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Partial dynamical SU(3) symmetry and the nature of the lowest $K=0$ collective excitation in deformed nuclei

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We discuss the implications of partial dynamical SU(3) symmetry (PDS) for the structure of the lowest $K=0^+(K=0_2)$ collective excitation in deformed nuclei. We consider an interacting boson model Hamiltonian whose ground and γ bands have good SU(3) symmetry, while the $K=0_2$ band is mixed. It is shown that the double-phonon components in the $K=0_2$ wave function arise from SU(3) admixtures which, in turn, can be determined from absolute $E2$ rates connecting the $K=0_2$ and ground bands. An explicit expression is derived for these admixtures in terms of the ratio of $K=0_2$ and γ bandhead energies. The SU(3) PDS predictions are compared with existing data and with broken-SU(3) calculations for ^{168}Er. [S0556-2813(99)50711-5]

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The nature of the lowest $K=0^+$ $[K=0_2]$ excitation in deformed nuclei is still subject to controversy. Recently its traditional interpretation as a vibration in the β degree of freedom [1] has been actively discussed and contested [2-5]. The preferential decay of some $K=0_2$ bands in deformed nuclei to the γ band rather than to the ground (g) band have led Casten and von Brentano to suggest that these bands should be understood as phonon excitations built on top of the γ band [2]. Such a decay pattern is consistent with calculations [6] in the interacting boson model [7] (IBM) and in the dynamic deformation model [3]. This new interpretation was subsequently questioned and challenged. Burke and Sood have claimed that the observed relative $E2$ strengths could arise from rather minor double-γ-phonon admixtures [4]. Günther *et al.* have argued that the empirical evidence presented in [2] involves higher-spin levels which are sensitive to K admixtures, and have shown that band mixing calculations can explain the $K=0_2\rightarrow\gamma$ transitions without the assumption of double-γ-phonon character [5]. The most relevant information needed to resolve the structure of the $K=0_2$ band lies in absolute transition rates. An important step in this debate was therefore the measurement of lifetimes of the lowest $2^+_{K=0_2}$ level [8] and the measurement via Coulomb excitation of $B(E2)$ values connecting the 2^+_g and 2^+_γ states with the $0^+_{K=0_2}$ level in ^{168}Er [9]. This nucleus was recently shown to be a good example of SU(3) partial dynamical symmetry (PDS), for which the ground and γ bands have good SU(3) symmetry, while the lowest excited $K=0_2$ band is mixed [10]. The purpose of this work is to study the nature of this band under the assumption of SU(3) PDS and to compare the predictions with the above-mentioned ^{168}Er data and with broken-SU(3) calculations in the IBM framework.

An IBM Hamiltonian with partial SU(3) symmetry has the form [10]

$$H=h_0P_0^\dagger P_0+h_2P_2^\dagger\cdot\tilde{P}_2. \tag{1}$$

Here $s^\dagger$ ($d^\dagger$) are monopole (quadrupole) bosons whose total number is N, the dot implies a scalar product and $P_0^\dagger=d^\dagger\cdot d^\dagger-2(s^\dagger)^2$, $P^\dagger_{2,\mu}=2\,s^\dagger d^\dagger_\mu+\sqrt{7}(d^\dagger d^\dagger)^{(2)}_\mu$ are boson-pairs, $\tilde{P}_{2,\mu}=(-1)^\mu P_{2,-\mu}$. For $h_0=h_2$ the above Hamiltonian is an SU(3) scalar related to the Casimir operator of SU(3), while for $h_0=-5h_2$ it is an SU(3) tensor, $(\lambda,\mu)=(2,2)$. Although H is not an SU(3) scalar, it has a subset of solvable states with good SU(3) symmetry. The solvable eigenstates belong to the ground and $\gamma^k_{K=2k}$ bands, and are simply selected members of the Elliott basis [11] with good SU(3) symmetry, $(\lambda,\mu)=(2N-4k,2k)K=2k$. States in other bands are mixed. The partial SU(3) symmetry of H is converted into partial dynamical SU(3) symmetry by adding to it $O(3)$ rotation terms which lead to an $L(L+1)$ splitting but do not affect the wave functions.

