $\sqrt{C_2'B_2'}\beta$, while the solutions for the γ-dependent part are not affected by the values of B_2' and C_2', and so the choice of $C_2'B_2'$ simply scales the basis functions in β. For a phonon expansion to reveal the presence of strong oscillatorlike components in a GCM wave function, the value of $C_2'B_2'$ used in generating the phonon basis must be chosen to produce basis states which approximately match the β extent of the wave function. In the decomposition of Fig. 4 the values $B_2'=61\times10^{-42}$ MeV s^2 and $C_2'=104$ MeV were used [12].

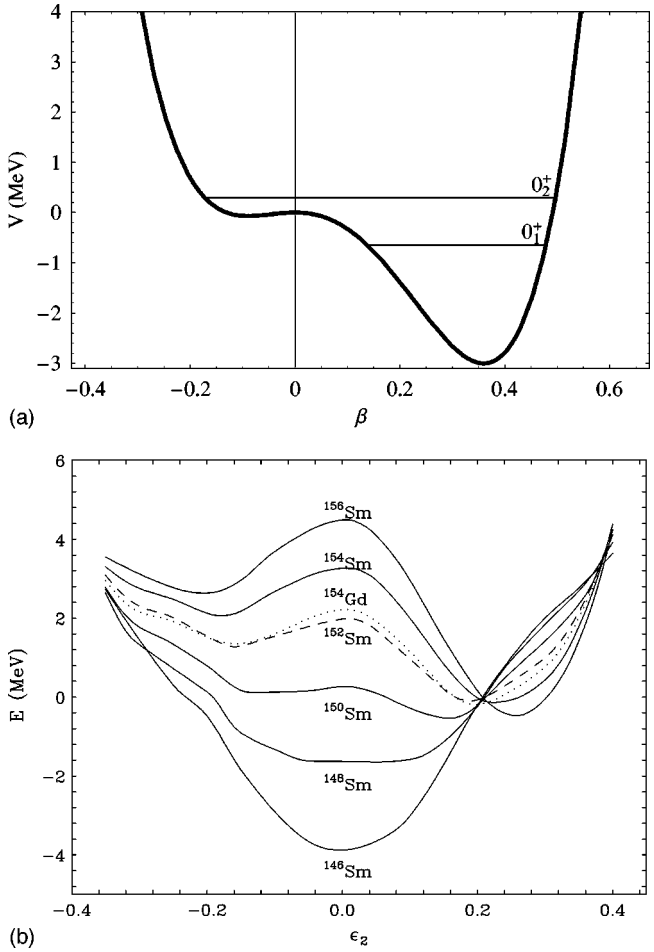

FIG. 5. (a) The potential in the GCM calculations along a cut with $\gamma=0°$. The energies of the ground state and the 0_2^+ state are shown to illustrate the different ranges of β values over which their respective wave functions are spread; (b) potential energy curves calculated with the Nilsson-Strutinsky-BCS model for $^{146-156}$Sm and ^{154}Gd.

The results for the key states are shown in Fig. 4 and support a coexistence picture. The yrast wave functions are broadly spread out in phonon space, a result characteristic of the expansion of a deformed state in a spherical vibrator basis. In contrast, the 0_2^+, 2_2^+, and 4_2^+ states have large components (albeit smaller than in the IBA) for zero, one, and two phonons, respectively.

Some of the higher levels of the 0_2^+-based family, such as the 2_3^+ and 4_3^+ levels, show ''phonon-diluted'' wave functions similar to those found in the IBA calculations. This is consistent with the reductions from vibrator $B(E2)$ values calculated for the decay of these levels as discussed above. An interesting feature of the GCM wave functions that does differ from those of the IBA is that some higher levels, such as the 0_3^+, 2_4^+, and 4_4^+ states (the latter two not shown) again have large probabilities for a single vibrator component. At least for the 0_3^+ level, this is probably a deficiency in the GCM calculations, as evidenced by the overly large 145 w.u. calculated $B(E2{:}0_3^+\rightarrow2_2^+)$ value. Finally, the phonon distributions also show the mixing of yrast and 0_2^+-based levels that led to the crossover transitions discussed earlier.

