Additional nucleon current contributions to neutrinoless double β decay

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We have examined the importance of momentum-dependent-induced nucleon currents such as weak-magnetism and pseudoscalar couplings to the amplitude of neutrinoless double beta decay ($0\nu\beta\beta$ decay) in the mechanisms of light and heavy Majorana neutrino, as well as in that of Majoron emission. Such effects are expected to occur in all nuclear models in the direction of reducing the light neutrino matrix elements by about 30%. To test this we have performed a calculation of the nuclear matrix elements of the experimentally interesting nuclei $A=76$, 82, 96, 100, 116, 128, 130, 136, and 150 within the proton-neutron renormalized quasiparticle random-phase approximation. We have found that indeed such corrections vary somewhat from nucleus to nucleus, but in all cases they are greater than 25%. In the case of a heavy neutrino the effect is much larger (up to a factor of 6). Combining our results with the best presently available experimental limits on the half-life of the $0\nu\beta\beta$ decay, we have extracted new limits on the effective neutrino mass (light and heavy) and the effective Majoron coupling constant. [S0556-2813(99)05610-1]

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I. INTRODUCTION

The neutrinoless double beta decay ($0\nu\beta\beta$ decay) is expected to occur if lepton number conservation is not an exact symmetry of nature. It is thus forbidden in the standard model (SM) of electroweak interaction. The recent Kamiokande results have given evidence that the neutrinos are massive particles and one has to go beyond the SM. To further understand neutrinos, we must know whether they are Dirac or Majorana particles, an issue which only double β decay can decide. The $0\nu\beta\beta$ decay can be detectable only if the ordinary β decay is forbidden or suppressed and the neutrino is a Majorana particle (i.e., identical to its own antiparticle) with nonzero mass [1–4]. The study of the $0\nu\beta\beta$ decay is stimulated by the development of grand unified theories (GUT's) and supersymmetric models (SUSY) representing extensions of the $SU(2)_L\otimes U(1)$ SM. The GUT's and SUSY offer a variety of mechanisms which allow the $0\nu\beta\beta$ decay to occur [5]. The best known possibility is via the exchange of a Majorana neutrino between the two decaying neutrons [1–4,6], but increased attention is being paid to more exotic processes, like the supersymmetric *R*-parity violating mechanisms of $0\nu\beta\beta$ decay [7–12]. Recent review articles [13,14] give a detailed account of the latest developments in this field.

In this contribution we shall discuss the role of induced currents such as weak-magnetism and pseudoscalar coupling in the calculation of the $0\nu\beta\beta$-decay amplitude, which enters the neutrino mass as well as the Majoron emission mechanisms. So far only the axial-vector and the vector parts have been considered systematically and in great detail [1–4,6,13,14].

The weak-magnetism and nucleon recoil terms have been considered in the extraction of the neutrino-mass-independent parameters associated with right-handed current mechanisms of $0\nu\beta\beta$ decay and were found to be very important in the case of the η parameter [6,15]. This is understood since the leading contribution in this mechanism is proportional to the lepton momenta. In the two-neutrino double beta decay ($2\nu\beta\beta$ decay) the weak-magnetism term [16] resulted in a renormalization of the Gamow-Teller matrix element independent of the nuclear model and lead to a reduction of only 10% to the half-life of medium heavy nuclei. In the case of the light neutrino mass mechanism of $0\nu\beta\beta$ decay there has been one attempt to include such effects resulting [17] from the recoil term and their contribution has been found very small. The weak-magnetism and induced pseudoscalar terms have been considered in connection with the heavy neutrino mass exchange mechanism in Ref. [18] and their importance has been manifested for $0\nu\beta\beta$ decay of ^{48}Ca.

To our knowledge the induced pseudoscalar term, which is equivalent to a modification of the axial current due to partially conserved axial vector current (PCAC), has been ignored in all calculations studying the light Majorana neutrino mass mechanism even though it provides a contribution, which is in fact greater than the (included in all calculations) vector contribution. This has perhaps happened because in the charged-current weak processes the current-current interaction, under the assumption of zero neutrino mass, leads to terms which, except the vector and axial-vector parts [19], are proportional to the lepton mass squared hence, i.e., they are small.

The induced pseudoscalar term, however, is a real function of the Lorenz scalar q^2, therefore, there is reason to expect it to be important. In fact we find that such correc-

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tions are of order $(\vec{q})^2/[(\vec{q})^2+m_\pi^2]$, i.e., they are important if the average momentum $\langle q \rangle$ of the exchanged neutrino cannot be neglected in front of the pion mass. In the case of a light intermediate neutrino the mean nucleon-nucleon separation is about 2 fm which implies that the average momentum $\langle q \rangle$ is about 100 MeV. This leads to corrections of about 30%. In the case of a heavy neutrino exchange the mean internucleon distance is considerably smaller and the average momentum $\langle q \rangle$ is supposed to be considerably larger.

We should mention that in the R-parity violating SUSY mechanism of $0\nu\beta\beta$ decay [11] one has scalar, pseudoscalar, and tensor couplings at the quark level, which, of course, induce analogous couplings at the nucleon level.

The correct nucleon current is important in any calculation of the nuclear matrix elements, which must be computed precisely in order to obtain quantitative answers for the lepton number violating parameters from the results of $0\nu\beta\beta$-decay experiments.

The goal of the present paper is to obtain reliable nuclear matrix elements by including the above refinements in the nucleon current in conjunction with the recent improvements of quasiparticle random-phase approximation (QRPA) (renormalization effects due to Pauli principle corrections [20,21]). In particular, we investigate what effects, if any, the weak-magnetism and pseudoscalar coupling terms will have on the neutrino mass mechanism as well as on the Majoron emission mechanism of $0\nu\beta\beta$ decay. To this end we have performed calculations, which cover most of the nuclear targets of experimental interest (^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, ^{136}Xe, ^{150}Nd).

The paper is organized as follows. In Sec. II, the basic elements of the theory of the $0\nu\beta\beta$ decay relevant to this work are presented. In Sec. III the contributions coming from these induced currents to the $0\nu\beta\beta$-decay amplitude are analyzed. Section IV summarizes the basic ingredients of the proton-neutron RQRPA (pn-RQRPA) method, which will be used for nuclear structure studies of the $0\nu\beta\beta$-decay transitions. In Sec. V we discuss the calculation of nuclear matrix elements and deduce limits on lepton-number violating parameters. Finally, in Sec. VI, we summarize the results and draw some conclusions.

II. THEORY

A. Majorana neutrino mass mechanism

We shall consider the $0\nu\beta\beta$-decay process assuming that the effective β decay Hamiltonian acquires the form

$$\mathcal{H}^{\beta}=\frac{G_F}{\sqrt{2}}[\overline{e}\gamma_\mu(1-\gamma_5)\nu_{eL}]J_L^{\mu\dagger}+\text{H.c.}, \tag{1}$$

where e and ν_{eL} are field operators representing electron and left-handed electron neutrino, respectively. We suppose that neutrino mixing does take place according to

$$\nu_{eL}=\sum_{k=\text{light}} U^L_{ek}\chi_{kL}+\sum_{k=\text{heavy}} U^L_{ek}N_{kL}, \tag{2}$$

where χ_k (N_k) are fields of light (heavy) Majorana neutrinos with masses m_k ($m_k \ll 1$ MeV) and M_k ($M_k \gg 1$ GeV), respectively, and U^L_{ek} is a unitary mixing matrix. In the first and second terms on the right-hand side of Eq. (2) the summation is only over light and heavy neutrinos, respectively. The fields χ_k and N_k satisfy the Majorana condition: $\chi_k\xi_k=C\overline{\chi}_k^T$, $N_k\hat{\xi}_k=C\overline{N}_k^T$, where C denotes the charge conjugation and $\xi,\hat{\xi}$ are phase factors.

We assume both outgoing electrons to be in the $s_{1/2}$ state and consider only $0_i^+\to 0_f^+$ transitions are allowed. For the ground-state transition restricting ourselves to the mass mechanism one obtains for the $0\nu\beta\beta$-decay inverse half-life [1–4,6,13,14],

$$[T_{1/2}^{0\nu}]^{-1}=G_{01}\left|\frac{\langle m_\nu\rangle}{m_e}M^{\text{light}}_{\langle m_\nu\rangle}+\eta_N M^{\text{heavy}}_{\eta_N}\right|^2. \tag{3}$$

The lepton-number nonconserving parameters, i.e., the effective neutrino mass $\langle m_\nu\rangle$ and η_N in Eq. (3) are given as follows:

$$\langle m_\nu\rangle=\sum_k^{\text{light}} (U^L_{ek})^2\xi_k m_k, \qquad \eta_N=\sum_k^{\text{heavy}} (U^L_{ek})^2\hat{\xi}_k\frac{m_p}{M_k}, \tag{4}$$

with m_p (m_e) being the proton (electron) mass. G_{01} is the integrated kinematical factor [2,6]. The derivation of the nuclear matrix elements associated with the exchange of light ($M^{\text{light}}_{\langle m_\nu\rangle}$) and heavy ($M^{\text{heavy}}_{\eta_N}$) Majorana neutrinos is outlined in the next section. However, Eq. (3) applies to any intermediate particle.