The Hamiltonian of Eq. (1) with $h_0=2h_2=0.008$ MeV was used in [10] to demonstrate the relevance of SU(3) PDS to the spectroscopy of ^{168}Er. The resulting SU(3) decomposition of the lowest bands is shown in Fig. 1, and compared to the conventional broken-SU(3) calculations of Warner Casten and Davidson (WCD) [12] where an $O(6)$ term is added to an SU(3) Hamiltonian, and to the consistent-Q formalism (CQF) [13], where the Hamiltonian involves a non-SU(3) quadrupole operator. In the WCD and CQF calculations all states are mixed with respect to SU(3). In the PDS calculation, states belonging to the ground $(K=0_1)$ and γ $(K=2_1)$ bands are pure Elliott states $\phi_E((2N,0)K=0,L)$ and $\phi_E((2N-4,2)K=2,L)$, respectively, while the $K=0_2$ band is mixed and has the structure

$$\begin{aligned}|L,K=0_2\rangle=&A_1\tilde{\phi}_E((2N-4,2)\tilde{K}=0,L)\\&+A_2\tilde{\phi}_E((2N-8,4)\tilde{K}=0,L)\\&+A_3\phi_E((2N-6,0)K=0,L).\end{aligned} \tag{2}$$

Here $\tilde{\phi}_E$ denote states orthogonal to the solvable $\gamma^k_{K=2k}$ Elliott states. For ^{168}Er ($N=16$) the $K=0_2$ band contains 9.6% (26,0) and 2.9% (24,4) admixtures into the dominant (28,2) irreducible representation (irrep). Using the geometric analogs of the SU(3) bands [14], $(2N-4,2)K=0\sim\beta$, $(2N-8,4)K=0\sim(\sqrt{2}\beta^2+\gamma^2_{K=0})$, $(2N-6,0)K=0\sim(\beta^2-\sqrt{2}\gamma^2_{K=0})$, the wave function of Eq. (2) can be expressed in

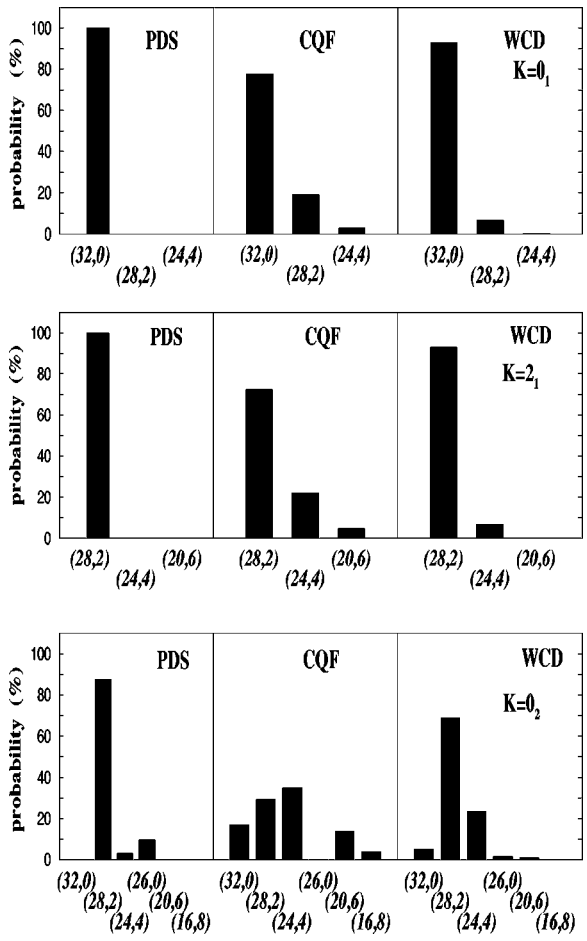

FIG. 1. SU(3) decomposition of wave functions of the ground ($K=0_1$), γ ($K=2_1$), and $K=0_2$ bands of ^{168}Er ($N=16$) in the SU(3) PDS calculation (present work), and broken-SU(3) calculations WCD [12] and CQF [13].