To see how the coexistence emerges in the GCM, we consider the potential energy surface corresponding to these calculations, which is shown in a cut along $\gamma=0°$ in Fig. 5(a). The energies of the 0_1^+ and 0_2^+ states are shown in the figure as horizontal lines. The ground state and low spin yrast levels are ''trapped'' within the deformed minimum, while the second minimum (actually a saddle point in the γ direction in the full $\beta\gamma$ plane) serves to spread the potential in β, so that the higher lying states have significantly larger fluctuations and smaller expectation values of β. A virtually identical energy surface results from a fit with the full GCM Hamiltonian.

The question naturally arises as to why this phase coexistence, arising within a single major shell, should appear in ^{152}Sm and not in other nuclei. In order to address this question, we have carried out microscopic Nilsson-Strutinsky-BCS calculations [15] of the potential energy for the Sm isotopes ($N=84-94$) and ^{154}Gd, an isotone of ^{152}Sm. The results, calculated at $\epsilon_4=0$, are shown in Fig. 5(b). Analogous calculations of the ground state of ^{152}Sm, using the microscopic total routhian surface with the ultimate-cranking approach [16], which includes the γ degree of freedom, show nearly identical results, with a saddle point in the γ direction near $\gamma=60°$ corresponding to the oblate minimum seen in Fig. 5(b).

Although such calculations do not take all degrees of freedom into account and, in particular, ignore any N dependence of single particle energies for an isotopic chain, they clearly show an evolution of structure which reflects the ideas of Ref. [4]. The light isotopes of Sm show rather deep spherical minima, the heavier ones a deformed minimum at about $\epsilon_2\sim0.25$. The essential point in the figure is that the minimum in the energy jumps from $\epsilon_2\sim0$ to $\epsilon_2\sim0.2$; one does *not* see a gradually shifting minimum. The only case at all comparable to ^{152}Sm is ^{154}Gd which is indeed similar in observed structure with $E(2_1^+)=123$ keV, almost the same as ^{152}Sm, and a small $B(E2{:}2_3^+\rightarrow0_2^+)$ value as well.

One might have expected similar phase coexistence in the $A\sim100$ region near $N=60$ but, there, the structural transition from spherical to deformed is so abrupt that there appears to be no single nucleus in which both spherical and deformed states coexist close in energy; this is an example of the ''integer nucleon number problem'' and the consequent discreteness of structural changes discussed in Ref. [4].

To summarize, we have studied the phase coexistence phenomenon in ^{152}Sm in the GCM, showing that (a) the GCM reproduces quite well an extensive set of $B(E2)$ values and branching ratios in ^{152}Sm — yrast $B(E2)$ values, ''0_2^+-yrast,'' and off-yrast $B(E2)$ values, and crossover $B(E2)$ values between these level sequences; (b) the GCM does so only for a very narrow region of parameters (given the discrete changes in structure that accompany changes in nucleon number in transitional regions, this suggests that phase/shape coexistence of this type will be a rare phenomenon); and (c) the GCM solution, like that of the IBA, and consistent with other empirical evidence in ^{152}Sm, shows (see the phonon expansion of the GCM wave functions) coexistence of a softly deformed set of yrast levels and a set of anharmonic vibrator levels built on the 0_2^+ state, as well as a gradual dissolution of the vibrator structure for low spin multiphonon states.

The fact that both the IBA and GCM give such strikingly similar results from different approaches, that both present the ^{152}Sm behavior as a rather isolated anomaly in parameter space, and that both show similar phonon-basis wave functions, points to a consistent structural interpretation of this unusual and pivotal nucleus. Along with the results and discussion in Ref. [4], these results suggest an evolution of structure in some spherical-deformed transition regions in which the global minimum jumps discretely from spherical to deformed at a given nucleus. Such a view is supported by microscopic calculations of the potential energy surfaces.

We would like to thank W. Nazarewicz for very useful discussions. The work was supported by the U.S. DOE under Contract Nos. DE-FG02-91ER-40609, DE-FG05-96ER-40983, and DE-FG02-88ER-40417.

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