B. Majoron mechanism

If the global symmetry associated with lepton number conservation is broken spontaneously, the models imply the existence of a physical Nambu-Goldstone boson, called a Majoron [22], which couples to neutrinos:

$$\mathcal{L}_{\phi\nu\nu}=\sum_{i\leq j}\overline{\nu_i}\gamma_5\nu_j(i\,\text{Im}\,\phi)P_{ij},$$

$$P_{i,j}=\sum_{\alpha,\beta=e,\mu,\tau}U^R_{i\alpha}U^R_{j\beta}g_{\alpha\beta}. \tag{5}$$

Here, ν_i denotes both light χ_i and heavy N_i Majorana neutrinos. We remind the reader that in analogy with Eq. (5) there is a unitary transformation for the right-handed electron neutrino to the mass eigenstates χ_k and N_k:

$$\nu_{eR}=\sum_{k=\text{light}}U^R_{ek}\chi_{kR}+\sum_{k=\text{heavy}}U^R_{ek}N_{kR}, \tag{6}$$

where $\chi_{kR}=P_R\chi_k$ and $N_{kR}=P_RN_k$ [$P_{R,L}=1/2(1\pm\gamma_5)$].

Equation (5) leads to Majoron production in the $0\nu\beta\beta$ decay ($0\nu\beta\beta\phi$ decay) [2,3,13,23]. We are interested in the light neutrino coupling and notice that the couplings U^R_{ek} are small in GUT models where the singlet neutrino is superheavy. We restrict our consideration of the $0\nu\beta\beta\phi$ decay only to light neutrinos $[m_{i,j}\ll q\approx p_F\approx\mathcal{O}(100\ \text{MeV})]$. Then the inverse half-life of the $0\nu\beta\beta\phi$ decay can be written:

$$[T_{1/2}^{\phi 0\nu}]^{-1}=|\langle g\rangle|^2|M_{\langle m_\nu\rangle}^{\text{light}}|^2 G_B. \tag{7}$$

Here $\langle g\rangle$ is the effective Majoron coupling constant

$$\langle g\rangle=\sum_{ij}^{\text{light}} U_{ei}^L U_{ej}^L P_{ij}. \tag{8}$$

The explicit form of the kinematical factor G_B can be found in Ref. [2].

III. EFFECTIVE TRANSITION OPERATOR

Within the impulse approximation the nuclear current J_L^ρ in Eq. (1) expressed with nucleon fields Ψ takes the form

$$J_L^{\mu\dagger}=\bar{\Psi}\tau^+\Big[g_V(q^2)\gamma^\mu-ig_M(q^2)\frac{\sigma^{\mu\nu}}{2m_p}q_\nu-g_A(q^2)\gamma^\mu\gamma_5+g_P(q^2)q^\mu\gamma_5\Big]\Psi, \tag{9}$$

where $q^\mu=(p-p')_\mu$ is the momentum transferred from hadrons to leptons (p and p' are four momenta of neutron and proton, respectively) and $\sigma^{\mu\nu}=(i/2)[\gamma^\mu,\gamma^\nu]$. $g_V(q^2)$, $g_M(q^2)$, $g_A(q^2)$, and $g_P(q^2)$ are real functions of a Lorenz scalar q^2. The values of these form factors in the zero-momentum transfer limit are known as the vector, weak-magnetism, axial-vector, and induced pseudoscalar coupling constants, respectively.

A. Effective transition operator in momentum space

For nuclear structure calculations it is necessary to reduce the nucleon current to the nonrelativistic form. We shall neglect small energy transfers between nucleons in the nonrelativistic expansion. Then the form of the nucleon current coincides with those in the Breit frame and we arrive at [24],

$$J^\mu(\vec{x})=\sum_{n=1}^{A}\tau_n^+[g^{\mu 0}J^0(\vec{q}^{\,2})+g^{\mu k}J_n^k(\vec{q}^{\,2})]\delta(\vec{x}-\vec{r}_n),\quad k=1,2,3, \tag{10}$$

with

$$J^0(\vec{q}^{\,2})=g_V(q^2),$$

$$\vec{J}_n(\vec{q}^{\,2})=g_M(\vec{q}^{\,2})i\frac{\vec{\sigma}_n\times\vec{q}}{2m_p}+g_A(\vec{q}^{\,2})\vec{\sigma}-g_P(\vec{q}^{\,2})\frac{\vec{q}\,\vec{\sigma}_n\cdot\vec{q}}{2m_p}. \tag{11}$$

$\vec{r}_n$ is the coordinate of the nth nucleon.

For the form factors we shall use the following parametrization: $g_V(\vec{q}^{\,2})=g_V/(1+\vec{q}^{\,2}/\Lambda_V^2)^2$, $g_M(\vec{q}^{\,2})=(\mu_p-\mu_n)g_V(\vec{q}^{\,2})$, $g_A(\vec{q}^{\,2})=g_A/(1+\vec{q}^{\,2}/\Lambda_A^2)^2$ and the induced pseudoscalar coupling is given by the partially conserved axial-vector current hypothesis (PCAC) [25]

$$g_P(\vec{q}^{\,2})=2m_p g_A(\vec{q}^{\,2})/(\vec{q}^{\,2}+m_\pi^2)\left(1-\frac{m_\pi^2}{\Lambda_A^2}\right), \tag{12}$$

where $g_V=1$, $g_A=1.254$, $(\mu_p-\mu_n)=3.70$, $\Lambda_V^2=0.71\ (\text{GeV})^2$ [29] and $\Lambda_A=1.09$ GeV [25]. In previous calculations only one general cutoff $\Lambda_V=\Lambda_A\approx0.85$ GeV was used. In this work we take the empirical value of Λ_A deduced from the antineutrino quasielastic reaction $\overline{\nu_\mu}p\to\mu^+n$. A larger value of the cutoff Λ_A is expected to increase slightly the values of the corresponding nuclear matrix elements. It is worth noting that with these modifications of the nuclear current one gets a new contribution in the neutrino mass mechanism, namely the tensor contribution.

As we have already mentioned in the Introduction, momentum-dependent terms, in particular the weak-magnetism term, have been considered previously in the $\beta\beta$ decay by Tomoda *et al.* [15] and Pantis *et al.* [6], but in connection with the η term. This term is proportional to the mixing between the vector bosons W_L and W_R, which mediate the left- and right-handed weak interaction, respectively. They are dominant since, due to their momentum structure, they can proceed via the s-wave electron wave function, while the standard terms in this case require p-wave electron wave functions. The pseudoscalar term is not accompanied by parity change and thus it is not important in the extraction of η. To our knowledge this term has not been considered in connection with the usual light Majorana neutrino mass term of the $0\nu\beta\beta$ decay.

Under the PCAC hypothesis [see Eq. (12)] the two-body effective transition operator takes in momentum space the form

$$\Omega=\tau_+\tau_+(-h_F+h_{GT}\sigma_{12}-h_T S_{12}), \tag{13}$$

where the three terms correspond to Fermi (F), Gamow-Teller (GT), and Tensor (T). One finds that

$$S_{12}=3(\vec{\sigma}_1\cdot\hat{q}\,\vec{\sigma}_2\cdot\hat{q})-\sigma_{12},\quad \sigma_{12}=\vec{\sigma}_1\cdot\vec{\sigma}_2. \tag{14}$$

$$h_F=g_V^2(\vec{q}^{\,2})$$

$$h_{GT}(\vec{q}^{\,2})=g_A^2(\vec{q}^{\,2})\left[1-\frac{2}{3}\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\pi^2}+\frac{1}{3}\left(\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\pi^2}\right)^2\right]+\frac{2}{3}\frac{g_M^2(\vec{q}^{\,2})\vec{q}^{\,2}}{4m_p^2},$$

$$h_T(\vec{q}^{\,2})=g_A^2(\vec{q}^{\,2})\left[\frac{2}{3}\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\pi^2}-\frac{1}{3}\left(\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\pi^2}\right)^2\right]+\frac{1}{3}\frac{g_M^2(\vec{q}^{\,2})\vec{q}^{\,2}}{4m_p^2}, \tag{15}$$

The exact results will depend on the details of the nuclear model, since the new operators have different momentum (radial) dependence than the traditional ones and the tensor component is entirely new. We can get a crude idea of what is happening by taking the above average momentum $\langle q \rangle = 100$ MeV. Then we find that the GT matrix element is reduced by 22%. Then assuming that T matrix element is about half the GT one, we find that the total reduction is 28%. This will be compared below with the results of our detailed calculations.