terms of the probability amplitudes for single- and double-phonon $K=0$ excitations

$$A_\beta = A_1, A_{\gamma^2} = (A_2 - \sqrt{2}A_3)/\sqrt{3}, \quad A_{\beta^2} = (\sqrt{2}A_2 + A_3)/\sqrt{3}. \tag{3}$$

It follows that, in the PDS calculation, the $K=0_2$ band of ^{168}Er contains admixtures of 12.4% $\gamma^2_{K=0}$ and 0.1% β^2 into the β mode, i.e., 12.5% double-phonon admixtures into the dominant single-phonon component.

General properties of the $K=0_2$ band can be studied by examining the general SU(3) PDS Hamiltonian of Eq. (1). In Fig. 2 we show the results (filled symbols connected by solid lines) of an exact diagonalization ($N=16$) as a function of h_0/h_2. The empirical value of the ratio of $K=0_2$ and γ bandhead energies $E(0_2^+)/[E(2_\gamma^+)-E(2_g^+)]=0.8-1.8$, in the rare-earth region [2,6] constrains the parameters of H to be in the range

$$0.7 \leqslant \frac{h_0}{h_2} \leqslant 2.4. \tag{4}$$

In general, the $K=0_2$ wave function retains the form as in Eq. (2) and, therefore, a three-band mixing calculation is sufficient to describe its structure. To gain more insight into this band mixing, we calculate the matrix elements of H (1) between large-N intrinsic states [15]

$$|\beta\rangle = b^\dagger_\beta |c; N-1\rangle, \quad |\beta^2\rangle = (1/\sqrt{2})(b^\dagger_\beta)^2 |c; N-2\rangle,$$

$$|\gamma^2_{K=0}\rangle = d^\dagger_2 d^\dagger_{-2} |c; N-2\rangle, \quad |c; N\rangle = (N!)^{-1/2}(b^\dagger_c)^N |0\rangle,$$

$$b^\dagger_c = (1/\sqrt{3})(s^\dagger + \sqrt{2}d^\dagger_0), \quad b^\dagger_\beta = (1/\sqrt{3})(d^\dagger_0 - \sqrt{2}s^\dagger). \tag{5}$$

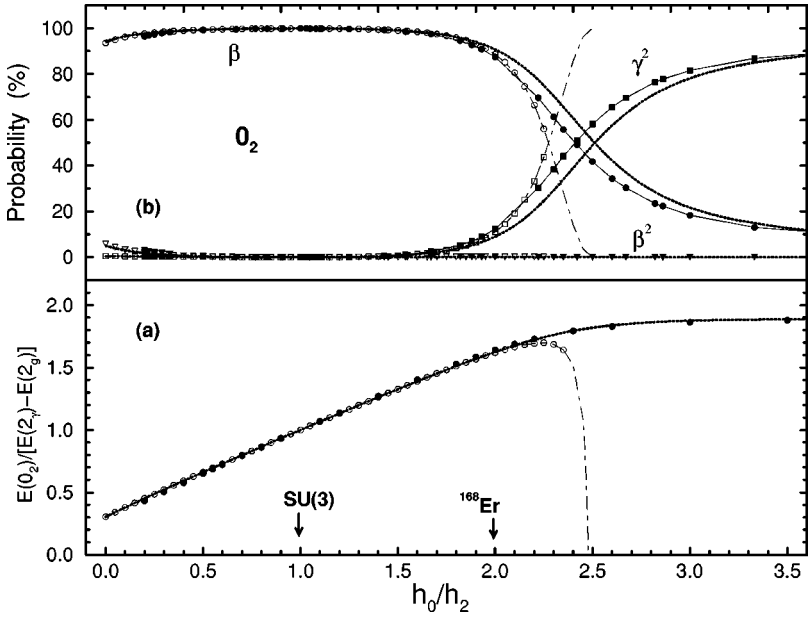

FIG. 2. Properties of the $K=0_2$ band as a function of h_0/h_2, parameters of the SU(3) PDS Hamiltonian, Eq. (1), $N=16$. (a) Ratio of $K=0_2$ and γ bandhead energies obtained from an exact diagonalization (filled circles), three-band mixing calculation based on Eq. (6) (dotted line) and an approximation based on Eqs. (7) and (8) (open circles connected by a dot-dashed line). (b) Probability amplitudes squared, $(A_\beta)^2$ (circles), $(A_{\gamma^2})^2$ (squares), $(A_{\beta^2})^2$ (triangles down) for the $K=0_2$ wave function. Notation for the different curves as in part (a) with corresponding symbols.

To order $\sqrt{N}$, the symmetric matrix elements (M_{ij}) are

$$M_{\beta,\beta}=M_{\beta^2,\beta^2}/2=\epsilon_\beta, \quad M_{\gamma^2,\gamma^2}=2\epsilon_\gamma,$$

$$M_{\beta,\gamma^2}=-\sqrt{2}M_{\beta,\beta^2}=-4(h_0-h_2)\sqrt{N}, \quad M_{\gamma^2,\beta^2}=0,$$

$$\epsilon_\beta=4(2h_0+h_2)N, \quad \epsilon_\gamma=12h_2N. \tag{6}$$

Diagonalization of the 3×3 matrix M_{ij} provides a good estimate both for the bandhead ratio and for the single- and double-phonon probabilities $(A_\beta)^2$, $(A_{\gamma^2})^2$, $(A_{\beta^2})^2$, as shown by the dotted lines in Fig. 2. When the lowest eigenvalue of the matrix M_{ij} is smaller than both $2\epsilon_\beta$ and $2\epsilon_\gamma$, the eigenvalue equation simplifies, and we can derive the following expressions for the bandhead ratio:

$$\frac{E(0_2^+)}{E(2_\gamma^+)-E(2_g^+)}=1+y-\frac{1}{4N}y^2\frac{3+y}{1-y^2},$$

$$y=\frac{2}{3}\left[\left(\frac{h_0}{h_2}\right)-1\right]=\frac{\epsilon_\beta}{\epsilon_\gamma}-1, \tag{7}$$

and for the mixing amplitudes

$$A_\beta=\frac{1}{\sqrt{1+\Delta}}, \quad A_{\gamma^2}=-\frac{1}{\sqrt{2N}}\frac{y}{(1-y)}A_\beta,$$

$$A_{\beta^2}=\frac{1}{2\sqrt{N}}\frac{y}{(1+y)}A_\beta, \quad \Delta=\frac{1}{4N}y^2\left[\frac{2}{(1-y)^2}+\frac{1}{(1+y)^2}\right]. \tag{8}$$