B. Effective transition operator in coordinate space

The nuclear matrix elements entering the half-life formula of $0\nu\beta\beta$-decay process now take the form

$$M^I_{\langle m_\nu \rangle, \eta_N} = M^I_{VV} + M^I_{MM} + M^I_{AA} + M^I_{AP} + M^I_{PP} \quad (16)$$

with $I=$light, heavy. The partial nuclear matrix elements M^I_{VV}, M^I_{MM}, M^I_{AA}, M^I_{PP}, and M^I_{AP} have their origin from the vector, the weak-magnetism, the axial, the pseudoscalar coupling and the interference of the axial-vector and pseudoscalar coupling, respectively. They can be expressed in relative coordinates by using second quantization. We end up with formula

$$M^I_{\text{type}} = \langle H^I_{\text{type}-\text{F}}(r_{12}) + H^I_{\text{type}-\text{GT}}(r_{12})\sigma_{12} + H^I_{\text{type}-\text{T}}(r_{12}) S_{12} \rangle \quad (17)$$

with type$=VV, MM, AA, PP, AP$, and

$$\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2, \quad r_{12} = |\mathbf{r}_{12}|, \quad \hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_{12}}{r_{12}},$$

$$S_{12} = 3(\vec{\sigma}_1 \cdot \hat{\mathbf{r}}_{12})(\vec{\sigma}_2 \cdot \hat{\mathbf{r}}_{12}) - \sigma_{12}, \quad \sigma_{12} = \vec{\sigma}_1 \cdot \vec{\sigma}_2. \quad (18)$$

$\mathbf{r}_1$ and $\mathbf{r}_2$ are coordinates of the β decaying nucleons. The form of the matrix element $\langle O(1,2) \rangle$ within the pn-RQRPA will be presented in the next section.

The light and heavy neutrino-exchange potentials $H^{\text{light,heavy}}_{\text{type-}K}(r_{12})$ ($K=$F, GT, T) have the following forms:

$$H^{\text{light}}_{\text{type-}K}(r_{12}) = \frac{2}{\pi g_A^2} \frac{R}{r_{12}} \int_0^\infty \frac{\sin(q r_{12})}{q + E^m_J - (E^i_{\text{g.s.}} + E^f_{\text{g.s.}})/2} \times h_{\text{type}-K}(q^2) dq, \quad (19)$$

$$H^{\text{heavy}}_{\text{type-}K}(r_{12}) = \frac{1}{m_p m_e} \frac{2}{\pi g_A^2} \frac{R}{r_{12}} \int_0^\infty \sin(q r_{12}) h_{\text{type-}K}(q^2) q dq. \quad (20)$$

Here, $E^i_{\text{g.s.}}$, $E^f_{\text{g.s.}}$, and E^m_J are, respectively, the energies of the initial, final, and intermediate nuclear states. $R = r_0 A^{1/3}$ is the mean nuclear radius, with $r_0 = 1.1$ fm. The relevant couplings are

$$h_{VV}(\vec{q}^{\,2}) = -g_V^2(\vec{q}^{\,2}),$$

$$h_{MM-\text{GT}}(\vec{q}^{\,2}) = \frac{2}{3} \frac{g_M^2(\vec{q}^{\,2})\vec{q}^{\,2}}{4m_p^2},$$

$$h_{MM-T}(\vec{q}^{\,2}) = \frac{1}{3} \frac{g_M^2(\vec{q}^{\,2})\vec{q}^{\,2}}{4m_p^2},$$

$$h_{AA-\text{GT}}(\vec{q}^{\,2}) = g_A^2(\vec{q}^{\,2}),$$

$$h_{PP-\text{GT}}(\vec{q}^{\,2}) = \frac{1}{3} \frac{g_P^2(\vec{q}^{\,2})\vec{q}^{\,4}}{4m_p^2},$$

$$h_{PP-T}(q^2) = -\frac{1}{3} \frac{g_P^2(\vec{q}^{\,2})\vec{q}^{\,4}}{4m_p^2},$$

$$h_{AP-\text{GT}}(\vec{q}^{\,2}) = -\frac{2}{3} \frac{g_A(\vec{q}^{\,2}) g_P(\vec{q}^{\,2})\vec{q}^{\,2}}{2m_p},$$

$$h_{AP-T}(\vec{q}^{\,2}) = \frac{2}{3} \frac{g_A(\vec{q}^{\,2}) g_P(\vec{q}^{\,2})\vec{q}^{\,2}}{2m_p}. \quad (21)$$

The tensor form factor includes a sign change going from momentum to coordinate space.

The full matrix element is of the form:

$$M^{\text{light}}_{\langle m_\nu \rangle} = -\frac{M_F^{\text{light}}}{g_A^2} + M^{\text{light}}_{\text{GT}} + M^{\text{light}}_T. \quad (22)$$

We see that the Fermi component is unchanged, the Gamow-Teller is modified and the tensor component appeared due to the new terms.

IV. NUCLEAR STRUCTURE INGREDIENTS

As we have mentioned above, we would like to evaluate the changes in the $0\nu\beta\beta$-decay nuclear matrix elements due to the modifications of the nuclear current introduced above, relevant for the neutrino mass mechanism. It is clear that the $0\nu\beta\beta$ decay is a second-order process in the weak interaction and, thus, the corresponding nuclear matrix elements require the summation over a complete set of intermediate nuclear states. Even though the construction of these states is not needed, a closure approximation with a reasonable average energy denominator is very accurate [26–28], the initial and final states of the nuclear systems, which can undergo double β decay, are not easy to construct, since these nuclei are far removed from closed shells. Thus the introduction of additional approximations is necessary.

Thus at this point we will reduce the computational difficulty in evaluating the effects of the above-mentioned modifications, by using the proton-neutron quasiparticle random phase approximation (pn-QRPA) [30–33], which is an approximation to solve the nuclear many-body problem. Admittedly the intermediate states must be explicitly constructed in this case, but in this scheme it is a simple matter to include even a large number of such states, if necessary.

The crucial simplifying point of the QRPA is the quasiboson approximation (QBA) assuming nuclear excitations to be harmonic. It leads to a violation of the Pauli exclusion principle. The drawback of this approximation is that with increasing strength of the nucleon-nucleon force in the particle-particle channel the QRPA overestimates the ground-state correlations and the QRPA solution collapses [20,21,34,35].

The renormalized QRPA (RQRPA) [20,21] overcomes this difficulty by taking into account the Pauli exclusion principle in a more proper way by using the renormalized QBA. In this paper we calculate the nuclear matrix elements of the $0\nu\beta\beta$ decay within the proton-neutron RQRPA (pn-RQRPA) [20,21], which is an extension of the pn-QRPA by incorporating the Pauli exclusion principle for fermion pairs. For studying the relative importance of the new induced terms, which is the main thrust of our paper, the inclusion of other refinements like p-n pairing, which is computationally very involved, is not essential.

Furthermore p-n pairing is rigorously incorporated in the BCS ansatz only for the $T=1$ states, while $T=0$ p-n pairing effects are implicitly taken into account. This procedure seems to avoid the collapse of the QRPA within the physical region of the Hamiltonian but we are not sure whether in some cases it does not produce more ground state correlations which can lead to strong cancellations in the matrix element. This might have been the case of ^{100}Mo in our earlier work [6] in which one should have expected a minor influence of p-n pairing. In fact if the collapse of the QRPA reflects a nearby phase transition [36], i.e., a change of the ground state from being dominated by $T=1$ pair correlations to being dominated by $T=0$ pair correlations, further work needs to been done to be sure about the p-n pairing. On the other hand the renormalized solution does not lead to a collapse of the QRPA for physical values of the proton-neutron interaction strength and is tractable from the computational point of view.

The pn-RQRPA excited states $|m,JM\rangle$ are of the form [20,21]

$$|J^\pi Mm\rangle = Q^{m\dagger}_{JM^\pi}|0^+_{\rm RPA}\rangle = \sum_{pn} [X^m_{(pn,J^\pi)} A^\dagger(pn,JM) + Y^m_{(pn,J^\pi)}\tilde{A}(pn,JM)]|0^+_{\rm RPA}\rangle, \tag{23}$$

where $X^m_{(pn,J^\pi)}$ and $Y^m_{(pn,J^\pi)}$ are free variational amplitudes, respectively, and

$$A^\dagger(pn,JM) = \sum_{m_p,m_n} C^{JM}_{j_p m_p j_n m_n} a^\dagger_{pm_p} a^\dagger_{nm_n},$$

$$\tilde{A}(pn,JM) = (-1)^{J-M} A(pn,J-M). \tag{24}$$

Here, $a^+_{\tau m_\tau}$ ($a_{\tau m_\tau}$, $\tau=p,n$) is the quasiparticle creation (annihilation) operator for spherical shell-model states, which is related to the particle creation and annihilation ($c^+_{\tau m_\tau}$ and $c_{\tau m_\tau}$, $\tau=p,n$) operators by the Bogoliubov-Valatin transformation:

$$\begin{pmatrix} c^+_{\tau m_\tau} \\ \tilde{c}_{\tau m_\tau} \end{pmatrix} = \begin{pmatrix} u_\tau & -v_\tau \\ v_\tau & u_\tau \end{pmatrix} \begin{pmatrix} a^+_{\tau m_\tau} \\ \tilde{a}_{\tau m_\tau} \end{pmatrix}, \tag{25}$$

where the tilde $\sim$ indicates time reversal [$\tilde{a}_{\tau m_\tau} = (-1)^{j_\tau - m_\tau} a_{\tau - m_\tau}$]. The label τ designates quantum numbers n_τ, l_τ, j_τ. The occupation amplitudes u and v and the single quasiparticle energies E_τ are obtained by solving the BCS equation.

In the pn-RQRPA the commutator of two-bifermion operators fulfill the following relation (renormalized QBA)

$$\langle 0^+_{\rm RPA}|[A(pn,JM),A^+(p'n',JM)]|0^+_{\rm RPA}\rangle = \delta_{pp'}\delta_{nn'} \times \underbrace{\left\{1 - \frac{1}{\hat{j}_l}\left\langle 0^+_{\rm RPA}|[a^+_p \tilde{a}_p]_{00}|0^+_{\rm RPA}\right\rangle - \frac{1}{\hat{j}_k}\left\langle 0^+_{\rm RPA}|[a^+_n \tilde{a}_n]_{00}|0^+_{\rm RPA}\right\rangle\right\}}_{\mathcal{D}_{pn,J^\pi}}, \tag{26}$$

with $\hat{j}_p = \sqrt{2j_p+1}$. If we replace $|0^+_{\rm RPA}\rangle$ in Eq. (26) with the uncorrelated BCS ground state, we obtain the quasiboson approximation (i.e., $\mathcal{D}_{pn,J^\pi}=1$), which assumes that pairs of quasiparticles obey the commutation relations of bosons. We note that it is convenient to introduce amplitudes

$$\overline{X}^m_{(pn,J^\pi)} = \mathcal{D}^{1/2}_{pn,J^\pi} X^m_{(pn,J^\pi)}, \quad \overline{Y}^m_{(pn,J^\pi)} = \mathcal{D}^{1/2}_{pn,J^\pi} Y^m_{(pn,J^\pi)}, \tag{27}$$

which are orthonormalized in the usual way:

$$\delta_{mm'} = \sum_{pn} (\overline{X}^m_{(pn,J^\pi)} \overline{X}^{m'}_{(pn,J^\pi)} - \overline{Y}^m_{(pn,J^\pi)} \overline{Y}^{m'}_{(pn,J^\pi)}). \tag{28}$$

One can show that

$$\frac{\langle J^\pi m_i \| [c_p^+ \tilde{c}_n]_J \| 0_i^+ \rangle}{\sqrt{2J+1}} = (u_p^{(i)} v_n^{(i)} \overline{X}^{m_i}_{(pn,J^\pi)} + v_p^{(i)} u_n^{(i)} \overline{Y}^{m_i}_{(pn,J^\pi)}) \sqrt{\mathcal{D}^{(i)}_{pn,J^\pi}}, \tag{29}$$

$$\frac{\langle 0_f^+ \| \widetilde{[c_p^+ \tilde{c}_n]_J} \| J^\pi m_f \rangle}{\sqrt{2J+1}} = (v_p^{(f)} u_n^{(f)} \overline{X}^{m_f}_{(pn,J^\pi)} + u_p^{(f)} v_n^{(f)} \overline{Y}^{m_f}_{(pn,J^\pi)}) \sqrt{\mathcal{D}^{(f)}_{pn,J^\pi}}. \tag{30}$$

The index i (f) indicates that the quasiparticles and the excited states of the nucleus are defined with respect to the initial (final) nuclear ground state $|0_i^+\rangle (|0_f^+\rangle)$. The forward and backward amplitudes $\overline{X}^{m_i}_{(pn,J^\pi)}$ and $\overline{Y}^{m_i}_{(pn,J^\pi)}$ and the energies of the excited states $\Omega^{m_i}_{J^\pi} = E^{m_i}_{J^\pi} - E^i_{\rm g.s.}$ are obtained by solving the nonlinear set of RQRPA equations for the initial nucleus (A,Z) [20,21]. By performing the RQRPA diagonalization for the final nucleus $(A,Z=2)$ we obtain the amplitudes $\overline{X}^{m_f}_{(pn,J^\pi)}$ and $\overline{Y}^{m_f}_{(pn,J^\pi)}$ and the eigenenergies $\Omega^{m_f}_{J^\pi} = E^{m_f}_{J^\pi} - E^f_{g.s.}$ of the RQRPA state $|J^\pi m_f\rangle$.

Within the pn-RQRPA the $0\nu\beta\beta$-decay matrix elements given in Eqs. (17) take the following form:

$$M^I_{\rm type} = \sum_{\substack{pnp'n' \\ J^\pi m_i m_f \mathcal{J}}} (-)^{j_n + j_{p'} + J + \mathcal{J}} (2\mathcal{J}+1) \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix} \langle p(1), p'(2); \mathcal{J} | f(r_{12}) \tau_1^+ \tau_2^+ O^I_{\rm type}(12) f(r_{12}) | n(1), n'(2); \mathcal{J} \rangle \times \langle 0_f^+ \| \widetilde{[c_{p'}^+ \tilde{c}_{n'}]_J} \| J^\pi m_f \rangle \langle J^\pi m_f | J^\pi m_i \rangle \langle J^\pi m_i \| [c_p^+ \tilde{c}_n]_J \| 0_i^+ \rangle. \tag{31}$$

Here, $O^I_{\rm type}(12)$ represents the coordinate and spin-dependent part of the two-body transition operators of the $0\nu\beta\beta$-decay nuclear matrix elements in Eq. (31)

$$O^I_{\rm type}(12) = H^I_{\text{type-}F}(r_{12}) + H^I_{\text{type-GT}}(r_{12}) \sigma_{12} + H^I_{\text{type-}T}(r_{12}) \mathbf{S}_{12}. \tag{32}$$

The short-range correlations between the two interacting protons $[p(1)$ and $p'(2)]$ and neutrons $[n(1)$ and $n'(2)]$ are taken into account by the correlation function $f(r_{12})$ in the nonantisymmetrized two-body matrix element in Eq. (31). $f(r_{12})$ is given as follows:

$$f(r_{12}) = 1 - e^{-\alpha r_{12}^2}(1 - b r_{12}^2) \quad \text{with} \quad \alpha = 1.1 \ \text{fm}^2 \quad \text{and} \quad b = 0.68 \ \text{fm}^2. \tag{33}$$

For the overlap matrix of intermediate nuclear states generated from the initial and final ground states we write [35]

$$\langle J^\pi M m_f | J^\pi M m_i \rangle \approx \sum_{pn} (\overline{X}^{m_i}_{(pn,J^\pi)} \overline{X}^{m_f}_{(pn,J^\pi)} - \overline{Y}^{m_i}_{(pn,J^\pi)} \overline{Y}^{m_f}_{(pn,J^\pi)})(u_p^{(i)} u_p^{(f)} + v_p^{(i)} v_p^{(f)}) \times (u_n^{(i)} u_n^{(f)} + v_n^{(i)} v_n^{(f)}). \tag{34}$$

V. CALCULATION, DISCUSSION, AND OUTLOOK

In order to test the importance of the new momentum-dependent terms in the nucleon current, we applied the pn-RQRPA to calculate the $0\nu\beta\beta$ decay of the $A=76$, 82, 96, 100, 116, 128, 130, 136, and 150 systems. To this end the considered single-particle model spaces both for protons and neutrons have been as follows: (i) For $A=76{,}82$ the model space consists of the full $2-4\hbar\omega$ major oscillator shells. (ii) For $A=96{,}100{,}116$ we added to the previous model space $1f_{5/2}$, $1f_{7/2}$, $0h_{9/2}$, and $0h_{11/2}$ levels. (iii) For $A=128{,}130{,}136$ the model space comprises the full $2-5\hbar\omega$ major shells. (iv) For $A=150$ the model space extends over the full $2-5\hbar\omega$ shells plus the $0i_{11/2}$ and $0i_{13/2}$ levels.

The single-particle energies were obtained by using a Coulomb-corrected Woods-Saxon potential. The interaction employed was the Brueckner G matrix which is a solution of the Bethe-Goldstone equation with the Bonn one-boson exchange potential. Since the model space considered is finite, the pairing interactions have been adjusted to fit the empirical pairing gaps according to [37]. In addition, we renormalize the particle-particle and particle-hole channels of the G matrix interaction of the nuclear Hamiltonian H by introducing the parameters g_{pp} and g_{ph}, respectively. The nuclear matrix elements listed in Tables I and II have been obtained for $g_{ph}=0.8$ and $g_{pp}=1.0$. With respect to the g_{pp} we wish to make the following statement: Our numerical results do not show significant variations (do not exceed 20%) in the physical region of $g_{pp}(0.8 \leq g_{pp} \leq 1.2)$.

A detailed study of Fermi, Gamow-Teller, and Tensor contributions to the full nuclear matrix element $M^{\rm light}_{\langle m_\nu \rangle}$ in Eq. (22) for the two representative $0\nu\beta\beta$-decay nuclei ^{76}Ge and ^{130}Te is presented in Table I. One notices significant additional contributions to GT (*AA* and *PP*) and tensor (*AA* and *PP*) nuclear matrix elements coming from the induced current terms. By glancing at the Table I we also see that Fermi and GT matrix elements are strongly suppressed due to the nucleon short-range correlations.

TABLE I. The Fermi, Gamow-Teller, and Tensor nuclear matrix elements for the light Majorana neutrino exchange of the $0\nu\beta\beta$ decay of ^{76}Ge and ^{130}Te with and without consideration of short-range correlations (s.r.c.).