These expressions are valid for $|y|<1-1/\sqrt{2N}$. The corresponding results of this approximation are shown in Fig. 2 as open symbols connected by dot-dashed lines. For ^{168}Er, ($h_0=2h_2$, $y=2/3$, $N=16$), Eq. (7) yields an estimate of 1.62 for the bandhead ratio as compared with the exact value 1.64. From Eq. (8) we obtain a mixing of 11.1% $\gamma^2_{K=0}$ and 0.2% β^2 into the β mode in good agreement with the exact results mentioned above. The quantity y in Eq. (7) measures, for large N, the extent to which the $K=0_2$ band is above ($y>0$) or below ($y<0$) the γ band, and signals the deviation from SU(3) symmetry. In the SU(3) limit $y=0$ ($h_0=h_2$, $\epsilon_\beta=\epsilon_\gamma$), there is no SU(3) mixing hence no mixing of double-phonon excitations into the $K=0_2$ band ($[A_{\gamma^2}=A_{\beta^2}=0]$ in Eq. (8)). In general, the SU(3) mixing $(1-A_\beta^2)$ is $(1/N)$ suppressed, but the mixing can be large when $|y|\to1$ ($h_0/h_2\to2.5$), corresponding to $\epsilon_\beta/\epsilon_\gamma\to2$. The SU(3) breaking and double-phonon admixture is more pronounced for $y>0$ ($h_0/h_2>1$, $\epsilon_\beta>\epsilon_\gamma$). This can be understood from the expression for Δ in Eq. (8), which is not symmetric about $y=0$. Near the SU(3) limit (small y), $(1-A_\beta^2)\sim\Delta\sim(1/4N)y^2[\,3+2y\,]$, which is larger for $y>0$. This implies that the two-phonon admixtures are expected to be larger when the $K=0_2$ band is above the γ band. As seen from Fig. 2, for most of the relevant range of h_0/h_2, Eq. (4), corresponding to bandhead ratio in the range $0.8-1.65$, the double-phonon admixture is, at most, $\sim15\%$. Only for higher values of the bandhead ratio can one obtain larger admixtures and even dominance of the $\gamma^2_{K=0}$ component in the $K=0_2$ wave function.

An important clue to the structure of $K=0_2$ collective excitations comes from $E2$ transitions. The relevant operator is

$$T(E2)=\alpha\,Q^{(2)}+\theta\,\Pi^{(2)}, \tag{9}$$

where $Q^{(2)}$ is the quadrupole SU(3) generator and $\Pi^{(2)}=(\,d^\dagger s+s^\dagger\tilde{d}\,)$ is a (2,2) tensor under SU(3). Since the wave functions of the solvable states are known, it is possible to obtain analytic expressions for the $E2$ rates between them [10]. If we recall that only the ground band has the SU(3) component $(\lambda,\mu)=(2N,0)$, that $Q^{(2)}$, as a generator, cannot connect different SU(3) irreps, and that the $\Pi^{(2)}$ term can connect the $(2N,0)$ irrep only with the $(2N-4,2)$ irrep, we obtain the following expressions for $B(E2)$ values of $\gamma\to g$ and $K=0_2\to g$ transitions:

$$B(E2;\gamma,L\to g,L')=\theta^2\frac{|\langle\phi_E((2N,0)K=0,L')||\Pi^{(2)}||\phi_E((2N-4,2)K=2,L)\rangle|^2}{(2L+1)},$$

$$B(E2;K=0_2,L\to g,L')=A_\beta^2\,\theta^2\frac{|\langle\phi_E((2N,0)K=0,L')||\Pi^{(2)}||\tilde{\phi}_E((2N-4,2)\tilde{K}=0,L)\rangle|^2}{(2L+1)}. \tag{10}$$

Here $\tilde{\phi}_E(\tilde{K}=0,L)$ is the state orthogonal to the solvable Elliott state $\phi_E(K=2,L)$ in the irrep $(2N-4,2)$. The Elliott states in Eq. (10) can be expressed in terms of the Vergados basis [16] for which the reduced matrix elements of $\Pi^{(2)}$ are known [17,18]. The $E2$ parameter θ in Eq. (10) can be determined from the known $2_\gamma^+\to0_g^+E2$ rates, and for ^{168}Er is found to be $\theta^2=2.175$ W.u. As seen from Eq. (10), the $B(E2)$ values for $K=0_2\to g$ transitions are proportional to $(A_\beta)^2$, hence, they provide a direct way for extracting the amount of SU(3) breaking and the admixture of double-phonon excitations in the $K=0_2$ wave function. In Table I we compare the predictions of the PDS and broken-SU(3) calculations with the $B(E2)$ values deduced from a lifetime measurement of the $2^+_{K=0_2}$ level in ^{168}Er [8] (the indicated range for the $B(E2)$ values correspond to different assumptions on the feeding of the level) and with the $B(E2)$ values

TABLE I. Comparison of theoretical and experimental absolute $B(E2)$ values [W.u.] for transitions from the $2^+_{K=0_2}$ level [8] and to the $0^+_{K=0_2}$ level [9] in ^{168}Er.