Transition	s.r.c.	Gamow-Teller			Tensor		M_F^{light}	M_{GT}^{light}	M_T^{light}
		AA	AP	PP	AP	PP			
^{76}Ge→ ^{76}Se	Without	5.132	-1.392	0.302	-0.243	0.054	-2.059	4.042	-0.188
	With	2.797	-0.790	0.176	-0.246	0.055	-1.261	2.183	-0.190
^{130}Te→ ^{130}Xe	Without	4.158	-1.173	0.258	-0.329	0.074	-1.837	3.243	-0.255
	With	1.841	-0.578	0.134	-0.333	0.075	-1.033	1.397	-0.258

The relative importance of the different contributions to the nuclear matrix elements $M^{light}_{\langle m_\nu\rangle}$ and $M^{heavy}_{\eta_N}$ [see Eq. (16)] is displayed in Fig. 1 for the $A=76$ and 130 nuclear systems. As expected from our discussion in Sec. III we see that for light neutrinos the pseudoscalar term, in particular the *AP* contribution, is important. It is in fact as important as the usual vector contribution but in the opposite direction (see Table II). Our calculations verify our above estimate, i.e., the new terms in the hadronic current, and in particular the induced pseudoscalar term, tend to increase the average neutrino mass $|\langle m_\nu\rangle|$ and the average Majoron coupling $\langle g\rangle$ from experiment by about 30%. They are much more important in the exchange of heavy neutrinos leading the suppression of $M^{heavy}_{\eta_N}$ by about factor of 3–6 (see Table II). The contributions from previously neglected M^{heavy}_{MM} and M^{heavy}_{AP} to $M^{heavy}_{\eta_N}$ are much more important as that from M^{heavy}_{VV}. A large

TABLE II. Nuclear matrix elements for the light and heavy Majorana neutrino exchange modes of the $0\nu\beta\beta$ decay for the nuclei studied in this work calculated within the renormalized pn-QRPA. G_{01} and G_B are the integrated kinematical factors for the $0^+\to 0^+$ transition. $\zeta_{\langle m_\nu\rangle}(Y)$, $\zeta_{\eta_N}(Y)$, and $\zeta_{\langle g\rangle}(Y)$ denote, according to Eq. (34), the sensitivity of a given nucleus Y to the light neutrino mass, heavy neutrino mass, and Majoron signals, respectively.

	$(\beta\beta)_{0\nu}-decay{:}0^+\to 0^+$ transition								
M. E.	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd
	Light Majorana neutrino (I=light)								
M^I_{VV}	0.80	0.74	0.45	0.82	0.50	0.75	0.66	0.32	1.14
M^I_{AA}	2.80	2.66	1.54	3.30	2.08	2.21	1.84	0.70	3.37
M^I_{PP}	0.23	0.22	0.15	0.26	0.15	0.24	0.21	0.11	0.35
M^I_{AP}	-1.04	-0.98	-0.65	-1.17	-0.69	-1.04	-0.91	-0.48	-1.53
$M^I_{VV}+M^I_{AA}$	3.60	3.40	1.99	4.12	2.58	2.96	2.50	1.02	4.51
$M^I_{\langle m_\nu\rangle}$	2.80	2.64	1.49	3.21	2.05	2.17	1.80	0.66	3.33
	Heavy Majorana neutrino (I= heavy)								
M^I_{VV}	23.9	22.0	16.1	28.3	17.2	25.8	23.4	13.9	39.4
M^I_{MM}	-55.4	-51.6	-38.1	-67.3	-39.8	-60.4	-54.5	-31.3	-92.0
M^I_{AA}	106.	98.3	68.4	123.	74.0	111.	100.	58.3	167.
M^I_{PP}	13.0	12.0	9.3	16.1	9.1	14.9	13.6	7.9	23.0
M^I_{AP}	-55.1	-50.7	-41.1	-70.1	-39.0	-64.9	-59.4	-34.8	-101.
$M^I_{VV}+M^I_{AA}$	130.	120.	84.5	151.	91.1	137.	123.	72.3	206.
$M^I_{\eta_N}$	32.6	30.0	14.7	29.7	21.5	26.6	23.1	14.1	35.6
	Sensitivity to neutrino mass signal								
$G_{01}\times 10^{15}y$	7.93	35.2	73.6	57.3	62.3	2.21	55.4	59.1	269.
$\zeta_{\langle m_\nu\rangle}(Y)$	2.49	4.95	4.04	7.69	5.11	1.02	4.24	1.60	17.3
$\zeta_{\eta_N}(Y)$	2.90	5.64	3.98	7.10	5.36	1.25	5.45	3.43	18.5
	Sensitivity to Majoron signal								
$G_B\times 10^{17}y$	7.40	62.3	159.	106.	104.	0.59	79.6	82.8	640.
$\zeta_{\langle g\rangle}(Y)$	2.41	6.59	5.93	10.5	6.60	0.53	5.08	1.90	26.7

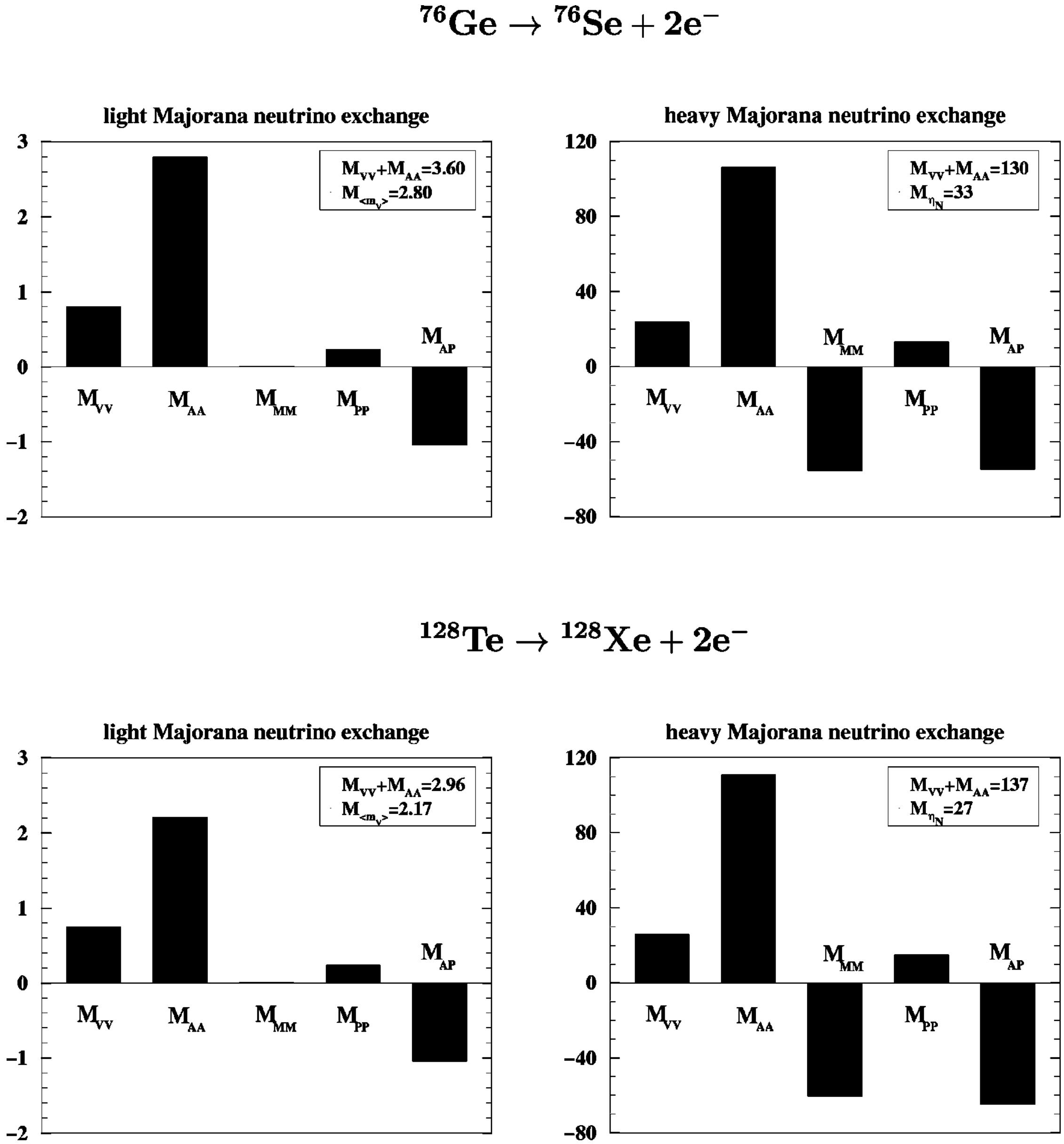

FIG. 1. Calculated light and heavy neutrino exchange $0\nu\beta\beta$-decay nuclear matrix elements for the $A=76$ and 128 systems. The partial matrix elements M_{VV}, M_{AA}, M_{MM}, M_{PP}, and M_{AP} originate from vector, axial-vector, weak magnetism, pseudoscalar coupling, and the interference of the axial-vector and induced pseudoscalar coupling, respectively. $M_{\langle m_\nu \rangle}$ and M_{η_N} are $0\nu\beta\beta$-decay matrix elements associated with the $\langle m_\nu \rangle$ and η_N parameters, respectively.

value of M_{MM}^{heavy}, which has its origin in weak magnetism, indicates that the average neutrino momentum is large, i.e., about the order of magnitude of the nucleon mass.

We present in Fig. 2 the nuclear matrix elements $M_{\langle m_\nu \rangle}^{\mathrm{light}}$ and $M_{\eta_N}^{\mathrm{heavy}}$ calculated within pn-RQRPA for the A = 76, 82, 96, 100, 116, 128, 130, 136, and 150 nuclear systems. We see that the inclusion of the induced pseudoscalar interaction and of weak magnetism in the calculation results in considerably smaller nuclear matrix elements for all nuclear systems. The numerical values of $M_{\langle m_\nu \rangle}^{\mathrm{light}}$ and $M_{\eta_N}^{\mathrm{heavy}}$ can be found in Table II. The largest matrix elements of $M_{\langle m_\nu \rangle}^{\mathrm{light}}$ for $A=150,\ 100$, and 76, are 3.33, 3.21, and 2.80, respectively. For $A=150,\ 76$, and 82 the largest values of $M_{\eta_N}^{\mathrm{heavy}}$ are 35.6, 32.6, and 30.0, respectively. We notice that the $A=136$ system has the smallest nuclear matrix elements: $M_{\langle m_\nu \rangle}^{\mathrm{light}}=0.66$ and $M_{\eta_N}^{\mathrm{heavy}}=14.1$. We suppose that it is connected with the fact that ^{136}Xe is a closed-shell nucleus for neutrons ($N=82$). The sharp Fermi level for neutrons yield smaller $0\nu\beta\beta$-decay matrix elements. We note that in calculating the matrix elements involving the exchange of heavy neutrinos, the treatment of the short-range repulsion and nucleon finite size is crucial. We have found that the consideration of short-range correlation effects reduces the values of $M_{\eta_N}^{\mathrm{heavy}}$ by about factor of 20–30. As we mentioned already the

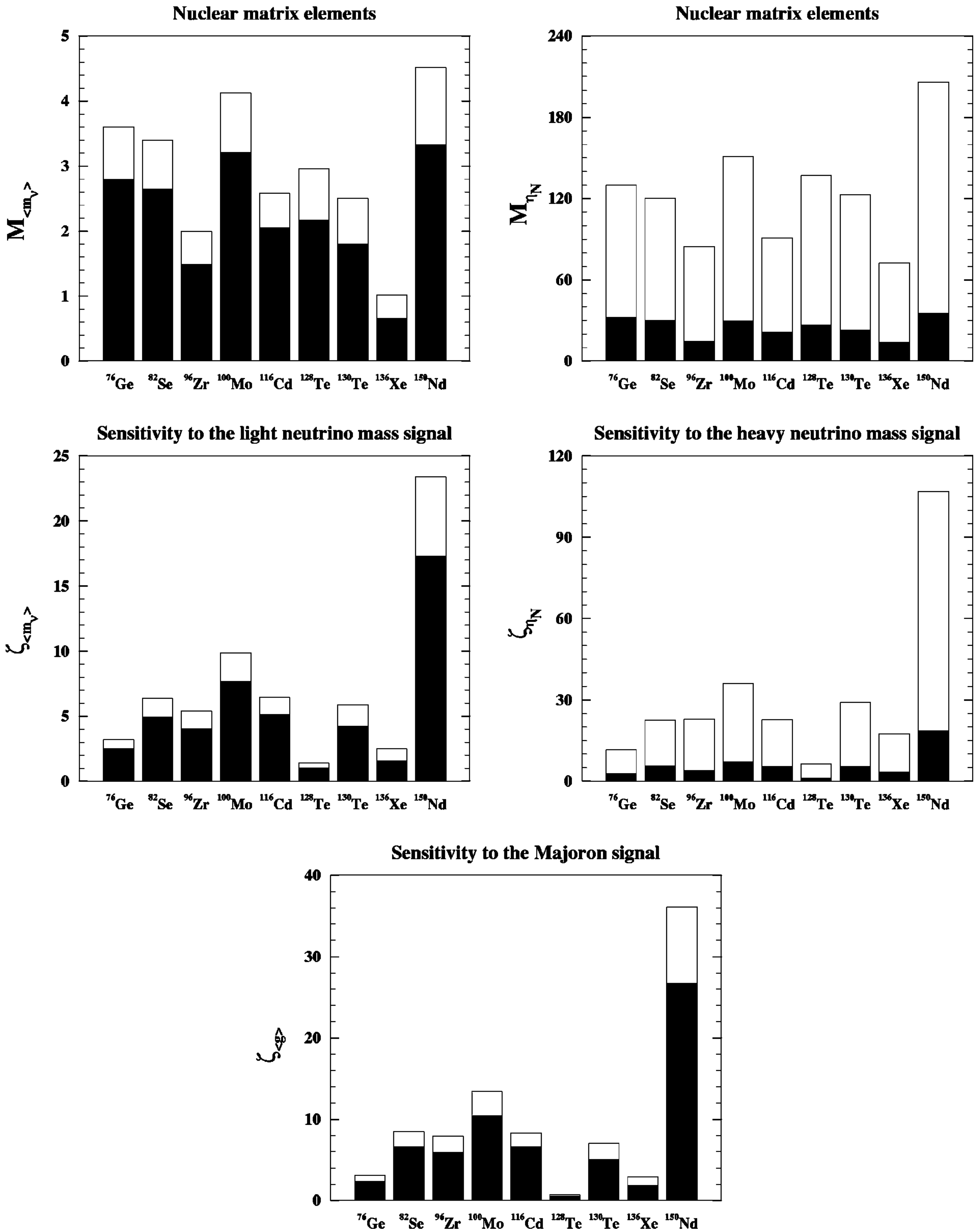

FIG. 2. Calculated nuclear matrix elements $M_{\langle m_\nu \rangle}$, M_{η_N} [see Eqs. (3) and (13)–(18)], sensitivities $\zeta_{\langle m_\nu \rangle}$, ζ_{η_N}, and $\zeta_{\langle g \rangle}$ for the experimentally interesting $A=76$, 82, 96, 100, 116, 128, 130, 136, and 150 nuclear systems. $\zeta_{\langle m_\nu \rangle}$, ζ_{η_N}, and $\zeta_{\langle g \rangle}$ are sensitivities to light neutrino mass, heavy neutrino mass and Majoron signal [see Eq. (34)], respectively. The open and black bars correspond to results obtained without and with the inclusion of the pseudoscalar interaction and of weak magnetism.

nucleon finite size has been taken into account through the phenomenological form factors and the PCAC hypothesis. However, the choice of the form factor can influence the results significantly as it was manifested in Ref. [18] performing the calculations with both phenomenological and quark confinement model form factors.

The limits deduced for lepton-number violating parameters depend on the values of nuclear matrix element, of the kinematical factor, and of the current experimental limit for a given isotope [see Eqs. (3) and (7)]. It is expected that the experimental constraints on the half-life of the $0\nu\beta\beta$ decay are expected to be more stringent in future. Thus it is useful to introduce sensitivity parameters for a given isotope to the effective light and heavy Majorana neutrino mass and Majoron signals, which depend only on the characteristics of a given nuclear system. They are as follows:

$$\zeta_{\langle m_\nu\rangle}(Y)=10^7|M^{\text{light}}_{\langle m_\nu\rangle}|\sqrt{G_{01}\ \text{year}},$$

$$\frac{\langle m_\nu\rangle}{m_e}\leqslant\frac{10^{-5}}{\zeta_{\langle m_\nu\rangle}}\sqrt{\frac{10^{24}\ \text{years}}{T_{1/2}^{0\nu-\exp}}},$$

$$\zeta_{\eta_N}(Y)=10^6|M^{\text{heavy}}_{\langle m_\nu\rangle}|\sqrt{G_{01}\ \text{year}},$$

$$\eta_N\leqslant\frac{10^{-6}}{\zeta_{\eta_N}}\sqrt{\frac{10^{24}\ \text{years}}{T_{1/2}^{0\nu-\exp}}},$$

$$\zeta_{\langle g\rangle}(Y)=10^8|M^{\text{light}}_{\langle m_\nu\rangle}|\sqrt{G_B\ \text{year}},$$

$$\langle g\rangle\leqslant\frac{10^{-4}}{\zeta_{\langle g\rangle}}\sqrt{\frac{10^{24}\ \text{years}}{T_{1/2}^{0\nu-\exp}}}. \qquad (35)$$

The normalization of 10^{24} years was chosen so that the ζ's are of order unity. The numerical values of these parameters for the nuclear systems considered in this work are listed in Table II and can be used in predicting the desired limits on lepton number violation with changing experimental data (see the above expressions). These characteristics estimate also the prospects for searches of light and heavy Majorana neutrinos and of the Majoron. The larger values of these parameters determine those $0\nu\beta\beta$-decay isotopes, which are the most promising candidates for searching for the corresponding lepton number violating signal. They give information on the requirements of a $0\nu\beta\beta$ detector.