Transition	Expt. $B(E2)$	Range	PDS	Calc. WCD [12]	CQF [13]
	Lifetime measurement [8]				
$2^+_{K=0_2}\rightarrow 0^+_g$	0.4	0.06–0.94	0.65	0.15	0.03
$2^+_{K=0_2}\rightarrow 2^+_g$	0.5	0.07–1.27	1.02	0.24	0.03
$2^+_{K=0_2}\rightarrow 4^+_g$	2.2	0.4–5.1	2.27	0.50	0.10
$2^+_{K=0_2}\rightarrow 2^+_\gamma$ [a]	6.2 (3.1)	1–15 (0.5–7.5)	4.08	4.16	4.53
$2^+_{K=0_2}\rightarrow 3^+_\gamma$ [a]	7.2 (3.6)	1–19 (0.5–9.5)	7.52	7.90	12.64
	Coulomb excitation [9]				
$2^+_g\rightarrow 0^+_{K=0_2}$	0.08 ± 0.01		0.79	0.18	0.03
$2^+_\gamma\rightarrow 0^+_{K=0_2}$	0.55 ± 0.08		3.06	3.20	5.29

[a] The two numbers in each entry correspond to an assumption of pure $E2$ and (in parenthesis) 50% $E2$ multipolarity.

connecting the 2^+_g and 2^+_γ states with the $0^+_{K=0_2}$ level, measured in Coulomb excitation [9]. It is seen that the PDS and WCD calculations agree well with the lifetime measurement, but the CQF calculation under predicts the $K=0_2\rightarrow g$ data. This may be due to the fact that the CQF parameters are triggered to spectral properties of the ground and γ bands. On the other hand, all calculations show large deviations from the quoted $B(E2)$ values measured in Coulomb excitation. It should be noted, however, that there are serious discrepancies between the above two measurements. First, Härtelin *et al.* [9], based on their Coulomb excitation measurement and use of generalized Alaga rule, predict a value of 0.058 ± 0.007 (W.u.) for the $2^+_{K=0_2}\rightarrow 0^+_g$ transition, which is marginally within the extreme range of the lifetime measurement of Lehmann *et al.* [8]. The latter refers to an extreme and, therefore, highly unlikely feeding scenario. Second, the quoted Lehmann [8] value of 6.2 W.u., (or 3.1 W.u., assuming 50% $E2$ multipolarity) for the $2^+_{K=0_2}\rightarrow 2^+_\gamma$ transition, translates via the Alaga rule to a value of 21.7 (or 10.85) W.u. for the $0^+_{K=0_2}\rightarrow 2^+_\gamma$ transition. The latter is a factor of 7.8 (or 3.9) larger than the value 2.8 ± 0.4 W.u. of Härtelin [9]. An independent measurement of the lifetime of the $0^+_{K=0_2}$ in ^{168}Er is highly desirable to clarify this issue.

To summarize, we have investigated the nature of the lowest collective $K=0$ excitation in deformed nuclei under the assumption of SU(3) partial dynamical symmetry (PDS). We have presented three types of calculations: an exact diagonalization, a three-band mixing calculation using intrinsic states, and an analytic approximation to the latter. In this framework, the SU(3) breaking and double-phonon admixture in the $K=0_2$ wave function are intertwined. The mixing is of order $(1/N)$ but depends critically on the ratio of the $K=0_2$ and γ bandhead energies. It can be obtained directly from the knowledge of absolute $E2$ rates connecting the $K=0_2$ band with the ground band. The PDS predictions agree with the lifetime measurement of the $2^+_{K=0_2}$ level in ^{168}Er [8], but a noticeable discrepancy remains with respect to the $B(E2)$ values measured via Coulomb excitation [9]. For the $K=0_2$ wave function in ^{168}Er, we find 12.5% of double-phonon admixtures into the dominant single-phonon component. These findings support the conventional single-phonon interpretation for this band with small but significant double-γ-phonon admixture.

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