Figure 2 shows that the most sensitive isotope to all three studied lepton number violating parameters is ^{150}Nd. It is mostly due to the large phase-space integral and to some extent also to the larger nuclear matrix elements. We caution the reader here that our matrix elements are not model independent in the sense that all the nuclei considered in this work are treated as spherical. The nucleus ^{150}Nd, is deformed and our results may not be the same quantitatively, were we to perform calculations taking into account effects of nuclear deformation.

The purpose of this work, however, is to study the effects of the induced currents. It is thus reasonable to do this by comparing calculations within the same model. This means that the model itself will play a minor role, if any, in the investigation of such effects. We are thus satisfied that for light neutrinos the effect is almost the same throughout the periodic table. In addition, as we see from Fig. 2, the inclusion of weak magnetism and induced pseudoscalar coupling in our calculations leads to a significant reduction of the parameters of $\zeta_{\langle m_\nu\rangle}(Y)$, $\zeta_{\eta_N}(Y)$, and $\zeta_{\langle g\rangle}(Y)$.

The present experimental situation in terms of the accessible half-life and the corresponding upper limit on $\langle m_\nu\rangle$, η_N, and $\langle g\rangle$ is given in Table III. Thus, the most restrictive limits are as follows:

$$\langle m_\nu\rangle^{\text{best}}<0.62\ \text{eV},\quad [^{76}\text{Ge},\quad \text{Ref. [38]}],$$

$$\eta_N^{\text{best}}<1.0\times10^{-7},\quad [^{76}\text{Ge},\ \text{Ref. [38]}],$$

$$\langle g\rangle^{\text{best}}<6.9\times10^{-5},\quad [^{128}\text{Te},\quad \text{Ref. [43]}]. \qquad (36)$$

The sensitivity of different experiments to $\langle m_\nu\rangle$, $\langle\eta_N\rangle$, and $\langle g\rangle$ is drawn in Fig. 3. Currently, the Heidelberg-Moscow experiment [38] offers the most stringent limit for effective light and heavy Majorana neutrino mass and the ^{128}Te experiment [43] for the effective Majoron coupling constant. By assuming $\langle m_\nu\rangle=\langle m_\nu\rangle^{\text{best}}$, $\eta_N=\eta_N^{\text{best}}$, and $\langle g\rangle=\langle g\rangle^{\text{best}}$ in Eqs. (3) and (7) we calculated half-lifes of the $0\nu\beta\beta$ decay $T_{1/2}^{\exp-0\nu}(\langle m_\nu\rangle^{\text{best}})$, $T_{1/2}^{\exp-0\nu}(\eta_N^{\text{best}})$, and $T_{1/2}^{\exp-0\nu\phi}(\langle g\rangle^{\text{best}})$ for nuclear systems of interest using specific mechanisms with the ''best'' parameters. These corresponding numerical values are listed in Table III and shown by open bars in Fig. 3. Since the quantities $\langle m_\nu\rangle,\eta_N,\langle g\rangle$ depend only on particle theory parameters these quantities indicate the experimental half-life limit for a given isotope, which the relevant experiments should reach in order to extract the best present bound on the corresponding lepton number violating parameter from their data. Some of them have a long way to go to reach the ^{76}Ge target limit.

At present most attention is paid to the light Majorana neutrino mass because of the experimental indications for oscillations of solar (Homestake [49], Kamiokande [50], Gallex [51], and SAGE [52]), atmospheric (Kamiokande [53], IMB [54] and Soudan [55], Super-Kamiokande experiments [56]), and terrestrial neutrinos (LSND experiment [57]). One can use the constraints imposed by the results of neutrino oscillation experiments on $\langle m_\nu\rangle$. The predictions differ from each other due to the different input and structure of the neutrino mixing matrix and in particular the assumed Majorana condition phases. Bilenky *et al. [58] have shown that in a general scheme with three light Majorana neutrinos and mass hierarchy* $|\langle m_\nu\rangle|$ *is smaller than* 10^{-2} eV. *In another study outlined in Ref. [59] the authors end up with* $|\langle m_\nu\rangle|\approx0.14$ eV. *Bednyakov, Faessler, and Kovalenko considered neutrino oscillations within the minimal supersymmetric standard model with R*-parity breaking. They showed that Super-Kamiokande atmospheric data are compatible with $|\langle m_\nu\rangle|\leqslant0.8\times10^{-2}$ eV [60]. One sees that, the current limit on $\langle m_\nu\rangle$ in Eq. (36) is quite a bit higher than the neutrino oscillation data.

There is a new experimental proposal for measurement of the $0\nu\beta\beta$ decay of ^{76}Ge, which intends to use 1 ton (in an extended version 10 tons) of enriched ^{76}Ge and to reach the half-life limit $T_{1/2}^{0\nu-\exp}\geqslant5.8\times10^{27}$ and $T_{1/2}^{0\nu-\exp}\geqslant6.4\times10^{28}$ after one (and 10 years) of measurements, respectively. From these half-life values one can deduce [see Eq. (35) and Table

TABLE III. The present state of the Majorana neutrino mass and Majoron searches in $\beta\beta$-decay experiments. $T_{1/2}^{exp-0\nu}$(present) and $T_{1/2}^{exp-0\nu\phi}$ (present) are the best presently available lower limit on the half-life of the $0\nu\beta\beta$ decay and $0\nu\beta\beta\phi$ decay for a given isotope, respectively. The corresponding upper limits on lepton number nonconserving parameters $\langle m_\nu \rangle$, $\langle g \rangle$, and η_N are presented. $T_{1/2}^{exp-0\nu}(\langle m_\nu \rangle^{best})$, $T_{1/2}^{exp-0\nu}(\eta_N^{best})$, and $T_{1/2}^{exp-0\nu\phi}(\langle g \rangle^{best})$ are calculated half-lifes of $0\nu\beta\beta$ decay, assuming $\langle m_\nu \rangle = \langle m_\nu \rangle^{best}$, $\eta_N = \eta_N^{best}$, and $\langle g \rangle = \langle g \rangle^{best}$, respectively. Here, $\langle m_\nu \rangle^{best} = 0.62$ eV, $\eta_N^{best} = 1.0\times10^{-7}$, and $\langle g \rangle^{best} = 6.9\times10^{-5}$ are the best limits deduced from the ^{76}Ge [38] and ^{128}Te [43] experiments.

Nucleus	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd
$T_{1/2}^{exp-0\nu}$(present) (y)	1.1×10^{25}	2.7×10^{22}	3.9×10^{19}	5.2×10^{22}	2.9×10^{22}
Ref.	[38]	[39]	[40]	[41]	[42]
$\langle m_\nu \rangle$ (eV)	0.62	6.3	203.	2.9	5.9
$T_{1/2}^{exp-0\nu}(\langle m_\nu \rangle^{best})$ (y)	1.1×10^{25}	2.8×10^{24}	4.2×10^{24}	1.2×10^{24}	2.6×10^{24}
η_N	1.0×10^{-7}	1.1×10^{-6}	4.0×10^{-5}	6.2×10^{-7}	1.1×10^{-6}
$T_{1/2}^{exp-0\nu}(\eta_N^{best})$ (y)	1.1×10^{25}	2.9×10^{24}	5.8×10^{24}	1.8×10^{24}	3.2×10^{24}
$T_{1/2}^{exp-0\nu\phi}$(present) (y)	7.9×10^{21}	1.6×10^{21}	3.9×10^{19}	5.4×10^{21}	1.2×10^{21}
Ref.	[47]	[48]	[40]	[41]	[42]
$\langle g \rangle$	4.7×10^{-4}	3.8×10^{-4}	2.7×10^{-3}	1.3×10^{-4}	4.4×10^{-4}
$T_{1/2}^{exp-0\nu\phi}(\eta_N^{best})$ (y)	3.7×10^{23}	4.9×10^{22}	6.0×10^{22}	1.9×10^{22}	4.9×10^{22}
Nucleus	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd	
$T_{1/2}^{exp-0\nu}$(present) (y)	7.7×10^{24}	8.2×10^{21}	4.2×10^{23}	1.2×10^{21}	
Ref.	[43]	[44]	[45]	[46]	
$\langle m_\nu \rangle$ (eV)	1.8	13.	4.9	8.5	
$T_{1/2}^{exp-0\nu}(\langle m_\nu \rangle^{best})$ (y)	6.6×10^{25}	3.8×10^{24}	2.7×10^{25}	2.3×10^{23}	
η_N	2.9×10^{-7}	2.0×10^{-6}	4.5×10^{-7}	1.6×10^{-6}	
$T_{1/2}^{exp-0\nu}(\eta_N^{best})$ (y)	5.9×10^{25}	3.1×10^{24}	7.9×10^{24}	2.7×10^{23}	
$T_{1/2}^{exp-0\nu\phi}$(present) (y)	7.7×10^{24}	2.7×10^{21}	1.4×10^{22}	2.8×10^{20}	
Ref.	[43]	[43]	[45]	[46]	
$\langle g \rangle$	6.9×10^{-5}	3.8×10^{-4}	4.5×10^{-4}	2.2×10^{-4}	
$T_{1/2}^{exp-0\nu\phi}(\eta_N^{best})$ (y)	7.7×10^{24}	8.2×10^{22}	5.9×10^{23}	3.0×10^{21}	

II] the possible future limits on the effective light neutrino mass 2.7×10^{-2} eV and 8.1×10^{-3} eV, respectively.

By comparing the above limits with those advocated by the neutrino oscillation phenomenology we conclude that the GENIUS [61] experiment will be able to reach similar limits, provided, of course, that the neutrinos are Majorana particles. We remind the reader that there is also a plethora of other $0\nu\beta\beta$-decay mechanisms predicted by GUT's and SUSY. One can show, however, that their presence implies that the neutrinos are massive Majorana particles even if the mass mechanism is not dominant [62,63]. Certainly, the experimental detection of the $0\nu\beta\beta$-decay process would be a major achievement with important implications on the field of particle and nuclear physics as well as on cosmology.

VI. CONCLUSIONS

The contributions coming from the induced currents at the nucleon level, such as the weak-magnetism and induced pseudoscalar coupling on the mass mechanism for the $0\nu\beta\beta$-decay transitions has been studied. The needed nuclear matrix elements, associated with the light and heavy Majorana neutrinos as well as the Majoron emission mechanisms, have been obtained in the context of pn-RQRPA, which is known to produce results more reliable than the standard QRPA. Our results are shown in Figs. 1–3 and listed in Tables I–III. One can see that the modification of the nuclear current due to the weak-magnetism and induced pseudoscalar coupling is important and results in considerable reductions of the $0\nu\beta\beta$-decay matrix elements. For the light neutrino exchange this reduction amounts to about 20–30% for all nuclei considered. The reductions for the heavy neutrino exchange are even more significant with factors ranging from 4 to 6.

The derived upper limits on $\langle m_\nu \rangle$, η_N, and $\langle g \rangle$ from the current experimental limits of the $0\nu\beta\beta$-decay lifetime for $A=76$, 82, 96, 100, 116, 128, 130, 136, and 150 are listed in Table III. This makes the extracted limits of the lepton number violating parameters less stringent yielding $\langle m_\nu \rangle^{best} \leqslant 0.62$ eV, $\eta_N^{best} \leqslant 1.0\times10^{-7}$, $\langle g \rangle^{best} \leqslant 6.9\times10^{-5}$ deduced from the ^{76}Ge [38] and ^{128}Te [43] data. Further, we introduced and evaluated useful sensitivity parameters for various lepton number violating signals for some nuclei of

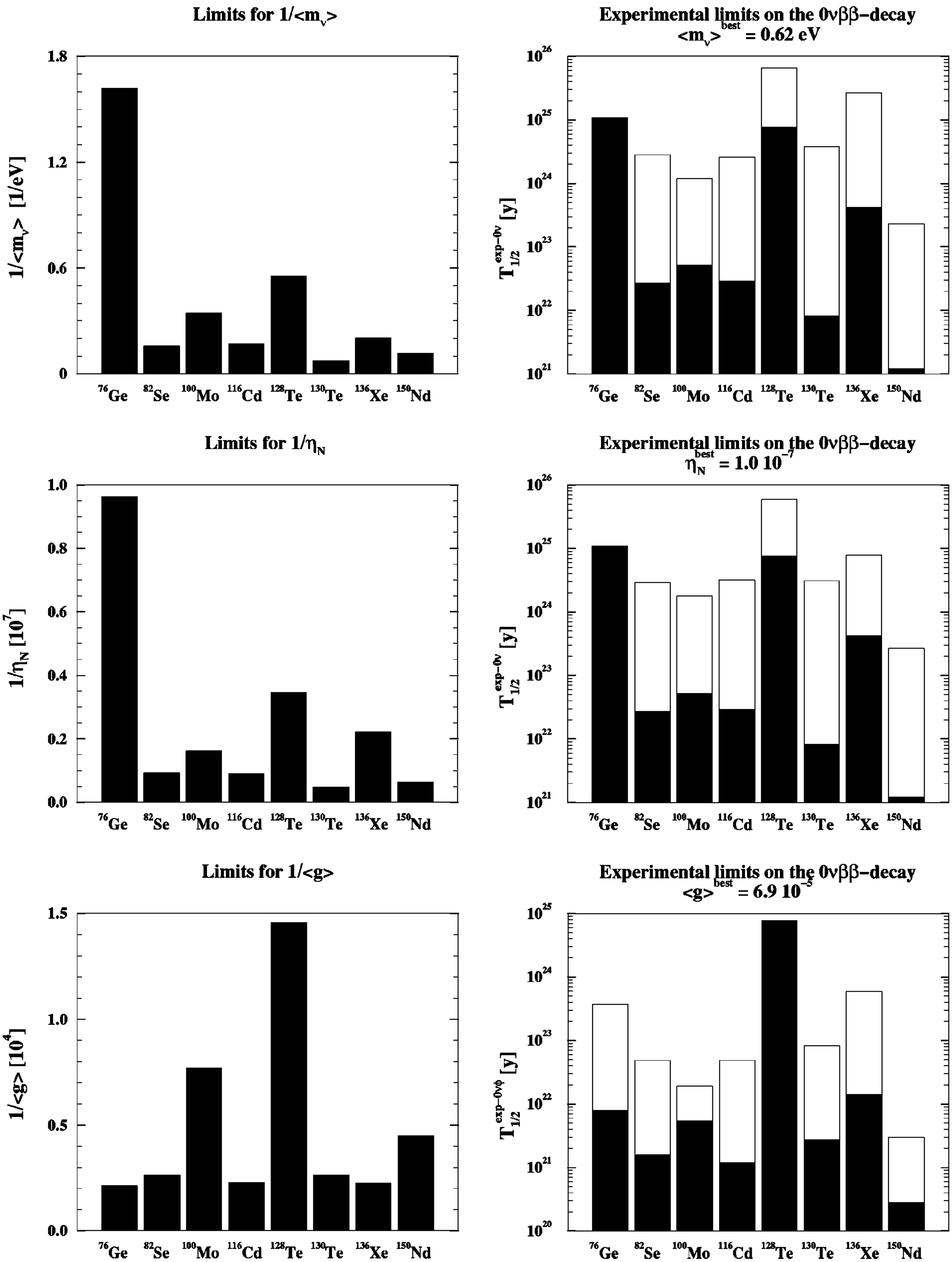

FIG. 3. The sensitivity of different experiments to the lepton-number violating parameters $\langle m_\nu \rangle$, η_N, and $\langle g \rangle$ are illustrated by histograms on the left side. The best presently available lower limits on the $0\nu\beta\beta$-decay half-life $T_{1/2}^{\exp-0\nu}$ and $0\nu\beta\beta\phi$-decay $T_{1/2}^{\exp-0\nu\phi}$ are displayed by black bars in histograms on the right side. The open bars in these histograms indicate the half-life limits $T_{1/2}^{\exp-0\nu}(\langle m_\nu \rangle^{\text{best}})$, $T_{1/2}^{\exp-0\nu}(\eta_N^{\text{best}})$, and $T_{1/2}^{\exp-0\nu\phi}(\langle g \rangle^{\text{best}})$ to be reached by a given experiment to reach the presently best limit on $\langle m_\nu \rangle$, η_N, and $\langle g \rangle$, respectively.

interest, which might be helpful in planning future $0\nu\beta\beta$-decay experiments.

The value of η_N extracted is, of course, associated with heavy Majorana neutrino. It can, however, be applicable in other processes involving the exchange of heavy particles, provided that the momentum structure of the relevant operators is not very different from that in Eqs. (19)–(21).

Admittedly there is a rather large spread between the calculated values of nuclear matrix elements within different nuclear theories (see, e.g., recent review articles [13,14]), which could be considered as a measure of the theoretical uncertainty. Between some of them there is no objective way to judge which calculation is correct. However, one can argue that the RQRPA method offers more reliable results than the QRPA primarily because of the collapse of the QRPA solution and the strong sensitivity of the QRPA results to the strength of particle-particle force. The only advantage of the QRPA over RQRPA is that it fulfills the Ikeda sum rule. However, the meaning of this fact is questionable because it is so close to the collapse of the QRPA, where the obtained solution is far from realistic. In the present calculations we are using the pn-RQRPA and we take into account also additional nucleon currents effects. Thus we consider the results of this paper more reliable in respect to the pn-QRPA results of our and other groups. In this work we did not deal with the problem of the proton-neutron pairing. The effects of proton-neutron pairing within the renormalized QRPA have been discussed for some nuclei of interest in Refs. [34].

Be that as it may, we find that in the case of the light neutrino the momentum-dependent terms in the nucleon current cause a more or less uniform reduction of the nuclear matrix elements by approximately 30% throughout the periodic table. We expect a similar reduction in almost any nuclear model. This will cause a corresponding increase of the extracted values for the neutrino mass.

We thus conclude that, with the best nuclear physics input, the extracted average neutrino mass is low, but quite a bit higher than that deduced from the present neutrino oscillation experiments. It will reach, however, similar levels, if the planned experiments reach the sensitivity aimed at by the GENIUS experiment [61]. In any case the neutrino oscillation data can neither set the absolute scale of the mass nor decide whether the neutrino is a Majorana particle. The latter issue can be decided only by the $0\nu\beta\beta$ decay.

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